How Should Transactions Services be Taxed? *

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Abstract

This paper considers the optimal taxation of transactions services, when the household can choose between cash and bank deposits. We analyze a version of the Freeman-Kydland model with a fully specified banking sector, and where cash has an "inconvenience cost" in the form of a time input. We show that while cash should be untaxed, the return on bank deposits should be should be taxed at a positive rate, reflecting the fact that deposits, by economizing on household time, are complementary with leisure in household production, but that this rate is generally different to the optimal rate of tax on consumption. A calibrated version of the model suggests that this transactions tax, and thus the tax on bank value-added, should be considerably lower than the tax on consumption.

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1 Introduction

This paper addresses a relatively neglected issue, the optimal structure and level of taxes on transactions services. By transactions services, we mean the services provided by fiat money (cash) and bank deposits, with their associated services such as debit cards. There is of course, a large literature on the optimal taxation of fiat money, the so-called inflation tax literature. However, this is only part of the story. In a modern economy, most transactions are conducted using bank deposits rather than cash. So, it is important to understand how the central results of the inflation tax literature extend to the case where the household has a choice between several transactions services.

Moreover, bank deposits are a particular form of financial intermediation service, and the question of how financial intermediation services should be taxed is a contentious one[1]. For example, there is a current policy debate on the taxation of banks, especially in Europe, where it is viewed by many, including the European Commission, that banks are undertaxed, because many of their services are exempt from VAT.

The current state of the literature is as follows. The inflation tax literature focusses on conditions for the zero taxation of cash, i.e. the Friedman rule, which says that the nominal interest rate should be zero. In this literature, however, it is assumed, without exception, that cash is the only medium of payment, or that some goods can be bought on credit, and so the issue of how intermediation services provided by banks should be taxed is not addressed[2].

On the other hand, there is hardly any literature on the taxation of transactions services provided by banks. There are a few papers which make the claim that these services should be taxed not at a zero rate, as in the Friedman rule, but at the same rate as consumption (Grubert and Mackie, 2000; Jack, 2000; Auerbach and Gordon, 2002). However, all these papers assume that there is only one transactions service, used in fixed proportion to aggregate consumption[3], and moreover, do not explicitly model the banking sector or consider second-best tax design issues, as explained in more detail in Section 2 below.

This paper attempts to fill the gap, by focusing on the optimal tax structure in a setting based on the well-known model of Freeman and Kydland (2000), originally used to study the relationship between money aggregates, inflation and output. In our model,

[1] There are technical difficulties in taxing financial intermediation when those services are not explicitly priced (so-called margin-based services), such as the intermediation between borrowers and lenders. However, conceptually, the problems can be solved, for example, by use of a cash-flow VAT (Hoffman et al., 1987; Poddar and English, 1997; Huizinga, 2002; Zee, 2005), and the increasing sophistication of banks' IT systems means that these solutions are also becoming practical.

[2] See for example, Correia and Teles (1996, 1999) which consider a transactions cost theory of money demand, or Chari et al. (1991, 1996), where some goods can be bought on costless credit.

[3] Chia and Whalley (1999), using a computational approach, reach the rather different conclusion that no intermediation services should be taxed, but their model is not directly comparable to these others, as the intermediation costs are assumed to be proportional to the *price* of the goods being transacted.

the household demands different varieties of goods in different quantities, and total consumption demand must be met by holdings of cash or deposits in a checking account. The inconvenience of holding large quantities of cash is explicitly modeled by assuming that it requires a time input, meaning that in equilibrium, there will be a "switch point" above which varieties will be bought using deposits. The household can economize on holdings of cash or deposits by making "trips" to the bank i.e. transferring funds from a savings account to a checking account or withdrawing cash, at some time cost.

Competitive banks can provide deposits (and the services associated with them, such as cheques and debit cards) at a cost, and the value-added of banks can be taxed. This tax, although notionally a value-added tax on banks, is also effectively a tax on the use of deposits to finance consumption, so it is also a bank transactions services tax, or *transactions tax* for short, and we use the latter terminology in what follows. The government can also levy a wage tax (or equivalently a general consumption tax) and an interest income tax, as in the dynamic optimal tax literature. The government then chooses these taxes, plus the rate of inflation (the inflation tax), to finance a public good in each period.

The solution to this tax design problem yields the following insights. First, irrespective of which transactions services are used, the optimal taxes on consumption and capital income take classic forms. The *effective tax* on consumption, which is the combination of the nominal consumption tax, plus the transactions taxes associated with consumption, satisfies a standard Ramsey-type formula. Also, in the steady state, the Chamley-Judd result holds, i.e. that the tax on capital income is zero (Chamley, 1986; Judd, 1985).[4]

Second, two special cases of our model relate closely to the existing literature. When there is no time cost of holding cash, it is optimal for the government to structure taxes so that households only use cash. Here, the Friedman rule holds i.e. a zero nominal interest rate is optimal. This relates to results in the existing literature on the optimality of the Friedman rule with a cash-in-advance constraint (Chari et al., 1991, 1996).

When the time cost of holding cash is high, it is optimal for the government to structure taxes so that households only use deposits. There are then two cases, depending on whether or not the number of trips to the bank is held fixed or chosen endogenously by the household. In the first case, there is an indeterminacy in the optimal tax structure; the optimal tax on transactions services is not independently determined, and a tax at the same rate as consumption is *an* optimal tax. This is essentially the result of Auerbach and Gordon (2002) and the other related papers, but established in a second-best setting[5].

[4] We make assumptions sufficient to ensure that the Straub and Werning (2014) critique of the Chamley-Judd result does not apply in our setting.

[5] Auerbach and Gordon (2002) consider a life-cycle model of the consumer where purchase of goods requires transactions services, which are assumed to be demanded in strict proportion to consumption. They show that if there is initially only a labor income tax imposed on the household, then this is equivalent to a value-added tax if and only if the transactions services consumed by the household are taxed at the same rate as other goods.

Our analysis makes clear, however, that the conditions for this result are very strong.

In the second case, if the number of trips to the bank is endogenous, then a positive tax on bank transactions services is optimal, but this is generally at a different rate to consumption. The intuition is a kind of Corlett-Hague one. In our set-up, as is usual, leisure is untaxed. Then, increasing the cost of holding deposits via a tax increases the number of trips to the bank, which reduces leisure, and is thus an indirect tax on leisure.

What about the general case where *both* cash and deposits are used? Here, it can be shown that the Friedman rule holds, and that a positive tax on bank transactions services is optimal, as in the case where only deposits as used. The intuition for this is similar to the intuition in the deposit-only case, with the additional twist that taxing deposits also encourages the use of cash, which implies a higher time cost for the household in managing cash, which is also an indirect tax on leisure.

Moreover, the transactions tax has a simple relationship to the tax on final consumption goods; the ratio between the two depends on the intertemporal elasticity of consumption and the elasticity of labor supply. In particular, if the sum of these two elements is above (below) unity, then transactions services should be taxed at a lower (higher) rate than final consumption. We then solve numerically for the steady-state taxes in a calibrated version of the model. We find that simulations give a transactions tax that is considerably lower than the consumption tax. For the central case, the transactions tax should only be 38% of the consumption tax, and this ratio can be much lower - even negative - if a wage tax at realistic levels is also included.

This finding has implications for the current policy debate on the taxation of banks, especially in Europe, where it is the view of many, including the European Commission, that banks are undertaxed, because many of their services are exempt from VAT.[6] In this debate, it is largely assumed that within a consumption tax system, such as a VAT, it is desirable to tax financial services at the standard rate of VAT, e.g. Ebrill et al. (2001).[7] Our result that the tax on bank value-added is generally lower than the consumption tax suggests that this implicit assumption may not be valid, and thus that this particular form of bank "under-taxation" may not be of great concern. Of course, there are other reasons for taxing banks, for example, to charge ex ante for the social costs of bailouts, or corrective taxes to discourage excessive risk-taking, and so on.

The remainder of the paper is organized as follows. Section 2 discusses related literature. Section 3 outlines the model, and Section 4 presents the main results. Section 5 studies a calibrated version of the model, and Section 6 concludes.

[6] Currently, within European Union countries, most financial intermediation services are exempt from VAT, notably financial services which are not explicitly priced (De La Feria and Lockwood, 2010; PWC, 2010; Buettner and Erbe, 2012).

[7] See also the recent IMF proposals for a Financial Activities Tax levied on bank profits and remuneration, one version of which - FAT1 - which would work very much like a VAT (IMF, 2010).

2 Related Literature

Our paper relates to several literatures. First, there is a small literature directly addressing the optimal taxation of borrower-lender intermediation and payment services (Auerbach and Gordon, 2002; Boadway and Keen, 2003; Grubert and Mackie, 2000; Jack, 2000). With the exception of Auerbach and Gordon (2002), these papers use a simple two-period consumption-savings model without an explicit production sector, and assume that payment services are consumed in fixed proportion to aggregate consumption[8]. In this setting, it is straightforward to show that if there is a pre-existing consumption tax at the same rate in both periods, the marginal rate of substitution between present and future consumption is left unchanged if payment services are taxed at the same rate as consumption.

Auerbach and Gordon (2002) consider a multi-period life-cycle model of the consumer where purchase of goods requires transactions services, which themselves are produced using other inputs. Transactions services are assumed to be demanded in strict proportion to consumption. They show that if there is initially only a labor income tax imposed on the household, then this is equivalent to a value-added tax if and only if the transactions services consumed by the household are taxed at the same rate as other goods[9].

There are, however, a number of restrictive assumptions implicit in these existing models. First, other taxes are assumed fixed, not optimized, and it is implicit that the existing taxes are non-distortionary, because the analysis proceeds by finding conditions under which taxation of transaction services does not introduce any further distortions. In turn, the only way in which a uniform consumption tax (or equivalently, a wage income tax) can be non-distortionary is if labor is in fixed supply, so it is arguable that this is a further implicit assumption of the above studies. By contrast, we take an explicit tax design approach to the question, assuming a household demand for leisure, and investigating the second-best tax structure, given that there is a government revenue constraint.

Second, and equally importantly, one can argue that the modeling of transactions services in the existing literature is at an abstract level, and not microfounded in any way; the papers above simply assume that the cost of these services is proportional to consumption. This corresponds to a very special case of our model where cash is prohibitively expensive, so the tax system cannot affect the choice of payment medium, and also the number of trips to the bank is fixed. In that special case, we find that taxing consumption and transaction services at the same rate is optimal, consistently with this existing literature.

[8] Chia and Whalley (1999), using a computational approach, reach the rather different conclusion that no intermediation services should be taxed, but their model is not directly comparable to these others, as the intermediation costs are assumed to be proportional to the *price* of the goods being transacted.

[9] In particular, they show that if there is initially a wage income tax at rate τ, which is replaced by a consumption tax at equivalent rate $\tau/(1-\tau)$, then the real equilibrium is left unchanged if and only if transactions services are also taxed at this equivalent rate.

The second related literature is on dynamic optimal taxation, in particular, that part of the literature focused on the optimal inflation tax[10]. This literature, building on the seminal contributions of Chamley (1986) and Judd (1985) is of course, large. However, as mentioned in the introduction, these models without exception, either assume (i) that cash is the only medium of payment, or (ii) assume that an exogenously specified subset of goods can be bought on credit, and are thus not subject to a cash-in-advance constraint. So, these contributions do not address the issue of how intermediation services provided by banks should be taxed. The most closely related contributions are Chari et al. (1996, 1991), who show that in a cash-in-advance model with credit goods, the optimal inflation tax is zero if utility is separable in consumption goods and leisure, and the consumption sub-utility function is homothetic.[11] Bhattacharya et al. (2005) explore the optimality of the Friedman rule in a similar model with two-period lived consumers. These papers, however, do not allow for a banking sector or costly transactions.[12]

A third related literature is the one on optimal taxation with household production (Sandmo, 1990; Piggott and Whalley, 2001; Kleven et al., 2000). In this literature, the complementarity of purchased inputs and household time in household production is an important determinant of the optimal tax structure, as in our analysis. Also, as in our results, generally, there is no household production efficiency i.e. the optimal taxes distort household input choices. The relationship of our results to theirs is further discussed in Section 4 below.

Finally, there has recently been a surge of literature[13] studying banks that engage in socially undesirable activities such as excessive risk-taking on both lending and deposit-taking margins. The main finding is that these should be corrected by Pigouvian taxes (or regulations) that apply directly to these decision margins, such as taxes on borrowing or lending. Our work is distinct from this line of inquiry, as bank lending has no external effects in our setting; we are concerned with the design of taxes to raise revenue. So, we are studying "boring banks" in the terminology of Aigner and Bierbrauer (2015), to which our paper is also related. They, however, focus on tax incidence issues, whereas we are concerned with tax design.

[10]We also assume linear income taxes, full commitment, and no information asymmetries, assumptions that are shared by most papers on the optimal inflation tax.

[11]There is also a less closely related literature which studies the optimal inflation tax with a money-in-the utility function approach to the demand for money (Kimbrough, 1986; Correia and Teles, 1996, 1999).

They also find optimality of the Friedman rule under certain conditions.

[12] Henriksen and Kydland (2010), who do have a banking sector, compare the marginal cost of public funds from an inflation tax to that from a labor tax, but they do not consider the taxation of transactions services.

[13]See e.g. Acharya et al. (2012); Bianchi and Mendoza (2010); Jeanne and Korinek (2010); Keen (2011); Perotti and Suarez (2011).

3 The Model

3.1 Set-Up

A single representative household consumes a number of different varieties of a consumption good, supplies labor to a competitive firm, and can also hold fiat money (cash), bank deposits and capital. The banks are competitive and hold capital assets. The government can use both the usual tax instruments (a wage income or consumption tax, an interest income tax) and can also impose a value-added tax on banks. It also sets an "inflation tax" via choice of the inflation rate.

3.2 Firms

In each period $t = 0, ..\infty$, a single competitive firm produces an intermediate good from labor and capital via the production function $f(k_t, h_t)$, where k_t is the capital stock, and h_t is hours of work supplied by the household. One unit of this intermediate good can be transformed into one unit of a continuum of different varieties $i \in [0, 1]$ of a consumption good, an investment good, a public good, and also into $1/\psi$ units of banking services. The nature of banking services is discussed in 3.4 below.[14] Capital depreciates at rate δ, so it follows the usual process:

$$k_{t+1} = \iota_t + (1 - \delta)k_t. \tag{1}$$

where ι_t is gross investment. Capital is rented from households and banks, at real rental rate $r_t = f_{kt} - \delta$. The real wage is determined by the usual condition $w_t = f_{ht}$. We use (here and below) the notation that for any any function $f(x_t, y_t)$, the partial derivative of f with respect to x_t is f_{xt}, the cross-derivative is f_{xyt} etc.

3.3 Households

There is a single infinitely lived household with preferences over levels of consumption goods, leisure, and a public good in each period $t = 0, ..\infty$ of the form:

$$\sum_{t=0}^{\infty} \beta^t(u(c_t,\ l_t) + v(g_t)), \quad c_t = \min_{i \in [0,1]} \{c_t(i)/i^{\omega}\} \tag{2}$$

where $c_t(j)$ is the level of consumption of variety j in period t, l_t is the consumption of leisure, and g_t is public good provision. Utilities $u(c, l)$, $v(g)$ are strictly increasing and strictly concave in their arguments and $0 < \beta < 1$ is a discount factor. We also assume $u_{cl} \geq 0$ and $\lim_{g \to 0} v_g(g) = \infty$.

[14] The assumption that labor is not needed to produce final goods, the investment good or banking services is for convenience only and could be relaxed at the cost of additional complexity, without changing the main results.

The fixed coefficients specification for the commodity index follows Freeman and Kydland (2000); it allows for consumption levels of the different varieties to vary in an analytically tractable way. In particular, all varieties will be consumed in fixed proportions to c i.e.

$$c(i) = ci^{\omega}, \; i \in [0,1], \quad \omega > 0 \tag{3}$$

So, ω determines the shape of the distribution of consumption goods; if $\omega < 1$, it is concave, and if $\omega > 1$, it is convex. In the macroeconomics literature, it is often assumed that $\omega < 1$ for household consumption goods e.g. Šustek (2010) sets $\omega = 0.5$. We will assume $\omega < 1$ in what follows.

Following Freeman and Kydland (2000), and Henriksen and Kydland (2010), we assume that the household goods must be purchased with either cash or deposits. In order to purchase a given amount of consumption goods in a period t, each household replenishes its money balances n_t times. So, cash balances multiplied by n_t held equals the amount of consumption financed by cash, and similarly for deposits. Each time a household replenishes its money balances but one unit, it spends φ units of time, interpreted as the time required to sell one unit of capital.[15] Total time spent on those transactions in a period then equals φn_t .

When does the household use cash? There is a substantial empirical literature on the use of cash versus other payment media, such as debit cards (Snellman et al., 2001; Lippi and Secchi, 2009; ten Raa and Shestalova, 2004). This literature finds that the choice between the two is determined by: (i) the relative opportunity cost of the two media; (ii) fees for the use of electronic payment media, and (iii) non-pecuniary costs, such as time and inconvenience; (iv) the risk of having cash lost or stolen. Opportunity costs alone would imply a corner solution where only cash or electronic media are used. This is inconsistent with what is observed in practice, where cash is used for small transactions, and cards for larger transactions[16].

To model this, we suppose that cash becomes increasingly costly for large transactions, by assuming a fixed time cost γi if variety i is bought with cash. This cost captures the fact that varieties with a higher i will be consumed in larger amounts (from (3)), and thus the cost of dealing with larger amounts of cash is higher.[17] This implies a cutoff j_t

[15] An alternative, mathematically equivalent interpretation of the model is offered by Šustek (2010): rather than holding capital directly, the household holds time deposits, which are backed by capital in the bank's balance sheet. Thus, replenishing holds of cash or funds in a checking account requires a trip to the bank to transfer funds from the time account to the checking account. This is the interpretation of the model presented in the introduction.

[16] For example, using a sample of Dutch retailers, ten Raa and Shestalova (2004) estimate that the point at which households switch from cash to electronic payment media is somewhere between 13 and 30 Euros.

[17] This specification is very close to Freeman and Kydland (2000). The key difference is that in their model, a non-trivial choice between cash and bank deposits is achieved by introducing a fixed cost of paying for variety j using deposits (e.g. writing a cheque), whereas in ours, it is generated by a time cost of using cash that is increasing in j. Our choice of specification further discussed below.

such that only goods $i \leq j_t$ will be bought with cash. The determination of the cutoff is further discussed below.

This discussion implies that the transactions constraints facing the household can be written

$$n_t m_t \geq \int_0^{j_t} c_t(i)di = K_t^m c_t, \tag{4}$$
$$n_t d_t \geq \int_{j_t}^1 c_t(i)di = K_t^d c_t$$

where m_t, d_t are real money balances and deposits held in period t, and from (3), $K_t^m = \frac{j_t^{\omega+1}}{\omega+1}$, $K_t^d = 1 - \frac{j_t^{\omega+1}}{\omega+1}$. The first of these is just a cash-in-advance constraint, and the second is similar, in that it requires that real deposits must be no less than the real value of goods purchased using bank deposits.

In each period, the household consumes goods and leisure, and can hold physical capital, cash, or deposits. So, labor supply is the time endowment minus leisure and the time cost of managing cash and deposits i.e.

$$h_t = 1 - l_t - 0.5\gamma j_t^2 - \varphi n_t. \tag{5}$$

Here, the overall time cost of managing cash is $\int_0^j \gamma i di = \gamma \frac{j^2}{2}$.

The budget constraint in real terms says that the cost of consumption, c_t, plus holdings of real cash m_{t+1} and deposits d_{t+1}, plus purchases of the capital good by the household, k_{t+1}^H, must be equal to after-tax wage income plus the real after-tax returns on initial holdings of real cash balances, real deposits, and capital.

$$c_t(1+\tau_t^c) + (1+\pi_{t+1})m_{t+1} + d_{t+1} + k_{t+1}^H = \tag{6}$$
$$w_t(1-\tau_t^w)h_t + m_t + (1+\tilde{r}_t(1-\tau_t^r))d_t + (1+r_t(1-\tau_t^r))k_t^H, \ t = 1, 2, ..$$

where $\pi_{t+1} = \frac{P_{t+1}}{P_t} - 1$, and τ_t^r is an interest income tax, and τ_t^c is a consumption tax. Moreover, $\tilde{r}_t$ is the real pre-tax return paid by banks on deposits, determined below. Finally, following Chari et al. (1996), we assume that $m_0 = d_0 = k_0 = 0$; if these initial conditions do not hold, then the government's problem is trivial[18].

As we will see, the returns on cash and deposits will be lower than the return on capital in equilibrium, implying that the household will wish to hold just the minimum stocks of m_t, d_t required to carry out transactions. So, we can suppose that the transactions constraints (4) hold with equality. Then, substituting out the k_t^H in (6), using (4) to

[18] As is well-known, if the initial stock $M_0 + D_0$ of nominal assets is positive (negative), then welfare is maximized by setting the initial price level to infinity (or sufficiently low). See Chari et al. (1996), p207. Similarly, if $k_0 > 0$, then the capital stock can be expropriated via a very high tax τ_0^r.

substitute out m_t, d_t, and also using (5), we obtain the present-value budget constraint:

$$\sum_{t=0}^{\infty} \chi_t c_t \left(1 + \tau_t^c + \frac{1}{n_t}(C_t^m K_t^m + C_t^d K_t^d)\right) = \sum_{t=0}^{\infty} \chi_t w_t (1 - \tau_t^w)(1 - l_t - 0.5\gamma j_t^2 - \varphi n_t) \quad (7)$$

where $\chi_t = \prod_{j=1}^{t} \frac{1}{1+r_j(1-\tau_j^r)}$. Here, C_t^m, C_t^d are the *tax-inclusive prices* of cash and deposits transactions services. Specifically, the prices are equal to the difference between the real post-tax return on capital and the post-tax return on cash or deposits respectively:

$$C_t^m = R_t, \; C_t^d = (r_t - \tilde{r}_t)(1 - \tau_t^r) \quad (8)$$

where

$$R_t = (1 + \pi_t)(1 + r_t(1 - \tau_t^r)) - 1$$

is the nominal interest rate.

Maximizing (2) subject to (7) gives us the following first-order conditions. First, we can write the first-order conditions for c_t, l_t respectively as:

$$\beta^t u_{ct} = \lambda \chi_t (1 + \tau_t^c + \frac{1}{n_t}(C_t^m K_t^m + C_t^d K_t^d)) \quad (9)$$

$$\beta^t u_{lt} = \lambda \chi_t w_t (1 - \tau_t^w) \quad (10)$$

where λ is the multiplier on (7). The condition (10) is standard. Condition (9) says that the marginal utility of consumption is proportional to the total unit cost of consumption, including tax τ_t^c and transactions costs $\frac{1}{n_t}(C_t^m K_t^m + C_t^d K_t^d)$.

As regards to n_t, we follow the literature in treating the number of trips to the asset market or bank (depending on the interpretation of the model) as a continuous variable, and we constrain it to be at least as great as unity. We will focus on the case where there is an interior solution for n_t at the solution to the government's tax design problem. In this case, from (7), the optimal n_t is given by the Baumol-Tobin condition that the marginal time cost of an additional trip equal the reduction in opportunity cost of holding cash or deposits relative to capital i.e.

$$\varphi w_t (1 - \tau_t^w) = \frac{c_t (C_t^m K_t^m + C_t^d K_t^d)}{n_t^2} \quad (11)$$

Finally, if both cash and deposits are used, the optimal choice of $j_t \in (0, 1)$ is given by;

$$w_t (1 - \tau_t^w) \gamma j_t = \frac{c_t}{n_t} j_t^{\omega} (C_t^d - C_t^m) \quad (12)$$

Condition (12) says that at an interior solution, the optimal choice of j_t balances the lower opportunity cost of holding cash, against the additional inconvenience cost of cash i.e. $w_t \gamma j_t$. In the case of j_t, we will also be interested in corner solutions. Allowing also for corner solutions, j_t can be written

$$j_t = \begin{cases} 0, & C_t^d \leq C_t^m \\ \min\left\{1, \left(\frac{c_t(C_t^d - C_t^m)}{w_t(1-\tau_t^w)n_t\gamma}\right)^{1/(1-\omega)}\right\}, & C_t^d > C_t^m \end{cases} \tag{13}$$

3.4 Banks

There are a large number of competitive banks who provide deposits to households, and use the funds to purchase capital. Without loss of generality, there is no reserve requirement, so the bank balance sheet in real terms can be written $d_t = k_t^B$, where k_t^B is the bank holding of capital in period t. We assume that payment services associated with a unit of real deposits require ψ units of the intermediate good[19]. The difference in real returns between capital and deposits i.e. $r_t - \tilde{r}_t$ is the value-added of the bank per unit of deposit, and is taxed at rate τ_t^b. As banks have a constant returns to scale technology, perfect competition implies that they make zero profit. So, the after-tax value added per unit of d_t must be equal to the cost of payment services per unit of d_t i.e.

$$\frac{r_t - \tilde{r}_t}{1 + \tau_t^b} = \psi \tag{14}$$

Finally, note from (14) and (8) that

$$C_t^d = (r_t - \tilde{r}_t)\left(1 - \tau_t^r\right) = \psi(1 + \tau_t^b)(1 - \tau_t^r) \tag{15}$$

So, ultimately, the opportunity cost of holding deposits for the household is $\psi(1 + \tau_t^d)$, where τ_t^d is the effective tax on deposits i.e.:

$$\tau_t^d \equiv (1 + \tau_t^b)(1 - \tau_t^r) - 1. \tag{16}$$

3.5 Government

In period t, the government finances the public good g_t from tax revenues generated from taxes $\tau_t^c, \tau_t^w, \tau_t^r, \tau_t^b$ and also real seigniorage revenues $m_{t+1}(1+\pi_{t+1}) - m_t$. In We solve the government's tax design problem using the primal approach, as discussed below.

3.6 Discussion

Our model is very close to the well-known Freeman and Kydland (2000) model. The key difference is that in their model, a non-trivial choice between cash and bank deposits is achieved by introducing a fixed cost of using deposits (specifically, a fixed cost of paying

[19]Our results go though unchanged, at the cost of additional algebra, if payment services are produced from the intermediate good and labor in fixed proportions.

for variety j via a checking account), whereas in ours, it is generated by a time cost of using cash that is increasing in j. Our reason for departing from the Freeman and Kydland specification is simply that at the second-best optimum, the Friedman rule holds i.e. the opportunity cost of cash is zero, and so with an additional cost of using deposits, at the optimum, the household would always be at a corner, using only cash, an uninteresting case for us. It seems to us that both assumptions are empirically plausible: there is undoubtedly some cost of setting up a bank account, but at the same time, carrying large amounts of cash is inconvenient and risky.

Also, as already discussed above, our model provides a general framework which encompasses the specific models of taxation of payment services (Auerbach and Gordon, 2002; Boadway and Keen, 2003; Jack, 2000; Grubert and Mackie, 2000) that have been developed so far.[20] Indeed, if one removes cash from the model, i.e. set $j_t = 0$, and fix n_t at n, we see that $K_t^d = 1$, and thus the overall price of c_t, excluding all taxes is $1 + \frac{\psi}{n}(1 + \tau_t^d)$; i.e. there is a direct cost of 1 unit of the intermediate good, plus an additional fixed transaction cost $\frac{\psi}{n}(1 + \tau_t^d)$. This is the specification considered in this literature. So, this literature effectively considers a special case of our model with no cash and a fixed number of trips to the bank.

Finally, we have retained a wage tax, even though given all the other tax instruments, it is redundant. This is mainly because in the simulations, we wish to explore the sensitivity of the optimal τ^c, τ^d to having a realistic level of wage taxation. To keep things simple, we will state our main analytical results for the case where $\tau^w = 0$.

4 Tax Design

4.1 The Tax Design Problem

We take a primal approach to the tax design problem. In this approach, an optimal policy for the government is a choice of all the primal variables in the model to maximize utility (2) subject to the resource constraint and the constraint that the household is making optimal choices of consumption, leisure, time use, and asset holdings. The resource constraint for the economy says that output of the intermediate good must be at least as great as the uses to which the intermediate good is put i.e. private and public consumption, investment, and banking services:

$$c_t + k_{t+1} - (1-\delta)k_t + g_t + \psi d_t \leq f\left(k_t, 1 - l_t - 0.5\gamma j_t^2 - \varphi n_t\right). \tag{17}$$

The constraint that household is making optimal choices implies an *implementation* constraint. This is obtained by substituting the household first-order conditions into the

[20] These papers also allow for savings intermediation,which can be taxed. The principles determining the tax on this spread are somewhat different, and are analyzed in a separate paper, Lockwood (2014).

present value budget constraint. Substituting (9), (10) into (7), and rearranging, we get:

$$\sum_{t=0}^{\infty} \beta^t \left(u_{ct}c_t - u_{lt}\left(1 - l_t - 0.5\gamma j_t^2 - \varphi n_t\right)\right) = 0 \tag{18}$$

Note that we have not used household first-order conditions (11) or (12) in constructing (18). Rather than impose (11), (12) directly on the government choice, we proceed by ignoring them, which creates a *less-constrained problem* for the government. We then show that the solution values $\{j_t, n_t\}_{t=1}^{\infty}$ can be decentralized by appropriate choice of taxes in the less-constrained problem. This of course, shows that the solution to the less-constrained problem is also the solution to the original problem for the government.

We now turn to the government's objective, which is (2). As is standard in the primal approach to tax design, we can incorporate the implementability constraint (18) into the government's maximand by writing an effective per period objective for the government of

$$W_t = u(c_t, l_t) + v(g_t) + \mu\left(u_{ct}c_t - u_{lt}(1 - l_t - 0.5\gamma j_t^2 - \varphi n_t)\right) \tag{19}$$

where μ is the Lagrange multiplier on (18). As we have assumed that $u_{cl} \geq 0, \lim_{g\to 0} v_g(g) = \infty$, it is possible to show, as is done in the Appendix, that $\mu > 0$ at the solution to this tax design problem. This is because it is always optimal to provide some public good, and this requires distortionary taxation.

So, to summarize, the tax design problem for the government is the choice of $\{c_t, l_t, m_t, d_t, j_t, k_t, g_t\}_{t=0}^{\infty}$ to maximize $\sum_{t=0}^{\infty} \beta^t W_t$ subject to (17). This problem is quite mechanical to solve, and the solution is given in the Appendix.

However, before we turn to a formal statement of the tax rules, we can develop a key intuition. From (19), we see

$$\frac{\partial W_t}{\partial n_t} = \mu\varphi > 0, \; \frac{\partial W_t}{\partial j_t} = \mu u_{lt}\gamma j_t > 0$$

This says that more trips to the bank, or a greater use of cash (a higher j_t) directly raises the government's objective. The the intuition for this is that an increase in either n_t or j_t directly reduces leisure, which is untaxed, and thus relaxes the implementability constraint. This means that any tax that raises n_t or j_t effectively taxes leisure; of course, τ^d is such a tax. Then, a Corlett-Hague argument implies that τ^d should be set at a positive value, and should be higher, the higher is μ, and this is exactly what we will find.

4.2 Optimal Tax Rules

We now move to a discussion of the optimal tax rules, which are derived formally in the Appendix. These rules are derived for the general case, but in our discussion, it is helpful to focus on the steady-state. So, in what follows, time subscripts are dropped for all variables, indicating steady-state values. In particular, a helpful simplification is that in the steady state, the Chamley result holds in our model, i.e. $\tau^r = 0$. This is proved formally in the Appendix.[21] This means, from (16), that in the steady state, the value-added tax on banks is also equal to the transactions tax τ^d, and so we focus entirely on the latter in what follows. Also, from now on, to simplify, we set $\tau^w = 0$ in the statement of analytical results. Our calibrated model allows for $\tau^w > 0$, as explained in the not-for-publication Appendix.

We begin with the optimal effective total tax rate on consumption. As a preliminary, define

$$H_c = \frac{1}{u_c}\left(u_{cc}c - u_{cl}h\right), \; H_l = \frac{1}{u_l}\left(u_{cl}c - u_{ll}h\right) \tag{20}$$

Here, u_{cl} etc. denote cross-partials of u w.r.t. c, l, and $h = 1 - l - \varphi n - \frac{1}{2}j^2$. So, H_c is what Atkeson et al. (1999) call the general equilibrium expenditure elasticity. Note that if there are no transactions costs, i.e. $\gamma = \varphi = 0$, H_l, H_c reduce to standard formulae found, for example, in the primal approach to the static tax design problem (Atkinson and Stiglitz, 2015).

Now we introduce the concept of the *total effective tax* on consumption, which measures the difference between the consumer and producer prices of consumption, taking into account all transactions costs. Formally, this is:

$$T = 1 + \tau^c + \frac{1}{n}\left(K^m R + K^d\psi(1+\tau^d)\right) - \left(1 + \frac{\psi K^d}{n}\right) = \tau^c + \frac{1}{n}\left(K^m R + K^d\psi\tau^d\right) \tag{21}$$

where $R = (1+\pi)(1+r) - 1$. Then, we have the following general characterization of T:

Proposition 1. *In the steady state, the optimal total effective tax on consumption, T satisfies*

$$\frac{T}{1 + K^d\psi/n + T} = \left(\frac{v_g - u_l/w}{v_g}\right)\left(\frac{H_l - H_c}{1 + H_l}\right) \tag{22}$$

On the left-hand side of (22), we have the overall effective tax rate on consumption, as a fraction of the total tax-inclusive price of consumption. The right-hand side of (22) is identical to the formula for the optimal consumption tax in the usual case without a transactions technology, when the primal approach is used (Atkinson and Stiglitz (2015)).

First, $(v_g - u_l)/wv_g$ is a measure of the social gain from additional taxation at the margin; note that by the assumption that v_g becomes infinite as g goes to zero, this is strictly positive. Second, H_l is a measure of the elasticity of labor supply. Third, by inspection of (20), $-H_c$ is positively related to u_{cl} and thus measures the degree of complementarity between consumption and leisure; the higher this is, other things equal, the higher the total effective tax on consumption, a well-known result.

[21] Because we have assumed that $k_0 = 0$, we do not have to impose any bounds on the capital tax in any period. So, the critique of the Chamley result by Straub and Werning (2014) does not apply.

We now turn to an initial characterization of the optimal taxes on cash and bank deposits.

Proposition 2. *In the steady state, if $j \in (0,1)$, the optimal taxes on cash and deposits satisfy:*

$$\frac{\psi}{1-A} = \psi(1+\tau^d) - R \tag{23}$$

In the steady state, if $n > 1$, the optimal taxes on cash and deposits satisfy:

$$\frac{\psi K^d}{1-A} = R + (\psi(1+\tau^d) - R)K^d \tag{24}$$

Here, $A = \frac{1}{1+H_l}\frac{v_g - u_l/w}{v_g} > 0$.

These conditions are derived from the conditions for the optimal choice of j_t, n_t respectively in the tax design problem. Specifically, they are the conditions on R_t, τ_t^d needed to ensure that the household choices of j_t, n_t described by (11),(12) are consistent with government choices. We will not attempt to interpret them here, but will give interpretations in various cases later on.

From now on, as our main focus is on the different possibilities for j, we assume throughout that $n > 1$. We now proceed in two steps. We first analyze the special cases where $j = 1$ or $j = 0$ at the solution to the tax design problem, and we call these cases the *cash economy* and the *bank deposit* economy respectively. We then move to the general case where $0 < j < 1$, which is our primary focus of interest.

4.3 Special Cases

The Cash Economy. This is where it is optimal in the tax design problem to set $j = 1$, so that deposits are not used. A sufficient condition for this case is $\gamma = 0$, because then cash dominates deposits, which have a real resource cost. Then, setting $K^m = 1$, $K^d = 0$ in (24), we see see from (24) that $R = 0$. This is of course, the Friedman rule.[22] This finding is related to Chari et al. (1991, 1996) who show that in a cash-in-advance model with credit goods, the optimal inflation tax is zero if utility is separable in consumption goods and leisure, and the consumption sub-utility function is homothetic. In our setting, these conditions are in fact satisfied, because $\min_j\{c_t(j)/j\}$ is a homothetic sub-utility function.[23]. Finally, substituting $R = 0$, $K^m = 1$, $K^d = 0$ into (21), (22), we get

[22] Specifically, this requires $1 + \pi = (1 + r(1 - \tau^r))^{-1} < 0$ i.e. deflation just offsets the real return on capital to make the cost of cash equal to zero.

[23] To complete the analysis, we must confirm that in (13), the household must choose $j = 1$. Note first that banks are inactive in this case, as there is no demand for deposits, so we can assume that $\tau^d = 0$. Then, from (13), $j = 1$ requires just $C^d > C^m$, or $\psi > R = 0$. But, this clearly holds.

$$\frac{\tau^c}{1+\tau^c} = \left(\frac{v_g - u_l/w}{v_g}\right)\left(\frac{H_l - H_c}{1+H_l}\right) \tag{25}$$

So, (25) uniquely determines τ^c in this case.

The Bank Deposit Economy. This is where it is optimal in the tax design problem to set $j = 0$, so that cash is not used. This arises, for example, if γ is very high. Then, setting $K^m = 0$, $K^d = 1$ in (21), (22) , we get

$$\frac{\tau^c + \frac{\psi}{n}\tau^d}{1+\tau^c + \frac{\psi}{n}(1+\tau^d)} = \left(\frac{v_g - u_l/w}{v_g}\right)\left(\frac{H_l - H_c}{1+H_l}\right) \tag{26}$$

There are then two sub-cases. The first is where n is fixed; the interest in this case is that then, our model is very close to Auerbach and Gordon (2002) and the other related literature discussed in Section 2. In this case, there is an indeterminacy, as (26) is the *only* condition determining the two taxes τ^d, τ^c. However, *one* optimal structure is to set $\tau^w = 0$, $\tau^c = \tau^d$ i.e. to tax transactions services and consumption at the same rate. This extends the results of the existing literature on taxation of transactions services, notably Auerbach and Gordon (2002), by showing exactly when a uniform tax on both the consumption good and the transactions technology is optimal, i.e. in a second-best environment where leisure is in elastic supply, and where there is a revenue constraint. The conditions are clearly very strong - they require no use of cash, and also that the number of trips n made by the household is fixed.

To see the last point, suppose instead that n is variable. Then, setting $K^d = 1$ in (24), and rearranging, we get

$$\frac{\tau^d}{1+\tau^d} = \frac{v_g - u_l/w}{v_g}\frac{1}{1+H_l} \tag{27}$$

In this case, τ^d is separately determined. Moreover, comparing (26) with (27), we see that there is no reason why we should have uniform taxation in this case. In fact, as we will see shortly, the relationship between τ^c and τ^d in this case is very similar to the relationship in the general case. We will compare τ^c and τ^d in detail in the general case.

4.4 The Economy with Both Cash and Bank Deposits

We are now focusing on the case when both payment services are used i.e. $1 > j > 0$. In this case, from Proposition 2, both (23), (24) must hold, and it is then very easy to see that this is a pair of linear simultaneous equations in R, $1+\tau^d$, which have a unique solution $R = 0$, $1+\tau^d = (1-A)^{-1}$. So, we have proved:

Proposition 3. *Assume that at the solution to the tax design problem, $1 > j > 0,\ n > 1$ in the steady state. Then, the Friedman rule $R = 0$ is optimal, and the optimal tax rate on deposits satisfies*

$$\frac{\tau^d}{1+\tau^d} = \left(\frac{v_g - u_l/w}{v_g}\right)\frac{1}{1+H_l} \tag{28}$$

So, unlike the bank deposit economy with fixed n, the taxes R, τ^d are uniquely determined. This is due to the fact that at the solution to the tax design problem, there are two household equilibrium conditions that must be satisfied, the Baumol-Tobin condition (11) and the condition for the optimal choice of j, (13), and two taxes are needed at specific values to ensure this.

From Proposition 3, a key feature of the optimal taxation of transactions services is that cash is not taxed, but deposits are taxed at a strictly positive rate. This means that the optimal tax structure distorts the household choice of transactions services, or to put it another way, we do not have household production efficiency. This result is reminiscent of Sandmo (1990)'s finding, in a rather different setting, that inputs to household production should be taxed (see also Piggott and Whalley (2001) and Kleven et al. (2000)).

We can now address the central question of the paper, the relationship of τ^d and τ^c. Setting $R = 0$ in (21), (22) , we get

$$\frac{\tau^c + \frac{1}{n}K^d\psi\tau^d}{1+\tau^c+\frac{1}{n}K^d\psi(1+\tau^d)} = \left(\frac{v_g - u_l/w}{v_g}\right)\left(\frac{H_l - H_c}{1+H_l}\right) \tag{29}$$

Also, it is helpful to simplify the interpretation of H_c, H_l by setting $u_{cl} = 0$. Then, we can write

$$H_c = \frac{u_{cc}c}{u_c} \equiv -\varepsilon_c,\ \ H_l = -\frac{u_{ll}l}{u_l}\frac{h}{l} \equiv \varepsilon_l\frac{h}{l}$$

where $\varepsilon_c, \varepsilon_l > 0$ are the elasticities of the marginal utility of consumption and leisure. Moreover, roughly speaking, $\varepsilon_l h/l$ measures the inverse of the elasticity of labor supply. This is an exact statement when in addition, u is linear in c.

Note that the left-hand side of (29) is the weighted combination of $\frac{\tau^c}{1+\tau^c}$ and $\frac{\tau^d}{1+\tau^d}$, and that the right-hand sides of (28) and (29) differ only by the factor $H_l - H_c = \varepsilon_c + \varepsilon_l h/l$. The following result is then immediate:

Proposition 4. *(a) If $\varepsilon_c + \varepsilon_l h/l = 1$, then taxes on transactions and consumption are the same i.e. $\tau^d = \tau^c$; (b) if $\varepsilon_c + \varepsilon_l h/l > 1,\ \tau^d < \tau^c$; (c) if $\varepsilon_c + \varepsilon_l h/l < 1,\ \tau^d > \tau^c$.*

To get a feel for the overall size of $\varepsilon_c + \varepsilon_l h/l$, note that a standard specification in the macroeconomics literature would be to take $\varepsilon_c, \varepsilon_l$ to be around 1 and 2, respectively, as discussed below. Moreover, one can think of working hours as being about 1/3 of the total time allocation, implying an h/l of about 1/2. Thus, $\varepsilon_c + \varepsilon_l h/l$ would certainly be greater than 1, and thus $\tau^d < \tau^c$. For a more in-depth comparison of τ^c, τ^d, we turn to a calibrated version of the model.

5 Calibration

Here, we solve numerically for the steady-state τ^c, τ^d and other endogenous variables of the model, using calibrated parameters. We solve for different fixed levels of government spending to GDP ratio, and so we will treat g - or more exactly, the ratio of g to GDP, $y = f(h, k)$ – as a parameter, in which case we do not have to model $v(g)$. The reason for varying the ratio g/y is that we want our calibration results to be applicable to all OECD countries, not just the US.

So, we assume a standard iso-elastic functional form for utility in (2) of the form:

$$u(c, l) = \frac{1}{1-\theta}(c^{1-\theta} - 1) + \frac{A}{1-\eta}(l^{1-\eta} - 1) \tag{30}$$

The production function is assumed to be Cobb-Douglas, i.e. $f(h, k) = h^{\alpha}k^{1-\alpha}$. Given these functional forms, and the focus on a steady state, the equilibrium conditions can be written as a number of simultaneous equations in unknowns $(c, l, h, k, \lambda, w, j, n, m, d, \tau^d, \tau^c)$, as described in the Not-For-Publication Appendix. The parameters of these equations are chosen as described in Table 1 below.

Table 1: Parameter values

Parameter	Description	Mean Value	Source
θ	Elasticity of utility w.r.t. consumption	1.0	Hall (1988); Gruber (2013) and others
η	Elasticity of utility w.r.t. leisure	2.0	Mankiw et al. (1985)
ψ	Cost of financial services	0.02	Philippon (2015)
β	Time discount factor*	0.93	Gomme and Rupert (2007)
α	Leisure parameter*	0.7	Gomme and Rupert (2007)
ω	Consumption distribution parameter	0.1	calibrated
γ	Cost-of-cash	0.007	calibrated
A	Share of labor	1.1	calibrated
δ	Depreciation*	0.05	calibrated
φ	Time cost to replenish money balances	0.000242	calibrated

*Parameters that were not randomly sampled.

In particular, the calibrated parameters are set as follows. We begin with a baseline, where $\tau^d = 0$, and where τ^c is chosen residually to satisfy the government budget constraint. The choice of $\tau^d = 0$ reflects the current situation where financial services are largely untaxed. The calibrated parameters are chosen to reproduce key macroeconomic ratios, as described in more detail below.

We discuss our baseline values in more detail, starting with the choice behind the preference parameters, θ, η, β. To begin with, θ is the inverse of the elasticity of intertemporal substitution (EIS). There are a very large range of estimates of θ, ranging from an early highly cited empirical study, Hall (1988), which concludes that EIS is not likely to be larger than 0.1, to more recent studies which give values of θ of well over 1 (Vissing-Jørgensen and Attanasio, 2003; Gruber, 2013). Given this range, we take a baseline value of 1, but as explained below, we conduct a sensitivity analysis with respect to changes in θ and other parameters.

Next, η is the inverse of the intertemporal elasticity of substitution of leisure. Empirical studies find η to be greater than 1 (Mankiw et al., 1985), where Smets and Wouters (2007, 2005) find the posterior of η to be near 2, and we therefore set $\eta = 2$. Next, note that $\beta = 1/(1+r)$ in the steady state. Following Gomme and Rupert (2007), we choose β so that $r = 0.075$, implying a value of β of 0.93.

On the production side, following Gomme and Rupert (2007), we set the labor share of national income to $\alpha = 0.7$. Then, given this value, we choose depreciation δ, to generate a capital output ratio of 2.5, as assumed by e.g. Freeman and Kydland (2000) and Henriksen and Kydland (2010). This requires $\delta = 0.05$, consistent with the range of values in Gomme and Rupert (2007). The final parameter on the production side is the cost of intermediation. Philippon (2015) argues that for the US, the unit cost of financial intermediation is very stable over time at about 2%, so we set $\psi = 0.02$.

The final exogenously fixed variables are g/y, fixed at 0.35, and $\tau^d = 0$. These last two choices are first, a baseline value for government expenditure, and second, a choice of τ^d reflecting the fact that most financial services are not subject to sales tax or VAT in practice - our baseline scenario. Then, τ^c was allowed to adjust to ensure that the government budget constraint was satisfied.[24]

Finally, $A, \omega, \gamma, \varphi, \delta$ were not specified exogenously, but were chosen to replicate some key ratios. These were; (i) a capital-output ratio k/y of 2.5, which is standard in the literature (Freeman and Kydland, 2000); (ii) a share of work hours from total of $h = 0.33$; (iii) a ratio of non-savings deposits to cash of $d/m = 9$, following Freeman and Kydland (2000); (iv) a ratio of the sum of d and m to the capital stock of 0.05, again following Freeman and Kydland (2000). As shown in Table 3 of the Not-For-Publication Appendix, we are able to closely approximate all these values.

Once the baseline parameters are chosen, we next solve the model for the optimal taxes using the parameter values presented in Table 1 - the counterfactual scenario. The results for τ^c, τ^d and τ^d/τ^c, for three different values of g/y, are reported in Table 2.

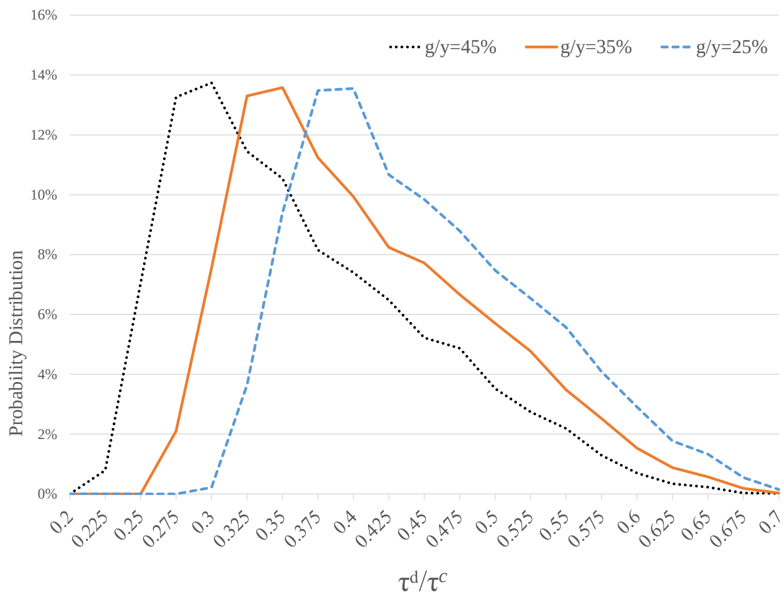
Table 2: Baseline results with varying g/y

g/y	τ^d	τ^c	τ^d/τ^c
25%	17%	40%	0.43
35%	25%	66%	0.38
45%	35%	105%	0.34

Note that τ^c is very high, as it is the single major source of government revenue in the economy, and so should not be interpreted as a realistic value. What is relevant is the ratio of τ^d/τ^c. It shows that the transactions tax is always less than half of the consumption tax and can be as low as a third of the consumption tax. This is consistent

[24]For the calibration, we also set $\tau^w = 0$, although due to indeterminacy of the complete set of taxes, this choice does not affect the calibration.

Figure 1: Monte Carlo simulation of τ^d/τ^c



Note: For varying levels of government expenditure shares, g/y, the graph shows the probability distribution of the model using a Monte Carlo method. E.g., for $g/y = 35\%$, the sample average is 0.44 with s.d. of 0.085.

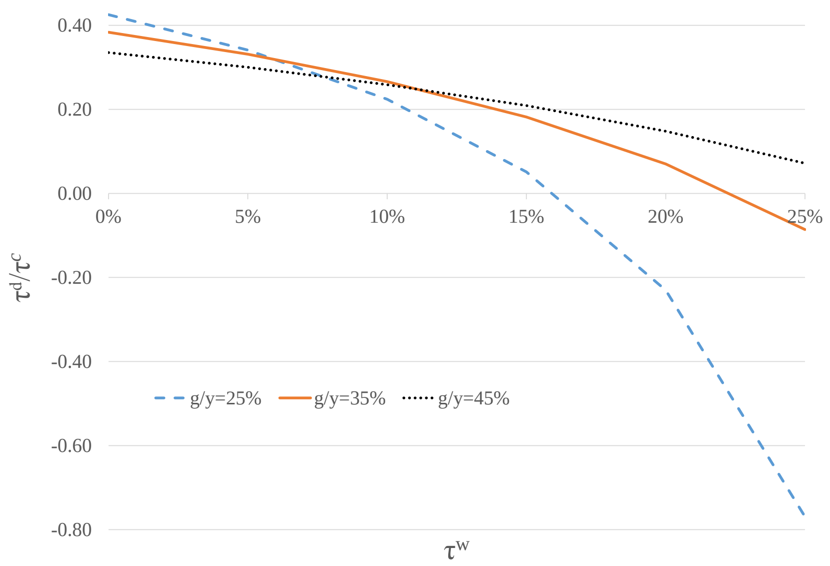
with Proposition 4, given our choices of θ and η.

We consider the robustness of these results in two ways. First, we allow each parameter in Table 1 to vary randomly. Specifically, we randomly draw a value from a uniform distribution with lower and upper values using an arbitrarily chosen value of 25% around the point-estimate for each parameter. For each simulation, the model is re-executed 10,000 times and the results are collected and analyzed. Figure 1 shows the probability distribution of the sample, for each of the three values of g/y. For $g/y = 35\%$, the average ratio of τ^d/τ^c is 0.4, with standard deviation 0.085. For $g/y = 25\%$ and 45%, the average ratios are 0.44 and 0.35, respectively. Note that the probability that $\tau^d > \tau^c$ is negligible, so our baseline finding that $\tau^d < \tau^c$ seems very robust.

A second exercise is to introduce a wage tax to make the simulations more realistic. For OECD countries, taxes on labor raise much more revenue than consumption taxes-on average, about 2.5 times as much.[25] The calibrated model also has τ^w as a parameter. Figure 2 shows how τ^d/τ^c changes as τ^w increases from zero. Obviously, both τ^c, τ^d fall as τ^w rises. However, what Figure 2 shows is that τ^d falls faster than τ^c and eventually

[25]This can be computed from OECD (2017) and OECD (2016) which shows that for 2015, the average OECD ratio of consumption tax revenue to GDP was about 10%, and that the average OECD tax wedge i.e. labor tax revenue to labor income is about 0.35. Combining this with a share of labor in GDP of 0.7 gives a ratio of labor tax revenue to GDP of about 25%.

Figure 2: τ^d/τ^c for varying levels of τ^w



Note: For varying levels of government expenditure shares, g/y, and income tax values, τ^w, the graph shows τ^d/τ^c.

becomes negative for reasonable values of τ^w of about 22% in the base case. So, it is certainly possible that banks should be subsidized.

However, our calibration results should not be over-interpreted. CGE models specifically designed for analysis of tax policy are generally highly disaggregated in terms of sectors, households and types of tax instruments.[26] In comparison to these, our model is very simple, necessarily so because the main purpose of the paper is to obtain analytical optimal tax results. In particular, other major sources of tax revenue, such as corporate taxes, are not modeled. The main point to take away from our numerical analysis is that it is hard to make the case for taxing the value-added of banks at the same or higher rate than the rate imposed on final consumption goods.

6 Conclusions

This paper has considered the optimal taxation of payment services when households can use both cash and deposits to finance consumption and can vary the stock of each

[26] A few examples are Böhringer et al. (2005) that analyze tax policy in Germany to reduce unemployment, Bhattarai (2007) assess equal-yield tax reforms for seven different taxes in the UK economy. Bettendorf et al. (2010) and Sørensen (2004) assess corporate tax harmonization in the EU and Radulescu and Stimmelmayr (2010) assess corporate tax reforms in Germany. The HMRC (2014), the UK's tax agency, developed a multi-regional UK CGE model for tax policy analysis. Pereira and Shoven (1988) provide a review of dynamic CGE tax models.

they need to hold via "trips to the bank", as in standard monetary models with the macroeconomics literature. We assume that cash is costless, but we also assume that the banking sector incurs real resource costs in providing deposits and the services associated with them. The question addressed in the paper is thus how to tax the transactions services provided by banks.

Our main finding is that transactions services should be taxed at a different rate to consumption goods. Theoretically, this rate could be higher or lower. However, using a calibrated version of the model, we find that the rate on transactions services should be lower, perhaps only a half to a third of the tax on consumption. This finding has implications for the current policy debate on the taxation of banks, especially in Europe, where it is a view of many, including the European Commission, that banks are undertaxed, because many of their services are exempt from VAT. Our results do not fully support this view, although there may be many other reasons why banks are under-taxed.

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A Proofs of Propositions and Other Results

First-Order Conditions for the Optimal Tax Problem. The Lagrangian for the government's tax design problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t (u(c_t, l_t) + v(g_t) + \mu \left(u_{ct} c_t - u_{lt}(1 - l_t - 0.5\gamma j_t^2 - \varphi n_t)\right) \tag{A.1}$$
$$+ \sum_{t=0}^{\infty} \xi_t \left(f\left(k_t, 1 - l_t - 0.5\gamma j_t^2 - \varphi n_t\right) - c_t - k_{t+1} + (1-\delta)k_t - g_t - \psi d_t\right)$$
$$+ \sum_{t=0}^{\infty} \left(\xi_t^m \left(m_t - c_t K_t^m / n_t\right) + \xi_t^d \left(d_t - c_t K_t^d / n_t\right)\right)$$

where ξ_t, ξ_t^d, ξ_t^m are the multipliers on (17) and the transactions constraints (4) respectively. From (A.1), the first-order conditions for all period t choice variables are:

$$c_t : \beta^t W_{ct} = \xi_t + \frac{1}{n_t}(\xi_t^d K_t^d + \xi_t^m K_t^m) \tag{A.2}$$
$$l_t : \beta^t W_{lt} = f_{ht} \xi_t \tag{A.3}$$
$$d_t : \xi_t^d = \xi_t \psi \tag{A.4}$$
$$m_t : \xi_t^m = 0 \tag{A.5}$$
$$k_t : \xi_t (f_{kt} + 1 - \delta) - \xi_{t-1} = 0 \tag{A.6}$$
$$n_t : (\beta^t \mu u_{lt} - f_{ht} \xi_t)\varphi + (\xi_t^m K_t^m + \xi_t^d K_t^d)\frac{c_t}{n_t^2} = 0 \tag{A.7}$$
$$j_t : \left(\beta^t \mu u_{lt} - f_{ht} \xi_t\right) \gamma j_t - \frac{1}{n_t}\left(\xi_t^m c_t j_t^\omega - \xi_t^d c_t j_t^\omega\right) = 0 \tag{A.8}$$
$$g_t : \beta^t v_{gt} = \xi_t \tag{A.9}$$

where we assume for the moment that j_t as an interior solution. Finally, note from (19) that the derivatives W_{ct}, W_{lt} are:

$$W_{ct} = u_{ct}\left(1 + \mu(1 + H_{ct})\right), \;\; W_{lt} = u_{lt}\left(1 + \mu(1 + H_{lt})\right) \tag{A.10}$$

where

$$H_{ct} = \frac{1}{u_{ct}}\left(u_{cct} c_t - u_{lc_t} h_t\right), \;\; H_{lt} = \frac{1}{u_{lt}}(u_{clt} c_t - u_{llt} h_t) \tag{A.11}$$

Proof that $\mu > 0$. From (A.9), (A.3), (A.10) we have:

$$u_{lt}(1 + \mu(1 + H_{lt})) = w_t v_{gt} \Longrightarrow \mu = \frac{1}{1 + H_{lt}} \frac{v_{gt} - u_{lt}/w_t}{u_{lt}/w_t} \tag{A.12}$$

Also, note that by the assumption that $u_{cl} \geq 0, u_{ll} < 0,\; H_{lt} > 0$. Suppose first that $\mu < 0$. Then, from (A.12), $v_{gt} < u_{lt}/w_t$. But then utility could be increased if 1\$ of spending on the public good were returned to the household as a lump-sum, contradicting the optimality of the policy. Next, suppose that $\mu = 0$. But then again from (A.12), $v_{gt} = u_{lt}/w_t$. Also, in this case, it is easily checked from the first-order conditions that all distortionary taxes are zero. So, $g_t = 0$ also. But, then $v_{gt} = \infty$, contradicting $v_{gt} = u_{lt}/w_t$. □

Proof that $\tau^r = 0$ in the Steady State. From (A.2), (A.10), we get

$$\frac{\beta^{t-1}W_{ct-1}}{\beta^t W_{ct}} = \frac{1}{\beta B_t}\frac{u_{ct-1}}{u_{ct}} = \frac{\xi_{t-1} + (\xi^d_{t-1}K^d_{t-1} + \xi^m_{t-1}K^m_{t-1})/n_{t-1}}{\xi_t + (\xi^d_t K^d_t + \xi^m_t K^m_t)/n_{t-1}} \tag{A.13}$$
$$= \frac{\xi_{t-1}(1 + \psi K^d_{t-1}/n_{t-1})}{\xi_t(1 + \psi K^d_t/n_t)}$$

using $\xi^d_t = \psi\xi_t, \xi^m_t = 0$ from (A.4),(A.5) in the second line, and where

$$B_t = \frac{1 + \mu(1 + H_{lt})}{1 + \mu(1 + H_{lt-1})}$$

Moreover, from (A.6) and $f_{kt} - \delta = r_t$, we have:

$$\frac{\xi_{t-1}}{\xi_t} = 1 + f_{kt} - \delta = 1 + r_t \tag{A.14}$$

Combining (A.13) and (A.14), we get:

$$\frac{u_{ct-1}}{u_{ct}} = \beta B_t \frac{(1 + \psi K^d_{t-1}/n_{t-1})}{(1 + \psi K^d_t/n_t)}(1 + r_t) \tag{A.15}$$

Finally, from (9), we get:

$$\frac{u_{ct-1}}{u_{ct}} = \beta(1 + (1 - \tau^r_t)r_t)\frac{1 + \tau^c_{t-1} + (C^m_{t-1}K^m_{t-1} + C^d_{t-1}K^d_{t-1})/n_{t-1}}{1 + \tau^c_t + (C^m_t K^m_t + C^d_t K^d_t)/n_t} \tag{A.16}$$

Combining (A.15), (A.16), and eliminating $\frac{u_{ct-1}}{u_{ct}}$, we get

$$B_t\frac{(1 + \psi K^d_{t-1}/n_{t-1})}{(1 + \psi K^d_t/n_t)}(1 + r_t) = (1 + (1 - \tau^r_t)r_t)\frac{1 + \tau^c_{t-1} + (C^m_{t-1}K^m_{t-1} + C^d_{t-1}K^d_{t-1})/n_{t-1}}{1 + \tau^c_t + (C^m_t K^m_t + C^d_t K^d_t)/n_t} \tag{A.17}$$

In the steady state, this reduces to $1 + r = 1 + (1 - \tau^r)r$ which of course, implies $\tau^r = 0$ as required. □

Proof of Proposition 1. (i) From (A.10), (A.2)-(A.5), we have:

$$\frac{W_{ct}}{W_{lt}} = \frac{u_{ct}}{u_{lt}}\frac{1 + \mu(1 + H_{ct})}{1 + \mu(1 + H_{lt})} = \frac{\xi_t + (\xi^d_t K^d_t + \xi^m_t K^m_t)/n_t}{f_{ht}\xi_t} \tag{A.18}$$
$$= \frac{1 + \psi K^d_t/n_t}{w_t}$$

And, from (9),(10):

$$\frac{u_{ct}}{u_{lt}} = \frac{1+\tau_t^c + (K_t^m C_t^m + K_t^d C_t^d)/n_t}{w_t(1-\tau_t^w)} \tag{A.19}$$

Combining (A.18), (A.19), we get

$$\left(1+\frac{\psi}{n}K_t^d + T_t\right)(1+\mu(1+H_{ct})) = \left(1+\frac{\psi}{n}K_t^d\right)(1+\mu(1+H_{lt})) \tag{A.20}$$

where

$$T_t = \frac{1+\tau_t^c + (K_t^m C_t^m + K_t^d C_t^d)/n_t}{1-\tau_t^w} - 1 - \frac{\psi}{n_t}K_t^d \tag{A.21}$$

is the effective total tax on consumption. Rearranging (A.20), we get:

$$T_t(1+\mu(1+H_{ct})) = \mu(H_{lt}-H_{ct})\left(1+\frac{\psi}{n}K_t^d\right) \tag{A.22}$$

Adding $T_t\mu(H_{lt}-H_{ct})$ to both sides, and and rearranging, we get

$$\frac{T_t}{1+\frac{\psi}{n}K_t^d + T_t} = \frac{\mu(H_{lt}-H_{ct})}{1+\mu(1+H_{lt})}$$

Then, using (A.12) to substitute out μ in (A.22), and rearranging, we get

$$\frac{T_t}{1+\frac{\psi}{n}K_t^d + T_t} = \left(\frac{v_{gt}-u_{lt}/w_t}{v_{gt}}\right)\frac{(H_{lt}-H_{ct})}{1+H_{lt}} \tag{A.23}$$

Evaluating (A.23) at the steady state, we get (22) as required. Finally, to get formula (21), set $\tau_t^w = 0$ in (A.21), and use the fact that in the steady state, $C^m = R$, $C^d = \psi(1+\tau^d)$ in (A.21) as well: making these substitutions gives (21) as required. □

Proof of Proposition 2. We assume an interior solution for j i.e. $0 < j < 1$. Evaluating all relevant first-order conditions at the steady state (dropping t subscripts), and combining (A.4), (A.5),(A.8), and using $\beta^t/\xi = 1/v_g$ from (A.9), $f_h = w$, the FOC for j can be written

$$\left(w - \frac{\mu u_l}{v_g}\right)\gamma = \frac{c}{n}j^{\omega-1}\psi \tag{A.24}$$

Also, setting $C^m = R$, $C^d = (1+\tau^d)\psi$ in (12), we get

$$w(1-\tau^w)\gamma = \frac{c}{n}j^{\omega-1}(\psi(1+\tau^d)-R) \tag{A.25}$$

Combining (A.24), (A.25), we get, in the steady state:

$$\frac{\psi(1-\tau^w)}{1-A} = \psi(1+\tau^d)-R,\ A = \frac{\mu u_l}{wv_g} \tag{A.26}$$

Finally, also, from (A.12) we get:

$$A = \frac{1}{1+H_l}\frac{v_g - u_l/w}{v_g} \tag{A.27}$$

Combining (A.26), (A.27), and setting $\tau^w = 0$ gives us (23) as required.

Now assume $n > 1$. By a similar argument, the FOC (A.7) for n can be written:

$$(w - \frac{\mu u_l}{v_g})\varphi = \psi K^d \frac{c}{n^2} \tag{A.28}$$

Also, setting $C^m = R$, $C^d = (1+\tau^d)\psi$ in (11), and using $K^m = 1 - K^d$, we get

$$\varphi w(1-\tau^w) = \frac{c(R + (\psi(1+\tau^d) - R)K^d)}{n^2} \tag{A.29}$$

Combining (A.28), (A.29), we get:

$$\frac{\psi(1-\tau^w)K^d}{1-A} = R + (\psi(1+\tau^d) - R)K^d \tag{A.30}$$

Combining (A.30), (A.27), and setting $\tau^w = 0$ gives us (24) as required. □

Not-For-Publication Appendix: Details of the Calibrated Model

Output and Factor Prices. First, from (17) and (5), the aggregate resource constraint can be written;

$$c + \delta k + g + \psi d = h^{\alpha} k^{1-\alpha} \tag{B.1}$$

Second, the firm's first-order conditions for labor and capital determine the factor prices. Also, in the steady state, $r = \frac{1}{\beta} - 1$, where β is a parameter, so we treat r as a parameter also. So, the factor price conditions give:

$$w = \alpha \left(\frac{k}{h} \right)^{1-\alpha} \tag{B.2}$$

$$r = (1-\alpha) \left(\frac{h}{k} \right)^{\alpha} - \delta = \frac{1}{\beta} - 1 \tag{B.3}$$

Household Constraints. To derive the steady-state household budget constraint from (6), set all variables independent of t, and set $\tau^r = 0$. This gives, after some cancellations:

$$c(1+\tau^c) + m\pi = w(1-t^w)h + \tilde{r}d + rk^H \tag{B.4}$$

But, by definition, $k^H = k - d$, from (14), $r - \tilde{r} = \psi(1+\tau^d)$, and from $R = 0$, $\pi = -\frac{r}{1+r}$, so we get from (B.4) that

$$c(1+\tau^c) + \psi(1+\tau^d)d - \frac{r}{1+r}m = w(1-t^w)h + rk \tag{B.5}$$

Also,

$$h = 1 - l - \frac{\gamma j^2}{2} - \varphi n$$

Finally, the transaction constraints (4) are

$$m = \frac{cK}{n}, \; d = \frac{c(1-K)}{n}, \; K = \frac{j^{\omega+1}}{\omega+1} \tag{B.6}$$

Household Optimization Conditions. Using utility function (30), the household first-order conditions (9)-(10) reduce to:

$$c^{-\theta} = \lambda(1+\tau^c+\psi(1+\tau^d)(1-K)) \tag{B.7}$$

$$Al^{-\eta} = \lambda w(1-t^w) \tag{B.8}$$

$$n = \left(\frac{c(1-K)\psi(1+\tau^d)}{\varphi w(1-\tau^w)}\right)^{0.5} \tag{B.9}$$

$$w(1-t^w)\gamma j = \frac{c}{n} j^\omega \psi(1+\tau^d) \tag{B.10}$$

Optimal Taxes. Finally, using (30), and allowing τ^w to be non-zero, and fixing g, it can easily be shown, following the proof of Propositions 1, 3 that the optimal tax rules can be written:

$$\frac{\tau^d+\tau^w}{1+\tau^d} = Z\frac{1}{1+\eta h/l} \tag{B.11}$$

$$\frac{(\tau^c+\tau^w)n+\psi(1-K)(\tau^d+\tau^w)}{(1+\tau^c)n+\psi(1-K)(1+\tau^d)} = Z\frac{\theta+\eta h/l}{1+\eta h/l} \tag{B.12}$$

where $Z = \frac{\xi_0/\lambda_0 - Al^{-\eta}/w}{\xi_0/\lambda_0}$, where ξ_0/λ_0 is the ratio of the social to the private marginal utility of income at time 0, and is thus a measure of the marginal cost of public funds.

Therefore, we obtain 13 equations (B.1)-(B.3), (B.5)-(B.12) in 13 unknowns:

$(c, l, h, k, Z, \lambda, w, n, j, m, d, \tau^d, \tau^c)$, and the parameters $(\theta, \eta, r, \alpha, \psi, A, \omega, \gamma, \varphi, \delta, \tau^w, g)$. We solve this system of equations using a Mixed Complementarity Problem (MCP) algorithm, which allows for a combinatorial relationship between variables, and thus also for corner solutions (*e.g.*, for variables x and y: $x \cdot y = 0, x \geq 0, y \geq 0$).[27]

We first calibrate the model by fixing $g/y = 0.35$, and $\tau^d = 0$, and choosing parameters $(A, \omega, \gamma, \varphi, \delta)$ to match some key ratios, as shown in Table 1 below. To do this, we drop equations (B.11), (B.12).

Table 3: Target values versus baseline model results

	Target	Model
k/y	2.5	2.4
d/m	9.0	9.0
$(d+m)/k$	5.0%	5.2%
g/y	35%	35%
h	0.33	0.33

Then, with these calibrated parameters, as described in Table 1 in the paper, we solve for the optimal τ^c, τ^d, for various values of the parameters, as described in the paper.

[27]We used the General Algebraic Modeling System (GAMS), which is a high-level modeling system for mathematical programming and optimization. The GAMS code can be provided by the authors upon request.

The GAMS code

Below is the GAMS code for the simulation. All data in excel format can be obtain upon request.

```
$TITLE: Optimal Tax model

* Number of Monte Carlo runs  and names of paramters
set mcall      All possible cases      /mc0*mc30000/,
    mc0(mcall) Point Estimate
    mc(mcall)  model runs              /mc0,mc1*mc30000/
    param   type  / A_mc,eta_mc, theta_mc, kappa_mc, psi_mc, omega_mc, gamma_mc, pi_mc
;

$ontext
Gamma = time cost of cash
Omega = pins down the size distribution of commodity purchases.
Theta = elasticity of MU of c
Eta   = elasticity of MU of leisure
Kappa = elasticity of MU of g
A     = shifter in leisure demand
Psi   = cost of financial intermediation
$offtext

$CALL GDXXRW MC_inputs.xlsx  O=MC_inputs  par=MC_inputs rng=sample!A4:x30100 Cdim=1 Ro
$GDXIN MC_inputs.gdx
Parameter MC_inputs(mc,param) balanced matrix;
$LOAD MC_inputs
$GDXIN


Parameters
A_mc(mc), eta_mc(mc), theta_mc(mc), kappa_mc(mc), psi_mc(mc),omega_mc(mc),
gamma_mc(mc), pi_mc(mc), varphi_mc(mc), tauw_mc(mc), GS_mc(mc);

A_mc(mc)       = MC_inputs(mc,"A_mc");
eta_mc(mc)     = MC_inputs(mc,"eta_mc");
theta_mc(mc)   = MC_inputs(mc,"theta_mc");
kappa_mc(mc)   = MC_inputs(mc,"kappa_mc");
psi_mc(mc)     = MC_inputs(mc,"psi_mc");
omega_mc(mc)   = MC_inputs(mc,"omega_mc");
gamma_mc(mc)   = MC_inputs(mc,"gamma_mc");
pi_mc(mc)      = MC_inputs(mc,"pi_mc");
varphi_mc(mc)  = MC_inputs(mc,"varphi_mc");
tauw_mc(mc)    = MC_inputs(mc,"tauw_mc");
GS_mc(mc)      = MC_inputs(mc,"GS_mc");
```

```
PARAMETERS
A         leisure parameter
eta       elasticity of utility w.r.t leisure
theta     elasticity of utility w.r.t consumption
psi       payment services normalized to one labour unit
varphi    baumil tobin time cost
gamma     coefficient for time lost to transacting in cash
omega     shoping time cost exponent (greater than 1)
beta      time preference
alpha     share of labour in cobb-douglas
delta     depreciation
r         interest rate
pi        inflation
tauw     income tax
GS       Government spending
;

A       = A_mc("mc0");
eta     = eta_mc("mc0");
theta   = theta_mc("mc0");
psi     = psi_mc("mc0");
varphi  = varphi_mc("mc0");
tauw    = tauw_mc("mc0");
GS      = GS_mc("mc0");
gamma   = gamma_mc("mc0");
omega   = omega_mc("mc0");
beta    = 0.93;
r       = (1/beta - 1);
alpha   = 0.70;
delta   = 0.05;

VARIABLES
n      number of runs to the bank
c       consumption
k       capital
KK     consumption using cash
h       labour
l       leisure
d       deposits
m       cash
lambda  lagrange
taud    tax on deposits
tauc    tax on goods
w       wage
jstar   mid-point between cash and credit good
g       government spending
CC       optimal rule parameter
```

```
y       ouput
Util    Utility
;

EQUATIONS
E1      Aggregate resource constraint
E3      marginal productivity condition
E4      full consumption price equals purchase price plus cost of transaction
E2      labour market clearance
E5      Optimal consumption rule
E9      full consumption equals deposits plus money
E10     Optimal deposit tax
E11
E6      optimal labour rule
E7      Optimal shopping rule
E8      transaction constraint
E12     taud
E13     Optimal consumption
E14
E18      production
E12
E19
E20
;
*Output and factor Prices
E1..    c + delta*k + g + psi*d =E= h**alpha * k**(1-alpha);
E3..    w =E= alpha * (k/h)**(1-alpha);
E4..    r =E= (1-alpha) * (h/k)**alpha - delta;

*Household Constraints
E2..    h + varphi*n =E= 1-l-(gamma*(jstar**2)/2);
E5..    c * (1+tauc) + psi*(1+taud)*d - m*(r/(1+r)) =E= w*(1-tauw)*h + r*k;
E9..    m =E= c*KK/n;
E10..    d =E= c*(1-KK)/n;
E11..   KK * (omega+1) =E= (jstar**(omega+1));

*Household Optimization
E6..    c**(-theta) =E= lambda*(1+ tauc + psi*(1+taud)*(1-KK));
E7..    A*l**(-eta) =E= lambda*w*(1-tauw);
E8..    w*gamma*(1-tauw) * n =E= c*(jstar**(omega-1))*psi*(1+taud);
E12..   (n**2)*(varphi*w*(1-tauw)) =E=   c*(1-KK)*psi*(1+taud)     ;

*Optimal Taxes
E13..   (taud+tauw)/(1+taud) =E= CC*(1/(1+(eta*h)/l) );
E14..   ((tauc + tauw)*n + psi * (1-KK) * (taud+tauw))
           / ((1+tauc)*n + psi*(1-KK)*(1+taud))
          =E= ((theta + (eta*h)/l)/(1+ (eta*h) /l)) * CC;
```

```
*Extra
E18..   y =E= h**alpha * k**(1-alpha);
E19..   Util =E= 1/(1-theta)*((c)**(1-theta)-1)+ A/(1-eta)*((l)**(1-eta)-1);
E20..   g =E= GS*y;

MODEL tax_FS
/
E1.CC, E3.h, E4.k, E2.l, E5.c, E6.w, E7.lambda, E8.jstar,
E9.m, E10.d, E11.KK, E13.taud, E14.tauc, E18.y, E12.n, E19.Util, E20.g
/;

***************************************
* LOOPS           *********************
***************************************
PARAMETERS
RES_TAB(mc,*)      Main Report;

Loop(mc,
A       = A_mc(mc);
eta     = eta_mc(mc);
theta   = theta_mc(mc);
psi     = psi_mc(mc);
gamma   = gamma_mc(mc);
omega   = omega_mc(mc);
varphi = varphi_mc(mc);
tauw   = tauw_mc(mc);
GS   = GS_mc(mc);

* Initial values
n.L = 4.9; c.L = 0.3; k.L = 1.13; h.L = 0.32; l.L = 0.67;
d.L = 0.05; m.L = 0.006; lambda.L = 2.37; taud.L = 0.25;
tauc.L = 0.66; w.L = 1.0177; jstar.L = 0.15; KK.L = 0.1158;
CC.L = 0.4; g.L = 0.17; y.L = 0.47;

* conditions
c.LO = -INF; k.LO = -INF; h.LO = -INF; l.LO = -INF; d.LO = -INF; m.LO = -INF; lambda.L
taud.LO = -INF; tauc.LO = -INF; w.LO = -INF; jstar.LO = -INF; KK.LO = -INF;
g.LO = -INF; y.LO = -INF; Util.LO = -INF;

SOLVE tax_FS USING MCP;

*endogenous  variables
RES_TAB(mc,"tauw")     = tauw;
RES_TAB(mc,"taud")     = taud.L;
RES_TAB(mc,"tauc")     = tauc.L;
RES_TAB(mc,"taud/tauc")       = taud.L/tauc.L;
RES_TAB(mc,"jstar")           = jstar.L;
RES_TAB(mc,"c/y")$(y.L>0)     =  c.L/y.L;
```

```
RES_TAB(mc,"g/y")$(y.L>0)     =  g.L/y.L;
RES_TAB(mc,"d/m")$(m.L>0)     =  d.L/m.L;
RES_TAB(mc,"m/y")$(y.L>0)     =  m.L/y.L;
RES_TAB(mc,"d/y")$(y.L>0)     =  d.L/y.L;
RES_TAB(mc,"(d+m)/k")         =  (d.L+m.l)/k.L;
RES_TAB(mc,"k/y")             = k.L/y.L;
RES_TAB(mc,"h*w/y")$(y.L>0)    = h.L*w.L/y.L;
RES_TAB(mc,"n")       = n.L;
RES_TAB(mc,"c")        = c.L;
RES_TAB(mc,"k")        = k.L;
RES_TAB(mc,"KK")      = KK.L;
RES_TAB(mc,"h")        = h.L;
RES_TAB(mc,"l")        = l.L;
RES_TAB(mc,"d")        = d.L;
RES_TAB(mc,"m")        = m.L;
RES_TAB(mc,"lambda")   = lambda.L;
RES_TAB(mc,"w")        = w.L;
RES_TAB(mc,"g")        = g.L;

* exogenous parameters
RES_TAB(mc,"A")       = A;
RES_TAB(mc,"eta")     = eta;
RES_TAB(mc,"theta")   = theta;
RES_TAB(mc,"varphi")  = varphi;
RES_TAB(mc,"psi")     = psi;
RES_TAB(mc,"omega")   = omega;
RES_TAB(mc,"gamma")   = gamma;
RES_TAB(mc,"alpha")   = alpha;
RES_TAB(mc,"delta")   = delta;

* other variables
RES_TAB(mc,"KK")      = KK.L;
RES_TAB(mc,"y")       =  y.L;
RES_TAB(mc,"Util")       =  Util.L;
RES_TAB(mc,"CC")    = CC.L;
RES_TAB(mc,"r")    = r;
);

display RES_TAB;

execute ’xlstalk.exe -S RESULT.xlsx’ ;
execute_unload ’RESULT.gdx’, RES_TAB;
execute ’gdxxrw.exe RESULT.gdx O=RESULT.xlsx par=RES_TAB rng=MC45!a4  Rdim=1 Cdim=1 ’
execute ’xlstalk.exe -O RESULT.xlsx’ ;
```