Proposing peace to belligerents with interdependent valuations

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In many disputes interdependent valuations of the winner’s prize are a more realistic assumption than private valuations.

Private beliefs of involved parties about the war payoff structure shape any peaceful settlement.

There is no “natural” game form for crisis bargaining with the timing of actions or bargaining protocol.

When and how unmediated communication reduces the ex ante probability of bilateral conflict in the case of interdependent valuations?
To understand how the total cost of interstate wars might be related to the fighting capacities of belligerents.

To estimate the degree to which fighting capacities are a noisy predictor of the outcome of war, captured by the probability of a win for the stronger side.
Estimations for linear regression models for battle-related fatalities in inter-state wars for the period 1816-2007

<table>
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$t$ statistics in parentheses

$p < 0.05$, $p < 0.01$, $p < 0.001$
Empirical results 2

- Estimates for the probability of a win for the stronger side, measured by the Composite Indicator of National Capability (CINC). The indicator is assembled by Singer et al. (1972) and expanded by Singer in the 4.0 version of the COW dataset.

Table: Outcomes of interstate wars for the period 1816-2007

<table>
<thead>
<tr>
<th>Type</th>
<th>Frequency</th>
<th>Fraction</th>
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<tr>
<td>Higher CINC side wins</td>
<td>49</td>
<td>.52</td>
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<tr>
<td>Lower CINC side wins</td>
<td>28</td>
<td>.30</td>
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<tr>
<td>Stalemate</td>
<td>8</td>
<td>.08</td>
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<tr>
<td>Tie</td>
<td>2</td>
<td>.02</td>
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<tr>
<td>Transformed</td>
<td>8</td>
<td>.08</td>
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Related literature 1

- Banks (1990), Fearon (1995), Warneryd (2003), Bester and Warneryd (2006), Fey and Ramsay (2009) study the design of political and economic institutions to prevent or resolve any type of destructive conflict.


- Horner, Morelli and Squintani (2010) study mediation, arbitration and unmediated communication in a model of conflict with exogenous cost of war and two types of opponents.
Fey and Ramsey (2011) investigate crisis bargaining games where either the relative power of opponents is uncertain or the win probabilities are common knowledge but there is uncertainty about the opponent’s fixed cost of war.

Results about the optimal design of mechanisms in the independent values setting do not extend automatically to the interdependent values setting.
Prize is of common value. War shrinks the value of the prize.

Probability of winning the war prize is determined by the relative strength of players. Fighting capacities of opponents (types) are privately observed.

Types are independently and uniformly distributed. The joint distribution of types is symmetric and it is a common knowledge.

Cost of war is uncertain and endogenously determined by types of opponents.
Model 1

- $t_i, i = 1, 2,$ are independently and uniformly distributed on $[0, 1]
- $p(t_i, t_j)$ is player $i$'s probability of winning the war prize

\[
p(t_i, t_j) = \begin{cases} 
  p & \text{for } t_i > t_j \\
  \frac{1}{2} & \text{for } t_i = t_j, \\
  1 - p & \text{for } t_i < t_j 
\end{cases}
\]

where $p > 1 - p$, that is, $p > \frac{1}{2}$. 

Model 2

- $\theta(t_i, t_j)$ is the value of the prize allocated to the winner
- the destruction technology

\[
\theta(t_i, t_j) = 1 - \alpha t_i t_j, \quad \text{where } 0 < \alpha < 1.
\]
Solution concept

- Bayesian-Nash equilibrium of the games induced by the mechanism
- Sets of Bayesian Nash equilibria that are achievable in the induced games are ranked. The Pareto ranking is based on the \textit{ex ante} probability of peace in the best separating equilibrium of the game.
Split proposal game

- Set of efficient outcomes $Y^e = \{(y_1, y_2) : y_1 + y_2 = 1\}$
- Split proposal $(x, 1 - x) \in Y^e$
- Pure strategy $\sigma_i(x, t)$ of player $i$,
  $\sigma_i : [0, 1] \times [0, 1] \rightarrow \{'accept', 'reject'\}$
- The expected war payoff for player $i$ of type $t_i$ is $\pi(t_i)$,
  $\pi(t_i) = \int_0^1 p(t_i, t_j)\theta(t_i, t_j) \, dt_j$
- $\pi(0) = 1 - p < \frac{1}{2}$, $\pi(1) = p \left( 1 - \frac{\alpha}{2} \right) > \frac{1}{2}$. 
Methods

Proposition

For parameter values satisfying condition $p(1 - \alpha/3) \leq 1/2$, the ex ante probability of peace is 1 subject to opponents playing a Bayesian Nash equilibrium of the split proposal game with offer $(1/2, 1/2)$.

Lemma

Expected war payoff of type $t$ is strictly increasing in $t$ if and only if the parameter values satisfy condition

$$a \leq \frac{4p - 2}{5p - 2}.$$  

(2)
Proposition

In the Bayesian Nash equilibrium of the split proposal game where condition (2) holds, the ex ante probability of peace with equal split is

\[ P(1/2, 1/2) = -\frac{4}{3} r \sin^2 \left( \frac{\pi}{6} - \frac{1}{3} \arccos \left( \frac{3\sqrt{3}q\sqrt{-\frac{1}{r}}}{2r} \right) \right) \]

where \( r \) and \( q \) are given by

\[ r = \frac{-2 - \alpha + 4p + \alpha p}{\alpha(1 - 2p)} \quad \text{and} \quad q = \frac{1}{\alpha}. \] (3)

\((p, \alpha) = (3/4, 1/4) : P(1/2, 1/2) \approx 0.310,\]
\((p, \alpha) = (5/6, 1/8) : P(1/2, 1/2) \approx 0.267,\]
\((p, \alpha) = (3/4, 1/6) : P(1/2, 1/2) \approx 0.286.\]
Proposition

The probability of peace $\mathcal{P}(x, 1 - x)$ in the Bayesian Nash equilibrium of the split proposal game satisfies inequality

$$\mathcal{P}(x, 1 - x) \leq \frac{(2p - 1)^2}{(4p - 2 - \alpha p)^2}$$

for parameter values that satisfy condition (2).
Split proposal game (Ex ante welfare 2)

**Figure**: A set of parameter values for which the ex ante probability of acceptance of any split proposal is less than 1/2
Unmediated peace talk game 1

- $M$ is a message space
- Set of acceptable splits $X \subset Y^e$, $X \equiv [1 - p, p]
- Probability of recommendation of acceptable split $x \in [1 - p, p]$ when the message profile is $m$ is $q(m) \in [0, 1]$

Definition

A peace talk mechanism is a pair $(M \times M, f)$ where the decision rule $f : M \times M \rightarrow D$ maps each pair of messages into a decision in $D = [1 - p, p] \times [0, 1]$. 
Utilities from a messages profile $m = (m_1, m_2)$ are

$$u_1(t, m) = q(m)x(m) + (1 - q(m))\pi_1(t)$$

$$u_2(t, m) = q(m)(1 - x(m)) + (1 - q(m))\pi_2(t)$$

Given the belief $b_i$, the expected payoff for player $i$ of strategy profile $s$ is

$$U_{i}^{b_i}(t_i) = \int_{0}^{1} u((t_1, t_2), s(t_1, t_2)) \, db_i(t_j),$$

Definition

A direct mechanism $(M, f)$ is interim incentive compatible (IC) if it has a Bayesian Nash equilibrium $(s_1^*(t_i), s_2^*(t_2))$ at the interim stage such that $s_1^*(t_2) = t_1$ and $s_2^*(t_1) = t_2$. 
Functions $q(m)$ and $x(m)$ are symmetric across messages:

$$x(m_1, m_2) = x(m_2, m_1) \quad \text{and} \quad q(m_1, m_2) = q(m_2, m_1).$$

Hence,

$$x(y, y) = 1/2 \quad \text{for any } y \in M.$$

Designer’s objective in full disclosure peace talk game

$$\mathcal{P} = \max_{x(m), q(m)} \int_0^1 \int_0^1 q(m_1, m_2) \, dm_2 \, dm_1 \quad (4)$$
Proposition

Let the direct mechanism be interim incentive compatible. Then the probability of recommendation of a peaceful split, expected by player of type $t_i$, decreases when $t_i$ increases.

Proposition

Let the direct mechanism be interim incentive compatible. Then the probability of recommendation of peace inducing split is not constant in reported types.
Peace talk game with partial revelation of types 1

Definition

Let \( \{ T_0, T_1, \ldots, T_n \} \) be a finite set of messages such that \( T_0 = 0, T_n < 1 \), and \( T_i < T_{i+1} \). We say that a report \( T_i, 0 \leq i \leq n - 1 \), of player \( j \) is truthful if the interval \( [T_i, T_{i+1}] \) contains player’s true type \( t_j \). Report \( T_n \) is truthful for player \( j \) if \( T_n \leq t_j \leq 1 \).

- Message space \( M = [h, l] \)
- \( x(l, l) = x(h, h) = 1/2 \) and \( x \equiv x(h, l) = 1 - x(l, h) \)
- Unknown probabilities

\[ q_l \equiv q(l, l), \quad q_h \equiv q(h, h), \quad q_m \equiv q(h, l) = q(l, h). \]
Lemma

The expected war payoffs in the mechanism \((M, (x, q))\), conditional on the type of the opponent, are strictly increasing in agent’s own type for parameter values satisfying condition (2). In particular,

1. the expected payoff \(\pi_l(t_i)\) of a low type \(t_i \leq 1/2\) from a war with low type is increasing and concave in \(t_i\) for \(t_i \in [0, 1/2]\),

2. the expected payoff \(\pi_h(t_i)\) of a high type \(t_i \geq 1/2\) from a war with high type is increasing and concave in \(t_i\) for \(t_i \in [1/2, 1]\).
Peace talk game with partial revelation of types 3

Objective function and constraints

\[ \max_{x,q_l,q_h,q_m} \left\{ \frac{1}{4} q_l + \frac{1}{2} q_m + \frac{1}{4} q_h \right\}, \]  \hspace{1cm} \text{(5)}

subject to the probability constraints

\[ 0 \leq q_l \leq 1, \ 0 \leq q_m \leq 1, \ 0 \leq q_h \leq 1 \]  \hspace{1cm} \text{(6)}
Methods

Peace talk game with partial revelation of types 4

Constraints

- The low type and the high type \textit{ex post} IR constraints

\begin{equation}
1 - x \geq \frac{1}{2}(1 - p), \quad x \geq p\left(\frac{1}{2} - \frac{\alpha}{16}\right),
\end{equation}

(7)

- The high type \textit{interim} IC constraint for \( t_i \geq 1/2 \)

\begin{equation}
q_h\left(\frac{1}{2} - \pi_h(t_i)\right) \geq \frac{q_l}{2} - q_l p\left(\frac{1}{2} - \frac{\alpha t_i}{8}\right) + q_m p\left(\frac{1}{2} - \frac{\alpha t_i}{8}\right) + q_m(1-2x) - q_m \pi_h(t_i)
\end{equation}

(8)

- The low type \textit{interim} IC constraint for \( t_i \leq 1/2 \)

\begin{equation}
q_l\left(\frac{1}{2} - \pi_l(t_i)\right) \geq q_m(1-p)\left(\frac{1}{2} - \frac{3\alpha t_i}{8}\right) + q_m(2x-1) - q_m \pi_l(t_i) + \frac{q_h}{2} - q_h(1-p)\left(\frac{1}{2} - \frac{3\alpha t_i}{8}\right)
\end{equation}

(9)
Peace talk game with partial revelation of types (Solution)

The value of the split

\[ x^* = \frac{8 - 8p + \alpha p - 3\alpha q_h - 8pq_h + 3\alpha pq_h + 8q_m + 3\alpha q_m + 16pq_m - 4\alpha pq_m}{32q_m} \]

- \( q_l = 1, \ 0 < q_h < q_m < 1 \)
- Inequalities for \( q_m \) and \( q_h \)

\[ p(1/2 - \alpha/16) \leq x^* \leq 1/2 + p/2, \]

\[ q_m - \frac{(1 - q_m)p\frac{\alpha}{8}}{2p - 1 - \frac{\alpha(1+p)}{8}} \geq q_h, \]

\[ q_m - (1 - q_m)\frac{2p - 1 + \frac{\alpha}{8}(2 - 5p)}{(1 - p)\frac{3\alpha}{8}} < q_h. \]
Methods

Peace talk game with partial revelation of types (Example)

- $\alpha = 0.2$ and $p = 0.7$
- The ex ante probability of peace in the split proposal game is bounded from above by

$$P \leq \frac{(2p - 1)^2}{(4p - 2 - \alpha p)^2} \approx 0.367.$$  

- $q_l = 1$, $q_m = p$, $q_h = 1 - p$
- $x^* = 0.6372$
- The ex ante probability of peace in the unmediated cheap talk game $P = 0.675$
Methods

Peace talk game with partial revelation of types (Ex ante welfare)

Proposition

There is no separating equilibrium of the game $G$ where high type dyads never fight. The peace talk game induced by mechanism where mixed type dyads never fight is equivalent to the split proposal game.

Proposition

There is an unique second best separating equilibrium of the game $G$ where parameter values satisfy condition (2). The ex ante probability of peace in this equilibrium is $P > \frac{1}{2}$.
Results

- Incorporates the uncertainty about the cost of war in a model of conflict where agents have independent types but interdependent values.
- Calculates an upper bound for the ex ante probability of peaceful split in the Bayesian equilibrium of the split proposal game.
- Constructs a Bayesian incentive compatible and ex post individual rational cheap talk mechanism with partial revelation of private types.
- Finds a range of parameter values for which the second best separating equilibrium in the induced game improves chances for peace compared to the split proposal game.
Conclusion

- The model specification does not yield any closed form solution for some range of parameter values.
- Estimated the efficiency of two peace promoting mechanisms by the *ex ante* probability of peaceful resolution of dispute.
- The comparison of the *ex ante* probability of peace for the peace talks mechanism and the split proposal is consistent with the conventional view that communication is efficient in promoting peace.
- The direction for further research is to assess the welfare properties for different types of mediation programmes.