Secession Clauses: Let them decide (to stay)*

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Abstract

This paper studies how central governments can deter secessionist movements by changing the decision structure of independence votes. Secession clauses — which promise to allow secession whenever a region likes — deter secession by making the decision to remain day-to-day rather than once-and-for-all, as is the case under a single vote. Surprisingly, the optimal decision structure from the government's perspective follows an alternating pattern: provide a once-and-for-all vote when the region's desire to secede is high or low, provide a clause when the desire is intermediate, and costly waiting in between. Greater pro-independence conflict, government instability, and uncertainty about the future benefits of secession can all contribute to the introduction of a secession clause. The impact of secession clauses on stability of a union is ambiguous: allowing clauses (cf. prohibiting them) increases the probability that secession eventually occurs, but can still improve the stability of the union in the sense of the union being longer-lived. The analysis highlights the surprising potential of secession clauses as deterrents.

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1 Introduction

Secession imposes costs on the residual country, for example through lost tax revenue, lost access to strategic resources, reduced geopolitical standing, and increased trade frictions due to new borders. The threat of secessionist movements is widespread, with over 60 active cases as of 2017 including Catalonia, New Caledonia, Kurdistan, Puerto Rico, and Tibet (Griffiths, 2021, 2016). Central governments possess a number of tools to deter independence movements to try to avoid these costs. Most transparent among them is inter-regional financial transfers, which make remaining in the union more attractive. For example, Scotland and Northern Ireland receive high government spending relative to their tax contributions — their net fiscal deficits are about £2,800 and £5,500 per person higher than the United Kingdom average, respectively (Office for National Statistics, 2023). Similarly, Canada gives CA$14 billion (CA$1,500 per person) annually to Quebec through its Equalization Program to ensure similar levels of provincial government services (Department of Finance Canada, 2025). Governments can also seek to reduce support for independence by fostering a joint national identity through shared experiences (Depetris-Chauvin et al., 2020), education (Chen et al., 2023) and interregional contact (Bagues and Roth, 2023; Cáceres-Delpiano et al., 2021). Governments may further attempt to block secession through force and violent conflict with the aid of police and military forces.

Despite investing considerable resources into making secession unattractive, many governments also selectively choose to permit votes on secession. In 2014, the UK government allowed a Scottish independence referendum, though it has rejected subsequent calls for a second vote. In 1998, the UK government was even more accommodative with Northern Ireland, promising to recognize all future secession votes in the Good Friday Agreement, which helped to end the violence of The Troubles. Canada was similarly accommodative by enacting the Clarity Act in 2000, which sets out the conditions for recognizing a provincial independence vote and entering into negotiations for the terms of secession. This law followed the 1995 Quebec independence referendum which was narrowly defeated (50.6% to 49.4%), though the Canadian Supreme Court held that the unsanctioned vote would have had no legal effect. Strikingly, despite the narrow loss in 1995, and Clarity Act shortly after, Quebec has not since voted on independence.

In this paper, I argue that the decision structure of secession can also be used to deter secession. In particular, a *secession clause* — a promise from the central government to allow future independence votes like in the Good Friday Agreement — means the region can wait and gather information about the benefits of independence before seceding, deterring secession in the interim. These promises are typically formalized in a statute or constitutional provision, but the essential element is a credible promise, not formality. In contrast to a secession clause, a single vote like Scotland faced forces the region to make the decision at a date specified by the

central government with no option to delay. This may be attractive for the government since they can point to the vote as settling the question of independence and securing the union. However, the region must be more convinced that they prefer the union, since the decision to remain is permanent, while under a secession clause it is merely day-to-day. Waiting for the independence movement to cool off sufficiently can be costly to the government if conflicts arise, and there is no guarantee that the region would ever choose to remain permanently.

To study how and when a central government optimally manipulates the decision structure of secession, I model the problem as optimal stopping in infinite, continuous time. The central government G decides when to take a single policy action — either a take-it-or-leave-it *ultimatum vote* on independence or a *secession clause*. Implementing either of these can be delayed, but inaction is costly due to conflict and protests. Once G takes a policy action, it is binding — they cannot take a further policy action nor revoke a secession clause. A representative regional voter V decides when to exercise a secession clause to hold a vote, and whether to leave or remain (permanently) when a vote occurs. V's flow benefit of independence — net of any financial transfers or other factors not explicitly modeled — serves as a state variable and follows a Brownian motion (the continuous-time limit of a random walk). This generates uncertainty and the potential for regret for both parties.[1] For example, V might choose to secede and later find out (as the flow payoff changes) that they would have been better off remaining. Brownian motions have previously been used in political economy to study secession decisions (Polborn and Willmann, 2009; Huysmans and Crombez, 2019).[2]

I find that when waiting is sufficiently costly, G's optimal policy action falls into one of three cases. When V prefers to remain, G holds an ultimatum vote, forcing them to decide when circumstances are favorable to the union. When V strongly desires independence, G gives up on the union, calling an ultimatum vote that leads to secession. However, when V's desire for independence is intermediate, G gives a secession clause. Here, V would leave if forced to decide immediately under an ultimatum vote, but prefers to remain for now and decide later — the secession clause makes this possible, creating a deterrence effect.

Between these three regions in which G takes a policy action, G chooses to wait and endure the cost of conflict, for two distinct reasons. In the lower wait region, G hopes waiting will lead to the independence movement cooling off, so that they can secure the union forever with an ultimatum vote. In the upper wait region, G may be able to deter secession with a clause

[1] Empirical evidence supports the assumption that the benefits of independence are uncertain and fluctuate (Reynaerts and Vanschoonbeek, 2021; Monastiriotis and Zilic, 2020).

[2] Brownian motions have also been used to study other political economy problems with uncertainty over time (Keppo et al., 2008; Gul and Pesendorfer, 2012; Carpenter, 2002, 2004; Carpenter et al., 2009, 2010; Henry et al., 2022), and policy experimentation and expertise (Callander, 2011, 2008; Callander and Hummel, 2014; Callander and McCarty, 2023; Aybas and Callander, 2024; Glick, 2012).

today, but waits in the hope of later implementing a clause that is not exercised so soon.

The model shows that a number of features of independence movements and unions contribute to the introduction of secession clauses. More violent and disruptive independence movements make inaction more costly, making clauses a more likely outcome. Conversely, if the region is more valuable to the union, secession clauses are less likely. Government instability or the prospect of being voted out soon can also contribute to the implementation of clauses by making G less patient and so less concerned about the eventual exercise of a secession clause. When V is more patient or the future benefits of independence are more uncertain and volatile, the deterrence effect of clauses is stronger, again making them a more attractive option for G.

I also consider the impact of secession clauses on the stability of the union. Some argue that they only cause instability and thus should never be implemented (e.g., Sunstein, 1991; Horowitz, 2003). This presents a question of constitutional design — should governments be barred from giving clauses? I first show that secession is indeed more likely to occur (eventually) when secession clauses are permitted compared to when they are not. However, this masks the effectiveness of secession clauses in delaying secession. When considering a measure of stability that accounts for how long the union survives, I show that secession clauses can extend the life of a union. In particular, when V is sufficiently patient, the deterrence effect is large enough that clauses can be used to maintain the union when G would otherwise give up on it. This result suggests that a blanket constitutional ban on secession clauses is unnecessary.

1.1 Related Literature

This paper builds on existing literature examining the impact of secession clauses. Previous papers show that secession clauses can deter secession by imposing costs on the seceding region, for instance by solving the coordination problem of punishment without commitment (Chen and Ordeshook, 1994) or by implementing socially optimal penalty structures (Huysmans and Crombez, 2019). I provide a novel explanation of how secession clauses can deter secession. Delegating the timing of the decision to the region preserves the option value of waiting relative to a one-off ultimatum, leading the region to remain in circumstances where they otherwise would not. This mechanism has empirical implications: Northern Ireland's Good Friday Agreement and Canada's Clarity Act, which do not impose explicit penalties for seceding, may nonetheless act as effective deterrents.

A subset of the secession clause literature focuses on how exit options affect intra-union bargaining and stability. Horowitz (2003) and Sunstein (1991) argue that regions will exploit the threat of secession to improve their bargaining position, heightening the risk of secession. Anesi and De Donder (2013) formalize this concern, while others show that regions could strategically

elect pro-independence leaders to further capitalize on this (Gradstein, 2004; Olofsgård, 2004). More generally, Bordignon and Brusco (2001) study the trade-off between clauses spurring secession and making the process less costly for all parties. I abstract away from concerns about intra-union bargaining. By focusing instead on the structure of the decision problem, I identify a deterrence channel through which secession clauses can improve stability, rather than undermine it.

This paper also contributes to the literature on rules governing *when* political decisions take place, such as cloture rules in committees (Anesi and Safronov, 2022) and the strategic timing of elections by incumbents (Balke, 1990; Smith, 1996; Keppo et al., 2008; Schleiter and Tavits, 2016; Smith, 2003).

Within this literature, the closest paper to the present one is Polborn and Willmann (2009). An agenda-setter dictates when a voter makes a once-and-for-all decision to deviate from the status quo. A pro-change agenda-setter calls the vote when the voter would choose to wait, and the voter approves the change because the single vote destroys the option value of waiting. Their approval is therefore only a weak check on timing power. I instead study an agenda-setter opposed to the change: a central government facing secession. In Polborn and Willmann (2009), such an agenda-setter can simply wait because inaction is costless. I make inaction costly and allow the agenda-setter to delegate timing to the region through a secession clause. Delegating timing preserves the voter's option value of waiting, which can benefit the government by delaying secession. In Polborn and Willmann (2009), dictating timing destroys this option value, benefiting the pro-change agenda-setter by hastening the change. I therefore show that agenda-setting power is realized not only by dictating timing, as in the papers above, but also by delegating it. This delivers the deterrence and stability results above.

2 Model

The model is in infinite-horizon, continuous time ($t \geq 0$). There is a central government G who controls when a single vote on secession may take place, and a representative voter V from a region that is considering independence.

2.1 Actions

G must decide when to respond to the secessionist movement, and when it responds, what form the policy response takes. G's policy may be to implement a vote on independence at the chosen response time, called an *ultimatum vote*. Alternatively, G can delegate the timing of a future vote by implementing a *secession clause*. After choosing to respond, and the form of that response, G takes no further actions. A secession clause, once granted, is binding and

irreversible: G cannot retract it, nor subsequently force an ultimatum vote. An ultimatum vote, once called, is similarly final — G cannot call another, nor subsequently grant a secession clause.

If G chooses to implement a secession clause, V decides when to exercise the clause, at some point after it is granted. Exercising a clause triggers a vote on independence at that time, just like an ultimatum vote. When a vote takes place (whether it be via a secession clause or an ultimatum vote), V decides between *leave* and *remain*. The decision is permanent — no further actions by either player take place after a vote. Until a vote takes place, V remains within the union.

2.2 Payoffs

G receives a flow payoff of 1 while V remains in the union, and 0 beginning from when they leave. Prior to taking a policy response to the secession movement, G also incurs a constant flow cost of waiting $k > 0$. This flow cost ceases as soon as an ultimatum vote is held, or a secession clause is granted. This captures the various costs of not settling the problem of an independence movement, e.g., protests and conflict. Since the benefit of the union is normalized to 1, k reflects the cost of waiting relative to the benefit of the union — therefore, if the union becomes more valuable to the government, k decreases. G applies continuous discount rate $r_g > 0$. Thus, G's payoff is:

$$\int_0^{T_L} \exp\left(-r_g t\right)\, dt - \int_0^{T_S} \exp\left(-r_g t\right) k\, dt, \tag{1}$$

where T_L is the time at which V leaves the union, and T_S is the time at which G takes the policy response.

V receives a flow payoff of 0 while remaining in the union, and a flow payoff of B_t beginning from when they leave the union. B_t evolves as a driftless Brownian motion with volatility parameter $\sigma > 0$ and initial value $B_0 \in \mathbb{R}$. At time t, the value B_t is publicly known, but its future values are unknown. We can interpret B_t as the relative benefit of independence for the region, net of financial transfers from the government. This includes any identity-based benefits of independence, and the benefits of having a more homogeneous population (Alesina and Spolaore, 1997). The volatility parameter σ captures how volatile the benefits of secession are, or how uncertain future values are. For example, if an independent Scotland is highly reliant on revenues from its North Sea oil and gas reserves, then the benefit of independence will fluctuate with the prices of those commodities, and σ will reflect this volatility. I assume that if V is indifferent between leaving and remaining, they choose to remain.[3] V applies

[3]It can be shown that this assumption does not affect the set of equilibria.

continuous discount rate $r_v > 0$. Thus, V's payoff is:

$$\int_{T_L}^{\infty} \exp(-r_v t) B_t \, dt. \tag{2}$$

2.3 Equilibrium

For simplicity, I restrict strategies to be conditioned only on payoff relevant information — specifically, the current value of B_t. A strategy for G specifies, for each value of B_t, whether to take the policy response and, if so, which form (ultimatum vote or secession clause). A strategy for V specifies, for each value of B_t, whether to exercise a secession clause if one has been granted, and whether to leave or remain if a vote is held. The equilibrium concept is Markov perfect equilibrium (henceforth, equilibrium).

I restrict attention to equilibria in which G chooses an ultimatum vote over a secession clause when indifferent.[4]

3 Analysis

3.1 Baseline without Secession Clauses

First, suppose G cannot provide a secession clause. If the ultimatum vote is held at some time t, V leaves if and only if independence has a higher expected payoff, i.e., $B_t > 0$. If V remains, the union is secured forever. From a time-t perspective, G's corresponding discounted sum of payoff from holding the vote is then $\frac{1}{r_g} \times \mathbb{1}_{B_t \leq 0}$ (Figure 1a). G must decide whether to implement an ultimatum vote now and receive this stopping value, or delay its policy response.

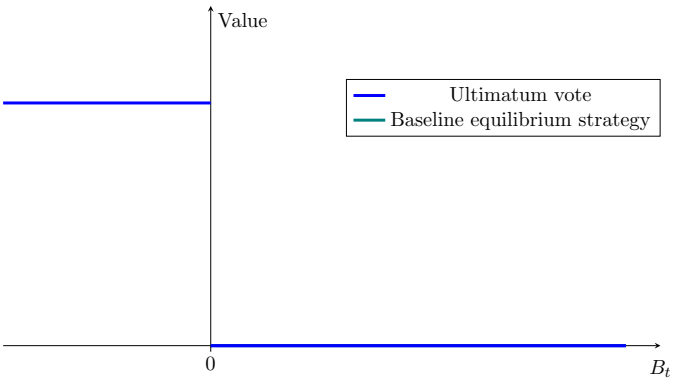
Proposition 1 (Baseline Equilibrium) *In the unique baseline equilibrium, G holds an ultimatum vote at the first time that $B_t \leq 0$ or $B_t \geq \bar{b}$, where*

$$\bar{b} = \begin{cases} \frac{\sigma}{\sqrt{2r_g}} \ln\left[\frac{k+\sqrt{2k-1}}{k-1}\right] & \text{if } k > 1, \\ \infty & \text{if } k \leq 1. \end{cases} \tag{3}$$

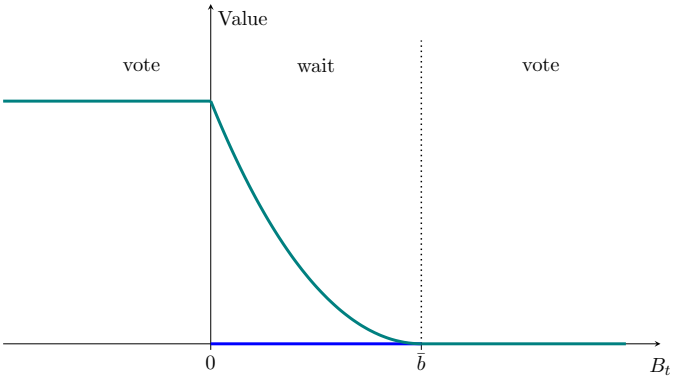
All proofs are located in the Appendix.

Clearly, if $B_t \leq 0$, G can do no better than to force V to decide now by holding an ultimatum vote — they secure the union forever and avoid costly waiting. For $B_t > 0$, G retains the ability to delay the vote, allowing B_t to change over time. The cost of doing so, k, is constant. Waiting has the benefit that V's desire to secede might eventually cool off, i.e., B_t drop to 0, resulting

[4] In particular, G is indifferent when both an ultimatum vote and clause lead to immediate secession. While South Sudan is an example of a clause being exercised as soon as it is granted, the use of a clause in this context is only nominally different from an ultimatum vote.



(A) G'S VALUE OF HOLDING ULTIMATUM VOTE



(B) G'S VALUE OF EQUILIBRIUM STRATEGY

FIGURE 1: BASELINE (NO SECESSION CLAUSES)

Parameter values: $r_g = r_v = 0.05$, $\sigma = 1$, $k = 3$. Value function of baseline equilibrium strategy shown in teal. Stopping value shown in blue. Where G takes the policy response, the value function coincides with the stopping value and is drawn on top.

in a discontinuously larger payoff for G. This is especially enticing when B_t is close to 0, since B_t does not need to move far. Here, G is willing to endure some conflict and unrest in the hope of eventually securing the union forever. However, if the cost of waiting is large and B_t is far from 0, G prefers to *give up* by holding a vote that leads to secession. If $k \leq 1$, waiting is always better than giving up. The value of waiting can be seen explicitly in Figure 1b — in particular, waiting is more valuable when B_t is closer to 0.

3.2 Deterrence

Let us now consider a secession clause. If V exercises the clause, then their voting behavior is as before — leave if and only if $B_t > 0$. However, V need not exercise the clause just because they would vote to leave.

Lemma 1 (Deterrence by Secession Clause) *In equilibrium, V exercises a secession clause at the first time that $B_t \geq b_v$, where*

$$b_v = \sigma/\sqrt{2r_v} > 0. \tag{4}$$

Given V's equilibrium play, if G takes the policy response, they strictly prefer a secession clause over an ultimatum vote if and only if $B_t \in (0, b_v)$.

Of course, there is no reason for V to use the clause when they intend to remain — waiting maintains this status quo without binding the future. On the other hand, they do not leave just because $B_t > 0$. Consider positive B_t close to 0. The cost of waiting, proportional to the forgone benefit of being independent (B_t), is low. Meanwhile, the risk that B_t could soon turn negative, and secession is ex-post suboptimal, is large. Waiting for B_t to increase reduces this risk. V can instead leave when they are more convinced that secession will be optimal. Thus as B_t increases, the risk of the potential downside of independence gets smaller while the forgone benefit of independence gets larger, at some point dominating the downside risk. This idea that the exercise threshold strictly exceeds the break-even point in net present value terms has its origins in the literature on irreversible investment under uncertainty (see, e.g., McDonald and Siegel, 1986; Dixit and Pindyck, 1994).

Lemma 1 states that for $(0, b_v)$, V would vote to leave if required to decide, but under a secession clause does not force themself to decide. Thus, clauses are an effective deterrent when V prefers secession but not too much, so that the option value of delaying the decision outweighs the forgone benefits of independence. In this case, when G takes the policy response, they strictly prefer it to be a clause rather than an ultimatum vote since the latter leads to immediate secession. Note that which of the thresholds b_v and $\bar{b}$ is larger depends only on k

and the relative patience of V and G (i.e., r_v/r_g). Importantly, for large k, we have $\bar{b} < b_v$, meaning that on the interval $(\bar{b}, b_v)$, G gives up on the union in the baseline for values of B_t that would not lead to immediate secession under a clause, reinforcing the potential value of clauses.

3.3 Equilibrium

Given Lemma 1, G's problem reduces to one of optimal stopping, where stopping entails a clause if $B_t \in (0, b_v)$, and an ultimatum vote otherwise. The stopping value is depicted in Figure 2a. The following result characterizes G's equilibrium strategy which solves this stopping problem.

Proposition 2 (Equilibrium) *There exists $\bar{k} \in (0, \infty)$, and stopping thresholds b_1, b_2, and b_3, such that for $k > \bar{k}$, G's strategy in the unique equilibrium is*

- *give an ultimatum vote at the first time that $B_t \leq 0$ or $B_t \geq b_3$, or*
- *give a secession clause at the first time that $B_t \in [b_1, b_2]$,*

whichever occurs first, with $0 < b_1 < b_2 < b_v < b_3$. When $k < \bar{k}$, G's strategy in the unique equilibrium is their baseline equilibrium strategy. When $k = \bar{k}$, both equilibria exist.

Intuition As in the baseline, there is an incentive to wait when B_t is just above 0, since the stopping value discontinuously jumps at 0. As B_t increases, this discontinuity gets further away, making waiting less attractive due to the cost k. If $k > \bar{k}$, G eventually prefers a secession clause. If $k < \bar{k}$, waiting to secure the union forever is more attractive than ever using a clause. The intuition for G waiting around b_v is more nuanced. At $B_t = b_v$, if G were to take a policy response now, secession would occur immediately. However, a brief wait may push B_t below b_v, preserving the union for some positive amount of time. For very small amounts of time, the variation in B_t is substantial, so waiting a short time increases the expected value of stopping considerably. In comparison, the costs of waiting are negligible.[5]

This means that slightly to the left of b_v, G does better by waiting in the hope of implementing a clause that is exercised later than by implementing one now that is likely to be exercised almost immediately, even though both would deter secession. The logic parallels that of V's exercise of the clause. V waits for B_t to rise well above 0 before leaving, reducing the risk that B_t soon turns negative and secession is ex-post suboptimal. Likewise, G waits for B_t to fall well below b_v before giving a clause, reducing the risk that V exercises it almost immediately, just before B_t falls below b_v again for a long period. With hindsight, G would have preferred to wait and give the clause during that long period below b_v. This unfortunate eventuality (V leaving

[5] Dixit and Pindyck (1994) offer a heuristic argument for this to justify the usual smooth pasting condition in optimal stopping problems. Décamps et al. (2006) formally prove that such stopping value kink points cannot be in the stopping region.

very soon) becomes less likely as B_t decreases away from b_v. Meanwhile, the cost of waiting remains constant, at some point dominating this risk. Thus, G endures costly conflict from the independence movement in two cases: when they hope they can secure the union forever, and when they hope they can implement a more effective clause.

The resulting value function is depicted in Figure 2. Note that as k increases, the value of waiting decreases. Then we can see the transition from Figure 2b to 2c, which reflects an increase in k, as pushing the waiting value in the baseline strategy down until it falls below the stopping value of a clause. The transition also shows that for large enough k, we have $\bar{b} < b_v$.

Proposition 2 indicates that two key features of an independence movement must be present for a secession clause to be used. First, the clause must be an effective deterrent — the benefit of independence must be positive but not too large. We would expect that clauses are particularly prevalent when voters are not too far from evenly split on the issue, as was the case in Quebec around the time of the 1995 referendum (evidenced by the 50.6–49.4 vote) and the subsequent introduction of the Clarity Act. Second, G's cost of doing nothing must be sufficiently high. If an independence movement is characterized by violence or disruptive protest, then it may receive a clause where a non-violent movement does not. This is consistent with the different treatment of Northern Ireland and Scotland.

Recall also that k represents the cost of waiting relative to a normalized benefit of maintaining the union. Therefore, a high k equally represents a low benefit of keeping the region in the union, and vice versa. The 2017 Catalan independence referendum likely faced significant opposition from Spanish police not because suppressing the vote was cheap, but because Catalonia (and Barcelona) has significant economic and cultural value for Spain. The Spanish government may also be concerned about strategic spillover to other independence movements like the Basque country, again lowering k.

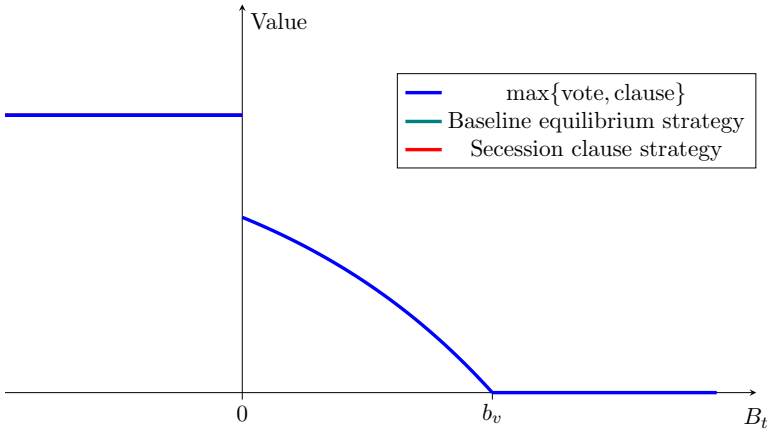
3.4 Properties of Optimal Secession Clauses

I now provide further analysis of the government's optimal strategy.

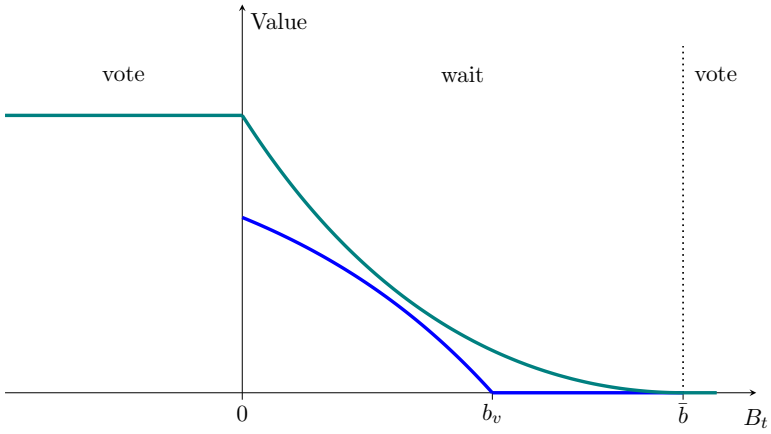
Proposition 3 (Comparative Statics) *Let $\Delta \coloneqq b_2 - b_1$ be the range of values of B_t for which a secession clause is given. Then Δ is:*

- *increasing in k,*
- *increasing in r_g,*
- *decreasing in r_v, and*
- *if $k > \bar{k}$, increasing in σ.*

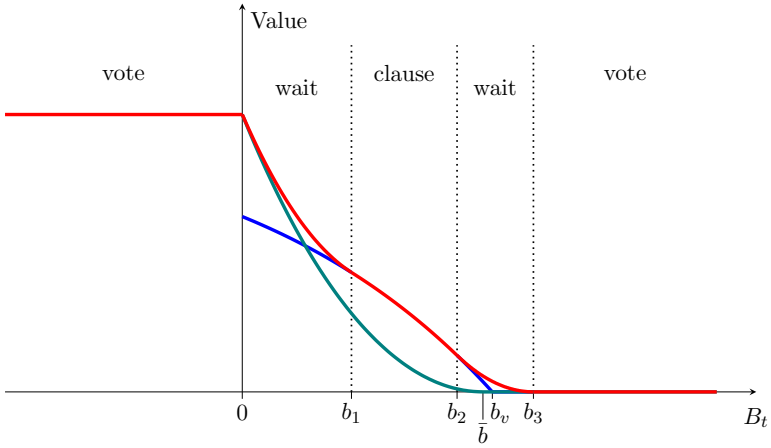
We have already discussed the importance of k — here, we see that secession clauses are more



(A) G'S VALUE OF STOPPING



(B) G'S VALUE OF EQUILIBRIUM STRATEGY ($k < \bar{k}$)



(C) G'S VALUE OF EQUILIBRIUM STRATEGY ($k > \bar{k}$)

FIGURE 2: FULL MODEL (WITH SECESSION CLAUSES)

Parameter values: All Panels: $r_g = r_v = 0.05$, $\sigma = 1$. Panel B: $k = 1.5$. Panel C: $k = 3.8$. Value function of baseline equilibrium strategy shown in teal. Value function with secession clauses shown in red. Stopping value shown in blue. Where G takes the policy response, the value function coincides with the stopping value and is drawn on top.

likely as G's cost of inaction increases. V's patience is also a key element — this determines how much V cares about the future downside risk of exercising a clause compared with the present benefits forgone by waiting. Intuitively, a more patient V will wait longer to be convinced of seceding (higher b_v), making the clause a more effective deterrent.

Conversely, a more patient G will be less willing to provide a clause, since they care more about its eventual exercise and less about the present cost of waiting for the independence movement to cool off. The patience of G may reflect the stability of the ruling party's power — a party that expects to hold government for a long time will be more patient than one which expects to lose office soon. Therefore, low government popularity and high churn may contribute to the introduction of secession clauses.

Greater volatility in the benefits of secession also contributes to G implementing secession clauses, as it increases the magnitude of the potential downsides for the region from leaving the union too soon. Volatility can reflect many things, such as exposure to international economic forces and commodity prices, or uncertainty about general economic success. There are a number of factors which we might expect to mediate this uncertainty. For instance, larger seceding regions may be able to diversify and hedge their economic activities. Similarly, forming a new union with another existing country, or joining a large common market like the European Union could help to mitigate unexpected trade shocks like the imposition of tariffs by large trading partners. Under this interpretation of σ, smaller and more economically isolated regions may be more likely to attract secession clauses.

3.5 Union Stability

An important question surrounding secession clauses is their impact on union stability, since this has considerable welfare implications for the broader public in non-seceding regions. A first result is that secession is indeed more likely when the government is permitted to give secession clauses, compared to when they are not.

Proposition 4 (Probability of Eventual Secession) *Suppose $k > \bar{k}$. Let $P(s)$ be the probability that secession ever occurs given strategy s, where s^* is G's equilibrium strategy in Proposition 2, and s_{bl} is G's baseline equilibrium strategy in Proposition 1. Then $P(s^*) \geq P(s_{bl})$ for all $B_0 \in \mathbb{R}$, and strictly for $B_0 \in (0, \bar{b})$.*

Thus, a unionist who cares only about the permanent survival of the union would never want to allow the government to use secession clauses. This result derives from the fact that $b_1 < \bar{b}$. Intuitively, giving a secession clause is at least as desirable to G as giving up on the union (calling an ultimatum vote that leads to a leave outcome), and strictly preferred when the

clause is not immediately exercised. Therefore, G will provide a clause sooner in the equilibrium of Proposition 2 than they would give up in the baseline equilibrium (Proposition 1), in the sense that they do it for lower values of B_t. In either case (secession clause or giving up on the union), secession is guaranteed to occur eventually.

This result, however, masks the effectiveness of secession clauses in *delaying* secession. We may instead be interested in how long the union is expected to survive. The expected length of the union under a clause is infinite (though secession occurs with probability 1). Intuitively, however, securing the union forever should provide even greater stability since there is no chance that secession ever occurs. Therefore, direct comparisons of expected union length would not be informative. To address this, I define a normalized measure of stability with discounting.

Definition 1 *The stability of G's strategy s is:*

$$L(s) \coloneqq \rho \mathbb{E}_0 \left[\int_0^{T_L(s)} \exp\left(-\rho t\right)\, dt \right] = \mathbb{E}_0 \left[1 - \exp\left(-\rho T_L(s)\right) \right], \tag{5}$$

where $T_L(s)$ is the time that secession occurs given strategy s and V's equilibrium strategy, and $\rho > 0$ is a generic discount rate.

The measure is equal to the expected discounted value of a flow of ρ from $t = 0$ until the time that secession occurs. Values range from 0 (immediate secession) to 1 (union forever). Unlike expected time of secession, this stability measure has the desirable property that as B_t decreases, the stability of giving a secession clause increases since the exercise threshold b_v is further away.

Note that for $k \leq 1$, the baseline stability is always 1 since G never gives up. For $k < \bar{k}$, clauses are never used. Therefore, I restrict attention to $k > \max\{1, \bar{k}\}$.

Proposition 5 (Stability) *Suppose $B_0 > 0$ and $k > \max\{1, \bar{k}\}$. Then:*

- *for a sufficiently patient V, $L(s^*) > L(s_{bl})$, and*
- *$L(s^*) - L(s_{bl})$ is weakly increasing in k and r_g (strictly when $B_0 < \bar{b}$), and weakly decreasing in r_v, for $B_0 \in (b_1, b_2)$.*

Proposition 5 states that under certain conditions, allowing secession clauses provides more stability than the alternative of never using them, even though there is a (weakly) greater probability that secession does eventually occur. This occurs when V is patient enough not to exercise the clause in circumstances where G would otherwise give up on the union. In this case, the union dissolves immediately unless clauses are available. Indeed, in the limit, an infinitely patient V never exercises a clause. This fact is neglected if we were to only consider the

probability that secession ever occurs. Therefore, stability concerns do not provide justification for a blanket constitutional ban on secession clauses.

Proposition 5 also highlights that many factors which increase G's propensity to give a secession clause also improve the stability benefit of a clause over the baseline. This is because high k and high r_g encourage stopping in both cases (with and without secession clauses), but in the baseline case stopping entails giving up on the union while a clause does not. Of course, a more patient voter (lower r_v) makes clauses more stable. It is these situations where it is particularly important from a stability perspective to allow the government to provide secession clauses, else they may be tempted to give up on the union with this region altogether.

4 Modeling Assumptions

The model abstracts away from a number of strategic elements of the secession problem to isolate the decision structure. In this section, I discuss taking k, B_t and credible commitment as given, as well as additional simplifications, and how these shape the model's conclusions.

4.1 Exogenous Cost of Waiting

The model considers a fixed flow cost of inaction for G, given by k. This represents the cost of conflict, protest and unrest that often come with independence movements. However, in reality these things do not arrive exogenously — they are the product of strategic decisions by the independence movement and the central government. Capturing how this interacts with the present model would require assumptions about how and why conflict is generated, with any conclusions resting on those assumptions. The natural reading of k is as a reduced-form parameter capturing the ability of the independence movement to strategically impose costs on G, with the results describing how G would respond to constant pressure of that kind. A first step may be to allow k to vary with B_t, though the essential element for these results is that there is some cost of inaction that a response removes. Implicit in this discussion and the model itself is that secession requires consent from the central government. De facto secession is possible, as in the case of Kosovo, but faces its own strategic contest over international recognition, which is not modeled here.

4.2 Exogenous Benefit of Independence

The stochastic process behind the benefit of independence for V is also taken as exogenous. In doing so, the model abstracts away from choices that affect B_t. These include interregional transfers, devolution, the dynamics of regional identity, and regional economic structure (which acts on the volatility σ). Allowing G to influence B_t would combine optimal control with optimal

stopping and would ask how these elements interact rather than isolating the decision structure. It would also require further assumptions about the control technology. Such assumptions are pivotal. Granting the region a fraction of the benefits of independence scales B_t and the thresholds but leaves the fundamental problem unchanged. A flow transfer might shift B_t without necessarily shifting the thresholds, leading to different secession timing. Policies that shape regional identity might add drift to B_t, without affecting the stochastic element that drives the option value. Economic diversification or improved trade relationships act instead on σ, moving every threshold with it. The results in this paper therefore hold the other instruments fixed, and whether clauses substitute for or complement them is beyond the scope of the paper.

4.3 Credibility of Commitment

Both policy responses studied in this paper rest on a commitment. The ultimatum vote commits to no future votes on secession; the secession clause commits to honoring the clause when exercised. The credibility of such commitments may itself be an outcome of strategic interactions, some of which lie outside the secession problem itself. The paper takes the ability to commit to these responses as given, making the option value mechanism as stark as possible — a decision to remain is permanent under an ultimatum vote and day-to-day under a clause. If future votes could not be ruled out, remaining under an ultimatum vote would carry a continuation value, raising the threshold for leaving. If a clause could be withdrawn, it would be exercised more readily. Both would shrink the wedge between the thresholds for leaving under an ultimatum vote and under a secession clause (0 and b_v). In a similar way, allowing the region to rejoin the union after seceding would shrink the wedge by reducing the downside risk that makes V wait. Importantly, the wedge remains as long as leaving carries some irreversibility, the two responses differ in how durable a decision to remain is, and the clause is sufficiently credible.

4.4 Other Simplifications

A number of other assumptions are used to simplify the model. First, there is no uncertainty about the current value of B_t. In reality, B_t could be observed with noise, or V might hold a private bias for or against independence. G's stopping values under both policy responses become expected values, smoothing the step at 0 and the kink at b_v and shifting the thresholds with the level of uncertainty. For modest levels of uncertainty of either kind, the wedge is expected to persist over a possibly smaller range of values. Second, V is a representative unitary decision maker for the region, whether they be the median (or super-median) voter, or some other regional decision maker. As others have shown, irreversible group decisions with uncertainty entail a number of complexities beyond the scope of the present paper (Gersbach,

1993b,a; Messner and Polborn, 2012; Moldovanu and Rosar, 2021).

5 Conclusion

Independence movements remain a constant threat to the stability of many countries. Pro-independence parties hold considerable influence in regional parliaments: the Scottish National Party has held government in Scotland since 2007; pro-independence parties in Catalonia presently account for almost half of all parliament seats.[6] While direct strategies of quelling independence movements, both carrots and sticks, have well-understood implications, the role of the structure of the decision problem has been understudied.

In this paper, I analyzed a model in which the dynamic elements of the independence decision problem could be used by the central government to influence the region's choices. Specifically, I considered how secession clauses could be used to deter secession. I characterized the conditions under which a central government finds it optimal to provide secession clauses. I also showed that clauses can improve the stability of a union by delaying secession, for instance when the region is especially patient. The analysis provides guidance to governments who may not have considered the intrinsic deterrent effect of secession clauses, and are seeking a less resource-intensive deterrent to independence movements.

The analysis also casts a different light on tactics that governments and independence movements have actually used. Each acts on an element of the problem that the model takes as given. Consider the Provisional Irish Republican Army's campaign during the Troubles. The expected outcome of a successful campaign may have been to force the British government to allow Irish unification. However, in the model, raising the cost of inaction does not necessarily force a vote. It moves the government from waiting to taking an action, and when support for secession is intermediate, that action is a secession clause which the region prefers not to exercise for now. Consistent with this, the result was not unification but a promise to allow it, in the Good Friday Agreement, and the promise has not been exercised.

France's response in New Caledonia combined features of both policies contemplated in this paper. The Nouméa Accord (1998) promised multiple opportunities to secede (like a secession clause) but limited them in number and timing (like an ultimatum vote). More importantly, it also introduced a new element, promising a vote on independence up to twenty years later, rather than immediately. The delay subdues conflict now while leaving time for the desire to secede to decrease, whereas a clause granted in 1998 might have been exercised immediately. Deferred votes of the same kind were granted in South Sudan and Bougainville, and were written into the union of Serbia and Montenegro. The New Caledonian example also surfaces another

[6] As of July 2025.

dimension on which the two sides may disagree — who is eligible to vote in the region. A 2024 bill to expand the provincial electoral roll to include a greater share of the non-Kanak population sparked riots. The identity of the decisive voter can itself be contested, and can affect whether conflict subsides.

Finally, another tactic beyond the model is boycotting referendums, as in Northern Ireland in 1973 and in New Caledonia at the final vote promised by the Nouméa Accord. Faced with an ultimatum vote it expects to lose, the movement boycotts in an attempt to keep the option to wait.

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A Proofs

A.1 Proof of Proposition 1.

I construct a candidate solution from conjectures, then verify optimality and uniqueness. Let $\mathcal{S}$ be the stopping region, $\tau_{bl} \coloneqq \inf\{t \geq 0 : B_t \in \mathcal{S}\}$ the stopping time, and $v(B, \tau)$ the value attained by stopping at time τ.

I conjecture the following conditions hold for G's value function — denote the candidate value function as f.

Conjectures Value matching at 0:

$$f(0) = 1/r_g. \tag{6}$$

For $B_t \in (0, \bar{b})$, the following Hamilton-Jacobi-Bellman (HJB) equation holds:

$$r_g f = 1 - k + \frac{\sigma^2}{2} f''. \tag{7}$$

For $k \leq 1$:

$$\bar{b} = \infty,$$
$$\lim_{B \to \infty} f(B) = \frac{1-k}{r_g}, \tag{8}$$

For $k > 1$, the following smooth pasting and value matching conditions hold:

$$f(\bar{b}) = 0, \tag{9}$$

$$f'(\bar{b}) = 0, \tag{10}$$

with $\bar{b} > 0$.

From (7), we get

$$f = C \exp(B\sqrt{2r_g}/\sigma) + D \exp(-B\sqrt{2r_g}/\sigma) + \frac{1-k}{r_g} \tag{11}$$

where C and D are unknown constants. For $k \leq 1$, the limit condition (8) implies that $C = 0$, so from value matching at 0 (6) we get $D = \frac{k}{r_g}$.

For $k > 1$, summing the value matching and smooth pasting conditions at $\bar{b}$, (9) + (10), along with substituting in (11) gives $C = \frac{k-1}{2r_g} \exp(-\bar{b}\sqrt{2r_g}/\sigma)$, while value matching at 0 (6) gives $D = \frac{k}{r_g} - C$. Using these expressions in (10) yields the solution $\bar{b} = \frac{\sigma}{\sqrt{2r_g}} \ln\left[\frac{k+\sqrt{2k-1}}{k-1}\right]$.

This in turn pins down C:

$$\begin{aligned} C &= \frac{k-1}{2r_g}\frac{k-1}{k+\sqrt{2k-1}} \\ &= \frac{k-\sqrt{2k-1}}{2r_g} \end{aligned}$$

and D:

$$\begin{aligned} D &= \frac{k}{r_g} - C \\ &= \frac{k+\sqrt{2k-1}}{2r_g}. \end{aligned}$$

We have the following useful results:

Lemma A.1 *$\bar{b}$ is decreasing in k, strictly for $k > 1$.*

Proof We have $\partial\bar{b}/\partial k < 0$ for $k > 1$ from differentiating equation (3). For $k \leq 1$, $\bar{b}$ is constant and $\lim_{k\to 1^+} \bar{b}(k) = \infty = \bar{b}(1)$. ■

Claim 1 *$f > 0$ and $f'' > 0$ for $B \in (0, \bar{b})$.*

Proof For $k \leq 1$, it is clear $f > 0$ and $f'' > 0$ on $(0, \bar{b})$, since the solution has $f = \frac{k}{r_g}\exp\left(-B\sqrt{2r_g}/\sigma\right) + \frac{1-k}{r_g}$. For $k > 1$, it follows from value matching and smooth pasting, and that $f \geq 0 \Rightarrow f'' > 0$ by HJB (7). ■

The solution defines f on $(0, \bar{b})$. For $B_t \leq 0$, let $f = 1/r_g$ and $f = 0$ for $B_t \geq \bar{b}$. Then f satisfies:

$$0 = \max\{-r_g f + 1 - k + \frac{\sigma^2}{2}f'', \mathbb{1}_{B\leq 0} - r_g f\} \tag{12}$$

Verification I apply a generalized Ito rule (Karatzas and Shreve (1998) Theorem 3.6.22, Corollary 3.6.23 and Problem 3.6.24) for any stopping time τ:

$$\begin{aligned} \exp(-r_g\tau)f(B_\tau) = f(B_0) &+ \int_0^\tau \exp(-r_g s)\left(-r_g f + \frac{\sigma^2}{2}f''\right)ds + \int_0^\tau \exp(-r_g s)f' dB_s \\ &+ \int_0^\tau \exp(-r_g s)\Delta_f(0)dL_s^0 + \int_0^\tau \exp(-r_g s)\Delta_f(\bar{b})dL_s^{\bar{b}} \\ = f(B_0) &+ \int_0^\tau \exp(-r_g s)\left(-r_g f + \frac{\sigma^2}{2}f''\right)ds \\ &+ \int_0^\tau \exp(-r_g s)f' dB_s + \int_0^\tau \exp(-r_g s)f'^{+}(0)dL_s^0 \end{aligned}$$

where $\Delta_f(B)$ is the change in first derivative at B, f'^{+} the right derivative, and L_s^x the appropriately defined local time at level x accrued up to time s. The second equality comes

from smooth pasting for $k > 1$, and that for $k \leq 1$ there is no discontinuity in the second derivative. Taking time-0 expectations:

$$\mathbb{E}_0\left[\exp\left(-r_g\tau\right)f(B_\tau)\right] = f(B_0) + \mathbb{E}_0\left[\int_0^\tau \exp\left(-r_g s\right)\left(-r_g f + \frac{\sigma^2}{2}f''\right)ds\right] + \mathbb{E}_0\left[\int_0^\tau \exp\left(-r_g s\right)f'^{+}(0)dL_s^0\right].$$

The dB_s term vanishes as it is martingale with 0 expected increment. Adding flow payoffs accrued until τ:

$$\begin{aligned}\mathbb{E}_0\left[\int_0^\tau \exp\left(-r_g s\right)(1-k)ds + \exp\left(-r_g\tau\right)f(B_\tau)\right] &= f(B_0) \\ &+ \mathbb{E}_0\left[\int_0^\tau \exp\left(-r_g s\right)\left(-r_g f + 1 - k + \frac{\sigma^2}{2}f''\right)ds\right] \\ &+ \mathbb{E}_0\left[\int_0^\tau \exp\left(-r_g s\right)f'^{+}(0)dL_s^0\right]. \qquad (13)\end{aligned}$$

Consider τ_{bl}. By construction, $-r_g f + 1 - k + \frac{\sigma^2}{2}f'' = 0$ on $(0, \bar{b})$, i.e., for $t < \tau_{bl}$, so the first integrand on the right-hand side is 0. Additionally, no local time accrues at 0 prior to stopping since stopping occurs at 0, so the second integrand is 0. Lastly, $\exp\left(-r_g\tau_{bl}\right)f(B_{\tau_{bl}}) = \int_{\tau_{bl}}^\infty \exp\left(-r_g s\right)\mathbb{1}_{B_{\tau_{bl}}\leq 0}ds$. Therefore, the value of stopping at τ_{bl} is given by f, so $f(B_0) = v(B_0, \tau_{bl})$.

Now, return to arbitrary stopping time τ. Since f satisfies (12), the first right-hand side integrand of (13) is non-positive. Additionally, $f'^{+}(0) \leq 0$ since $v(B, \tau) \leq 1/r_g$ for all τ. Thus:

$$\begin{aligned}&\mathbb{E}_0\left[\int_0^\tau \exp\left(-r_g s\right)(1-k)ds + \int_\tau^\infty \exp\left(-r_g s\right)\mathbb{1}_{B_\tau\leq 0}ds\right] \\ \leq &\mathbb{E}_0\left[\int_0^\tau \exp\left(-r_g s\right)(1-k)ds + \exp\left(-r_g\tau\right)f(B_\tau)\right] \\ \leq &f(B_0),\end{aligned}$$

so $v = f$, and τ_{bl} is an optimal stopping time.

Uniqueness Suppose $\tau^* \neq \tau_{bl}$ is optimal. We cannot have $\tau^* > \tau_{bl}$ with positive probability, otherwise the first right-hand side integral of (13) would be strictly negative while the second integral remains non-positive, so G would be worse off than using τ_{bl}.

We cannot have $\tau^* < \tau_{bl}$ with positive probability since for any such optimal stopping time we have $v(B_0, \tau^*) = f(B_0)$ iff:

$$\mathbb{E}_0\left[\int_0^{\tau^*} \exp\left(-r_g s\right)(1-k)ds + \int_{\tau^*}^\infty \exp\left(-r_g s\right)\mathbb{1}_{B_{\tau^*}\leq 0}ds\right]$$

$$= \mathbb{E}_0\left[\int_0^{\tau^*} \exp(-r_g s)(1-k)ds + \exp(-r_g\tau^*)f(B_{\tau^*})\right]$$

$$\iff \mathbb{E}_0\left[\frac{1}{r_g}\mathbb{1}_{B_{\tau^*}\leq 0}\right] = \mathbb{E}_0\left[\exp(-r_g\tau^*)f(B_{\tau^*})\right]$$

which cannot be true if $\tau^* < \tau_{bl}$ with positive probability, since $f > 0$ on $(0, \bar{b})$, which is the case for times $t < \tau_{bl}$. Thus, τ_{bl} is unique almost surely. ■

A.2 Proof of Lemma 1.

For the first statement, see Polborn and Willmann (2009) with $b = 0$, $\rho = r_v$, $\gamma = \sigma$ and $C = 0$.

G's expected discounted payoff from a clause is the solution ϕ to the following system on $(-\infty, b_v]$:

$$\phi(b_v) = 0$$
$$\lim_{B\to-\infty} \phi(B) = 1/r_g$$
$$r_g\phi = 1 + \frac{\sigma^2}{2}\phi'' \text{ for } B < b_v,$$

with extension of value 0 for $B \in (b_v, \infty)$. The system yields $\phi(B) = \frac{1}{r_g}\left[1 - \frac{\exp(B\sqrt{2r_g}/\sigma)}{\exp(b_v\sqrt{2r_g}/\sigma)}\right]$ on $(-\infty, b_v]$. The value of ϕ can be verified similar to f above. Notice that $\phi(B) > 0 \iff B < b_v$, and $\phi(B) < 1/r_g$, $\forall B$. ■

A.3 Proof of Proposition 2.

By Lemma 1, G's value of stopping is $\hat{\phi}(B) = \max\left\{\frac{1}{r_g}\left[1 - \exp\left(-\frac{\sqrt{2r_g}}{\sigma}(b_v - B)\right)\right], \frac{1}{r_g}\mathbb{1}_{B_t\leq 0}\right\}$.

I derive monotonicity of f w.r.t. k, then show that for some $\bar{k}$, $f \geq \hat{\phi}$ for $k \leq \bar{k}$.

Claim 2 *f is continuous and weakly decreasing in k, strictly decreasing for $B \in (0, \bar{b})$.*

Proof For $B \leq 0$ this is obvious. For $B > 0$, from Lemma A.1, $\exists\kappa > 1$ such that $k \geq \kappa \iff B \geq \bar{b}$. For $k \geq \kappa$, we have $f = 0$. Lastly, for $k < \kappa$, f is given by

$$\begin{cases} \frac{k}{r_g}\exp(-B\sqrt{2r_g}/\sigma) + \frac{1-k}{r_g} & \text{if } k \leq 1, \\ \frac{k-\sqrt{2k-1}}{2r_g}\exp(B\sqrt{2r_g}/\sigma) + \frac{k+\sqrt{2k-1}}{2r_g}\exp(-B\sqrt{2r_g}/\sigma) + \frac{1-k}{r_g} & \text{otherwise,} \end{cases}$$

from the proof of Proposition 1. For $k \leq 1$, f is decreasing since $\exp(-B\sqrt{2r_g}/\sigma) < 1$ for $B > 0$.

For $1 < k < \kappa$, consider k_l and k_h such that $1 < k_l < k_h < \kappa$. Suppose by contradiction there exists some $B \in (0, \bar{b}(k_l))$ such that $f(B, k_h) \geq f(B, k_l)$. Since $\bar{b}(k_h) < \bar{b}(k_l)$ (Lemma

A.1), then $f(B, k_h) = 0 < f(B, k_l)$ on $(\bar{b}(k_h), \bar{b}(k_l))$ (Claim 1). Since f is continuous in B, $f(k_h)$ and $f(k_l)$ must intersect on $(0, \bar{b}(k_h))$. Let $\tilde{b}$ be the last such intersection. So $f(\tilde{b}, k_h) = f(\tilde{b}, k_l)$ and $f'(\tilde{b}, k_h) \leq f'(\tilde{b}, k_l)$. From HJB (7), $f(B, k_h) \geq f(B, k_l) \Rightarrow f''(B, k_h) > f''(B, k_l)$. Then $f(B, k_h) > f(B, k_l)$ and $f'(\tilde{b}, k_h) < f'(\tilde{b}, k_l)$ on $(0, \tilde{b})$. But then $f(0, k_h) > f(0, k_l)$, which contradicts $f = 1/r_g$ (value matching at 0). Therefore, $f(B, k_h) < f(B, k_l)$ on $(0, \bar{b}(k_l))$.

We have that f is continuous at $k = 1$ by substituting $k = 1$ above. Continuity at $k = \kappa$ follows from the definition of κ and value matching at $\bar{b}$. ■

Claim 3 *For small enough k, f does not intersect $\hat{\phi}$ on $(0, b_v)$.*

Proof We have $\lim_{k\to 0} f = 1/r_g > \phi$ on $(0, b_v)$. Since f is continuous in k, there exists $\varepsilon > 0$ such that for $k = \varepsilon$, $f > \phi$ on $(0, b_v)$. ■

Claim 4 *For large enough k, f does intersect $\hat{\phi}$ on $(0, b_v)$.*

Proof From Lemma A.1, we have $\bar{b}$ decreasing in k. Additionally, $\lim_{k\to\infty} \bar{b} = 0$. Then $\exists k'$ such that $\bar{b}(k') < b_v$. On $(b(k'), b_v)$ we have $f = 0$ and $\phi > 0$, while at $B = 0$ we have $f = 1/r_g > \phi$. Since both are continuous functions, they intersect on $(0, b_v)$. ■

Claim 5 *There exists $\bar{k}$ such that:*

- *$k = \bar{k} \Rightarrow f$ touches $\hat{\phi}$ exactly once on $(0, b_v)$ at a tangency point,*
- *$k < \bar{k} \Rightarrow f$ does not touch $\hat{\phi}$ on $(0, b_v)$, and*
- *$k > \bar{k} \Rightarrow \exists B \in (0, b_v) : f(B) < \hat{\phi}(B)$.*

Proof Let $\bar{k}$ be the infimum of values of k such that f intersects $\hat{\phi}$ on $(0, b_v)$. By the previous claims it exists and is attained. When $k = \bar{k}$, it must be that $\bar{b} > b_v$ else either $f(\bar{b}) = 0 < \phi(\bar{b})$ if $\bar{b} < b_v$, or if $\bar{b} = b_v$ then for some $\varepsilon > 0$, we have $f(\bar{b} - \varepsilon) = 0 < \phi(\bar{b} - \varepsilon)$ since $\phi'(\bar{b}) < 0$ and $f(\bar{b}) = 0$. In either case, we could decrease k and maintain an intersection between the functions, contradicting the definition of $\bar{k}$. Indeed, we must have $f \geq \phi$.

On $(0, b_v)$, $\hat{\phi}$ is strictly concave while f is strictly convex, both with continuous first and second derivatives, so they must only touch once and it must be at a point of tangency.

Since f is decreasing in k, and strictly so on $(0, b_v)$, the claim follows. ■

Claim 6 *The baseline strategy is optimal given V's equilibrium behavior if and only if $k \leq \bar{k}$. It is uniquely optimal for $k < \bar{k}$.*

Proof Since $f \geq \hat{\phi}$ on $\mathbb{R}$, f satisfies this HJB equation:

$$0 = \max\{-r_g f + 1 - k + \frac{\sigma^2}{2} f'', r_g \hat{\phi} - r_g f\}.$$

The remainder of the proof is analogous to the proof of Proposition 1 so is omitted. ■

I now consider $k \geq \bar{k}$. I will conjecture a secession clause strategy which I show exists iff $k \geq \bar{k}$, and is optimal when it exists.

A.3.1 Conjecture

Denote the candidate value function as g.

Ordering: $0 < b_1 \leq b_2 < b_v < b_3$

Value matching at 0:

$$g(0) = 1/r_g.$$

HJB equation for $B \in (0, b_1) \cup (b_2, b_3)$:

$$r_g g = 1 - k + \frac{\sigma^2}{2} g''.$$

Value matching and smooth pasting:

$$g(b_i) = \hat{\phi}(b_i)$$
$$g'(b_i) = \hat{\phi}'(b_i)$$

for $i = 1, 2$.

For $k \leq 1$:

$$b_3 = \infty,$$
$$\lim_{B \to \infty} g(B) = \frac{1-k}{r_g},$$

and for $k > 1$:

$$g(b_3) = 0,$$
$$g'(b_3) = 0.$$

Assuming for now that $b_1 \leq b_2$, the conjectures yield

$$b_1 = b_v - \frac{\sigma}{\sqrt{2r_g}} \ln \left[\frac{1 + k \exp\left(\frac{\sqrt{2r_g}}{\sigma} b_v\right) - \sqrt{1 + 2k \exp\left(\frac{\sqrt{2r_g}}{\sigma} b_v\right)}}{k} \right]$$

$$b_2 = \begin{cases} b_v - \frac{\sigma}{\sqrt{2r_g}} \ln \left[\frac{2k}{2k-1} \right] & \text{if } k > 1, \\ b_v - \frac{\sigma}{\sqrt{2r_g}} \ln \left[\frac{2}{k} \right] & \text{if } k \leq 1, \end{cases}$$

$$b_3 = \begin{cases} b_v + \frac{\sigma}{\sqrt{2r_g}} \ln \left[\frac{2k-1}{2k-2} \right] & \text{if } k > 1, \\ \infty & \text{if } k \leq 1. \end{cases}$$

As in the baseline case, a corollary of this construction is that $g > \hat{\phi}$ and $g'' \geq 0$ for $B \in (0, b_1) \cup (b_2, b_3)$, assuming $b_1 \leq b_2$.

A.3.2 Existence

Let $k = \bar{k}$. By the proof of Claim 5, $\hat{\phi}(b') = f(b')$ and $\hat{\phi}'(b') = f'(b')$ for some $b' \in (0, b_v)$. From the uniqueness of b_1 and b_2, value matching and smooth pasting for both are satisfied at b', i.e., $b_1 = b_2 = b'$, verifying existence at $k = \bar{k}$.

Now, b_1 is strictly decreasing in k since $\partial b_1 / \partial k < 0$. We have that $\partial b_2 / \partial k$ is piecewise positive, and $\lim_{k \to 1^+} b_2 = b_2(k = 1)$ so b_2 is continuous in k.

Therefore $k < \bar{k} \Rightarrow b_1 > b_2$, while $k > \bar{k} \Rightarrow b_1 < b_2$, ensuring existence.

A.3.3 Verification and uniqueness

Assume $k \geq \bar{k}$. Uniqueness of b_1, b_2 and b_3 implies the existence of a unique g on $(0, b_1)$ and (b_2, b_3). We consider g with extensions of value $1/r_g$ for $B \leq 0$, value 0 for $B \geq b_3$, and taking values $\hat{\phi}(B)$ for $B \in (b_1, b_2)$. Thus, g satisfies:

$$0 = \max\{-r_g g + 1 - k + \frac{\sigma^2}{2} g'', r_g \hat{\phi} - r_g g\}.$$

The remainder of the proof is analogous to the proof of Proposition 1 so is omitted. ■

A.4 Proof of Proposition 3.

For k, see Section A.3.2.

We have $\partial \Delta / \partial b_v < 0 \Rightarrow \partial \Delta / \partial r_v > 0$.

Also:

$$\frac{\partial \Delta}{\partial r_g} = \frac{\sigma}{2r_g}\left[\frac{b_v k \exp\left(b_v\frac{\sqrt{2r_g}}{\sigma}\right)\left(1-\frac{1}{\sqrt{1+2k\exp\left(b_v\frac{\sqrt{2r_g}}{\sigma}\right)}}\right)}{1+k\exp\left(b_v\frac{\sqrt{2r_g}}{\sigma}\right)-\sqrt{1+2k\exp\left(b_v\frac{\sqrt{2r_g}}{\sigma}\right)}} - \Delta\right]$$

$$> \frac{\sigma b_v}{2r_g\sqrt{1+2k\exp\left(b_v\frac{\sqrt{2r_g}}{\sigma}\right)}}$$

$$> 0$$

since $\Delta < b_v$. Finally, $\partial\Delta/\partial\sigma = \Delta/\sigma > 0$ for $k > \bar{k}$. ■

A.5 Proof of Proposition 4.

From the standard properties of Brownian motions, we have:

$$P(s^*) = \begin{cases} 0 & \text{if } B_0 \le 0, \\ B_0/b_1 & \text{if } B_0 \in (0, b_1), \\ 1 & \text{if } B_0 \ge b_1, \end{cases}$$

and

$$P(s_{bl}) = \begin{cases} 0 & \text{if } B_0 \le 0, \\ B_0/\bar{b} & \text{if } B_0 \in (0, \bar{b}), \\ 1 & \text{if } B_0 \ge \bar{b}. \end{cases}$$

So it is sufficient to show $b_1 < \bar{b}$ for $k > 1$. This is true iff

$$\Delta_P \coloneqq \frac{k+\sqrt{2k-1}}{k-1} - \frac{k\exp\left(b_v\sqrt{2r_g}/\sigma\right)}{1+k\exp\left(b_v\sqrt{2r_g}/\sigma\right)-\sqrt{1+2k\exp\left(b_v\sqrt{2r_g}/\sigma\right)}} > 0,$$

for all $k > 1$. Now $\partial\Delta_P/\partial b_v > 0$ for $k > 1$, and noting that $b_v > 0$, we have

$$\lim_{b_v\to 0^+}\Delta_P = \frac{2k-\sqrt{4k^2-1}+(1+k)\sqrt{2k-1}-k\sqrt{1+2k}}{(k-1)(1+k-\sqrt{1+k-\sqrt{1+2k}})}$$

which is positive for $k > 1$. Therefore, $b_1 < \bar{b}$. ■

A.6 Proof of Proposition 5.

For $B \in [0, \bar{b}]$, the stability of s_{bl} is the solution to:

$$\begin{aligned} \psi(\bar{b}) &= 0 \\ \psi(0) &= 1 \\ \rho\psi &= \rho + \frac{\sigma^2}{2}\psi'' \text{ for } B \in (0, \bar{b}). \end{aligned}$$

This yields:

$$\psi(B) = 1 - \frac{\exp\left(B\sqrt{2\rho}/\sigma\right) - \exp\left(-B\sqrt{2\rho}/\sigma\right)}{\exp\left(\bar{b}\sqrt{2\rho}/\sigma\right) - \exp\left(-\bar{b}\sqrt{2\rho}/\sigma\right)}$$

Then $L(s_{bl}) = \max\{\psi, 0\}$ on $B_t \geq 0$. Under a clause, stability is $\rho\phi$ but with $r_g = \rho$: let $\tilde{\phi} = 1 - \exp\left(-\frac{\sqrt{2\rho}}{\sigma}(b_v - B)\right)$, then $L(s^*) = \tilde{\phi}$ on (b_1, b_2). Clearly, $L(s^*) \geq \tilde{\phi}$ on $(0, b_1)$—$\tilde{\phi}$ gives the stability when secession occurs the next time $B_t = b_v$, while under s^*, the union is also secured forever if $B_t = 0$ sooner.

Fix any $B_0 > 0$. We have $\lim_{r_v \to 0} b_2 = \infty$, $\lim_{r_v \to 0} L(s^*) = 1$ and $L(s_{bl}) < 1$. Then by continuity of $\tilde{\phi}$ and b_v w.r.t. r_v, there exists small r_v such that $L(s_{bl}) < \tilde{\phi} \leq L(s^*)$.

For $\tilde{\phi}$, we have $\partial\tilde{\phi}/\partial k = 0$, $\partial\tilde{\phi}/\partial b_v > 0$, $\partial\tilde{\phi}/\partial r_g = 0$, for $B \in (b_1, b_2)$.

For ψ, we have $\partial\psi/\partial\bar{b} > 0$, and $\partial\psi/\partial b_v = 0$ for $B \in (0, \bar{b})$. For $\max\{\psi, 0\}$, weak inequalities hold. ■