Retrospective Screening in Education and Curriculum Choice

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March 2017

ABSTRACT

I model ex-post screening of teachers by pupils based on student experience. If screening costs for pupils are sufficiently low, the only equilibrium features imperfect screening and suboptimal curriculum choices. Restricting ex-post screening then raises pupils' welfare.

KEY WORDS: Curriculum Choice; Student Experience

JEL CLASSIFICATIONS: D8; A2

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1 Introduction

The education literature has emphasized moral hazard with respect to effort in education provision. A widely adopted remedy for the lack of direct observability of teaching quality is standardized testing.[1]

A separate dimension of education provision that has received much attention is student choice, evidence showing that competition amongst schools can boost performance (see, e.g., Hoxby, 2003). This is often understood as also implying that retrospective assessment of teaching by students promotes efficient selection.

I show that retrospective screening of teachers by students can lead to worse outcomes for students. When students can screen out poor teachers on the basis of student experience, this can improve outcomes for students when the relationship between the costs students incur and student or teacher ability is orthogonal to the relationship between observed costs and curriculum choice. Otherwise, retrospective screening can promote inefficient curriculum choices – a problem that may be particularly serious in contexts where standardized testing is not widely used (such as higher education).

2 Curriculum choice and ex-post screening

I formalize my arguments in terms of a game of sequential curriculum choices by teachers and screening choices by pupils.

2.1 Setup

Assume a large number of teachers and an even larger number of students. A teacher may have more than one pupil, and there is a sufficiently large number of teachers that, if teachers and students are randomly matched, the probability of a teacher being matched to any particular pupil is approximately zero.

Teachers and pupils each have ability $a_T \in \{0,1\}$ and $a_P \in \{0,1\}$, respectively. The proportions of high-ability teachers ($a_T = 1$) and high-ability pupils ($a_P = 1$) are respectively q_P and q_T, and are known to both teachers and pupils.

There are two possible curriculum choices, $h \in \{0,1\}$, yielding a gross payoff to pupils equal to $v(h) = \alpha h$, with $\alpha > 0$; i.e. $v(1) \equiv \alpha > v(0) = 0$. Tackling curriculum h involves a cost to pupils that depends on h and on the ability types of both the pupil and the teacher, taking the form $c(h, a_P, a_T) = (1 - a_P a_T)\, \beta h$, with $\beta < \alpha$; i.e., a choice $h = 0$ never entails any learning cost, whereas a choice $h = 1$ entails no cost only if both teacher and pupil are of high ability. The assumption $\beta < \alpha$ implies that $h = 1$ always delivers a higher payoff to pupils than $h = 0$, irrespective of the pupil's ability. Thus, $h = 1$ is the efficient choice.

[1] See Neal (2011) for a discussion of accountability and performance systems in education.

The payoff to teachers if they deliver the curriculum h is $1+\gamma h$, with $\gamma > 0$; i.e., other things equal, teachers of all types prefer $h=1$ to $h=0$.

Teachers observe their own ability, a_T, and are able to distinguish between curricula – this is what makes them teachers. Pupils cannot observe either a_P or a_T, and cannot tell $h=0$ and $h=1$ apart.[2]

The sequence of actions is as follows:

(i) Teachers and pupils are matched at random.

(ii) The teacher observes a_T and chooses h.

(iii) The pupil observes a perfectly informative signal of cost, $\sigma = c(h, a_P, a_T)$, without directly observing h, a_P or a_T, and without actually incurring the cost.

(iv) The pupil can either stick to the initially assigned teacher or switch to another, newly sampled teacher. If the pupil does not switch the game ends, the realized payoff for the pupil is $v(h) - c(h, a_P, a_T) = \big(\alpha - (1 - a_P a_T)\,\beta\big)\,h$ and the realized payoff for the teacher is $1+\gamma h$. If the pupil decides to switch, the game proceeds to the next stage.

(v) The new randomly assigned teacher chooses h and the game ends, with a realized payoff for the pupil of $\delta\big(v(h^R) - c(h^R, a_P, a_T^R)\big) = \delta\big(\alpha - (1 - a_P a_T^R)\,\beta\big)\,h^R$, where R refers to the new randomly sampled teacher, and $\delta \in [0,1]$ is a discount factor, reflecting a cost of switching for the pupil. The realized payoff for the newly assigned teacher is $1+\gamma h^R$ and the realized payoff for the teacher initially assigned to the pupil is zero.

Given that actions in stage (iv) are made on the basis of beliefs that vary in dependence of an informative signal that depends on preceding actions, the relevant solution concept is Perfect Bayesian Equilibrium.

2.2 Actions

Starting from (v), selecting $h=1$ is a dominant strategy for teachers of both ability types.

Moving backwards to (iv), choices at (v) mean that the continuation payoff to the pupil from switching is $\delta\big(\alpha - (1 - q'_P(\sigma, \pi(0), \pi(1))\,q_T)\,\beta\big) \equiv \delta\,\omega_S(\sigma, \pi(0), \pi(1))$, where $\pi(a_T)$ are the probabilities with which teachers of each ability type select $h=1$, and where $q'_P(\sigma, \pi(0), \pi(1))$ is the pupil's updated belief about a_P after observing the signal, σ, at (iii). Bayesian updating gives

$$q'_P\big(\beta, \pi(0), \pi(1)\big) = q_P\,\frac{\Pr(\sigma = \beta \mid a_P = 1)}{\Pr(\sigma = \beta)} < q_P, \tag{1}$$

[2] In the real world, students accumulate experiences that inform them about their own abilities. Previous learning experiences, however, cannot fully remove a student's uncertainty about her suitability to new learning opportunities (e.g. a new stage in a sequence of learning stages).

$$q'_P(0, \pi(0), \pi(1)) = q_P \frac{1 - \Pr(\sigma = \beta \mid a_P = 1)}{1 - \Pr(\sigma = \beta)} > q_P, \tag{2}$$

where

$$\Pr(\sigma = \beta \mid a_P = 1) = (1 - q_T)\pi(0) \tag{3}$$

is the probability of a signal $\sigma = \beta$ being received by a high-ability pupil, and

$$\Pr(\sigma = \beta) = q_P(1 - q_T)\pi(0) + (1 - q_P)\big((1 - q_T)\pi(0) + q_T\pi(1)\big) \tag{4}$$

is the probability of such a signal being received by a randomly selected pupil.

Let $z(\sigma, \pi(0), \pi(1))$ denote the pupil's posterior belief that, having observed σ, the curriculum choice is $h = 1$. Since a positive cost can only be experienced if $h = 1$, the pupil's posterior belief that, having observed $\sigma = \beta$, curriculum choice is $h = 1$ is $h = 1$ is $z(\beta, \pi(0), \pi(1)) = 1$, and so the expected payoff from not switching if $\sigma = \beta$ is $\alpha - \beta \equiv \rho(\beta)$. On the other hand, the pupil's posterior belief that, having observed $\sigma = 0$, curriculum choice is $h = 1$ is $q_T q_P \pi(1)/((1 - q_T)(1 - \pi(0)) + q_T(1 - \pi(1) + \pi(1)q_P) \equiv z(0, \pi(0), \pi(1))$, and so the expected payoff from not switching if $\sigma = 0$ is $z(0, p(0), p(1))\,\alpha \equiv \rho(0)$.

The switching decision given σ depends on a comparison of $\delta\omega_S(\sigma, \pi(0), \pi(1))$ and $\rho(\sigma)$. Noting that $\omega_S(\sigma, \pi(0), \pi(1))$ equals δ times a convex combination of $\rho(\beta)$ and $\rho(0)$, and given that $\delta < 1$, we can exclude that $\rho(\sigma) < \delta\omega_S(\sigma, \pi(0), \pi(1))$ for both $\sigma = \beta$ and $\sigma = 0$, i.e. switching can only occur for one of the two signal types. If δ is sufficiently small, switching never occurs. If the pupil is indifferent between switching and not switching for one of the two signal types, switching can occur with some probability $\pi_S(\sigma)$ between zero and unity.

In stage (ii), the expected payoff to a teacher of ability type a_T from choosing h is

$$(1 + \gamma h)\big(1 - \Pr(\beta \mid h, a_T)\big)\,\pi_S(\beta) \equiv V(h, a_T), \tag{5}$$

where $\Pr(\beta \mid h, a_T) = (1 - a_T q_P)h$ is the probability of generating a signal $\sigma = \beta$ for a given curriculum choice and teacher ability type. The teacher then chooses the curriculum h that delivers the higher $V(h, a_t)$ given her type, or mixes between the two if indifferent.

2.3 Equilibria

An equilibrium where the pupil switches upon observing $\sigma = 0$ with any positive probability cannot exist: anticipating this, selecting $h = 1$ is a dominant strategy for both teacher types, implying $\pi(0) = \pi(1) = 1$; this in turn means that switching when receiving $\sigma = 0$ is a dominated strategy for the pupil – because, given these beliefs about play and since the cost realization is zero, not switching delivers α, the maximum possible payoff for the pupil.

An equilibrium with no switching is possible if $\delta \leq \bar{\delta}$, where $\bar{\delta}$ is the discount rate that makes the pupil indifferent between switching and not switching upon receiving $\sigma = \beta$, i.e. the value of δ that solves $\rho(\beta) = \delta\omega_S(\beta, 1, 1)$:

$$\bar{\delta} = \frac{\rho(\beta)}{\omega_S(\beta, 1, 1)} = \frac{\alpha - \beta}{\alpha - \beta(1 - 2q_T q_P + q_T^2 q_P)/(1 - q_T q_P)}. \tag{6}$$

I.e. if the pupil believes that both teacher types always select $h = 1$, and δ is sufficiently low that switching upon receiving a signal $\sigma = \beta$ is not profitable under these beliefs, then selecting $h = 1$ is a dominant strategy for all teachers; and so this is an equilibrium.

If $\delta > \bar{\delta}$ then beliefs $\pi(0) = \pi(1) = 1$ would induce switching upon receiving $\sigma = \beta$; but then a low-ability teacher (who can never induce a signal $\sigma = 0$ by choosing $h = 1$) would always gain from selecting $h = 0$ over $h = 1$. In turn, if the pupil holds beliefs $\pi(0) = 0$, switching cannot be a best response for her, since a signal $\sigma = \beta$ would reveal a choice $h = 1$ by a high-ability teacher, and also reveal that the pupil is of low-ability type, implying a payoff $\alpha - \beta$ with no switching and a continuation payoff $\delta(\alpha - \beta)$ with switching. The only possible equilibrium in this case is a mixed-strategy equilibrium with $\pi_S(\beta)$ strictly between zero and one and

$$\rho(\beta) = \delta\omega_S(\beta, \pi(0), \pi(1)). \tag{7}$$

Since, for any given $\pi_S(\beta)$, the expected payoff from choosing $h = 1$ is higher for high-ability teachers than it is for low-ability teachers but the expected payoff from choosing $h = 0$ is the same for both ability types, an equilibrium where both types play mixed strategies is not possible. If the high-ability teacher type is indifferent between the two, then the low-ability teacher type must strictly prefer $h = 0$; but, as was noted above, this cannot be an equilibrium because it makes no switching a dominant strategy for the pupil. So, the only possible equilibrium in this case involves high-ability teachers selecting $h = 1$ with probability $\pi(1) = 1$ and low-ability teachers playing a mixed strategy with $\pi(0)$ strictly between zero and one, and

$$V(1, 0) = V(0, 0). \tag{8}$$

Since $\pi(1) = 1$, (7) becomes

$$\rho(\beta) = \delta\omega_S(\beta, \pi(0), 1). \tag{9}$$

Mixed-strategy equilibrium probabilities $\pi(0)$ and $\pi_S(\beta)$ are identified by (8) and (9), giving

$$\pi_S(\beta) = \frac{\gamma}{1 + \gamma}, \qquad \pi(0) = \frac{(1 - q_P)q_T/(1 - q_T)}{q_T q_P \delta\beta/((1 - \delta)(\alpha - \beta)) - 1}. \tag{10}$$

2.4 Welfare and policy implications

I focus on welfare implications for pupils, both because this is what education policies are typically concerned with, and because, with individual teachers being matched to multiple students, welfare effects for teachers are less important from a utilitarian perspective (becoming increasingly negligible as the student-to-teacher ratio increases).

With full information about teacher ability, making it possible to select high-ability teachers ex ante, the mean surplus for pupils is $\alpha - (1 - q_P)\beta \equiv U^F$. This is the maximum level that can be theoretically achieved. Pupils' expected surplus in an equilibrium with private information in a scenario with $\delta > \bar{\delta}$ and where (9) holds is $(1 - q_T)\pi(0)(\alpha - \beta) + q_T\big(q_P\alpha + (1 - q_P)(\alpha - \beta)\big) \equiv U^N$. A ban on ex-post screening – or, equivalently, an increase in switching costs as represented by an exogenous fall in δ below $\bar{\delta}$ – yields an expected surplus to pupils equal to $\alpha - (1 - q_T q_P)\beta \equiv U^B$. Comparing this with U^N gives $U^B - U^N = (1 - \pi(0))(1 - q_T)(\alpha - \beta) \geq 0$; i.e. restricting switching raises pupils' expected welfare. Finally, a curriculum standard that directly enforces $h = 1$ but does not restrict switching gives $q_T q_P \alpha + (1 - q_T q_P)\delta(\alpha - (1 - q_T q_P)\beta) \equiv U^S$. A ban on switching also results in a choice of $h = 1$ by all teacher types, but a standard does so without restricting the ability to screen out low-ability teachers, and so it must produce a level of expected welfare for pupils that is at least as high as that under a switching ban.

A parameterized example can help illustrate these comparisons. Assume $\alpha = 2$, $\beta = 1$, $q_T = q_P = 1/2$, $\delta = 1/1.05 = 20/21$, $\gamma = 1/2$. We then have $\bar{\delta} = 6/7 < \delta$ (and so we have a mixed-strategy equilibrium), $\pi_S(\beta) = 1/3$, $\pi(0) = 1/8$, $U^N/U^F = 13/24 \simeq 0.54$, $U^B/U^F = 5/6 \simeq 0.83$, $U^S/U^F = 13/14 \simeq 0.93$.

If a direct curriculum standard can be enforced, restrictions on ex-post student choice become redundant. Still, if a standard cannot be perfectly enforced, it can be dominated by restrictions on ex-post student choice. To see this, suppose that, rather than directly imposing $h = 1$, students receive, in addition to σ, a separate noisy signal, τ, about curriculum choice through standardized testing that equals the curriculum actually chosen with probability $\eta > 1/2$ and the other curriculum with probability $1 - \eta$. If η is sufficiently close to $1/2$, then τ will have no effect on the switching choice. On the other hand, for $\eta = 1$ testing is equivalent to a standard. Then, by continuity, there must be a value $\widetilde{\eta}$ lying between $1/2$ and unity for which pupils' expected welfare in an equilibrium with noisy standardized testing equals U^B; and so for values of η strictly between $1/2$ and $\widetilde{\eta}$ restricting ex-post screening dominates testing.

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