Skills Scarcity and Export Intensity*

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This version: August 2022

Abstract

We describe a model of trade with skills-based product differentiation and non-proportional trade costs that predicts a positive correlation between firms' export intensity, the price of their exports, and the wages they pay to their workers. In equilibrium, firms that employ workers with comparatively scarcer skills export a larger proportion of their output, pay higher wages and charge higher prices. In line with empirical evidence, the model predicts that trade liberalization can cause the distribution of earnings to become more polarized, with patterns that reflect the heterogeneous effects of trade liberalization on firms' export performance.

KEY WORDS: Product Differentiation, Exporter Wage Premia, Wage Polarization

JEL CODES: F11, F12, F16, J24, J31

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*We are grateful to Costas Arkolakis, Lorenzo Caliendo, Carsten Eckel, Sharat Ganapati, Elhanan Helpman, Gianmarco Ottaviano, Andrés Rodríguez-Clare, other participants at seminars and conference presentations at CEPII in Paris, CESifo in Munich, HSE University in St. Petersburg, ETH in Zurich, London School of Economics and Polytechnic of Milan, and two anonymous reviewers for comments and suggestions. Carlo Perroni gratefully acknowledges the support from Warwick's CAGE. Davide Suverato gratefully acknowledges the support from the Excellence Initiative of the German Research Foundation. The authors thank the INSEE and the assistance of the Centre d'accès sécurisé aux données (CASD) for the access to the data used in the project.

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1 Introduction

Firms are heterogeneous in many dimensions. While workhorse models of international trade focus on factor-neutral productivity differences, recent studies have shown that specific factor content and input-output linkages are by far a more important driver of firm heterogeneity.[1] In particular, studies that focus on firms' exports find that factor content matters because it confers specific characteristics to exported products, concluding that input differentiation is a key determinant of output quality.[2] This conclusion is remarkably robust for labor: the specific varieties of skills employed are a first-order predictor of both output quality and export conduct.[3]

This paper develops a model of international trade where firms behave heterogeneously because the labor skill types they employ are differentially scarce and confer distinct characteristics to their products—what we call *input-based product differentiation*. In this model, comparative skills scarcity is sufficient to generate a rich set of predictions that can account for the observed patterns between prices that firms charge and the wages that they pay to their employees. We keep the structure of our model simple to highlight its distinctive features relative to existing models where prices, wages and export conduct respond to productivity differentials between firms, to quality choice and to a skill-biased composition of the labor force.[4] In contrast to those models, ours does not hinge on exogenous productivity differentials and does not need to assume an ex-ante ranking of products or worker types in terms of their intrinsic quality. The association of high-price products with high-wage workers at firms with greater measured productivity emerges in the model as an equilibrium pattern: under monopolistic competition and increasing returns to scale, comparatively scarcer skills are concentrated in fewer firms, which are larger, pay higher wages and charge higher prices.

We extend Krugman's (1980) model of trade between symmetric countries with homogeneous, monopolistically competitive firms by introducing multiple types of output-differentiating skill types. Firms each employ a composite intermediate input and one specific skill type. Products are horizontally differentiated both in relation to the skill types used to produce them and to the firms that produce them. The labor market is

[1]Hottman et al. (2016) show that less than 25% of observed firm heterogeneity can be accounted for by differences in factor-neutral marginal costs. Between 50% and 70% of firm heterogeneity is attributable to product characteristics that match consumer taste, conditional on price. Roberts et al. (2018) find that firm heterogeneity is mostly accounted for by product differentiation, as opposed to differences in marginal or fixed costs. The prominence of input-output linkages as a determinant of firm heterogeneity is starting to emerge more clearly as more evidence becomes available; as an example, see Barrot and Sauvagnat (2016), Atalay (2017) and Oberfield (2018).

[2]The importance of differences in factor usage has been documented in many studies, such as Manova and Zhang (2012) and Feenstra and Romalis (2014), among others, and it has been recently quantified by Dingel (2017) as accounting for half of the total variation in quality.

[3]A number of studies trace back exports to the implied demand of labor skills; starting from the seminal work by Bombardini et al. (2012) to more recent contributions by Brambilla et al. (2017a), Brambilla et al. (2017b), Dai and Xu (2017) and Lichter et al. (2017). Other studies document the linkages between skills, product quality and characteristics of the destination market; see Bastos and Silva (2010), Brambilla and Porto (2016) and Macis and Schivardi (2016).

[4]See Epifani and Gancia (2006), Bernard et al. (2007), Crinò and Epifani (2012), Kugler and Verhoogen (2012), Emami Namini et al. (2015), Gervais (2015) and Harrigan and Reshef (2015).

perfectly competitive; and so, with horizontal product differentiation between products arising from the type of input used to produce them, skill types in comparatively shorter supply will earn higher wages in equilibrium. Moreover, since fixed costs involve labor inputs, they will be larger for those firms that employ scarcer, higher-wage skill types (and that, because of this, charge higher prices), implying that those firms will have to be larger in order to break even, and that there will be fewer of them. Intuitively, the greater concentration of scarcer skills at fewer firms causes those firms to benefit comparatively more from economies of scale, which in turn translates into a higher measured productivity of labor at firms that employ scarcer and more highly paid skill types.

As in Krugman (1980), producers sell both at home and internationally, with exports incurring trade costs. But we depart from the standard assumption that transportation costs are of the iceberg type, an assumption that implies that export intensity (the ratio of a firm's domestic revenues to its total revenue) should be independent of its size and of the prices it chargers. Those patterns are consistently rejected in the data; in particular, a well-documented feature of trade data is that the incidence of trade costs in final export prices tends to be smaller for higher-priced products—the so-called "Alchian-Allen effect" (Alchian and Allen, 1972).[5] Our model replicates this feature by positing that exporting requires export services, delivered through a combination of a firm's own output (which would be the only component of cost in an iceberg specification) and of an intermediate composite input that is independent of the firm's output.[6] This implies that the factor content of a product at destination, including export services, is different from the factor content of the product itself and that export services for higher-priced products employ the firm's own inputs comparatively less intensively than lower-priced products. The price of a product in export markets is then increasing with its producer price but less than proportionally so. Thus, products that are comparatively higher-priced in the domestic market remain comparatively higher-priced also in the export market; but the relative price gap between higher-priced and lower-priced products is lower in export markets than it is in the domestic market. As a result, trade liberalization raises the relative demand for higher-priced products, which in turn raises the relative demand for scarcer skill types.

The combination of skills-based product differentiation and Alchian-Allen effect is sufficient to account for the positive correlation between wages, prices and sales in export markets that is observed in the data—the basic empirical patterns that give rise to a large literature focusing on trade with heterogeneous firms, exporter wage premia, a skill-biased production and quality choice.

[5]Starting with the seminal contribution by Hummels and Skiba (2004), to more recent studies, such as Martin (2012), Feenstra and Romalis (2014), Gervais, 2015), and Irarrazabal et al. (2015), the empirical literature on trade costs has documented the prominence of an additive (as opposed to proportional) component of unit costs. The Alchian-Allen effect has received renewed attention in the literature on export conduct and quality choice (Feenstra and Romalis, 2014, Bekkers et al., 2016 and Flach and Unger, 2016.

[6]The assumption of a production function for export services is aimed to capture in a reduced form the evidence discussed in Labanca et al. (2014), Bastos et al. (2016), and Brambilla et al. (2019). The authors document how exporting requires tasks that do not perfectly overlap with the ones of production. This idea is reflected in our setup, where the intensity of the specific skill types employed by the firm is greater in production for domestic sales than in that for export sales.

Despite its parsimoniousness, the model structure can also generate a rich set of predictions about the effect of trade openness on the distribution of earnings. Every skill type gains from trade. However, due to the interplay between increasing returns to scale and the incidence of trade costs on products that are produced with differentially scarce inputs and command different prices, the model predicts that international trade can make the distribution of earnings more concentrated at the upper end of the distribution of earnings and less so at its lower end, resulting in wage polarization—in line with a large body of evidence.[7] This heterogeneity in distributional effects across workers who are paid different wages mirrors the heterogeneity in the size and export intensity of the firms that they work for.

Evidence of a relationship between the scarcity of a firm's inputs and its export intensity predicted by the model can be found in the data: focusing on French exporters, we show that, after controlling for observable technology differentials and workers' demographic characteristics, the evolution of differences in export intensity across sectors is positively correlated with changes in the scarcity of the worker types they employ. Results of numerical simulations with a calibrated version of the model also show that, when compared with independent estimates of wage polarization, the wage polarization effect predicted by the mechanism we describe is quantitatively significant. More importantly, our predictions on trade-induced wage inequality mirror the observed qualitative sorting: reduction of inequality at lower wages and increase of inequality at higher wages.

Our study contributes to several strands of research. First, as already discussed, we contribute to the debate on the causes of the observed correlation between export conduct, wages and prices. The key contributions in this area combine firm heterogeneity in terms of factor-neutral productivity differentials with a mechanism generating a skill-biased composition of the labor force and a quality-biased product choice in comparatively more productive firms (Epifani and Gancia, 2006; Verhoogen, 2008; Crinò and Epifani, 2012; Kugler and Verhoogen, 2012; Feenstra and Romalis, 2014; Emami Namini et al., 2015; Harrigan and Reshef, 2015). Our model can account for the same patterns without the need to postulate a quality ranking for products or a ranking of labor types, and without the need to postulate that high-quality products are intensive in the employment of "high-skill" labor types.[8] The positive association between higher-wage skills, higher-priced products and greater export sales arises endogenously in our model as a result of differential skills endowments. Although the supply of product-specific skills has received less attention in the trade literature, there is evidence of a prominent role of specific labor characteristics in explaining firms' export conduct,[9] an

[7]See, for example, Autor and Dorn (2013) and Goos et al. (2014).

[8]This set of assumptions is typically validated by approximating unobserved quality with price rankings and the sorting of labor types with wage rankings. This approach is problematic, as it measures an exogenous characteristic in the model with its endogenous equilibrium outcome (Khandelwal, 2010; Hottman et al., 2016). Our approach has a simpler and less demanding interpretation: high-quality products are observationally equivalent to horizontally differentiated products that must be produced with comparatively scarcer inputs.

[9]Irarrazabal et al. (2013) show that more than half of the exporter wage premium is accounted for by characteristics of the labor force that are exogenous to the firm. The role of specific labor types for firm performances has received attention in two other recent contributions. Caliendo et al. (2015) show that

empirical finding that our theoretical model can shed light on.

Second, our model is related to the large and well-established body of literature on factor proportions, which has seen a wave of new theoretical and empirical contributions (Romalis, 2004; Bernard et al., 2007; Bombardini et al., 2012; Burstein and Vogel, 2017). As in Romalis (2004), our model combines a monopolistically competitive structure à la Krugman (1980) with factor intensity differentials and factor endowment differentials across producers; but in our model with symmetric countries and non-iceberg trade costs, factor endowment differentials give rise to comparative differentials in trade intensity across producers rather than to comparative advantage differentials across economies: products produced with comparatively scarcer skill types are predicted to be exported comparatively more, symmetrically so across countries. This is a markedly different mechanism from those highlighted in previous frameworks that combine comparative factor scarcity with output differentiation.[10]

Third, our study contributes to the literature seeking to explain the relationship between skill premia and exporter wage premia.[11] The predictions of our model are consistent with the findings in this literature, which show how skills rewarded with higher wages are more intensively employed in the production of products that are exported comparatively more. But while existing explanations focus on the role played by labor market frictions, our formalization only relies on the interplay between comparative skills scarcity and non-proportional trade costs. The effect of international trade on the wage distribution predicted by our model is consistent with growing evidence against the Stolper-Samuelson theorem and factor price equalization.[12,13]

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firms willing to expand pay more for certain types of occupations (higher in the organizational hierarchy, in their terminology), while they pay less for the others. Harrigan et al. (2016) investigate the rise of wage polarization in the French labor market and conclude that polarization is driven by the changes in the wages of certain specific worker types (those with technology-related occupations, in their terminology). Our interpretation here is consistent with those findings.

[10]It should be noted that firm heterogeneity is neither necessary nor sufficient to reach this conclusion. In Bernard et al. (2007), firm heterogeneity in productivity introduces a Ricardian channel that reinforces the classical Heckscher-Ohlin mechanisms. By comparison, in our framework (which abstracts from productivity differentials across firms), international trade affects prices, wages and export performance because the relative demand for higher-priced goods (which are produced with scarcer skill types) is comparatively greater in export markets than in the domestic market.

[11]Helpman and Itskhoki (2010), Helpman et al. (2010), Felbermayr et al. (2011), Gopinath et al. (2012) and Montagna and Nocco (2013) focus on the role of rent-sharing mechanisms; Amiti and Davis (2012), Davis and Harrigan (2011), Egger et al. (2013) and Egger et al. (2017) put forward efficiency wage-based arguments. Positive assortative matching between high-productivity, export-intensive firms and high-skill workers is discussed in Yeaple (2005) and Sampson (2016).

[12]Burstein and Vogel (2017) develop a quantitative general equilibrium trade model that extends Heckscher-Ohlin framework to allow for firm heterogeneity, with more productive firms characterized by more skill-intensive technologies. They show that the skill premium tends to increase in all countries, as reductions in trade costs raise the relative demand for high skills. Our model arrives at the same conclusion but through a different mechanism (and with different implications): comparative skills scarcity dictates which skill types and which products have a higher price in equilibrium; hence, less-than-proportional trade costs determine the increase in relative demand for more expensive products.

[13]The idea of a "race between demand and supply" of skills shaping the distribution of earnings has been originally proposed by Tinbergen (1956), and since then it has influenced the development of economic thoughts in many fields, prominently in the literature on human capital and endogenous growth—see Acemoglu and Autor (2011) and Heckman (2018) for a review.

The interplay between factor proportions and non-iceberg costs in accounting for inter-sectoral trade differentials has previously been described by Matsuyama (2007) and by Bekkers et al. (2016).[14] In those models the relationship between export conduct and prices is driven by an Alchian-Allen effect, as it is in our model, but the mechanism that generates the effect is different. Trade costs in our model respond endogenously and less than proportionally to variation in producer prices because of the complementarity between produced goods and other intermediate inputs in the production of export services.

The rest of the paper is structured as follows. Section 2 describes the model. Section 3 derives autarky and open-economy equilibria and presents our main theoretical predictions. Section 4 discusses some reduced-form evidence and a quantification. Section 5 concludes.

2 Trade with skills-based product differentiation

Endowments

There are m countries. In each country $i \in \{1, \ldots, m\} \equiv M$, individuals are endowed with one unit each of indivisible labor. There exist S different varieties of labor inputs, which we refer to as *skill types*, each indexed by $s \in \{1, \ldots, S\}$.[15] The supply of skill type s in a country i (the number of individuals endowed with that skill type) is exogenously given and equal to $L_i(s) > 0$.

Technology, preferences and markets

There are S good types, each being produced with labor of a certain labor skill type, s, and for each good type, s, there is a measure $N_i(s)$ of horizontally differentiated varieties of that type being produced in country i.[16] Varieties of a certain good type are aggregated into a composite for that good type through a constant-elasticity-of-substitution technology:

$$X_i(s) = \left(\sum_{h \in M} \int_0^{N_h(s)} x_{ih}(j,s)^{\frac{\eta-1}{\eta}} \, \mathrm{d}j \right)^{\frac{\eta}{\eta-1}}, \tag{1}$$

[14] Matsuyama (2007) considers a many-factors Ricardian model where production technologies feature different factor intensities depending on where products are sold, leading to a link between factor endowments, factor prices, and market integration that cannot be captured by an iceberg cost specification. Bekkers et al. (2016) extend Krugman (1980) with exogenous, per-unit trade costs and heterogeneous labor types by skill level. The distinctive role of non-iceberg trade costs in an otherwise standard workhorse trade model (without a specific role for quality or skill types) is discussed in Sørensen (2014).

[15] Higher-s skill types need not necessarily be thought of as representing "higher skill level" types, although an interpretation in terms of vertical differentiation is possible (and gives rise to an equivalent formulation and identical predictions).

[16] The skill types in the model should be interpreted as varieties of workers within the same occupation, but the same modeling approach could be extended to include multiple occupations.

where $x_{ih}(j,s)$ is the quantity of variety j of good type s produced in country h and used in country i, and $\eta > 1$ is the elasticity of substitution across different varieties of the same good type $X_i(s)$. Goods of different types $s \in \{1,\ldots,S\}$ produced in country i are then combined to obtain a composite good through a Cobb-Douglas technology:

$$Y_i = \prod_{s=1}^{S} X_i(s)^{\alpha} \ , \quad \alpha = 1/S. \tag{2}$$

The structure formalized by (2) and (1) implies that varieties produced by different firms but with the same skill type are closer substitutes with each other than are varieties produced with different skill types. We refer to this property as *input-based product differentiation*, although it can equivalently be thought of as sector-specificity of differentiated labor inputs.[17] The composite good is used as an intermediate input in production and is consumed by individuals.[18]

Varieties of good s are produced combining labor inputs of skill type s and the composite good. Specifically, a firm producing a variety j of good type s in country i employs $l_i(j,s)$ units of differentiated labor of skill variety s and $y_i(j,s)$ units of the product composite of country i as intermediate input, transforming them into output, $q_i(j,s)$, according to a Cobb-Douglas technology:

$$q_i(j,s) = \frac{l_i(j,s)^{\lambda}\, y_i(j,s)^{1-\lambda}}{\lambda^{\lambda}(1-\lambda)^{1-\lambda}} \ , \tag{3}$$

where $\lambda \in (0,1]$ is the share of labor in variable production costs. In addition to variable costs, every active firm incurs a fixed production cost consisting of $f > 0$ units of labor of the same skill type as that used to produce its output.[19]

Individuals of a country i derive utility form consuming units of the composite product: the utility of representative worker of type s in country i is

$$u_i(s) = y_i^c(s), \tag{4}$$

[17] The assumption of a symmetric Cobb-Douglas aggregator in (2) is expositionally convenient, and implies a unit elasticity of substitution among goods produced with different skill types. However, our analysis and results go through with any aggregation that features a constant elasticity of substitution greater than unity and lower than η. They would also go through if we allowed asymmetric demand shares, $\alpha(s)$, and ordered skill types in terms of their scarcity relative to demand, i.e., in terms of the ratios $L(s)/\alpha(s)$. Indeed, what characterizes skills scarcity is a comparatively shorter supply—not in an absolute sense, but relative to demand.

[18] This is as in Eaton and Kortum (2002) and in many other related models.

[19] A more general specification of sector-specificity of labor inputs would be one where the production of varieties of type s uses a composite $\check{l}$ defined as $\check{l}(s)^{(\varsigma-1)/\varsigma} = \kappa\, l(s)^{(\varsigma-1)/\varsigma} + ((1-\kappa)/S) \sum_{s' \neq s} l(s')^{(\varsigma-1)/\varsigma}$, $\varsigma > 0$, $\kappa > 1/S$. Our specification coincides with the case $\kappa = 1$, and can be viewed as limit case of a Roy assignment model where goods with different characteristics are produced by different sectors and different worker types are each best matched with a different sector (Roy, 1951; Costinot and Vogel, 2015; Lee, 2020; Gola, 2021; Galle et al., 2020); we are thankful to an anonymous referee for pointing this out. Allowing for symmetrically differentiated substitution possibilities between different skill types in the production of each good variety would have a qualitatively analogous effect on the demand for skill types as having a higher elasticity of substitution of demand in our more restrictive setting.

where $y_i^c(s)$ is the quantity of country-i's composite good consumed by that worker.

Trade costs

Firms sell the variety they produce both in the domestic market and in foreign markets. Markets are segmented internationally. In order to sell differentiated goods in export markets, firms must combine the differentiated good they produce with export services, such as market-specific marketing, establishing and handling distribution channels, complying with export-market specific laws, regulations, and standards. We assume that the exported goods and export services are perfect complements in the production of these export services. One unit of exported output from country i to country h requires b_{ih} units of export services, where the parameter b_{ih} captures the level of trade barriers faced by a firm producing in country i when exporting to country h, such that $b_{ii} = 0$ but $b_{ih} > 0$ for every $h \neq i$.

The technology used to produce export services is analogous to (3), but employs the firm's output according to a share $\xi \in (0, 1)$,

$$\chi_i(j,s) = \frac{q_i(j,s)^{\xi}\, y_i(j,s)^{1-\xi}}{\xi^{\xi}(1-\xi)^{(1-\xi)}}\,, \tag{5}$$

where $\chi_i(j,s)$ is the output of export services produced with the employment of $q_i(j,s)$ units of firm's output, and $y_i(j,s)$ units of aggregate good. Denoting with $c_i(j,s)$ the unit cost of firm's output and with P_{Yi} the price of aggregate input in country i, the cost per unit exported (the c.i.f./f.o.b. margin) is equal to $b_{ih}\, c_i(j,s)^{\xi}\, P_{Yi}^{1-\xi}$.

The limit case $\xi = 1$ corresponds to the standard iceberg costs case where each unit of exported variety requires $1 + b_{ih}$ units of shipment of that variety, implying a corresponding total export unit cost of $c_i(j,s)(1 + b_{ih})$. For $\xi \in (0, 1)$, however, the factor content of exports differs from the factor content of domestic sales. The feature of this formulation that is crucial to our results is that there is an element of trade costs that does not vary with the price of the good traded.[20] This implies that the effective share of variety-specific skill type, s, in trade costs is less than that in production, i.e., $\xi\lambda < \lambda$, which in turn makes trade costs less-than-unit-elastic with respect to the f.o.b. price of the exported good. Strong empirical support for this representation of trade costs is presented in Hummels and Skiba (2004) and in Feenstra and Romalis (2014).[21,22]

[20]This would also be true if, instead of trade costs consisting of a combination of goods and the composite input in (5), they consisted of a combination of goods and differentiated labor inputs (be they of a low-s or high-s variety), as long as the composition of those differentiated labor inputs (and thus its composite price) did not vary with s, the type of good being exported.

[21]While they do not provide a micro-foundation of the export technology, the estimation approach they use to study the empirical relationship between price and trade costs is directly consistent with (5). Their central estimate of the elasticity of the c.i.f./f.o.b. margin to price is $\xi = 0.6$.

[22]Arkolakis (2010) proposes a theory of marketing costs in which the employment of labor required to enter a foreign market is increasing and convex in the fraction of consumers reached, with the result that sales of products with low volumes prior to trade liberalization grow relatively more when trade barriers decline. The same prediction arises naturally from a specification where trade costs are less-than-unit elastic with respect to price, as in the formulation that we adopt here.

Equilibrium

Consumers in country i derive utility from consumption of the overall product composite for country i. Firms maximize profits. As individual firms are atomistic, the market for varieties of any given good type s is characterized by a monopolistically competitive structure. The structure of preferences (1) is such that the price elasticity of demand faced by individual firms is constant and equal to $-\eta$, and therefore marginal revenue is proportional to the price charged by a constant factor $(\eta - 1)/\eta$. Profit maximization then implies a constant markup factor (price over marginal cost) equal to $\eta/(\eta - 1)$ in each destination market.

There are no restrictions to firm entry and exit, and no costs for entering or exiting. With symmetric firms producing horizontally differentiated varieties for each good type, s, free entry and exit implies that, in equilibrium, all active firms exactly break even (total revenues equal total costs), with $N_i(s)$ being determined endogenously.[23]

Skill types are fully observable. Since only labor of skill type s can be employed to produce goods of type s, equilibrium wages will be skill-type specific. Labor markets are competitive, which implies price taking behavior by firms and workers with respect to wages and full employment.

An equilibrium for this economy consists of, for each skill/good type s, a measure of firms $N_i(s)$, a wage level $w_i(s)$, an allocation of output $q_i(j,s)$, inputs $(l_i(j,s), y_i(j,s))$, an allocation of per-capita consumption levels $y_i^c(s)$, and prices $p_{ih}(s)$ for differentiated varieties in each origin-destination market pair $i \in M$, $h \in M$; and prices P_{Yi} for the composite good in each originating market, $i \in M$, such that:

(i) consumption $y_i^c(s)$ by each worker satisfies the budget constraint $P_{Yi}\, y_i^c(s) = w_i(s)$;

(ii) prices $p_{ih}(s)$ maximize profits for every firm $j \in [0, N_i(s)]$;

(iii) total revenue equals total cost for every firm $j \in [0, N_i(s)]$ that is active in every country;

(iv) markets for the composite good Y_i clear in every country;

(v) product markets clear for every produced variety $j \in [0, N_i(s)]$;

(vi) labor markets clear;

for every skill variety $s \in S$ produced in every country $i \in M$.

3 Autarky and open-economy equilibria

3.1 Autarky equilibrium

We first describe an equilibrium for an economy under conditions of autarky (i.e., when trade barriers are prohibitive), omitting origin and destination market indicators.

[23]This also means that the sorting of firms across skill/good types is irrelevant in this context—which would not be the case if firms were heterogeneous with respect to their technologies, e.g., if they had different productivity levels.

Cost minimization given the technology (3) implies a marginal cost $w(s)^{\lambda} P_Y^{1-\lambda}$ for a variety of good type s, where P_Y is the price of the aggregate Y, and thus a price

$$p(s) = \frac{\eta}{\eta - 1} \, w(s)^{\lambda} \, P_Y^{1-\lambda}. \tag{6}$$

Utility maximization yields the level of demand for the composite good s, equal to

$$X(s) = \alpha Y \frac{P_Y}{P(s)}, \tag{7}$$

where $P(s)$ is the composite price of varieties of good type s, corresponding to the CES aggregation (1). Equilibrium price symmetry across the different varieties of any given good type implies that $P(s)$ can be written as

$$P(s) = N(s)^{\frac{1}{1-\eta}} \, p(s). \tag{8}$$

This yields the following equilibrium level of demand for a variety of good s:

$$x(s) = X(s) \left(\frac{P(s)}{p(s)} \right)^{\eta} = \frac{\alpha Y}{N(s)} \, \frac{P_Y}{p(s)}. \tag{9}$$

As shown in Appendix A.1, market clearing in product markets and in input markets, in conjunction with a zero-profit condition for producers under conditions of free entry, give a firm-level output quantity for varieties of type s equal to

$$q(s) = x(s) = (\eta - 1) \left(\frac{w(s)}{P_Y} \right)^{1-\lambda} f. \tag{10}$$

The associated levels of revenue and demand for inputs, and the equilibrium number of firms producing product varieties of type s are

$$r(s) \; = p(s) \, q(s) = \eta \, w(s) \, f, \tag{11}$$

$$l(s) \; = f + \lambda \, \frac{\eta - 1}{\eta} \, \frac{r(s)}{w(s)} = \big(1 + \lambda(\eta - 1)\big) \, f, \tag{12}$$

$$y(s) \; = (1 - \lambda) \, \frac{\eta - 1}{\eta} \, \frac{r(s)}{w(s)} = (1 - \lambda)(\eta - 1) \, \frac{w(s)}{P_Y} \, f, \tag{13}$$

$$N(s) = \frac{\alpha \, P_Y Y}{r(s)} = \frac{\alpha \, P_Y Y}{\eta \, w(s) \, f}. \tag{14}$$

Using the labor market clearing condition $N(s) \, l(s) = L(s)$, we can then characterize the equilibrium wage level for every skill type s as

$$\frac{w(s)}{P_Y} = \big(1 + \lambda(\eta - 1)\big) \, \frac{\alpha Y}{\eta L(s)}, \tag{15}$$

where Y—the equilibrium level of output of the composite good—is independent of s and is derived in the Appendix A.1.

The last variable to be determined is the price of the aggregate good P_Y. This can be obtained as a Cobb-Douglas price aggregation corresponding to (2), i.e.,

$$P_Y = \frac{1}{\alpha} \prod_{s=1}^{S} P(s)^{\alpha}. \tag{16}$$

As the system of equilibrium conditions is homogeneous of degree zero with respect to prices, we can normalize prices arbitrarily. Without loss of generality, we consider the composite product Y to be the numeraire good, imposing $P_Y = 1$. Individual consumption of the composite product is thus equal the real wage (15), and so differences in wages, $w(s)$, correspond to differences in levels of welfare across different labor types.

Conditions (6) and (15) together then deliver a full characterization of the relationship between skills scarcity, wages and prices:

Proposition 1 *Skill varieties that are comparatively scarcer, i.e., lower $L(s)$, are remunerated with higher wages and firms that employ them earn higher revenues and charge higher prices for their products.*

(Proofs of propositions are presented in Appendix A.2.)

The equilibrium composite price index, $P(s)$, and the equilibrium revenue per firm, $r(s)$, are increasing in the wage $w(s)$, and thus in the price $p(s)$; instead, the equilibrium number of firms $N(s)$ is decreasing with the price. Therefore, the price index of goods produced with scarcer skills is higher; there are fewer firms employing comparatively scarcer skills, and these firms make greater revenues. Moreover, measured labor productivity, defined as the ratio of output to labor input, $q(s)/l(s)$, is proportional to $(w(s)/P_Y)^{1-\lambda}$, and thus greater at firms employing comparatively scarcer skill types that earn higher wages in equilibrium.

Despite its simplicity, the model produces a rich set of reduced-form predictions that are consistent with the observed patterns for the correlation between product quality, the sorting of workers by wage levels, and the sorting of firms by productivity levels. Input-based product differentiation alone is sufficient to account for these patterns in a multi-factor general equilibrium framework based on a Krugman (1980)-type closed economy.

3.2 Equilibrium in open economy

We next characterize an equilibrium for an open economy with $m > 1$ identical countries. The symmetry assumption implies that countries have the same endowments and share the same bilateral trade barriers. As a consequence, the price indexes, and the consumption composites, are the same in every country, and so source and destination market indicators can be omitted. Thus, m can be readily interpreted as the global market size relative to the domestic market. We use a "$\tilde{}$" to refer to values of variables in the open-economy case, and assume $\tilde{P}_Y = 1$ as in the autarky case.

Given the production technology (3) and the trade technology (5), cost minimization implies a marginal cost $\tilde{w}(s)^{\lambda}$ for a variety of good s produced in a country and sold in the domestic market; if the same variety is exported to a foreign market, the corresponding marginal cost, inclusive of transportation costs, is

$$\tilde{w}(s)^{\lambda} + b\,\tilde{w}(s)^{\xi\lambda} = \tau(s)\,\tilde{w}(s)^{\lambda}, \qquad \tau(s) \equiv 1 + b\,\tilde{w}(s)^{\xi\lambda-\lambda}, \tag{17}$$

where $\tau(s)$ is the relative unit cost (and price) of exports relative to that of domestically sold goods, i.e., the c.i.f./f.o.b. unit price ratio. Note that since the labor intensity in export services is lower than the labor intensity in goods production, i.e., $\xi\,\lambda \equiv \delta < \lambda$, the c.i.f./f.o.b. unit price ratio is lower the higher the wage, $\tilde{w}(s)$, of the skill type employed. The elasticity of trade costs with respect to the wage is $\big(d\tau(s)/d\tilde{w}(s)\big)\big(\tilde{w}(s)/\tau(s)\big) = -\lambda\,(1-\xi)\,b\,\tilde{w}(s)^{\delta-\lambda}/\big(1+b\,\tilde{w}(s)^{\delta-\lambda}\big)$; this is bounded in the interval $(-\lambda,\,0)$. Thus, the elasticity of the trade cost to the producer price is negative and bounded in the interval $(-1,\,0)$.[24]

In the open-economy case, the product composite in every country incorporates both domestic and imported varieties. As in the autarky case, profit-maximizing prices feature a constant markup over marginal cost and so the price of imported varieties is $\tau(s)$ times the price of domestic varieties, that is $\tilde{p}(s)$ in open economy. The price of individual varieties and the quantity and price of the composite of product varieties of type s are then

$$\tilde{p}(s) = \frac{\eta}{\eta-1}\,\tilde{w}^{\lambda}; \tag{18}$$

$$\tilde{X}(s) = \frac{\alpha\tilde{Y}}{\tilde{P}(s)}; \tag{19}$$

$$\tilde{P}(s) = \Big(1 + (m-1)\,\tau(s)^{1-\eta}\Big)^{\frac{1}{1-\eta}}\,\tilde{N}(s)^{\frac{1}{1-\eta}}\,\tilde{p}(s)\,. \tag{20}$$

Proceeding as we did earlier to derive (9), we can obtain expressions for the level of demand originating in the domestic market and that originating in the $m-1$ export markets, which are respectively equal to

$$\tilde{x}(s) \;= \tilde{X}(s)\left(\frac{\tilde{P}(s)}{\tilde{p}(s)}\right)^{\eta} = \frac{\alpha\tilde{Y}}{\big(1+(m-1)\,\tau(s)^{1-\eta}\big)\,\tilde{N}(s)}\,\tilde{p}(s)^{-1}, \tag{21}$$

$$\tilde{x}^{*}(s) = (m-1)\,\tilde{X}(s)\left(\frac{\tilde{P}(s)}{\tau(s)\tilde{p}(s)}\right)^{\eta} = (m-1)\,\tau(s)^{-\eta}\,\tilde{x}(s). \tag{22}$$

Equilibrium levels of output, revenue and demand for inputs, and the equilibrium

[24] The limit case $\xi \to 0$ corresponds to pure additive trade cost $\tau(s)\tilde{p}(s) = \tilde{p}(s) + b$. The limit case $\xi \to 1$ corresponds to pure iceberg trade cost $\tau(s)\tilde{p}(s) = (1+b)\tilde{p}(s)$.

number of firms producing varieties of type s are

$$\tilde{q}(s) = \tilde{x}(s) + \tilde{x}^*(s) = (\eta - 1)\left(\xi + (1-\xi)\,\mu(s)\right)\tilde{w}(s)^{1-\lambda} f, \tag{23}$$

$$\tilde{r}(s) = \eta\,\tilde{w}(s)\,f, \tag{24}$$

$$\tilde{l}(s) = \Big(1 + \lambda\,(\eta-1)\left(\xi + (1-\xi)\,\mu(s)\right)\Big) f, \tag{25}$$

$$\tilde{y}(s) = \Big(1 - \lambda\left(\xi + (1-\xi)\,\mu(s)\right)\Big)(\eta-1)\,\tilde{w}(s)\,f, \tag{26}$$

$$\tilde{N}(s) = \frac{\alpha\tilde{Y}}{\eta\,\tilde{w}(s) f}; \tag{27}$$

where

$$\mu(s) = \frac{1 + \tilde{x}^*(s)/\tilde{x}(s)}{1 + \left(\tilde{x}^*(s)/\tilde{x}(s)\right)\left(\tilde{p}^*(s)/\tilde{p}(s)\right)} = \frac{1 + (m-1)\,\tau(s)^{-\eta}}{1 + (m-1)\,\tau(s)^{1-\eta}} \in \left(\tau(s)^{-1}, 1\right) \tag{28}$$

is the price charged in the domestic market relative to the average price charged in all markets (domestic and foreign) weighted by demand shares.

The relative price $\mu(s)$ is central to the characterization of the effects of trade liberalization that we discuss below. It depends on two equilibrium effects. First, the total export price relative to the domestic price, $\tilde{p}^*(s)/\tilde{p}(s) = \tau(s) > 1$, is comparatively lower for those goods that have a comparatively higher price in the domestic market (a *relative trade cost* effect). Second, relative demand weights in (28), $\tilde{x}^*(s)/\tilde{x}(s) = (m-1)\,\tau(s)^{-\eta}$ (foreign demand relative to domestic demand) decrease with $\tau(s)$, and thus they increase with the domestic price (a *relative demand* effect). When the relative trade cost effect dominates, $\mu(s)$ is decreasing in $\tau(s)$, while the opposite relationship applies when the relative demand effect dominates. As a result of these two effects, and of the fact that $\tau(s)$ is monotonically decreasing with the wage, from (17), the relative price $\mu(s)$ is a non-monotonic function of $\tilde{w}(s)$. This relationship is depicted in the left panel of Figure 1.

Specifically, for given market barriers and market size (b, m) there exists a unique wage threshold, w_+, such that, for $\tilde{w}(s) < w_+$, the second effect dominates and thus $\mu(s)$ is a decreasing function of the wage, and such that, for $\hat{w}(s) \geq w_+$ the first effect dominates and $\mu(s)$ is an increasing function of the wage.[25] A greater global market size, m, (relative to the domestic one) makes the relative demand effect comparatively stronger, lowering the threshold w_+. Higher trade barriers, b, make the relative trade cost effect comparatively stronger, raising the threshold w_+. With iceberg trade costs ($\xi = 1$), $\tau(s)$ and $\mu(s)$ do not vary with $\hat{w}(s)$.

The wage support is determined by the labor market clearing condition $\tilde{N}(s)\,\tilde{l}(s) = L(s)$, which, in conjunction with (27) and (25), yields the following relationship:

$$\frac{\tilde{w}(s)}{1 + (\eta-1)\,\lambda\left(\xi + (1-\xi)\mu(s)\right)} = \frac{\alpha\tilde{Y}}{\eta L(s)}. \tag{29}$$

[25] A more detailed analysis of the shape of $\mu(s)$ is presented in the proof of Proposition 3 in Appendix A.2.

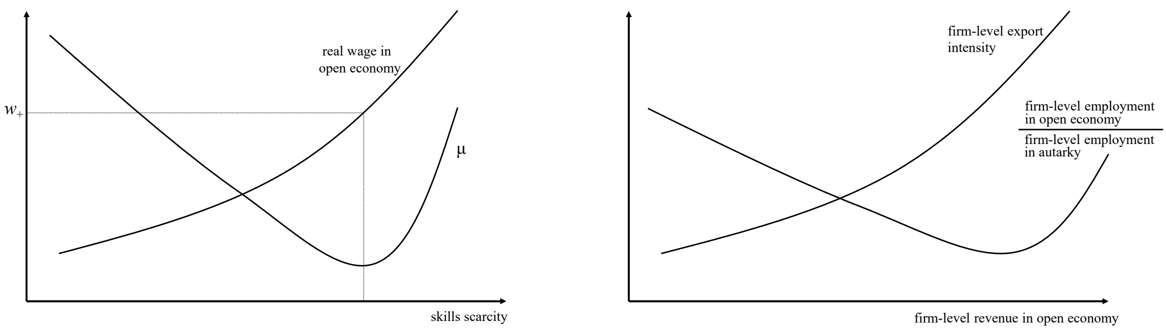

Figure 1: Firm size, wages, and export intensity

The right-hand side of (29) is decreasing in $L(s)$; and it can be shown that the left-hand side is increasing in the wage. This implies a negative equilibrium relationship between $\tilde{w}(s)$ and $L(s)$ as in autarky (Proposition 1).

The relationship between wage and scarcity for a skill type arises here through labor market clearing as in autarky. However, as can be seen by inspecting equations (25) and (26), in open economy the total demand for each input includes not only the demand for inputs required to meet consumption demand and the demand for intermediates in production, but also the demand for those inputs that are required to produce export services; and so, with non-iceberg trade costs ($\xi < 1$), the relationship between wage and scarcity is also shaped by a relative price response associated with export demand (as reflected in $\mu(s)$).

3.3 Export intensity, wages, and employment

The model offers predictions about the intensive margin of export at the firm level, i.e., about *export intensity*—the ratio of export revenues to total revenues, $\tilde{r}^*(s)/\tilde{r}(s)$. In turn, if we define the ratio export revenues to domestic revenues as

$$\rho(s) \equiv \frac{\tilde{r}^*(s)}{\tilde{r}(s) - \tilde{r}^*(s)} = (m-1)\,\tau(s)^{1-\eta}, \tag{30}$$

then export intensity can be expressed as $\rho(s)/(1+\rho(s))$, which is monotonically increasing in $\rho(s)$.

This dimension of heterogeneity remains typically overlooked in studies based on Melitz (2003), since in that framework is the same for all exporting firms and is exogenously determined by the size of iceberg trade costs. In contrast, in our framework export intensity varies across firms producing different good types. Specifically, it relates to the wage firms pay to the workers they employ:

Proposition 2 *In an open-economy equilibrium in which trade costs are less-than-unit elastic to the f.o.b. price of the exported good (i.e., where $\xi < 1$), skill types that are comparatively scarcer are remunerated with higher wages; and firms that pay comparatively higher wages earn greater total revenues and earn a greater proportion of their total revenues in export markets.*

Proposition 2 implies that exporters that charge higher prices in the domestic market earn a greater portion of their total revenues in the export market, a prediction that matches the salient empirical patterns (see Manova and Zhang, 2012). In our Krugman-style model with input-based product differentiation, trade costs that are less-than-unit elastic with respect to the domestic price (an assumption well supported by the data; see Hummels and Skiba, 2004) are sufficient to match the observed correlation between price and export intensity.

Note that (25) is increasing in $\mu(s)$. Therefore, for skill types for which $\tilde{w}(s) < w_+$ (for which $\mu(s)$ and $\tilde{w}(s)$ are negatively related), employment at the firm level and export intensity are negatively related; whereas for skill types for which $\tilde{w}(s) \geq w_+$ (for which $\mu(s)$ and $\tilde{w}(s)$ are positively related) employment at the firm level and export intensity are positively related. These relationships are depicted in the right panel of Figure 1.

In conjunction with Propositions 1 and 2, this implies the following result:

Proposition 3 *There exists a unique firm size threshold, r_+ (corresponding to a wage level w_+), such that for firms with size $\tilde{r}(s) < r_+$ employment at the firm level and export intensity are negatively related, and for firms with size $\tilde{r}(s) \geq r_+$ they are positively related.*

In the model, firm size (revenue), export intensity, and the wage paid by the firm are always positively related. The result in Proposition 3 thus reflects the combined effect of how firm size increases with export intensity, how employment increases with firm size, and how firms substitute between labor and other inputs as the price of labor inputs increases, with the overall effect being reflected in the evolution of $\mu(s)$. Firms that pay higher wages and export more earn higher revenues and produce more output (requiring more inputs), but also substitute more expensive labor with intermediate inputs. The elasticity of output to revenue is comparatively larger at high-wage, high-export intensity firms, and so the first effect comes to dominate the second at higher levels of revenue but not at low levels of revenue.[26]

3.4 Gains from trade

We now turn our attention to the predictions of the model for the effects of trade on welfare.

Given the normalization $P_Y = \tilde{P}_Y = 1$, the welfare effect of trade liberalization for workers who supply labor of skill type s is measured by the ratio of their wage in open economy to their autarky wage. Using (15) and (29), we can express this ratio as

$$\frac{\tilde{w}(s)}{w(s)} = k(s)\,\frac{\tilde{Y}}{Y}, \tag{31}$$

[26]The prediction of a positive correlation between firm-level employment and export intensity for the pool of larger, higher-price and higher-wage firms has been documented by a number of empirical studies (e.g., Manova and Zhang, 2012). When it comes to the predicted relationships at low wages, the evidence is weaker. Lichter et al. (2017) use linked employer-employee German data to show that the pattern of wage elasticity of labor demand within the group of exporting firms, specifically in correspondence to an increase in export intensity, varies substantially by skill level (see Table 5 in their paper). The authors find that greater export intensity is associated with a more elastic labor demand for low-wage workers than for high-wage workers. Although the empirical exercise in that study is not directly comparable to our prediction (which holds in equilibrium), it is suggestive of a reduction of employment at low-wage firms as their export intensity increases.

where

$$k(s) \equiv \frac{\tilde{l}(s)}{l(s)} = \frac{1 + \lambda\,(\eta - 1)\,\left(\xi + (1 - \xi)\,\mu(s)\right)}{1 + \lambda\,(\eta - 1)} \tag{32}$$

is the relative employment of specific labor type s in open economy compared to autarky, with $\mu(s) < 1$ implying $\xi + (1 - \xi)\,\mu(s) < k(s) < 1$.

Equation (31) shows that the change in individual welfare is a fraction, $k(s)$, of the change in the aggregate output of the composite good, $\tilde{Y}/Y$. Using the expressions for the equilibrium levels of output of the aggregate composite good in autarky and in open economy, (46) and (48) in Appendix A.1, the latter can be expressed as a function of $\rho(s)$ and $k(s)$:

$$\frac{\tilde{Y}}{Y} \;=\; \prod_{s=1}^{S} \left(\frac{1 + \rho(s)}{k(s)^{1+\lambda\,(\eta-1)}} \right)^{\frac{\alpha}{\lambda\,(\eta-1)}} . \tag{33}$$

Since $\rho(s) > 0$ and $k(s) < 1$ for every labor type $s \in \{1, \ldots, S\}$, then the model predicts a greater output of the composite good in open economy: $\tilde{Y}/Y > 1$, unambiguously.

Combining (31) and (33), the welfare gain for a certain skill type $\tilde{w}(s)/w(s)$ can be expressed as the product of two factors, both greater than one: the expansion in the output of the composite good that comes from the contribution of skill types other than s, i.e., $\prod_{n \neq s} \left((1 + \rho(n))/\left(k(n)^{1+\lambda\,(\eta-1)}\right)\right)^{\frac{\alpha}{\lambda\,(\eta-1)}} > 1$, and a multiplier, $\left((1 + \rho(s))/k(s)\right)^{\frac{\alpha}{\lambda\,(\eta-1)}}$ > 1, that is specific to skill type s. It follows that the welfare effects of trade liberalization are positive for all skill types:

Proposition 4 *Every skill type earns a higher real wage in open economy than in autarky.*

An implication of Proposition 4 is that, relative to wages, the price of the composite good is lower in open economy than it is in autarky. Since labor and the composite goods are substitute inputs both in the production of goods and the production of export services, this also implies that trade liberalization lowers production costs and labor intensity: firms substitute labor with the composite good. Trade liberalization in this model then generates consumption diversification gains, arising from consumers being able to access additional product varieties, as well as production rationalization gains (a reduction in average production costs), the latter arising from the reduction in the cost of the intermediate inputs associated with firms having access to more product varieties (Costinot and Rodríguez-Clare, 2014).

However, unlike in Krugman (1980), which also considers CES demand but assumes iceberg trade costs, in our model trade liberalization increases the number of available varieties not only through the availability of additional imported varieties, $(m - 1)\,\tilde{N}(s)$, but also because of an increase in the number of varieties produced in each country: the prediction $\tilde{N}(s) > N(s)$ follows directly from the labor market clearing condition, $\tilde{N}(s)\,\tilde{l}(s) = N(s)\,l(s) = L(s)$, which yields

$$\frac{\tilde{N}(s)}{N(s)} = \frac{l(s)}{\tilde{l}(s)} = \frac{1}{k(s)} > 1. \tag{34}$$

Contrast this with the case where $\xi = 1$ (iceberg trade costs); then $k(s)$ in (32) is unity and $\tilde{N}(s) = N(s)$.[27]

3.5 Trade liberalization and wage polarization

We have shown that all skill types benefit from trade liberalization: trade liberalization brings about an expansion in the number of varieties of all good types (as demonstrated by (34)), which, through a variety effect, results in an increase in the real wage for all skill types. However, because of comparative skills scarcity and non-iceberg trade costs, welfare changes are asymmetric between skill types, depending on the relative labor share, $k(s)$.

To assess how the welfare effects of trade liberalization are distributed across different skill types, we can focus on a comparison between $\tilde{w}(s')/w(s')$ and $\tilde{w}(s'')/w(s'')$ for two skill types s' and s'' such that $L(s') < L(s'')$ (and thus $\tilde{w}(s') > \tilde{w}(s'')$); i.e., on the ratio $\left(\tilde{w}(s')/w(s')\right)/\left(\tilde{w}(s'')/w(s'')\right)$. If this ratio is greater than unity, then trade liberalization raises welfare inequality between s' and s'', and it lowers it if it is less than unity. The key determinant of these effects is $k(s)$, the change in the skill-specific labor share. From (31), skill types that experience a comparatively smaller (greater) reduction in the labor share gain disproportionately more (less):

$$\frac{\tilde{w}(s')/w(s')}{\tilde{w}(s'')/w(s'')} = \frac{k(s')}{k(s'')} \,. \tag{35}$$

By studying how the right-hand side of (35) varies with s, we obtain the following result:

Proposition 5 *If the wage threshold w_+ lies in the interior of the wage support, trade liberalization lowers welfare inequality at the bottom of the wage distribution, i.e., when comparing two wage levels w' and w'' such that $\tilde{w}(s'') < \tilde{w}(s') \leq w_+$; but, widens it at the top of the wage distribution, i.e., when comparing two wage levels w' and w'' such that $\tilde{w}(s') > \tilde{w}(s'') \geq w_+$.*

Proposition 5 says that, although there are gains from trade for all skill types, they are not evenly distributed. Trade liberalization lowers the relative demand for labor by comparison with other inputs. In equilibrium, the labor share falls and real wages rise, for every skill type. But the real wages of skill types that are at the bottom or at the top of the wage distribution (i.e., those that are very abundant and those that are very scarce) increase disproportionately more compared to skill types with wages in the neighborhood of w_+. These effects are illustrated in Figure 2. This also implies an increase in income concentration at the top of the distribution—i.e., a higher income Gini index for a truncation of the income that is limited to incomes above w_+—but not necessarily for the whole distribution.[28]

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[27] In discussing his results, Krugman (1980) notes that his model's prediction that trade does not change the number of varieties produced in each country "... depends on the assumed form of the transport costs, which shows at the same time how useful and how special the assumed form is".

[28] The pattern predicted for incomes above w_+ amounts to a prediction of *ratio dominance*. Adapting Preston (2006)'s definition, we say that a discrete distribution f_1 of a variable, w, for J individuals indexed by $j = 1, \ldots, J$, ratio dominates distribution f_0 of the same variable if and only if $\ln w_1^{j+1} - \ln w_0^{j+1} > \ln w_1^j - \ln w_0^j, \ \forall j \in \{1, \ldots, J-1\}$. In other words, when individuals are ordered in terms of (increasing) wages, the new distribution shows a greater proportional increase in the income of the $(j+1)$th individ-

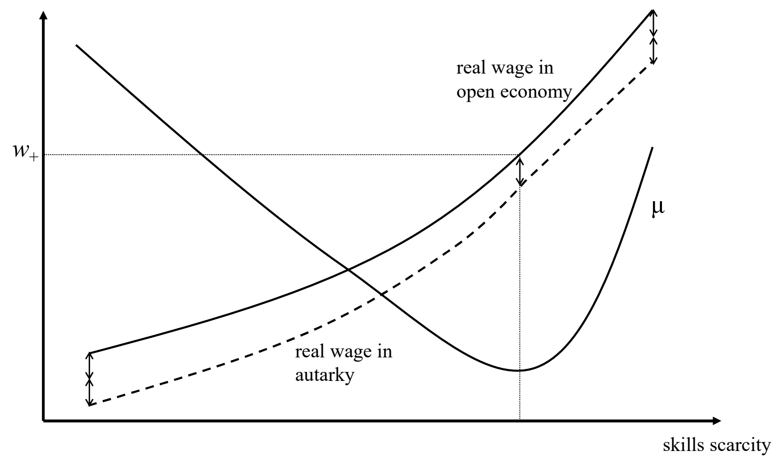

Figure 2: Gains from trade and wage polarization

The model thus predicts that trade liberalization may induce wage polarization, exacerbating disparities over a certain range of the wage support while mitigating them over a different range of the wage support, a prediction that finds strong support in empirical studies.[29] In contrast with other models that explicitly design a mechanism for polarization (e.g., task specialization as in Autor and Dorn, 2013, or biased technical change, as in Acemoglu and Restrepo, 2020), in our theory polarization is a by-product of the features that underlie the model's main predictions.

The effect flows through the two channels that we have previously highlighted: an expansion in the relative demand for higher-priced goods—a relative demand effect favoring the scarcest skill types—and an expansion in the demand for cheaper goods—a relative trade cost effect favoring the most abundant skill types. The first effect is the direct demand effect of trade liberalization in the presence of the Alchian-Allen effect, which is to raise demand comparatively more for scarcer skill types. However, when transportation costs are not of the iceberg type, trade liberalization also raises the demand for trade services, which translate into a comparatively greater demand increase for those skill types whose price increases comparatively less as a result of the direct effect. If this latter channel is sufficiently strong, wage differentials for the more abundant skill types become compressed relative to autarky.

Polarization in the distribution of earnings occurs if and only if the wage threshold w_+ lies in the interior of the wage support. Otherwise, gains from trade proportionately increase moving in one direction only: either toward higher wages (if w_+ lies to the left of the wage support) or toward lower wages (if w_+ lies to the right of the wage support). In Appendix A.3 and in Section 4, we show that all these cases are possible

ual compared to that of the jth individual, for any j. Ratio dominance implies Lorenz dominance, i.e., a higher Gini index for the new distribution relative to the initial distribution.

[29] Keller and Utar (2016) find that within the sample of mid-wage workers adjusting after a trade shock, the relatively more skilled workers (those earning a higher wage to begin with) end up in higher-wage jobs, whereas less skilled workers (those earning a lower wage to begin with) reallocate to lower-wage positions. Furthermore, the authors show that the effect of job polarization through trade-induced worker adjustments in the US labor market is as strong as the documented effects of technical change and offshoring.

and that polarization occurs for plausible parameter values.

The wage support is determined by (29) for an arbitrary distribution of endowments $(L(s),\ s \in \{1, \ldots, S\})$; the wage threshold w_+, on the other hand, does not depend on the distribution of the endowments of skill types. We can thus obtain theoretical insights about the conditions that produce particular scenarios by studying how the equilibrium value of w_+ varies with the model's parameters—specifically with b, which determines the size of trade costs, with m, which determines the relative size of export revenues relative to domestic revenue, and with $\eta > 1$, the elasticity of substitution across varieties of the same type:[30]

$$\frac{\mathrm{d}w_+}{\mathrm{d}b} > 0, \qquad \frac{\mathrm{d}w_+}{\mathrm{d}m} < 0, \qquad \frac{\mathrm{d}w_+}{\mathrm{d}\eta} > 0\,. \tag{36}$$

The wage threshold w_+ is located comparatively more to the right of the wage support—implying a larger portion of the wage support characterized by proportionally decreasing trade gains and a smaller portion characterized by proportionally increasing trade gains as wages increase—when trade barriers are high and the export market is small compared to the domestic market; and the effect is stronger the greater is the substitutability among varieties of the same good type. And so, for example, if liberalization starts by being limited to a small set of countries (i.e., m is small), it may produce systematically higher gains for higher-wage workers; but as market integration progresses (i.e., m becomes larger), the gains might become more polarized.

The wage threshold w_+ also responds to the share of own output in trade costs, ξ (the fraction of trade costs that consist of iceberg costs); and it does so in a way that depends on whether w_+ is greater or lower than the price of composite good:

$$\begin{cases} \dfrac{\mathrm{d}w_+}{\mathrm{d}\xi} > 0, & \text{if } w_+ > \tilde{P}_Y, \\ \dfrac{\mathrm{d}w_+}{\mathrm{d}\xi} \leq 0, & \text{if } w_+ \leq \tilde{P}_Y. \end{cases} \tag{37}$$

This prediction can be understood as follows. For a given $\xi < 1$, the c.i.f./f.o.b. price ratio (as measured by (17)) is always decreasing in $\tilde{w}(s)$; and, for a given $\lambda > 0$, it is decreasing in ξ (the share of non-iceberg costs in total trade costs) when $\tilde{w}(s) > \tilde{P}_Y$, and increasing in ξ when $\tilde{w}(s) < \tilde{P}_Y$. It follows that, if $w_+ > \tilde{P}_Y$, the threshold w_+ (the wage level for which the wage change induced by trade liberalization reaches a minimum) moves to the left of the wage support when exports use comparatively more general-purpose inputs, i.e., when the Alchian-Allen effect becomes stronger, while the opposite occurs if $w_+ < \tilde{P}_Y$.

Propositions 2 and 5 taken together also imply that relative welfare effects for workers are directly related to the size of the firms they work for:

Proposition 6 *Trade liberalization widens wage inequality among worker types employed at larger firms (those with revenues above a threshold r_+, paying wages higher than w_+) and reduces it among workers employed at smaller firms (those with revenues below r_+, paying wages lower than w_+).*

[30]These are derived in section A.3 of the appendix.

So, for workers employed by firms with revenues above a certain threshold, welfare gains are comparatively larger for higher-wage workers than for lower-wage workers, while for workers employed by smaller firms the opposite applies. And as firm size is in turn related to export intensity, this also means that relative welfare effects for workers are directly related to the export intensity of the firms that employ them.

The above results also have implications for the distribution of firm sizes as measured by their revenues. Using the expressions (11) and (24) derived earlier, the ratio of the revenue of a firm producing varieties of type s in open economy to the corresponding revenue under autarky, $\tilde{r}(s)/r(s)$, can be expressed simply as $\tilde{w}(s)/w(s)$. Proposition 5 then directly translates into a prediction on the polarization in firm sizes by firm type.

The overall effect of trade liberalization on the distribution of firm sizes also depends on how the number of firms changes for each firm type. As shown, the number of firms increases for each firm type—because $\tilde{N}(s)/N(s) = l(s)/\tilde{l}(s) > 1$; but since for $\tilde{w}(s) < w_+$ wages and employment at the firm level are negatively related (Proposition 3), the ratio $\tilde{N}(s)/N(s)$ is increasing in $\tilde{w}(s)$ at the bottom of the wage distribution and decreasing in $\tilde{w}(s)$ at the top of the distribution (while always remaining above unity), which will make the distribution of firm sizes more concentrated. In light of Proposition 5, we then conclude that international trade strengthens the correlation between firm size and the wages firms pay to their employees.

4 Evidence and a quantitative assessment

The key predictions of our model can be summarized as follows:

(i) Sectors that employ comparatively scarcer skill types earn a comparatively greater proportion of their total revenues in export markets;

(ii) Greater trade openness lowers inequality at the bottom of the wage distribution but widens it at the top.

Crucially, while previous studies describing these patterns in the data link them to Hicks-neutral productivity differentials, our theory links them to variation in the characteristics of labor inputs. Thus, in looking for evidence in the data of the mechanism we describe, we need to focus on the component of cross-sectoral variation that cannot be ascribed to technological differences which are neutral with respect to characteristics of labor inputs.

In this section, we first use French administrative data on employer-employee contracts and on firm balance sheets to document that at least about half of the cross-sectoral variation in wage, employment and export intensity is not due to labor-neutral technological differences. We show that the cross-sectoral variation in export intensity strongly correlates with the sectoral intensity in the employment of scarce skills, with a positive and robust relationship between the two measures that is consistent with prediction (i). Second, we offer a quantitative assessment of prediction (ii) in a calibrated version of the model that matches the moments of the wage distribution in the sample and incorporates evidence from independent studies on c.i.f./f.o.b. price ratios.

4.1 Evidence from French exporter data

Data

Our data comes from two different sources: the declaration of social security data (DADS) and the consolidated balance sheet data (Ficus-FARE). DADS is an administrative dataset that collects mandatory information from all businesses with employees located in France. The Ficus-FARE dataset contains exhaustive information on the balance sheets of firms paying taxes in France. We merge the two via the unique identifier for the employer (in DADS) and the firm (in Ficus-FARE), obtaining a dataset that contains information on the universe of workers' contracts and firms balance sheets in France at yearly frequency for two years, 2010 and 2013.

We aggregate balance sheet information at the 2-digit sector-level NACE classification and restrict our analysis to manufacturing sectors.[31] To compare the workforce composition between years, we aggregate the information on contracts by seven characteristics: gender of the employee, age spell of the employee (from age 21 to 65 with 5 year interval), province of residence of the employee, province where the working place is located, occupational task of the employee, main sector of activity of the employer and employment status of the employee in the previous year (whether employed in manufacturing, employed in non-manufacturing or not employed).[32]

The resulting dataset consists of 43,892 distinct worker types employed in 22 manufacturing sectors, for which we observe "full-time equivalent units" and "yearly gross wage" (per full-time equivalent unit), in 2010 and 2013. We can also observe "total revenues", "revenues abroad", "cost of intermediates", "cost of raw materials" and "labor cost" as reported in firms' income statements, and "stock of fixed asset" as reported in firms' balance sheets.

Other observable sources of variation

To isolate sources of cross-sectional variation in labor and output market outcomes that relate to relative input scarcity, we proceed in the spirit of Costinot (2009) and first derive value residualized for other observable sources of variation, namely variation on workers' demographic characteristics and variation in technology across firms.[33]

[31]These correspond to paragraphs 10 to 33 in Section C of the NACE-Rev. 2 classification. Out of the 24 sectors, we exclude Paragraph 12, "Manufacture of products from tobacco", and Paragraph 33, "Repair and installation of machinery and equipment", the former because it is tightly linked to the specific raw material, the latter because it consists of service activities.

[32]The dataset is particularly rich in the description of occupations. This information is coded according to the classification *Professions et Catégories Socioprofessionnelles*. We exclude from the analysis categories that do not apply to employees of private business, such as: "Business owners", "Clergy", "Farmers", "Police and military", "Public service professions". We use the version *Postes* of the DADS dataset, which has the advantage to provide the universe of contracts employer-employee in a year. A disadvantage is that the employee identifier is unique within the same year, but not across years; therefore, we can only exploit cross-sectional information for the employees. Thus, by pooling the information on a fine grid of observable characteristics, we are able to compare the characteristics of the workforce over time.

[33]Labor market outcomes vary systematically across workers by demographic characteristics—see Helpman et al., 2010 for a discussion in the context of international trade. As discussed in the introduction, there is also overwhelming evidence of Hicks neutral technological differences playing an important

To control for the first channel, we regress wage and employment at the level of a worker type on: gender, age, province of residence and province of the working place, separately for 2010 and 2013. This allows us to focus on the source of variation that is not attributable to demographic characteristics, which is a substantial component: e.g., wages residualized by demographic characteristics account for 68.3% of the total variation of wages across worker types in 2013, and residualized employment levels account for 53.9% of the total variation of employment levels across worker types in 2013.

To account for the second channel, we then regress wages and employment levels (residualized by demographic characteristics) at the level of skill type and sector pairs against sectoral stock of fixed assets (as a proxy of economies of scale) and sectoral ratio of asset per unit of labor (as a proxy of capital intensity, and, more broadly, of average labor productivity unconditional on skill). The resulting residualized variables still account for a large component of total variation: e.g., in 2013, wages and employment level residualized by technological differences account, respectively, for 49.1% and 68.8% of the total variation across sectors in wage and employment levels. Export intensity residualized by technological differences (using the same procedure) accounts for 76% of the total variation in export intensity across sectors in 2013.

Sector-level changes in skills scarcity and export intensity

Prediction (i) links the relative scarcity of skill types to the export performance of the sectors that employ them. This requires defining what a skill type is in the data, measuring the relative scarcity of a certain skill type, and measuring how a sector is intensive in the use of relatively scarce skills.

We define a skill type as the intersection of occupational task (classified in 21 categories), experience (junior if age is below 40, senior otherwise) and previous employment status (3 categories, described above). The final dataset consists of 75 distinct skill types. The common practice in the literature to associate low-qualified and non-technical occupations with low wages, while scientific and managerial occupations are associated with high wages, finds support in our evidence; although, experience and previous employment status also play a role.[34]

Measuring the comparative scarcity of skill types is inherently challenging. In the model, different skill types vary in their level of supply but are exposed to the same demand patterns (because of our symmetric input differentiation assumption), which simplifies the analysis of the role of comparative skill scarcity. In the data, however, employment levels by skill type are an equilibrium outcome resulting from the combination of heterogeneous demand and supply patterns across skill types—i.e., employment levels cannot be taken as a measure of relative skills scarcity. However, under the hypothesis that the time window 2013-2010 is narrow enough to assume no changes in preferences and technologies, observed relative changes in the equilibrium wages and

role in the variation of output and labor market outcomes across sectors and firms.

[34]E.g., *Business administrative and commercial executives* earn on average the highest wage (€ 56k in 2013) followed by *Tradesmen* (€ 55.9k) and *Engineers* (€ 54.9k), whereas workers employed in *Personal services* earn the lowest wage (€ 28k) on average. Moreover, greater experience is associated with a statistically significant wage premium (€ 7.6k).

employment levels of different skill types reveal unobserved changes in their supply relative to demand. We can thus exploit changes in the labor market equilibrium to construct a measure of relative *changes* in skills scarcity that is fully consistent with the theory and use it to construct sector-aggregate measures of changes in skills scarcity. Comparing these measures with measures of sector-aggregate changes in export intensity effectively implements a difference-in-differences empirical strategy.

We classify a skill type as having become relatively scarcer if both the associated wage change between 2010 and 2013 is comparatively higher than that of other skill types *and* the associated change in the level of employment is comparatively lower than that of other skill types:[35]

$$Scarcer_s = \begin{cases} 1 \quad \text{if} \begin{cases} \ln w_{s,2013} - \ln w_{s,2010} > \text{ Median}\left[\ln w_{2013} - \ln w_{2010}\right] \\ \text{and} \\ \ln fte_{s,2013} - \ln fte_{s,2010} < \text{ Median}\left[\ln fte_{2013} - \ln fte_{2010}\right], \end{cases} \\ 0 \quad \text{otherwise,} \end{cases} \tag{38}$$

where $w_{s,t}$ and $fte_{s,t}$ indicate, respectively, the residualized wage and full-time equivalent units of skill type $s \in \{1, ..., 75\}$ in year $t = \in \{2010, 2013\}$. In our sample, 22 skill types out of 75 (29.33% of all skill types) have become relatively scarcer. Their yearly gross wage is higher (by €7.9k on average) and their employment in the entire French workforce is smaller (by −5.1k full time equivalent units on average) than those of skill types that are not classified as having become relatively scarcer, both differences being statistically significant at the 1% level. Therefore, this classification criterion is also broadly consistent with the more commonly adopted distinction between high-skill and low-skill occupations (Autor, 2015).[36]

We then aggregate this information at the sector level weighting the contribution of each skill type to the sectoral composite labor input by that skill type's input value share:

$$SI_k = \sum_s \left(\frac{Payroll_{sk,2010}}{\sum_j Payroll_{jk,2010}} \right) Scarcer_s \ ,$$

where $Payroll_{sk,2010}$ is the labor cost of skill type s in sector k in 2010. Thus, $SI_k \in (0, 1)$ is a measure of sectoral intensity with respect to those skill types that have become comparatively scarcer between 2010 and 2013 based on the 2010 sectoral composition of labor inputs by skill types.

The second variable of interest is export intensity. Consistently with our theory,

[35]Under the model's assumptions, a comparative increase in the wage would be sufficient, alone, to indicate a decrease in supply relative to demand at equilibrium prices. Classifying skill types has having become scarcer based on the combined criterion of a higher wage and a lower employment level is more restrictive, i.e., it implies a more conservative choice in relation to the theory. Across the 75 skill types, the correlation coefficient between percentage change in wage and the corresponding percentage change in employment is negative and statistically significant at the 1% level, suggesting that the direction of wage changes in the data reflects the direction of supply changes (rather than just demand changes).

[36]The two criteria are not fully equivalent, however. For example, skill types including occupations such as *Business administrative and commercial executives* and *Engineers* are at the top of the wage distribution but are not classified by our criterion as having become comparatively scarcer.

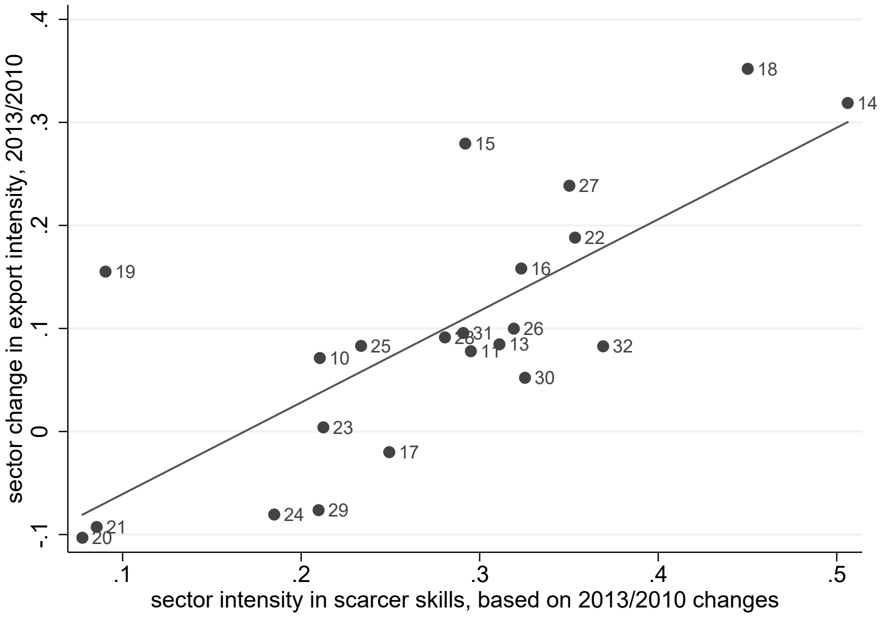

Figure 3: Export intensity versus intensity in the use of scarce skills

Note: Each point corresponds to one of 22 manufacturing sectors, each labeled by its chapter number in the NACE classification. On the horizontal axis is a sectoral measure of input intensity in skill types whose wage has increased by comparatively more and whose employment has increased by comparatively less between 2010 and 2013. On the vertical axis is a sectoral measure of change export intensity. Both measures are constructed with using observations residualized by sectoral stocks of fixed assets and sectoral ratios of asset per unit of labor. The line is the linear fit, with slope $\tilde{\beta} = 0.889$, p-value$= 0.0004$ and $R^2 = 0.561$.

export intensity is the fraction of total revenue due to export revenue. We construct sectoral measures of changes in export intensity as

$$EI_k = \ln\left(\frac{Export\_Revenue_{k,2013}}{Total\_Revenue_{k,2013}}\right) - \ln\left(\frac{Export\_Revenue_{k,2010}}{Total\_Revenue_{k,2010}}\right), \tag{39}$$

where $Export\_Revenue_{k,t}$ and $Total\_\ Revenue_{k,t}$ are sector aggregates respectively for residualized export revenues and total revenues.

Figure 3 shows the relationship between sectoral intensities in those skill types that have become comparatively scarcer between 2010 and 2013 (SI_k, on the horizontal axis) and the corresponding sectoral measures of changes in export intensity, (EI_k, on the vertical axis), with the label next to each point indicating the sector k by its chapter number in the NACE classification. The relationship is positive and robust, with a p-value lower than 1%. More than 56% of the residualized cross-sectional variation with respect to how export intensity changes across sectors between 2010 and 2013 is explained by the residualized cross-sectional variation across sectors in the intensity of use of those skill types that have become comparatively scarcer over the same period. Although no causal link can be inferred from this pattern, it is in line with our key prediction (i).[37]

[37] Two sectors are at the bottom of the distribution for both changes in export intensity and for intensity in the employment of skills that have become scarcer—sector 20 ("Manufacture of chemical products")

4.2 A quantification of the model's wage polarization predictions

Model calibration

We next calibrate our model for a quantitative assessment of our qualitative prediction (ii). This requires to assign a value to six parameters: the elasticity of substitution across varieties, η, the elasticity of export services to firm's output, ξ, the labor share in production, λ, the fixed cost, f, a size of trade barriers, b, and a measure of the international market relative to the domestic one, m. Table 1 reports the parameter values we take from the literature and the observed ranges of values we take from evidence on French manufacturers to calibrate the model.

Estimates of the elasticity of substitution η between varieties within a commodity group are, typically, in the range of 2 to 10, with Feenstra and Romalis (2014) giving a median estimate of $\eta = 6.07$. Following Coşar et al. (2016), we set the fixed cost of operating a business at $f = 7.84$ units of labor. Deriving the expression for c.i.f./f.o.b. margin in our model yields a (negative) constant elasticity of $1 - \xi$. Hummels and Skiba (2004) estimate the elasticity of c.i.f./f.o.b. margins with respect to the f.o.b. price and accounting for their results in our expression implies a value of $\xi = 0.6$, thus lending empirical support to the assumption $\xi < 1$ at the core of our assumed export technology.

In our theory, the composite good serves as numeraire. In the data, we observe nominal wages. Without loss of generality, we let the price of one unit of the composite good be equal to the mean residualized nominal wage, $\bar{\omega}$. This means that, if ω is an observed nominal wage, the corresponding relative wage is $\tilde{w} = \omega/\bar{\omega}$.[38] Under this normalization, evidence on trade costs can be directly used to quantify the parameters capturing trade barriers, b, and relative size of the international market, m.

An observed c.i.f./f.o.b. price ratio is the empirical counterpart of the trade cost $\tau(s)$ in the model. Inverting the theoretical expression for trade costs yields $b = (\tau - 1)\, w^{\lambda(1-\xi)}$, given a relative wage, w, and a corresponding trade cost, τ. In the data there is no clear indication on which wage level should be assigned to which trade cost. However, the theory predicts negative sorting: goods produced with lower-wage skill types incur relatively higher c.i.f./f.o.b. ratios.

Miao and Fortanier (2017) estimate trade-weighted values for c.i.f./f.o.b. margins controlling at the product and shipment level for distance, country characteristics, quality of the infrastructure, insurance cost and oil price.[39] The upper bound for the av-

and 21 ("Manufacture of pharmaceutical products". Symmetrically, two sectors are at the top of the distribution for both changes in export intensity and intensity in the employment of skills that have become scarcer: 14 ("Manufacture of apparel") and 18 ("Printing and reproduction of recorded media"). Sector 19 ("Manufacture of coke and refined petroleum products") is the one that mostly deviates from the linear fit, but it is also a peculiar tradable manufacturing sector given its linkages with energy production.

[38]Rewriting the cost structure of the model in nominal wages shows that the nominal marginal cost of production is $\omega^{\lambda}\tilde{P}_y^{1-\lambda}$ and the nominal marginal cost of export services per unit of exported variety is $b\,(\omega^{\lambda}\tilde{P}_y^{1-\lambda})^{\xi}\tilde{P}_y^{1-\xi}$. Thus, the total nominal marginal cost of an exported variety is $\tilde{P}_y(\omega/\tilde{P}_y)^{\lambda}\big(1 + b\,(\omega/\tilde{P}_y)^{-\lambda(1-\xi)}\big)$, where the expression in large brackets is the trade cost, τ, and $\tilde{w} = \omega/\tilde{P}_y$ is the relative wage. If $\tilde{P}_y = \bar{\omega}$ then $\tau = 1 + b$ is exactly the trade cost charged on goods produced with skill types paid at the mean wage.

[39]The inclusion of these controls makes these estimates well-suited to our calibration exercise, as it implies that the variation captured in the estimated trade costs can be plausibly attributed to characteristics

Table 1: Calibration parameters

	Value	Source
Elasticity of substitution across varieties	$\eta = 6.07$	Feenstra and Romalis (2014)
Fixed cost (units of labor)	$f = 7.84$	Coşar et al. (2016)
Elasticity of export services to firm's output	$\xi = 0.60$	Hummels and Skiba (2004)
Trade barriers (c.i.f./f.o.b. margin)	$b = 0.26$	Miao and Fortanier (2017)
Relative size of the international market	$m = 1.48$	2013 French manufacturing data
Labor share in production	$\lambda = 0.31$	2013 French manufacturing data

erage c.i.f./f.o.b. margin across products within the same HS 2-digit sector is 28%, hence $\tau_{ub} = 1.28$. In the French 2013 manufacturing data, the 5th percentile of the observed distribution of full time equivalent yearly wage (residualized by demographic and technological differences) corresponds to €18,890 with a mean of €38,752. We take the ratio of these wage levels as a proxy for the lower bound of the relative wage $w_{lb} = 0.487$. French manufacturing data also contain information about the share of labor in variable cost (measured as the sum of labor cost, purchases of raw materials and intermediates), giving $\lambda = 31.1\%$. The expression for trade costs then implies $b = (\tau_{ub} - 1)\, w_{lb}^{\lambda(1-\xi)} = 0.256$.

The size of the international market relative to the domestic one (m) can be inferred from evidence on export intensity. The expression for export intensity is isoelastic with respect to trade cost, and a first-order Taylor approximation around $\tau = 1$ is $\rho \approx (m - 1)(1 - (\eta - 1)(\tau - 1))$. The value of m is obtained by evaluating this expression at the mean trade cost, $\bar{\tau}$ and mean export intensity, $\bar{\rho}$. The estimated c.i.f./f.o.b. margin for all countries over the period 1995 to 2014 reported by Miao and Fortanier (2017) is 6.2%, which implies $\bar{\tau} = 1.062$. In manufacturing French data, the (revenue weighted) average export intensity in 2013 (residualized by differences in technologies) is equal to $\bar{\rho} = 35.3\%$. The implied size of the international market is $m = 1 + \bar{\rho}/(1 - (\eta - 1)(\bar{\tau} - 1)) = 1.479$.[40]

In the model, the distribution of wages across different skill types follows from the distribution of endowments across those skill types, given symmetric expenditure shares. In the 2013 French manufacturing data, where we observe employment levels for 75 skill types, the assumption of endowments of skill types following an exponential distribution cannot be rejected. Without loss of generality, we normalize these observations so that they add up to unity, and use the 10th and 90th percentiles of the resulting empirical distribution, respectively L_{10} and L_{90}, to compute the parameter of the approximating exponential distribution as $\big(\ln(1 - 10\%) - \ln(1 - 90\%)\big)/(L_{90} - L_{10}) =$

of the exported product, as opposed to variation in transportation services or geography.

[40]In the theory, m is the relative size of the international market when foreign markets are symmetric to the domestic one. When taking the model to the data, asymmetries in country size and non-prohibitive trade barriers would be reflected in the size of m. In this interpretation, $m = 1.479$ means that, for France, the size of the foreign market measured in equivalent units of the domestic market is 47.9%. This is in line with the observation that the share of foreign sales over total sales is below that threshold.

71.879. This gives an exponential distribution with a mean of 0.0139.[41]

Given the above parameterization, the threshold wage level w_+ lies within the support of this wage distribution for the 75 skill types considered in this simulation exercise. As discussed in Section 3.5, the location of w_+ in the wage support is sensitive to the values of η, b and m. If we vary the value of η while holding all the other parameter values, the cutoff w_+ remains within the wage support for η lying approximately between 2.5 and 8, a range that covers essentially all relevant literature estimates. In line with (36), the cutoff w_+ is decreasing in m—i.e., a higher m reduces the range over which wage inequality is locally decreasing in the wage. In our simulation exercise, w_+ remains interior to the support for m lower than 5.2 (a threshold that is well above our central value of $m = 1.479$); for values of m above 5.2, trade liberalization would systematically increase wage inequality at all wage levels. The value of w_+ is also increasing in b, a parameter that determines the size of trade barriers. In our parameterization, trade costs average to 29.6% (the average of $\tau(s) - 1$ weighted by the values of exports for each good type, s). The cutoff level w_+ remains interior to the wage support for values of b between 0.16 and 0.31, corresponding to trade costs averaging between 18% and 35%. If trade costs lied below this range, trade liberalization would cause wage inequality to increase at all wage levels, whereas if they lied above this range, liberalization would cause inequality to decrease at all wage levels.

Results

Propositions 4 to 6 in the model predict trade-induced welfare gains and wage polarization, in line with a distributional trend that has been widely documented elsewhere (Autor, 2015). That literature attributes the effect primarily to a complementarity between trade liberalization and skill-biased technological change (Burstein and Vogel, 2017), a mechanism that also leads to heterogeneity in wage polarization by demographic characteristics (Helpman et al., 2017)). In our calibration exercise the contribution of that mechanism is absent, as the effect of trade on the wage distribution is channeled only through skills-based product differentiation and the Alchian-Allen effect (in line with the theory). Additionally, we focus exclusively on how this effect operates through occupational differences residualized by demographic characteristics.

We use the calibrated model to run 5,000 independent experiments. In each we feed the model $S = 75$ random draws from the calibrated exponential distribution of endowments of skill types, obtaining a sample corresponding to a distribution $L(s),\ s \in \{1, ..., 75\}$ of 75 skill types, sorted so that $s = 1$ is the most abundant type and $s = 75$ the scarcest, and normalized so that endowments add up to unity. In each particular experiment, what characterizes a skill type is its relative scarcity, as in the theory.

For each experiment, we solve for equilibrium values and compute two statistics, based on our theory, which aim to quantify welfare gains from trade (as predicted by Proposition 4) and the local change in inequality (as predicted by Proposition 5 and 6).

[41] As discussed earlier, we assume symmetric expenditure shares in the theory purely for simplicity; but, if we normalize employment levels for the observed input expenditure shares, the resulting empirical distribution of normalized employment levels also follows an exponential distribution that is very close to the distribution obtained without normalization. Calibration to the distribution of normalized employment levels leads to essentially identical results and conclusions.

Welfare gains from trade of a skill type s in experiment n are measured by the ratio of the wage in open economy over the wage in autarky:

$$WG^n(s) = \frac{\tilde{w}^n(s)}{w^n(s)}. \tag{40}$$

The local change in inequality in experiment n in the neighborhood of the wage of a skill type s is measured as the log difference in real wage in open economy versus autarky between the wage of skill types that are contiguous in the wage ranking:

$$LCI^n(s) = 100 \times \left(\ln\left(\frac{\tilde{w}^n(s+1)}{w^n(s+1)}\right) - \ln\left(\frac{\tilde{w}^n(s)}{w^n(s)}\right)\right) , \tag{41}$$

where 100 times log changes in employment approximates percentage points for small changes. Since both in open economy $\tilde{w}^n(s+1) > \tilde{w}^n(s)$ and in autarky $w^n(s+1) > w^n(s)$ skill types are sorted by their wage, then higher $LCI^n(s)$ means that in open economy as opposed to autarky there is a greater proportional increase in the wage of the skill type $s+1$ compared to that of the skill type s. Since ratio dominance implies Lorenz dominance, a positive $LCI^n(s)$ implies that income concentration computed locally around the wage of skill type s has increased in open economy as opposed to autarky, whereas a negative $LCI^n(s)$ implies that income concentration has locally decreased. We repeat the exercise for two different scenarios, a baseline scenario that corresponds to the calibrated economy, and a counterfactual scenario in which the elasticity of export services to firms' output is lowered from its baseline level of $\xi = 0.6$ by 10% down to $\xi = 0.54$. Comparing results from the two scenarios sheds light on the role of non-iceberg trade costs in determining both the level and the distribution of the gains from trade. In the baseline equilibrium with $\xi = 0.6$, gains from trade average to 8.29%.[42] When we decrease $\xi = 0.54$, that figure falls to 8.15%. Thus, an increase in the strength of the Alchian-Allen effect has a negative and small effect on the average gains from trade (the elasticity of the latter with respect to changes in ξ is 0.17 in our exercise).

Figure 4 shows the change in inequality due to trade liberalization in the baseline and in the counterfactual scenarios, reporting, for each skill type s, mean values of $LCI^n(s)$ across all 5,000 experiments and the corresponding normalized mean wage. With pure iceberg trade costs, i.e., for $\xi = 1$, the model would still predict gains from trade for all types but they would be equi-proportional across all skill types—and so there would be no effect on relative wage dispersion. When trade costs include a component whose price is independent of the price of the good exported and is produced with inputs other than a firm's own output (i.e., for $\xi < 1$), asymmetries in the relative distribution of the gains from trade arise.

Inequality decreases for more abundant, low-wage, skill types and it increases for scarcer, high-wage skill types. In the baseline calibrated economy, local changes in inequality range between -0.0089 and 0.0167. To put these results into perspective, deriving the same measure of local change in inequality from consensus evidence on the

[42]Costinot and Rodríguez-Clare (2014) report literature estimates of gains from trade for France ranging from 9.4% to 25.8%, based on a quantification of multi-sector models with monopolistic competition among firms that are not heterogeneously productive (as in our model). Estimates reach 32.1% with heterogeneous firms.

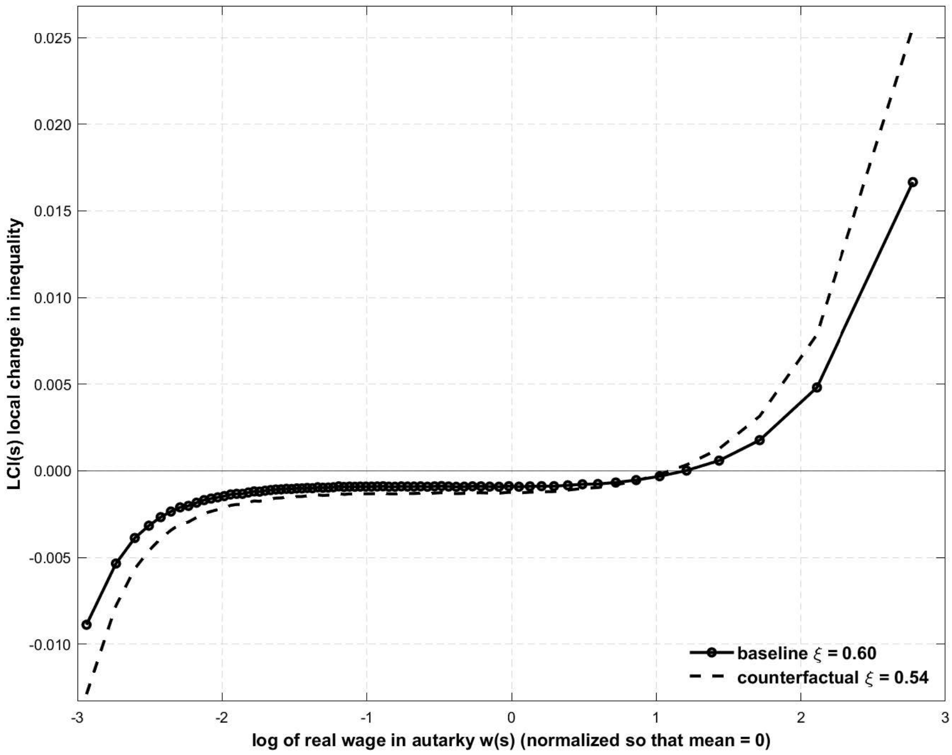

Figure 4: Change in inequality

Note: Points correspond to different skill types, $s \in \{1, ..., 75\}$ (more abundant types on the left, scarcer skill types on the right). Confidence intervals (not shown) are very tight for both specifications: at the 95% confidence level, $LCI(s) < 0$ for all skill types more abundant than the 69-th and $LCI(s) > 0$ for all skill types scarcer than the 70-th.

polarization in the wage distribution yields changes between -0.0789 (below the median of the distribution of skill types) and 0.0398 (above the median of the distribution of skill types).[43] Thus, the changes in inequality predicted by the mechanism we model are small but not at all negligible when compared with independent estimates of the overall changes that have occurred—they account for 11.2% of the documented local change in inequality at lower wages, and for 41.9% of the documented local change in inequality at higher wages—and they reproduce the observed qualitative sorting: reduction of inequality at lower wages and increase of inequality at higher wages.

When the export technology becomes more intensive in the use of inputs other than the firm's own output, making the Alchian-Allen effect stronger, this range widens in both directions: local changes in inequality now range from -0.0129 to 0.0256, corre-

[43]The figures we use for this calculation were taken from the replication files for Autor (2015), and correspond to the evidence presented in Figure 4 of that paper. For developed countries, and over a time period that overlaps with the years we focus on in our calibration, Autor (2015) documents that polarization in the wage distribution occurred but was quantitatively small by comparison with earlier waves.

sponding to 16.3% and 64.3% of the overall documented changes, respectively. The Alchian-Allen mechanism has a positive and sizable effect on local changes of inequality: across all skill types, the mean (weighted) elasticity of $LCI^n(s)$ with respect to a change in ξ is 4.54.

5 Conclusion

We have presented a model of international trade where goods are horizontally differentiated on the basis of the inputs that are required to produce them, as well as because different firms produce them. As is customary in the literature on international trade in differentiated products (starting with Krugman, 1979), the model assumes a monopolistically competitive market structure, with the important difference that trade costs are not of the iceberg type but fall with price in relative terms.

In the model, the relative scarcity of different skill varieties determines the wage firms pay to employ workers with a given skill variety as well as the price of the goods that are produced using that skill variety. The combination of non-iceberg export technologies, input-based product differentiation and comparative skills scarcity is then sufficient to predict a positive correlation between wages, prices and share of foreign sales on total sales, as we observe it in the data. The model also predicts market integration to produce wage polarization and an increase in the concentration of firm sizes.

Theoretical trade models that derive all effects from factor-neutral productivity differentials leave out a channel that is both in evidence in the data and in line with standard general-equilibrium thinking: firms are inevitably supply-constrained with respect to the characteristics of their labor force. The model can be re-interpreted as a framework where quality differentials across products arise from quality differentials across the inputs used to produce them. The model could also be readily extended to allow for firm heterogeneity in total factor productivity and to incorporate fixed exporting costs. Such extensions would of course produce a richer set of predictions, but all the results of the baseline version of the model would still apply, conditional on a given level of total factor productivity. Input-based product differentiation should therefore be thought of as a complementary mechanism that can work alongside factor-neutral productivity differentials, rather than as a competing paradigm.

The role played by input-based product differentiation in the export conduct of firms not only appears to be of first-order prominence in the evidence that we have quoted and documented, but also points to distinctive policy implications. With input-based product differentiation, trade liberalization produces effects on the distribution of wages, even in the absence of a trade-induced selection of firms (in either the domestic or the export market) and independently of the usual discriminating categories (such as firms that are ex-ante defined as low- or high-productivity firms, and workers that are ex-ante grouped as low- or high-skill), and these effects are linked to the skills composition of domestic labor market supply. This angle has been overlooked by the literature, and opens up new possibilities for understanding the interplay between trade policies and labor market outcomes.

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Appendix

A.1 Derivation of equilibrium relationships

Autarky

Market clearing in product markets requires that the level of output, $q(s)$, for a firm producing a variety of good type s must equal demand, $x(s)$. Given the technology (3), the differentiated labor input and composite input demands of a firm producing varieties of good type s in equilibrium are then

$$l(s) = \lambda \left(\frac{w(s)}{P_Y}\right)^{\lambda-1} x(s) + f, \tag{42}$$

$$y(s) = (1-\lambda) \left(\frac{w(s)}{P_Y}\right)^{\lambda} x(s); \tag{43}$$

and total revenue and cost for the firm are respectively

$$r(s) = \frac{\eta}{\eta-1} w(s)^{\lambda} P_Y^{1-\lambda} x(s), \tag{44}$$

$$c(s) = w(s)^{\lambda} P_Y^{1-\lambda} x(s) + w(s) f. \tag{45}$$

Using the zero-profit condition $r(s) = c(s)$ implied by free entry, substituting the equilibrium demand (10) into (42) and (43), substituting for the price from (6) in (9) and equating this with (10), we obtain the equilibrium relationships presented in the main text.

Substituting for (14) in (8) yields the equilibrium price index, which, after substitution in (7), yields the equilibrium level of demand for the composite good $X(s)$ as a function of the vector of real wages. Substituting in the definition of aggregate composite product (2), and normalizing P_Y to be unity, then yields[44]

$$Y = \left(\frac{(\eta f)^{\frac{1}{\eta-1}}}{(\eta-1)/\eta}\right)^{\eta-1} \prod_{s=1}^{S} \left(\alpha^{-\eta} w(s)^{1+\lambda(\eta-1)}\right)^{\alpha}. \tag{46}$$

Open economy

The supply of any given firm has to satisfy domestic demand, $x(s)$, foreign demand, $x^*(s)$, and the demand originating from the production of export services, according to the technology (5). For each unit of exported variety, the firm must produce b units of export services at a marginal cost $\tilde{w}(s)^{\delta}$. This requires $\xi\, b\, \tilde{w}(s)^{\delta-\lambda} = \xi\, \big(\tau(s)-1\big)$ units of output, each at a marginal cost $\tilde{w}(s)^{\lambda}$, and $(1-\xi)\, b\, \tilde{w}(s)^{\delta} = (1-\xi)\, \big(\tau(s)-1\big)\, \tilde{w}(s)^{\lambda}$ units of the aggregate good. Market clearing for products thus requires

$$\tilde{q}(s) = \tilde{x}(s) + \Big(1 + \xi\, \big(\tau(s)-1\big)\Big)\, \tilde{x}^*(s). \tag{47}$$

Proceeding as for the autarky case, we obtain the equilibrium relationships presented in the text.

Combining (27) with $\tilde{P}(s)$ yields an equilibrium price index. Substituting in (19) yields the quantity index for good type s. Substituting in (2) yields the equilibrium level of the good composite:

$$\tilde{Y} = \left(\frac{(\eta f)^{\frac{1}{\eta-1}}}{(\eta-1)/\eta}\right)^{\eta-1} \prod_{s=1}^{S} \left(\alpha^{-\eta} \frac{\tilde{w}(s)^{1+\lambda(\eta-1)}}{1+(m-1)\,\tau(s)^{1-\eta}}\right)^{\alpha}. \tag{48}$$

[44] We can also obtain an expression for the output of composite product as a function of the skill endowments, $L(s)$, as $Y = \theta \left(\prod_{s=1}^{S} L(s)^{\alpha}\right)^{\frac{1+\lambda(\eta-1)}{\lambda(\eta-1)}}$, where $\theta = ((\eta-1) f)^{-\frac{1}{\lambda(\eta-1)}} \left(\frac{\eta-1}{\eta} S\right)^{\frac{1-\lambda}{\lambda}} \left(\frac{\eta-1}{1+\lambda(\eta-1)}\right)^{\frac{1+\lambda(\eta-1)}{\lambda(\eta-1)}}$ is a positive constant.

A.2 Derivations and proofs of propositions

Proof of Proposition 1

Taking the ratio $w(s')/w(s'')$ from (15) for any pair of skill types s' and s'' such that $L(s') < L(s'')$ yields $w(s')/w(s'') = L(s'')/L(s')$. Therefore, $L(s') < L(s'') \iff w(s') > w(s'')$. For $\lambda > 0$, the equilibrium price in autarky (6) is an increasing function of the wage. Therefore, $L(s') < L(s'') \iff w(s') > w(s'') \iff p(s') > p(s'')$. From (11), the firm's equilibrium revenue is an increasing function of the wage. □

Monotonicity of the wage equation in open economy

The goal is to show that the left-hand side in (29) is increasing in the wage, i.e., that

$$\frac{d}{d\tilde{w}(s)}\left(\frac{\tilde{w}(s)}{1+(\eta-1)\lambda\big(\xi+(1-\xi)\mu(s)\big)}\right) > 0$$

is positive. The sign of the above expression coincides with the sign of

$$1+(\eta-1)\lambda\big(\xi+(1-\xi)\mu(s)\big)-(\eta-1)\lambda(1-\xi)\frac{d\mu(s)}{d\tilde{w}(s)}\tilde{w}(s) > 0\,.$$

Rearranging to express the elasticity of the relative price to the wage yields

$$\frac{d\mu(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\mu(s)} < 1+\frac{1}{\mu(s)}\frac{1+\xi(\eta-1)\lambda}{(\eta-1)\lambda(1-\xi)}\,.$$

Since $0 < \mu(s) < 1$, a sufficient condition is given by

$$\frac{d\mu(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\mu(s)} < 1+\frac{1+\xi(\eta-1)\lambda}{(\eta-1)\lambda(1-\xi)} \implies \frac{d}{d\tilde{w}(s)}\left(\frac{\tilde{w}(s)}{1+(\eta-1)\lambda\big(\xi+(1-\xi)\mu(s)\big)}\right) > 0\,.$$

The proof continues by showing that this sufficient condition is satisfied. Call $NUM = 1+(m-1)\,\tau(s)^{-\eta}$ and $DEN = 1+(m-1)\,\tau(s)^{1-\eta}$, where $\tau(s) > 1$ implies $DEN > NUM > 1$. Then, the elasticity of $\mu(s)$ with respect to the wage is given by

$$\begin{aligned}
\frac{d\mu(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\mu(s)} &= \frac{dNUM}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{NUM}-\frac{dDEN}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{DEN}\\
&= -\eta\frac{NUM-1}{NUM}\frac{d\tau(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\tau(s)}+(\eta-1)\frac{DEN-1}{DEN}\frac{d\tau(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\tau(s)}\\
&= -\frac{d\tau(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\tau(s)}\left(\eta\left(\frac{NUM-1}{NUM}-\frac{DEN-1}{DEN}\right)+\frac{DEN-1}{DEN}\right).
\end{aligned}$$

Let $\delta \equiv \xi\lambda$. Then, the elasticity of trade costs with respect to the wage is given by

$$\frac{d\tau(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\tau(s)} = -\lambda\frac{(1-\xi)b\tilde{w}(s)^{\delta-\lambda}}{1+b\tilde{w}(s)^{\delta-\lambda}} = -\lambda(1-\xi)\frac{\tau(s)-1}{\tau(s)}\,;$$

therefore:

$$\frac{d\mu(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\mu(s)} \quad = \quad \lambda(1-\xi)\frac{\tau(s)-1}{\tau(s)}\left(\eta\left(\frac{NUM-1}{NUM}-\frac{DEN-1}{DEN}\right)+\frac{DEN-1}{DEN}\right)\,.$$

Thus, if $\left(\eta\left(\frac{NUM-1}{NUM}-\frac{DEN-1}{DEN}\right)+\frac{DEN-1}{DEN}\right) < 0$, then $\frac{d\mu(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\mu(s)} < 0$, which proves the result.

Otherwise, if $\left(\eta\left(\frac{NUM-1}{NUM}-\frac{DEN-1}{DEN}\right)+\frac{DEN-1}{DEN}\right)>0$, then

$$\begin{aligned}\frac{d\mu(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\mu(s)} &< \lambda\left(\eta\left(\frac{NUM-1}{NUM}-\frac{DEN-1}{DEN}\right)+\frac{DEN-1}{DEN}\right),\\ \frac{d\mu(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\mu(s)} &< \lambda\frac{DEN-1}{DEN}<\lambda,\end{aligned}$$

where the last line is implied by $DEN > NUM > 1$ and $\frac{DEN-1}{NUM-1} = \tau(s) > 1$ for every $s = 1, ..., S$, implying $\frac{DEN-1}{DEN} > \frac{NUM-1}{NUM}$, which proves the result for every $\lambda \in (0,1)$. □

Proof of Proposition 2

The expression for trade costs (17) implies that $\tau(s)$ is greater than unity for any positive level of trade barriers and for $\xi \in (0,1)$ $d\tau(s)/d\tilde{w}(s) < 0$. The definition of export intensity (30) yields $d\rho(s)/d\tau(s) < 0$ for an elasticity of substitution $\eta > 1$. Therefore, for any pair of skill types, s' and s'', export intensity is greater the higher the wage $\tilde{w}(s') > \tilde{w}(s'') \iff \rho(s') > \rho(s'')$. The price charged in the domestic market is $\tilde{p}(s) = \left(\eta/(\eta-1)\right)\tilde{w}(s)^{\lambda}$. Therefore, $L(s') < L(s'') \iff \tilde{w}(s') > \tilde{w}(s'') \iff \tilde{p}(s') > \tilde{p}(s'') \iff \rho(s') > \rho(s'')$. Moreover, higher-price varieties sold in their domestic market at a price $\tilde{p}(s)$ remain more expensive when they are shipped to foreign markets at a price $\tau(s)\tilde{p}(s)$. In fact, the elasticity of trade cost to wage, $\frac{d\tau(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\tau(s)} > -\lambda$, is less than compensates the elasticity of domestic price to wage, $\frac{d\tilde{p}(s)}{d\tilde{w}(s)}\frac{\tilde{w}(s)}{\tilde{p}(s)} = \lambda$. Finally, substituting in (24) yields the equilibrium level of total revenue in open economy, $\tilde{r}(s) = \eta f \tilde{w}(s)$, which is an increasing function of the wage. □

Proof of Proposition 3

Given (28), the first-order derivative of $\mu(s)$ with respect to trade cost is given by

$$\frac{d\mu(s)}{d\tau(s)} = \frac{-\eta\tau(s)^{\eta-1}+(\eta-1)\tau(s)^{\eta}-(m-1)}{\left(\tau(s)^{\eta}+(m-1)\tau(s)\right)^{2}}.$$

Thus, (17) yields the change with respect to the wage:

$$\frac{d\mu(s)}{d\tilde{w}(s)} = -\lambda(1-\xi)\frac{\tau(s)-1}{\tilde{w}(s)}\frac{-\eta\tau(s)^{\eta-1}+(\eta-1)\tau(s)^{\eta}-(m-1)}{\left(\tau(s)^{\eta}+(m-1)\tau(s)\right)^{2}}.$$

Therefore, a stationary point $\tau(s) = \tau_+$ satisfies the condition:

$$-\eta\tau_+^{\eta-1}+(\eta-1)\tau_+^{\eta}-(m-1)=0\,, \tag{49}$$

where it should be noted that $-\eta\tau_+^{\eta-1}+(\eta-1)\tau_+^{\eta}>0$ implies $\tau_+ > \frac{\eta}{\eta-1}$.

The sign of the derivative of the left-hand side in (49) with respect to the trade cost is given by $\tau_+^{\eta-2}\left(-1+\tau_+\right) > 0$, and so the left-hand side is strictly increasing in the trade cost. Therefore, for a given market size m, there exists a unique real value τ_+, corresponding to a wage w_+, as given in (17), that solves the stationary condition and for which

$$\frac{d\mu(s)}{d\tilde{w}(s)} \leq 0 \iff \tau(s) \geq \tau_+ \iff \tilde{w}(s) \leq w_+\,.$$

Substituting for $1+(m-1)\,\tau_+^{-\eta} = \frac{\eta(\tau_+-1)}{\tau_+}$ and $1+(m-1)\,\tau_+^{1-\eta} = (\eta-1)\tau_+$, as implied by the stationary condition (49) in (28), yields the following minimum:

$$\mu(s_+) = \frac{\eta \tau_+^{-1}}{\eta - 1} ,$$

which shows that $\mu(s_+)$, and hence $k(s_+)$ from (32),

$$k(s_+) = 1 - \frac{\lambda(\eta-1)(1-\xi)(1-\mu(s_+))}{1+\lambda(\eta-1)} = 1 - \frac{\lambda(\eta-1)(1-\xi)}{1+\lambda(\eta-1)} \left(1 - \frac{\eta\tau_+^{-1}}{\eta-1}\right) ,$$

are decreasing functions of τ_+. Multiplying the stationary condition (49) by $\tau(s_+)^{1-\eta}$ yields the export intensity evaluated at w_+:

$$\rho_+ \quad = \quad (m-1)\tau_+^{1-\eta} = (\eta-1)\tau_+ - \eta .$$

Given (17) and (30), export intensity $\rho(s)$ is an increasing and invertible function of the wage $\tilde{w}(s)$. Given (25), employment is an increasing function of $\mu(s)$, and thus it is decreasing with the wage for $\tilde{w}(s) < w_+$ and increasing for $\tilde{w}(s) \geq w_+$. The monotonic relationship between wage and export intensity implies that the same patterns apply to the relationship between employment and export intensity. As shown in the proof of Proposition 2, total revenue is an increasing and invertible function of the wage. Therefore, there exists a unique threshold $r_+ = \eta f w_+$ such that $\tilde{w}(s) \geq \tilde{w}_+ \iff \tilde{r}(s) \geq r_+$ and $\tilde{w}(s) < \tilde{w}_+ \iff \tilde{r}(s) < r_+$. □

Proof of Proposition 4

The proof involves three steps. First, we show that the model predicts gains from trade for skills with a relative labor share greater than the geometric average. Second, we show that the model predicts aggregate gains from trade. Third, we show that even the skill with the worst predicted outcome gains from trade.

Rearranging (31) as

$$\frac{\tilde{w}(s)}{w(s)} \quad = \quad \prod_{n=1}^{S} \left(\frac{k(s)}{k(n)}\right)^{\alpha} \prod_{n=1}^{S} \left(\frac{1+\rho(n)}{k(n)}\right)^{\frac{\alpha}{\lambda(\eta-1)}} \tag{50}$$

shows that skills with a relative labor share greater than the geometric average $k(s) \geq \prod_{n=1}^{S} k(n)^{\alpha}$ necessarily gain from opening the economy. In fact, the second product in (50) is necessarily greater than one, since $k(s) < 1$ and $\rho(s) > 0$ for every skill type. Therefore, despite the overall drop in the labor share due to the employment of intermediate inputs in export services (i.e., $\xi < 1$), the model predicts gains from trade, in the form of greater consumption, for at least a subset of skill types.

Moreover, it should be noted that the second product in (50) corresponds to the geometric average of changes in individual welfare across skill types:

$$\prod_{s=1}^{S} \left(\frac{\tilde{w}(s)}{w(s)}\right)^{\alpha} = \prod_{n=1}^{S} \left(\frac{1+\rho(n)}{k(n)}\right)^{\frac{\alpha}{\lambda(\eta-1)}} > 1 . \tag{51}$$

Aggregate gains from trade are equal to the arithmetic average of changes in individual welfare across skill types. Since the arithmetic average is at least equal to the corresponding geometric average, the model predicts aggregate gains from trade:

$$\frac{\tilde{C}}{C} = \frac{1}{S} \sum_{s=1}^{S} \frac{\tilde{w}(s)}{w(s)} \geq \prod_{s=1}^{S} \left(\frac{\tilde{w}(s)}{w(s)}\right)^{\alpha} > 1 , \tag{52}$$

because the increase in output of composite product more than compensates the average reduction in labor share.

The skill type with the lowest possible change in real wage under market integration is the one that earns a wage $\tilde{w}(s_+) = w_+$, corresponding to the minimum of the relative price function $\mu(s_+) = \frac{\eta \tau_+^{-1}}{\eta - 1}$ and to a trade cost $\tau_+ > \frac{\eta}{\eta-1}$. The corresponding relative labor share $k(s_+)$ and export intensity ρ_+ are

$$\begin{aligned} k(s_+) &= 1 - c_0 + \frac{c_0 \eta}{\eta - 1}\tau_+^{-1} > 1 - c_0 > \frac{1}{1+\lambda(\eta-1)}, \\ 1 + \rho_+ &= (\eta - 1)(\tau_+ - 1) > 1\,, \end{aligned}$$

where $c_0 \equiv \frac{\lambda(\eta-1)(1-\xi)}{1+\lambda(\eta-1)} < \frac{\lambda(\eta-1)}{1+\lambda(\eta-1)} < 1$. Assume that s_+ belongs to the set of skill types $\{1, ..., S\}$. Then, rearranging (31) yields

$$\frac{\tilde{w}(s_+)}{w(s_+)} = \left(\prod_{n=1}^{S} \frac{k(s_+)}{k(n)} \left(\frac{1+\rho(n)}{k(n)} \right)^{\frac{1}{\lambda(\eta-1)}} \right)^{\alpha} = \left(\left(\frac{1+\rho_+}{k(s_+)} \right)^{\frac{1}{\lambda(\eta-1)}} \prod_{n \neq s_+}^{S} \frac{1}{k(n)} \left(\frac{1+\rho(n)}{k(n)} \right)^{\frac{1}{\lambda(\eta-1)}} \right)^{\alpha}.$$

Therefore $\frac{1+\rho_+}{k(s_+)} \geq 1$ is a sufficient condition for $\frac{\tilde{w}(s_+)}{w(s_+)} > 1$. The ratio $\frac{1+\rho_+}{k(s_+)}$ is increasing in τ_+, and for the lower extreme of the support we have $\tau_+ \to \frac{\eta}{\eta-1}$, and thus both $k(s_+)$ and $(1+\rho_+)$ approach unity. Consequently, $\frac{1+\rho_+}{k(s_+)} \geq 1$ is always met. Substituting for the lower bound $\tau_+ \to \frac{\eta}{\eta-1}$ reveals that

$$\frac{\tilde{w}(s_+)}{w(s_+)} \geq \prod_{n \neq s_+} \left(\frac{1+\rho(n)}{k(n)^{1+\lambda(\eta-1)}} \right)^{\frac{1}{S\lambda(\eta-1)}} > 1\,. \tag{53}$$

Therefore, there are no skill types that lose from market integration. The gain for a certain skill type is at least as large as the expansion in the output of the composite product caused by the expansion in the production of goods associated with all other skill types. □

Proof of Proposition 5

Equation (35) shows that relative gains from trade between two different skill types, s' and s'', are driven by changes in their labor share in open economy. For every skill type s, the trade induced change in the labor share, $k(s)$, is increasing in $\mu(s)$, as given in (32). Proposition 3 shows that $\mu(s)$, and hence $k(s)$, are non-monotonic functions of the wage, with a minimum at w_+. Proposition 2 implies that $L(s') < L(s'') \iff \tilde{w}(s') > \tilde{w}(s'')$. Therefore, the definition (32) and Propositions 2 and 3 together imply

$$\begin{aligned} &\text{either} \quad L(s') < L(s'') \iff \tilde{w}(s') > \tilde{w}(s'') \geq w_+ \implies \frac{k(s')}{k(s'')} > 1\,, \\ &\text{or} \quad L(s') < L(s'') \iff \tilde{w}(s'') < \tilde{w}(s') \leq w_+ \implies \frac{k(s')}{k(s'')} < 1\,. \end{aligned}$$

Thus, above the wage threshold w_+ scarcer and comparatively more highly paid skill types gain relatively more from trade liberalization; but below the wage threshold w_+ scarcer and more comparatively highly paid skill types gain relatively less from trade liberalization. □

Proof of Proposition 6

Proposition 6 is a corollary that follows from Propositions 2 and 5. □

A.3 Comparative statics on w_+

We start by showing that

$$\tau_+ > \frac{\eta}{\eta - 1} \qquad \frac{\mathrm{d}\tau_+}{\mathrm{d}m} > 0 \qquad \frac{\mathrm{d}\tau_+}{\mathrm{d}\eta} < 0\,. \tag{54}$$

The relationship between relative size of the international market and trade cost evaluated at the minimum of the relative price function (49), that is

$$m \quad = \quad 1 - \eta\tau_+^{\eta-1} + (\eta - 1)\tau_+^{\eta}\,,$$

is sufficient to determine that for a given pair (m, η), there exists a unique threshold of trade cost $\tau_+ > 1$. The right-hand side starts at zero for $\tau_+ \to 1$ and is increasing in τ_+. The left-hand side is a constant greater or than 1. Thus, there exists a unique value of τ_+, and it is increasing with the size of the international market relative to the domestic one. Moreover, we have already established that $m \geq 1 \implies \tau_+ > \dfrac{\eta}{\eta - 1}$. This is a sufficient condition for the right-hand side to be an increasing function of $\eta > 1$. Thus, a higher η for a given m implies a lower τ_+.

Note that the minimum of the relative price $\mu(s_+) = \dfrac{\eta\tau_+^{-1}}{\eta - 1}$ is decreasing in τ_+. Since trade-induced changes in wages follow the same pattern as $\mu(s)$ (from (32)), then, ceteris paribus, gains from trade for skill types in the neighborhood of the wage threshold are comparatively smaller in economies that are small compared to the global market.

Next, we show that

$$w_+ < ((\eta - 1)b)^{\frac{1}{\lambda(1-\xi)}}\,, \qquad \frac{\mathrm{d}w_+}{\mathrm{d}m} < 0, \qquad \frac{\mathrm{d}w_+}{\mathrm{d}\eta} > 0, \qquad \frac{\mathrm{d}w_+}{\mathrm{d}b} > 0\,. \tag{55}$$

Given (54), the expression for trade costs (17) evaluated at τ_+, that is

$$\tau_+ \quad = \quad 1 + bw_+^{-\lambda(1-\xi)}\,,$$

is sufficient to determine the upper bound to the wage cutoff such that $1 + bw_+^{-\lambda(1-\xi)} < \dfrac{\eta}{\eta - 1}$, and to establish the sign of the comparative statics with respect to m and η. Trade costs are decreasing in the wage, and so, holding constant τ_+, higher trade barriers imply a higher wage threshold. The comparative statics with respect to λ and ξ depend on whether w_+ is greater or lower than the price of the composite good:

$$\text{if } w_+ > \tilde{P}_Y \text{ then} \qquad \frac{\mathrm{d}w_+}{\mathrm{d}\lambda} < 0, \qquad \frac{\mathrm{d}w_+}{\mathrm{d}\xi} > 0; \tag{56}$$

$$\text{if } w_+ < \tilde{P}_Y \text{ then} \qquad \frac{\mathrm{d}w_+}{\mathrm{d}\lambda} > 0, \qquad \frac{\mathrm{d}w_+}{\mathrm{d}\xi} < 0\,. \tag{57}$$

Rearranging (29) and substituting for $\mu(s_+) = \dfrac{\eta\tau_+^{-1}}{\eta - 1}$ yields the fraction of gross output that consists of value added by the skill corresponding to the wage threshold:

$$\frac{w_+ L(s_+)}{\tilde{P}_Y \tilde{Y}} = \alpha\, \frac{1 + \lambda\left(\xi(\eta - 1) + (1 - \xi)\eta\tau_+^{-1}\right)}{\eta}\,.$$

Since $\frac{\tilde{w}(s)L(s)}{\tilde{P}_Y\tilde{Y}}$ is an increasing function of $\mu(s)$, $\frac{w_+L(s_+)}{\tilde{P}_Y\tilde{Y}}$ is the minimum fraction of gross output that consists of value added by a certain skill type. For higher wages, i.e., $\tilde{w}(s) > w_+$, this share increases with the wage despite the greater comparative scarcity of higher-s skill types ($L(s) < L(s_+)$); for lower wages, i.e., $\tilde{w}(s) < w_+$, it increases with the wage because of higher-s skill types being relatively more abundant ($L(s) > L(s_+)$). This is sufficient to locate the wage threshold w_+. If this level of earnings is neither the highest nor the lowest in the wage support, then both the effects predicted by the model occur: among skill types with wages below w_+, comparatively less highly paid labor types gain comparatively more; among skill types with wages above w_+ comparatively more highly paid labor types gain comparatively more.

A.4 Welfare effects by worker type

Figure 5 graphically depicts the ratios $\hat{w}(s)/w(s)$ in the numerical simulation of Section 4.2.

Figure 5: Welfare effects of trade liberalization

Note: Points correspond to different skill types, $s \in \{1, ..., 75\}$ (more abundant types on the left, scarcer skill types on the right).