Keynesian Search during the U.S. Great Depression

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Using historical data from the U.S during the Great Depression we use Bayesian techniques to estimate and compare three model specifications discussed in Farmer (2021). One specification corresponds to a Real Business Cycle (RBC) model driven by productivity shocks. The other two specifications are labelled Keynesian Search models as they display a continuum of non-stochastic steady-state unemployment rates (or steady-state indeterminacy) and are driven by shocks to beliefs. In the first Keynesian Search model, beliefs about investment expenditure follow an exogenous process. In the second, beliefs about the value of the stock market follow an exogenous process. The empirical results show that both Keynesian Search specifications outperform the RBC model in fitting the data. Among the Keynesian Search models, data favour the specification where shocks to beliefs about investment expenditure follow an exogenous process. This result suggests that an effective way to restore full employment in the aftermath of the Great Depression in the U.S was to replace private investment with public investment.

Keywords: Bayesian estimation, Beliefs, DSGE, Hysteresis, Indeterminacy, Search and matching, Sunspots, VAR.

I. Introduction

Disastrous economic events such as the Great Depression and the Great Recession pose critical challenges to dynamic general equilibrium models with rational expectations in explaining high and persistent unemployment.

A substantive amount of literature, mostly the New-Keynesian tradition, relies on the idea that involuntary unemployment can be explained because prices, wages, or in some settings, information, adjust slowly in response to monetary shocks.[1]

In a recent paper, Farmer (2021) argues that this is not a good characterization of the Great Depression in the U.S.: between 1929 and 1933 nominal wages and

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[1] See Clarida, Gali and Gertler (1999), Woodford (2011) and Galí (2015) for treatments of the New-Keynesian model, and Mankiw and Reis (2002) and Sims (2003) for New-Keynesian models with sticky information

nominal prices fell by around 25%, while employment and consumption both fell substantially at the same time that the real wage increased.[2]

One alternative explanation to the "stickiness" approach - the Keynesian Search Theory - has been developed in a series of papers in the last decade such as Farmer (2012, 2013), Plotnikov (2019), and Farmer (2021, 2020). This class of models uses the tools of incomplete factor models, such as the Diamond-Mortensen-Pissarides' search and matching model, but it differs from them (and from standard business cycles models) in that it displays both *static* and *dynamic* indeterminacy.

Static indeterminacy means there are many possible equilibrium steady-state unemployment rates, a property also called hysteresis.[3] In turn, *dynamic* indeterminacy refers to the existence of many dynamic equilibrium paths, all of which converge to a given steady state.[4]

Keynesian Search Theory exploits both forms of indeterminacy and solves them by specifying how agents form expectations about the future. Specifically, this class of models is closed with a forecasting rule from current and past observable variables to probability distributions over future economic variables. Therefore, this mapping -or *belief function*- pins down a unique rational expectations equilibrium.

In this way, Keynesian Search Theory provides an explanation of the causes of big economic crises and the high persistence of unemployment. In other words, beliefs - or "animal spirits" have an independent role in macroeconomic outcomes.

The objective of this study is threefold: First, it expands the class of models in Keynesian Search theory to study a historical event where high unemployment persisted for a long period of time, namely, the Great Depression. For this purpose, we close the Keynesian Search model in two alternative ways: In the first one, aggregate demand is driven by an exogenous investment process. In the second one, aggregate demand is driven by consumer confidence. Specifically, through self-fulfilling expectations about the future value of the asset market. In either case, aggregate demand determines the steady-state unemployment rate.

Second, we provide a framework to justify policy interventions. If investment is exogenous, the only way to restore full employment is by replacing private investment with public investment. In contrast, if consumer confidence is exogenous, then interventions in the asset market are more effective in restoring full employment than fiscal policy. This suggests that, in the face of permanently depressed beliefs about the value of private assets, government spending will not be effective at maintaining full employment.

Third, we develop an empirical exercise where the two Keynesian Search models and a standard Real Business Cycle (RBC) model are estimated using historical

[2] This is not to say that wage stickiness is irrelevant in this period, but that is unlikely to be the main reason for high and persistent unemployment. Keynes itself argued that the assumption that money wages per unit of labour employed are constant was "dispensable" in his *General Theory* (Keynes, 1936).

[3] See Cerra, Fatás and Saxena (2020) for a survey on hysteresis and business cycles.

[4] See Benhabib and Farmer (1999) for a survey on dynamic indeterminacy in macroeconomics.

data of the U.S. during the Great Depression (1929-1940).[5]

Using Bayesian estimation techniques, we find that the Keynesian Search specifications outperform the RBC model. The empirical performance of the Keynesian Search theory stems from its ability to account for persistent movements in the data.

Moreover, among the two Keynesian Search models, data favour the specification where aggregate demand is driven by an exogenous investment process. Therefore, the corollary of these results is that in terms of policy prescription an effective way to restore full employment is by replacing private investment with public investment.

The paper is organized as follows: Section 2 covers a brief detour with the relevant literature. Section 3 presents the structural form of the three specifications. Section 4 computes the steady-state equilibrium. Section 5 discusses the solution of dynamic general equilibrium models under static and dynamic indeterminacy. Sections 6 and 7 present the estimation exercise and the observed policies of the Keynesian Search models. Section 8 concludes.

II. Relation with Previous Literature

Our paper relates to several research programs in macroeconomics. First, it relates to studies on the causes and consequences of the Great Depression in the U.S, such as Keynes (1936), Friedman and Schwartz (1963), Bernanke (1983) and Cole and Ohanian (2004), by providing evidence that rescue and rationalizes two important ideas of the *General Theory*: "animal spirits" and involuntary unemployment.

Second, it connects with search and matching theory in environments with multiple equilibria and hysteresis. Because Keynesian Search theory displays a continuum of non-stochastic steady-state unemployment rates, it dispenses the idea of a natural rate of unemployment and takes distance from most of the RBC and New-Keynesian literature.[6]

Third, it provides empirical evidence of the agenda proposed originally in Farmer (1999) and surveyed in Benhabib and Farmer (1999) where beliefs (sunspot, sentiments or self-fulfilling prophecies) act as an independent driver of the business cycle.[7]

Fourth, it connects with the empirical literature that uses Bayesian techniques to estimate DSGE models using historical and recent data.[8]

[5] Because wage rigidity is difficult to square with the observed data, the standard New-Keynesian model is not included in the comparison. Other studies such as Cole and Ohanian (2004) focus on competition and institutional designs like the New Deal cartelization policies and are not included in this exercise either.

[6] See Kaplan and Menzio (2016), Michaillat and Saez (2015, 2019) and Fernández-Villaverde et al. (2019) for models with multiple unemployment rates. See Blanchard and Summers (1986, 1992), Farmer (2012, 2013), Plotnikov (2019), and Cerra, Fatás and Saxena (2020) for models that display hysteresis.

[7] Some recent empirical studies using sunspot shocks in DSGE models are Bhattarai, Lee and Park (2016), Borağan Aruoba, Cuba-Borda and Schorfheide (2018), Cuba-Borda and Singh (2020).

[8] See for instance Lubik and Schorfheide (2004), Farmer and Nicolò (2018), Nicolò (2018),

The present work is part of a body of literature that provides theory and evidence in favour of demand-side explanations for high and persistent unemployment rates following stressful economic events such as recessions and macroeconomic crises. It also presents a framework to prescribe realistic policies to restore full employment.

III. Structural Form of RBC and Keynesian Search Models

This section describes the structural models mentioned in the Introduction. All the specifications have a common structure composed of seven economic relations that are standard in most of the business cycle literature. However, each of these models is closed in a way that involves an alternative interpretation of how the economy works, and therefore, each model would suggest a different type of policy intervention.

We consider three alternative specifications: a standard **RBC** model and two Keynesian Search models. In the first Keynesian Search model (**KS1**), aggregate demand is determined by an exogenous investment expenditure stochastic process. In the second Keynesian Search model (**KS2**) instead, aggregate demand is driven by consumer confidence and the latter is modelled through an exogenous time-invariant forecasting rule about the future value of the stock market. In other words, in the Keynesian Search models, aggregate demand is determined by the self-fulfilling beliefs of market participants.

The canonical Keynesian Search model has been studied in a series of papers over the last decade.[9] Therefore, we stress here the two main differences between the basic business cycle model with leisure: the indeterminacy of the steady state and the role of beliefs in shaping macroeconomic outcomes.

A. *Steady-State Indeterminacy and the Role of Beliefs*

Standard neoclassical models such as the RBC describe the labour market as an auction where the Walrasian auctioneer can costlessly match those individuals who want a job with those looking to fill vacancies by proposing a wage to clear the labour market. If the desired trades are not satisfied (meaning the process whereby unemployed individuals find a vacancy), the auctioneer adjusts the proposed wage until the aggregate demand of labour equals the aggregate supply of labour. Therefore, absent frictions, there is no involuntary unemployment in this process.

The limitations to modelling unemployment as an involuntary phenomenon in neoclassical models has been addressed, among other theories, by the search and

Bianchi and Nicolò (2019) and Goren and Platonov (2021) for models with dynamic indeterminacy estimated with Bayesian techniques. The studies in the previous footnote are also estimated using these tools.

[9] See Farmer (2012), Farmer (2013), Plotnikov (2019), Farmer (2020), Farmer (2021).

matching theory.[10]

The key insight of search models is to assume away the Walrasian auctioneer that coordinates activity in neoclassical models. In contrast, Search models are non-Walrasian in assuming that trading is costly (whether these be time or monetary costs) and uses social resources. In addition, search theory assumes that the technology for matching unemployed workers with vacant job openings cannot be decentralized by markets (Farmer, 2021).

From long ago, it has been shown that search models may display multiple finite equilibria (Diamond, 1982, 1984) and under some conditions that a continuum of non-stochastic steady-state unemployment rates may exist (Howitt and McAfee, 1987).

This type of indeterminacy has been solved in most of the literature by assuming that workers and firms are not price takers. Instead, when there is a match, workers and firms bargain a wage where both the fundamentals of the economy and the relative negotiation power of each party interact. This selection mechanism, referred to as *Nash bargaining*, provides the missing equation to pin down a unique unemployment rate as a function of the fundamentals of the economy.[11]

In contrast, Keynesian Search exploits the state-indeterminacy appealing to independent and non-fundamental forces (beliefs) that represent different forms of market psychologies. In this setting, firms are price takers and they employ enough workers to produce the quantities demanded by consumers. The demand for goods depends on consumers' wealth, which is determined by the future value of their assets. Therefore, it is aggregate demand, through expectations about future wealth, that determines the steady-state unemployment rate.

When a model is closed in this way, equilibrium uniqueness is restored and every sequence of shocks is associated with a unique sequence of values for the endogenous variables. If beliefs about future wealth or the future value of the assets market are random, so does the unemployment rate. By exploiting this modelling device, Keynesian Search theory offers a framework able to replicate the low-frequency movements seen in macroeconomic data and specifically high and persistent unemployment rates.

Caveat: Observational equivalence. — This paper uses model estimation and comparison techniques to discriminate between determinate and indeterminate specifications. However, as discussed in Beyer and Farmer (2007), the distinction between determinate and indeterminate models is usually based on untestable restrictions about the dynamic structure of an economic model. Given a determinate model, Beyer and Farmer (2007) show that it is possible to construct families of indeterminate models that produce the same likelihood function as in the determinate case.

[10] A classic survey is Rogerson, Shimer and Wright (2005).
[11] The Search literature has studied several other selection mechanisms that we do not discuss here.

For example, it is possible to construct a determinate model that is observationally equivalent to the Keynesian Search model by using a Nash bargaining equation where the bargaining weights are time-varying. As discussed in Lubik and Schorfheide (2004), a weakness in testing determinate and indeterminate structural models is that model misspecification may bias posteriors distributions toward indeterminacy.

This is not to say that the comparison is useless, but to recognize the implicit identification assumptions behind model specification. How plausible is that the Great Depression or the Great Recession were explained by sudden changes in the relative bargaining power of workers and firms? Would an exogenous drop in confidence be a more plausible explanation? The answers to these questions complete the set of restrictions among specifications. In our case, we explore the latter.

B. Common Equations Across Models

This subsection describes the common economic relations across the three specifications explained before.

The core structural relations in this paper relate to a standard representative agent *RBC* model with labour augmenting technology such as King, Plosser and Rebelo (1988) and King and Rebelo (1999).

Technology is characterized by a Cobb-Douglas production function that uses capital, K_{t-1}, and labor, L_t, as inputs

$$Y_t = K_{t-1}^{\alpha}(\Gamma_t L_t)^{1-\alpha}$$

where $\alpha \in (0,1)$ represents the capital share of output. The parameter Γ_t represents the cumulative product of "growth" shocks and $\Gamma_t L_t$ are effective labor units. In particular,

$$\Gamma_t = e^{\gamma_t}\Gamma_{t-1} = \prod_{s=0}^{t} e^{\gamma_s}$$

and,

$$\gamma_t = \rho_\gamma \gamma_{t-1} + (1-\rho_\gamma)\mu_\gamma + \varepsilon_t^{\gamma}$$

where $\rho_\gamma < 1$ and ε_t^{γ} represents *i.i.d* draws from a normal distribution with zero mean and standard deviation σ_γ. The term μ_γ represents productivity's long-run mean growth rate. We refer to the realizations of γ as the growth shocks as they constitute the stochastic trend of productivity.

Because a realization of γ permanently influences Γ, output is non-stationary with a stochastic trend and both consumption and investment are cointegrated with output.

For any generic variable $X_t \in \mathbb{R}$ we can define its detrended counterpart as:

$$x_t = \frac{X_t}{\Gamma_t}$$

Denote the detrended variables with lowercase letters: y_t is output, c_t is consumption, i_t is investment, k_t is capital, g_t is (exogenous) government spending, and $1-\ell_t$ is the unemployment rate. The economy starts out with a capital stock $K_0 > 0$ and a level of productivity trend $\Gamma_0 > 0$.

The representative household maximizes the expected present value of a time-separable utility function discounted by its time preference, β:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta\, \mathcal{U}(c_t, \ell_t)$$

The symbol $\mathbb{E}_0$ denotes the expectation of future values of c and ℓ based on the information available at time zero. Individual are endowed with their time, which is normalized to unity. Period utility has an iso-elastic structure consistent with a balanced growth path:

$$\mathcal{U}(c_t, \ell_t) = \begin{cases} log(c_t) + \psi\ log(1-\ell_t) & \text{if } \varphi = 1 \\ \frac{\left(c_t(1-\ell_t)^{\psi}\right)^{1-\varphi}-1}{1-\varphi} & \text{if } \varphi \neq 1 \end{cases}$$

Where φ is the coefficient of risk aversion and ψ represents the preference weight of leisure in utility.

By solving the utility maximization problem of the representative agent, the detrended version of the equilibrium conditions in this economy is partially characterized by the following set of equations:

- Goods Market Clearing:

$$y_t = c_t + i_t + g_t \tag{1}$$

- Production Function:

$$y_t = \left(\frac{k_{t-1}}{e^{\gamma_t}}\right)^{\alpha} \ell_t^{1-\alpha} \tag{2}$$

- Law of Motion of Capital:

$$k_t = (1-\delta)\left(\frac{k_{t-1}}{e^{\gamma_t}}\right) + i_t \tag{3}$$

- Consumption Euler Equation:

$$(4)\qquad \frac{1}{c_t} = \beta\ \mathbb{E}_t\left[\frac{1}{e^{\gamma_{t+1}}c_{t+1}}\left(1-\delta+\alpha\left(e^{\gamma_{t+1}}\frac{y_{t+1}}{k_t}\right)\right)\right] \qquad \text{if } \varphi = 1$$

$$(4')\qquad \left(\frac{1-\ell_t}{1-\ell_{t+1}}\right)^{\psi(1-\varphi)} = \beta\ \mathbb{E}_t\left[\left(\frac{c_t}{e^{\gamma_{t+1}}c_{t+1}}\right)^{\varphi} R_t\right] \qquad \text{if } \varphi \neq 1$$

where $R_t = 1-\delta+\alpha\mathbb{E}_t\left[e^{\gamma_{t+1}}\frac{y_{t+1}}{k_t}\right]$ is the real interest rate between dates t and $t+1$.

It is important to note than in the Keynesian search models (***KS1*** and ***KS2***) employment is demand determined. This assumption collapses into $\psi = 0$ for this class of models.

- Productivity Growth process:

$$(5)\qquad \gamma_t = \rho_\gamma\gamma_{t-1} + (1-\rho_\gamma)\mu_\gamma + \varepsilon_t^\gamma$$

- Government Spending process:

$$(6)\qquad g_t = \rho_g g_{t-1} + (1-\rho_g)\mu_g + \varepsilon_t^g$$

Where δ is the depreciation rate of capital, μ_g is the long-run mean of government spending, ρ_g is the persistence of this process, and ε_t^g represents $i.i.d$ draws from a normal distribution with zero mean and standard deviation σ_g.

We incorporate the concept of the asset market using the following representation. Let V_t be the value of the corporate sector of the economy (or aggregate assets). V_t has two components: debt, B_t, and equity, Z_t. If we denote by λ the ratio between debt and the value of the corporate sector ($\lambda = \frac{B_t}{S_t}$), then the stock market can be defined as $Z_t = (1-\lambda)S_t$.

The value of the corporate sector, V_t, is defined to be the discounted present value of future returns to capital:

$$V_t = \mathbb{E}_t\left[\alpha Y_t + \frac{V_{t+1}}{R_t}\right]$$

Multiply both sides of this relation by $(1-\lambda)$ and use the definition of the stock market, Z_t:

$$Z_t = \alpha(1-\lambda)Y_t + \mathbb{E}_t\left[\frac{Z_{t+1}}{R_t}\right]$$

Then, our last common relation is the detrended version of the above equation, namely:

- Stock Market Value:

$$z_t = \alpha(1-\lambda)y_t + \mathbb{E}_t\left[e^{\gamma_{t+1}}\frac{z_{t+1}}{R_t}\right] \tag{7}$$

Equations (1) through (7) define the core relations of the three models analyzed: **RBC**, **KS1**, and **KS2**.

C. Different equations across models

Relations (1) through (7) do not completely characterize an economy. Each of the three models presented before is closed with a different equation that emphasizes alternative economic interpretations of what drives the business cycle.

Real Business Cycle. — The RBC model is closed with an optimal labor-leisure relation that solves a unique unemployment rate. This is derived from the optimality conditions of the representative household:[12]

- Labour-leisure Condition:

$$\psi\frac{c_t}{(1-\ell_t)} = (1-\alpha)\left(\frac{k_{t-1}}{e^{\gamma_t}}\right)^{\alpha}\ell_t^{-\alpha} \tag{8}$$

Keynesian Search 1. — The KS1 model is closed with an exogenous investment process that pins downs the unemployment rate through its effects on aggregate demand:

- Investment Process:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\mu_i + \varepsilon_t^i \tag{9}$$

where μ_i is the long-run mean of investment, ρ_i is the persistence of the process, and ε_t^i represents $i.i.d$ draws from a normal distribution with zero mean and standard deviation σ_i.

[12] Note that this condition is equivalent whether $\varphi = 1$ or $\varphi \neq 1$.

Keynesian Search 2. — The KS2 model, in turn, is completed with two equations: a) an expectation formation equation or *belief function* about the future value of the stock market, and b) a consistency condition that guarantees rational expectations:[13]

- Belief Function:

$$z_t^b = \rho_b z_{t-1}^b + (1-\rho_b)\mu_z + \varepsilon_t^b \tag{10}$$

- Rational Expectations condition:

$$z_t^b = \mathbb{E}_t\left[z_{t+1}\right] \tag{11}$$

Beliefs about the future value of the stock market are defined by z_t^b. The variable μ_z is the long-run value of the stock market and ρ_b measures how important are previous beliefs relative to their average value. The variable ε_t^b is a non-fundamental (sunspot) *i.i.d* shock drawn from a normal distribution with zero mean, standard deviation σ_b, and is uncorrelated with the innovations to the other fundamental shocks.

It is important to note that expectations are exogenous and driven by a backward-looking relation as shown in equation (11). If agents use this forecasting rule in every period, and if their current beliefs about future prices are functions of the current sunspot shock, those beliefs will be validated in a rational expectations equilibrium. Moreover, because of time invariability in its parameters, this rule is immune to the Lucas Critique (Farmer, 1999, 2021).

D. *Where is Search and Matching?*

A natural question at this point when looking at the complete models is where are specified the functions and parameters associated with an incomplete factor model such as search and matching.

As shown in Farmer (2021), if we compute the steady-state GDP as a function of employment for two different technologies with standard calibrations values; one with and one without search costs, these two functions are almost indistinguishable for levels of employment ranging from 0% to 90%.[14]

The similarity between these two technologies except at very high levels of capacity utilization implies that total employment (for producing and recruiting) is approximately equal to employment used to produce goods. We exploit this similarity by working directly with total employment and we abstract from the matching function.[15]

[13] The belief function is a mapping from current and past observable variables to probability distributions over future economic variables.

[14] See Figure 4 in Farmer (2021).

[15] See also Farmer (2012, 2013) for a formal derivation of the model without reproducible capital and

IV. Steady-State Properties across Models

In this subsection, we compute the steady state of the three specifications. As mentioned before, in the *RBC* model the non-stochastic steady-state unemployment rate is uniquely determined by the labour-leisure optimality condition. In contrast, both Keynesian Search models display static indeterminacy.

Define the steady-state values as lower-case letters *without* the time subscript, t. To solve the steady-state define the "big-ratios" common to each model as follows:

A. Common "Big-ratios"

From the Euler equation, we can derive the following expressions according to the definition of the utility function.

For $\varphi = 1$:

$$\frac{k}{y} = e^{\mu_\gamma}\left(\frac{\alpha}{\frac{e^{\mu_\gamma}}{\beta} - (1-\delta)}\right) \tag{12}$$

and

$$R = \frac{e^{\mu_\gamma}}{\beta} \tag{13}$$

while for $\varphi \neq 1$:

$$\frac{k}{y} = e^{\mu_\gamma}\left(\frac{\alpha}{\frac{e^{\mu_\gamma \cdot \varphi}}{\beta} - (1-\delta)}\right) \tag{12'}$$

and

$$R = \frac{e^{\mu_\gamma \cdot \varphi}}{\beta} \tag{13'}$$

In both cases, the two expressions derived from the Euler equation depend only on the structural parameters $\theta \in \Theta$. Note that the parameter φ appears in (12') and (13'). This structural ratio appears in the next expressions as well.

Plotnikov (2019) for the case with reproducible capital.

From the production function, equation (2):

$$\frac{\ell}{y} = \left(e^{\mu_\gamma}\frac{y}{k}\right)^{\frac{\alpha}{1-\alpha}} \tag{14}$$

and using (12) and (13),

$$\frac{k}{\ell} = e^{\mu_\gamma}\left(e^{\mu_\gamma}\frac{y}{k}\right)^{\frac{1}{\alpha-1}} \tag{15}$$

From the low of motion of capital, equation (1):

$$\frac{i}{y} = \left(\frac{1-(1-\delta)}{e^{\mu_\gamma}}\right)\frac{k}{y} \tag{16}$$

From the market clearing condition, equation (1):

$$\frac{c}{y} = 1 - \frac{i}{y} - \frac{g}{y} \tag{17}$$

Finally, from the stock market value relation, equation (7):

$$\frac{z}{y} = \frac{(1-\lambda)\alpha}{1-\frac{e^{\mu_\gamma}}{R}} \tag{18}$$

Equations (12) through (18) complete the common steady-state relations across models.

B. Real Business Cycle

Using the labour-leisure condition, equation (8), and the assumption that time is normalized to unity:

$$\frac{1-\ell}{\ell} = \left(\frac{\psi}{1-\alpha}\right)\frac{c}{y} \tag{19}$$

Thus,

$$\ell^{RBC} = \frac{1}{1 + \left(\frac{\psi}{1-\alpha}\right)\frac{c}{y}} \tag{20}$$

Replacing the "big ratios" in equation (20), the steady-state employment rate, ℓ^{RBC}, can be expressed in terms of structural parameters only. As shown in (20), the RBC model has a unique steady-state unemployment rate, $1 - \ell^{RBC}$.

C. Keynesian Search 1

In the Keynesian Search 1 model, the unemployment rate, $1 - \ell^{KS1}$, is determined by aggregate demand.

Aggregate demand, in turn, is determined by investment, which follows an exogenous process as in equation (9). Therefore, output is computed as:

$$\bar{y}^{KS1} \equiv i\ \frac{y}{i} = \mu_i\ \frac{y}{i} \tag{21}$$

Where the ratio, $\frac{y}{i}$, from equation (16), depends only on structural parameters and so does $\bar{y}^{KS1}$. Using equation (14), the steady-state labour supply in the Keynesian Search model 1 is:

$$\ell^{KS1} = \left(e^{\mu_\gamma}\frac{y}{k}\right)^{\frac{\alpha}{1-\alpha}}\ \bar{y}^{KS1} \tag{22}$$

This expression states that there is a continuum of non-stochastic steady-state unemployment rates, $1 - \ell^{KS1}$, each of them determined by the unconditional mean of investment expenditure, μ_i.

D. Keynesian Search 2

The Keynesian Search 2 model follows a similar logic. Aggregate demand is determined by beliefs about the future value of the asset market. Beliefs follow the exogenous process described by equation (10). Therefore, output is computed as:

$$\bar{y}^{KS2} \equiv z\ \frac{y}{z} = \mu_z\ \frac{y}{z} \tag{23}$$

Where the ratio, $\frac{y}{z}$, from equation (18) is expressed in terms of structural parameters only and so does $\bar{y}^{KS2}$.

Using equation (14), the non-stochastic steady-state labour supply under this specification is:

$$\ell^{KS2} = \left(e^{\mu_\gamma} \frac{y}{k}\right)^{\frac{\alpha}{1-\alpha}} \bar{y}^{KS2} \tag{24}$$

As before, the Keynesian Search 2 model displays a continuum of non-stochastic steady-state unemployment rates, $1-\ell^{KS2}$, determined by the unconditional mean of the stock market value, μ_z.

Note that because there are two definitions of the ratio $\frac{y}{k}$ depending on whether $\varphi = 1$ or $\varphi \neq 1$, the unemployment rates will differ in these two cases.

V. Model Solution and Steady State Properties

This section presents the methods to solve for the reduced form of a linear rational expectations (LRE) model. In addition, it shows the econometric interpretation of the model and its steady-state properties.

A. Reduced Form

Following Sims (2002), consider the general class of linear rational expectation (LRE) models,

$$\Gamma_0(\theta)X_t = C + \Gamma_1(\theta)X_{t-1} + \Psi(\theta)\epsilon_t + \Pi(\theta)\eta_t \tag{25}$$

where $X_t \in \mathbb{R}^n$ is a vector of endogenous variables that may or may not be observable. Matrices $\Gamma_0(\theta)$, $\Gamma_1(\theta)$, $\Psi(\theta)$, and $\Pi(\theta)$ collect the coefficients from the dynamic systems described in the previous section. The vector $\theta \equiv vec\left(\Gamma_0, \Gamma_1, \Psi, \Pi, \Omega_{\epsilon\epsilon}\right) \in \Theta$ contains the structural parameters of the model as well as the variance-covariance matrix of the exogenous shocks.[16] The variable ϵ_t is a vector of exogenous shocks, referred to as "fundamental", and η_t collects the one-step ahead forecast errors for the expectational variables of the system (also referred to as "non-fundamental" shocks).

Also, we impose,

$$\mathbb{E}_{t-1}(\epsilon_t) = 0$$

$$\mathbb{E}_{t-1}(\eta_t) = 0$$

[16] Where $\Gamma_0(\theta)$ and $\Gamma_1(\theta)$ are possibly singular.

$$\Omega_{\epsilon_t \epsilon_t} \equiv \mathbb{E}_{t-1}(\epsilon_t \epsilon_t')$$

Using this form, each model is identified by a superscript label with its respective abbreviation: RBC, $KS1$, $KS2$. Therefore, using the structure in (25), each model can be expressed accordingly as:

- Real Business Cycle (RBC): Equations (1)-(7) and (8).

$$(26)\quad X_t^{RBC} = \begin{bmatrix} y_t \\ c_t \\ i_t \\ g_t \\ \ell_t \\ k_t \\ \gamma_t \\ z_t \\ \mathbb{E}_t(y_{t+1}) \\ \mathbb{E}_t(c_{t+1}) \\ \mathbb{E}_t(\gamma_{t+1}) \\ \mathbb{E}_t(z_{t+1}) \end{bmatrix} ; \quad \epsilon_t^{RBC} = \begin{bmatrix} \varepsilon_t^g \\ \varepsilon_t^\gamma \end{bmatrix} ; \quad \eta_t^{RBC} = \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{bmatrix} = \begin{bmatrix} y_t - \mathbb{E}_{t-1}(y_t) \\ c_t - \mathbb{E}_{t-1}(c_t) \\ \gamma_t - \mathbb{E}_{t-1}(\gamma_t) \\ z_t - \mathbb{E}_{t-1}(z_t) \end{bmatrix}$$

The RBC has four expectational variables and two exogenous fundamental shocks: one to government spending, ε_t^g, and one to productivity, ε_t^γ.

- Keynesian Search 1 ($KS1$): Equations (1)-(7) and (9).

$$(27)\quad X_t^{KS1} = \begin{bmatrix} y_t \\ c_t \\ i_t \\ g_t \\ \ell_t \\ k_t \\ \gamma_t \\ z_t \\ \mathbb{E}_t(y_{t+1}) \\ \mathbb{E}_t(c_{t+1}) \\ \mathbb{E}_t(\gamma_{t+1}) \\ \mathbb{E}_t(z_{t+1}) \end{bmatrix} ; \quad \epsilon_t^{KS1} = \begin{bmatrix} \varepsilon_t^g \\ \varepsilon_t^\gamma \\ \varepsilon_t^i \end{bmatrix} ; \quad \eta_t^{KS1} = \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{bmatrix} = \begin{bmatrix} y_t - \mathbb{E}_{t-1}(y_t) \\ c_t - \mathbb{E}_{t-1}(c_t) \\ \gamma_t - \mathbb{E}_{t-1}(\gamma_t) \\ z_t - \mathbb{E}_{t-1}(z_t) \end{bmatrix}$$

The $KS1$ has four expectational variables and three exogenous fundamental shocks: on top of the government spending and the productivity shocks (ε_t^g and ε_t^γ, respectively), investment is driven by an additional exogenous shock, ε_t^i. Investment is therefore driven by exogenous beliefs.

- Keynesian Search 2 ($KS2$): Equations (1)-(7) and (10)-(11).

$$(28)\quad X_t^{KS2} = \begin{bmatrix} y_t \\ c_t \\ i_t \\ g_t \\ \ell_t \\ k_t \\ \gamma_t \\ z_t \\ \mathbb{E}_t(y_{t+1}) \\ \mathbb{E}_t(c_{t+1}) \\ \mathbb{E}_t(\gamma_{t+1}) \\ \mathbb{E}_t(z_{t+1}) \\ z_t^b \end{bmatrix} \;;\quad \epsilon_t^{KS2} = \begin{bmatrix} \varepsilon_t^g \\ \varepsilon_t^\gamma \\ \varepsilon_t^b \end{bmatrix} \;;\quad \eta_t^{KS2} = \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{bmatrix} = \begin{bmatrix} y_t - \mathbb{E}_{t-1}(y_t) \\ c_t - \mathbb{E}_{t-1}(c_t) \\ \gamma_t - \mathbb{E}_{t-1}(\gamma_t) \\ z_t - \mathbb{E}_{t-1}(z_t) \end{bmatrix}$$

The $KS2$ model is more involved since it displays *dynamic indeterminacy* as well. This model considers two additional equations, (10) and (11). It has four expectational variables and three shocks: two standard fundamental shocks, ε_t^g, and ε_t^γ, and a new redefined fundamental shock, ε_t^b. The first two are common to all models. The third one, ε_t^b, is a belief (sunspot) shock that perturbs the forecasting rule in equation (10) and is potentially correlated with the other fundamental shocks.

Algorithms to solve linear rational expectation models with indeterminacy have been explored in Lubik and Schorfheide (2003), Farmer, Khramov and Nicolò (2015), and Nicolò (2018) and Bianchi and Nicolò (2019). These solution methods rely on an expansion of the vector of fundamental shocks, ϵ_t, with a subset of the non-fundamental shocks, η_t, in order to satisfy the Blanchard-Khan conditions for boundedness and uniqueness of equilibrium. In the case of $KS2$, the vector ϵ_t^{KS2} includes the belief shock, ε_t^b.

This solution to indeterminacy considers the value of the stock market, z_t, as a predetermined variable, thus its contemporaneous deviations are only due to the belief (sunspot) shock. This means that equation (7) is lagged one period when representing the dynamic system in (25).

Finally, the parameters of the variance-covariance matrix of expanded fundamental shocks in the dynamically indeterminate model, $\Omega_{\epsilon_t^{KS2}\,\epsilon_t^{KS2}}$, are considered new fundamentals and as such, they may be calibrated or estimated in the same way as the parameters of a utility or the production function (Farmer and Nicolò, 2018).

Following Sims (2002), we use the generalized Schur decomposition to solve the LRE model in equation (25) for each specification. The reduced form of (25), popularized by the MATLAB code `GENSYS`, follows a VAR of the form,

$$X_t = \hat{C} + G_0(\theta)X_{t-1} + G_1(\theta)\epsilon_t \tag{29}$$

such that all stochastic sequences $\{X_t\}_{t=1}^{\infty}$ generated by this equation also satisfy the structural model in (25). The algorithm eliminates unstable generalized eigenvalues of the matrices $\Gamma_0(\theta)$ and $\Gamma_1(\theta)$ by finding expressions for the non-fundamental shocks, η_t, as functions of the fundamental shocks, ϵ_t.

When there are too few unstable generalized eigenvalues relative to the number of non-predetermined (or free) variables, there are multiple equilibria or dynamic indeterminacy. This is the case for $KS2$.

To solve and estimate the three models, we use an implementation of `GENSYS` programmed in `DYNARE` (Adjemian et al., 2011). First, for each specification, we find the reduced form of the structural model in (25). Second, we use Bayesian techniques for the estimation. Specifically, we construct a state-space representation of the model with a common set of measurement equations. We use the Kalman filter to generate the likelihood function and standard Markov Chain Monte Carlo (MCMC) algorithms to sample the posterior distribution of the model.

VI. Model estimation

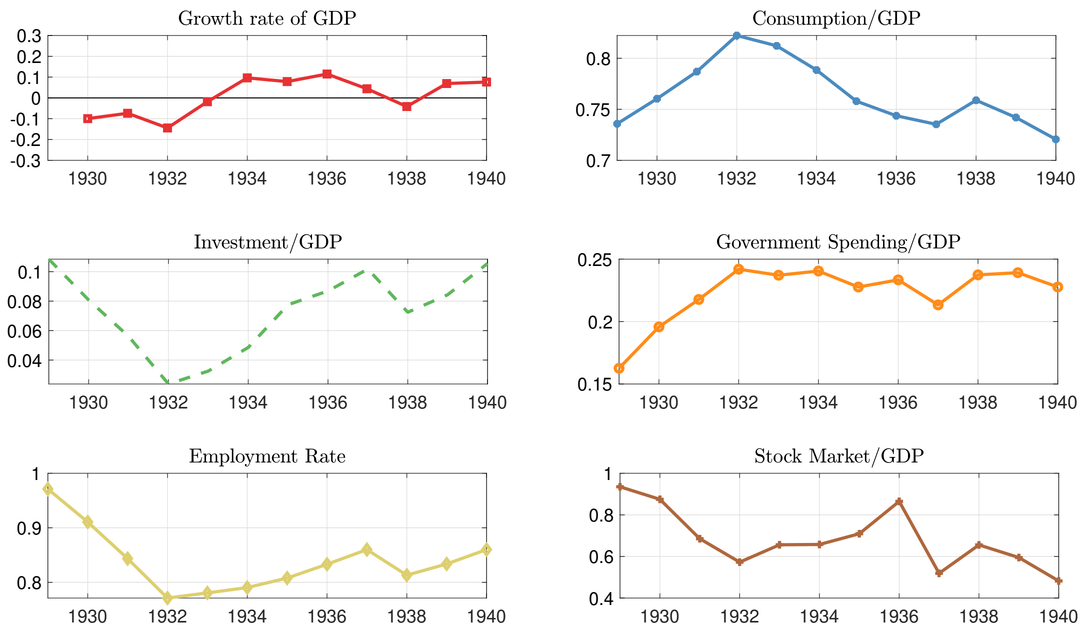
A. Data

In this section, we estimate the three specifications described above: RBC, $KS1$, and $KS2$.

Each model is estimated with annual data during the Great Depression in the United States for the period from 1929 to 1940. Figure 1 plots the data used for the estimation exercise. The data is produced by the Bureau of Economic Analysis (BEA) and includes annual data from the National Income and Product Accounts (NIPA). The main macroeconomic variables from NIPA are: output (Y_t), consumption (C_t), investment (I_t), and government spending, (G_t). Because we are modelling a closed economy, total consumption is defined as $CT_t = C_t + X_t - M_t$, where CT_t is total consumption, X_t is exports, and M_t represents imports. All variables are in real terms. (See the note in Figure 1 for definitions).

The other two relevant variables for this study, namely, the unemployment rate, (U_t), and the stock market value to GDP ratio (SY_t), are taken from two different sources. The unemployment rate is taken from Ramey and Zubairy (2018) and the stock market to GDP ratio is computed in Kuvshinov and Zimmermann (2020). The data is plotted in Figure 1.

Figure 1. : Macroeconomic data during the Great Depression in the U.S. (1929-1940)



Note: Figure 1 shows the macroeconomic variables used for model estimation and comparison. $\boldsymbol{Y_t}$ **:** is real gross domestic product and $\boldsymbol{\Delta\%Y_t}$ **:** is real GDP per capita growth. $\boldsymbol{CT_t}$ **:** is real personal consumption expenditures plus real exports minus real imports; $\boldsymbol{I_t}$ **:** is real gross private domestic investment; and $\boldsymbol{G_t}$ **:** is real government consumption expenditures and gross investment. These series are computed by the BEA. $\boldsymbol{U_t}$ **:** is the annual unemployment rate and is computed as the average of quarterly unemployment rates calculated in Ramey and Zubairy (2018); $\boldsymbol{SY_t}$ **:** represents the stock market capitalisation to GDP constructed with historical records by Kuvshinov and Zimmermann (2020).

In Figure 1 four facts are worth mentioning. First, the tremendous impact of the Great Depression induced negative growth rates until 1933, with a trough in 1932, and then a recovery until 1937. Second, this pattern is followed closely by the employment rate. The diamond line shows the dramatic increase in unemployment through 1932 and then a sluggish recovery with high and persistent unemployment rates until the beginning of World War II. Third, we can see the dramatic reduction in the stock market capitalisation to GDP ratio after 1929. Although it recovered between 1932 and 1936, the stock market continued persistently below the level it had before the crash. Finally, we can see that investment as a fraction of GDP consistently similarly fell until 1932 as the employment rate. The recovery in investment was weak, reaching its previous level 8 years after the crash. The comovement of the investment to GDP ratio and the employment rate will turn out important for our empirical results.

B. *Bayesian Estimation*

Measurement Equation. — The set of measurement equations that relates the macroeconomic data to the endogenous variables is defined as:

$$X_t^{obs} \equiv \begin{bmatrix} \Delta log\left(Y_t^{pc}\right) \\ log\left(CT_t\right) - log\left(Y_t\right) \\ log\left(I_t\right) - log\left(Y_t\right) \\ log\left(G_t\right) - log\left(Y_t\right) \\ log\left(1 - U_t\right) \\ log\left(SY_t\right) \end{bmatrix} = \begin{bmatrix} \Delta log\left(y_t\right) + \gamma_t \\ log\left(c_t\right) - log\left(y_t\right) \\ log\left(i_t\right) - log\left(y_t\right) \\ log\left(g_t\right) - log\left(y_t\right) \\ log\left(\ell_t\right) \\ log\left(z_t\right) - log\left(y_t\right) \end{bmatrix} + \omega_t \tag{30}$$

Where capital letters in the vector X_t^{obs} represent the observed variables described in the previous subsection and lowercase letters are the endogenous variables of the models described in Section 3 (Y_t^{pc} denotes GDP per capita). Each equation has a measurement error that is collected in the vector ω_t. These are $i.i.d$ and are normally distributed.

The state-space representation consists of the dynamic system in (25) augmented with the measurement equation (30). Note that stationarity is preserved through "big ratios" rather than by filtering or taking differences in the observed data (with the exception of the growth rate of GDP).

Prior Distributions. — Table 1 summarizes the prior distribution of common parameters across models. The table reports the prior shape, mean, mode, standard deviation and 90% probability interval for each parameter. The parameter for the ratio between debt and the value of the corporate sector, λ, is originally calibrated to 0.81, to make the steady-state value of the stock market to GDP equal to 68%, which is the average during the period studied. In each Keynesian Search model, the unconditional mean for the exogenous process, μ_i and μ_z for investment and the value of the stock market, respectively, is calibrated to make the steady-state unemployment rate equal to the average of the period studied, i.e, 15%. The same logic applies to the preference weight for leisure,psi, in the case of the RBC model. In order to facilitate the estimation, the capital share, α is fixed at 30%. The rest of the parameters follow standard values with loose prior distributions.

In the case of the $KS2$ model, we identified the variance-covariance matrix of shocks, $\Omega_{\epsilon_t^{KS2}\ \epsilon_t^{KS2}}$, by setting the covariance of ε_t^b with the other fundamental shocks to zero.

Table 1—: Prior distributions (Common parameters across models)

Parameter	Range	Distribution	Mean	Mode	Std. dev.	90% Interval Lower	90% Interval Upper
λ	$[0,1)$	Beta	0.7000	0.7049	0.0500	0.6150	0.7795
δ	$[0,1)$	Beta	0.0300	0.0165	0.0200	0.0059	0.0686
β	$[0,1)$	Beta	0.9500	0.9578	0.0200	0.9133	0.9780
σ_γ	$\mathbb{R}^+$	Inv. Gamma	0.2000	0.1450	0.1000	0.1055	0.3729
σ_g	$\mathbb{R}^+$	Inv. Gamma	0.2000	0.1038	0.3000	0.0736	0.4742
ρ_γ	$[0,1)$	Beta	0.5000	0.5000	0.2000	0.1718	0.8282
ρ_g	$[0,1)$	Beta	0.5000	0.5000	0.2000	0.1718	0.8282
$log(\mu_\gamma)$	$\mathbb{R}$	Gaussian	0.0100	0.0100	0.2000	-0.3190	0.3389
$log(\mu_g)$	$\mathbb{R}$	Gaussian	-1.3110	-1.3110	0.2000	-1.6400	-0.9820

Note: Table 1 shows the prior distribution for parameters that are common across models. These parameters are described in equations (1) through (7).

Table 2—: Prior distributions (Different parameters across models)

Parameter	Range	Distribution	Mean	Mode	Std. dev.	90% Interval Lower	90% Interval Upper
			RBC: Labour-leisure				
ψ	$\mathbb{R}^+$	Gamma	0.1500	0.0833	0.1000	0.0307	0.3429
			KS1: Investment is exogenous				
σ_i	$\mathbb{R}^+$	Inv. Gamma	0.2000	0.1038	0.3000	0.0736	0.4742
ρ_i	$[0,1)$	Beta	0.5000	0.5000	0.2000	0.1718	0.8282
$log(\mu_i)$	$\mathbb{R}$	Gaussian	-2.1611	-2.1611	0.2000	-2.4901	-1.8321
			KS2: Beliefs are exogenous				
σ_b	$\mathbb{R}^+$	Inv. Gamma	0.5000	0.3904	0.2000	0.2886	0.8615
ρ_b	$[0,1)$	Beta	0.7000	0.7222	0.1000	0.5242	0.8525
$log(\mu_z)$	$\mathbb{R}$	Gaussian	-0.2877	-0.2877	0.9000	-1.7681	1.1927

Note: Table 2 shows the prior distribution for parameters that differ across models. **RBC:** is the Real Business Cycle model and its additional parameter is in equation (8). **KS1:** is the Keynesian Search model where investment is exogenous. Its additional parameters are in equation (9). **KS2:** is the Keynesian Search model where beliefs about the value of the stock market are exogenous. Its additional parameters are in equation (10).

C. Results

Table 3 reports the posterior distribution for each parameter in each specification for $\varphi = 1$, i.e., for the separable logarithmic utility function. We can see how parameter values might change considerably when we compare the model with a unique non-stochastic steady-state against the models that display steady-state indeterminacy

For instance, when we compare posterior means we can see that the RBC model estimates a greater mean for the volatility of the fundamental shocks σ_γ, σ_g but it favours lower persistence parameters, ρ_γ and ρ_g, for these processes.

In contrast, the Keynesian Search models favour greater persistence for the fundamental shocks, especially for the growth of productivity. Regarding the beliefs shocks, the $KS1$ assigns a greater persistence to the investment process than the $KS2$ assigns to the belief formation process. This lack of persistence in the $KS2$ model relative to the $KS1$ might explain the better fit of the latter specification. The rest of the parameters show similar magnitudes in each specification and are in line with common calibrations and estimations in other studies.

Table 3—: Posterior estimates with logarithmic preferences

	RBC			**KS1**			**KS2**		
Parameter	Mean	90% CI		Mean	90% CI		Mean	90% CI	
λ	0.767	0.7094	0.8179	0.847	0.8146	0.8794	0.874	0.8408	0.907
δ	0.027	0.0087	0.0439	0.019	0.0034	0.0319	0.009	0.0008	0.0165
β	0.890	0.8654	0.9158	0.926	0.9119	0.9416	0.935	0.9220	0.9470
σ_γ	0.148	0.0930	0.2034	0.122	0.0831	0.1580	0.117	0.0816	0.1520
σ_g	0.145	0.0957	0.1914	0.074	0.0464	0.0981	0.077	0.0501	0.1049
ρ_γ	0.374	0.2138	0.5128	0.978	0.9628	0.9942	0.997	0.9935	0.9995
ρ_g	0.408	0.1597	0.6419	0.568	0.2671	0.8858	0.611	0.2997	0.9426
$log(\mu_\gamma)$	0.025	0.0019	0.0480	0.018	0.0016	0.0346	0.008	-0.0017	0.0187
$log(\mu_g)$	-1.278	-1.3987	-1.1603	-1.242	-1.3652	-1.1176	-0.992	-1.0862	-0.8881
ψ	0.177	0.1565	0.1989	-	-	-	-	-	-
σ_i	-	-	-	0.392	0.2730	0.5115	-	-	-
ρ_i	-	-	-	0.502	0.3100	0.7110	-	-	-
$log(\mu_i)$	-	-	-	-2.279	-2.4236	-2.1295	-	-	-
σ_b	-	-	-	-	-	-	0.350	0.2459	0.4492
ρ_b	-	-	-	-	-	-	0.232	0.0500	0.4000
$log(\mu_z)$	-	-	-	-	-	-	-0.042	-0.1286	0.0444

Note: Table 3 shows the posterior mean and the 90% credible intervals (Highest Posterior Density) for the model parameters in each specification when $\varphi = 1$, i.e, separable logarithmic utility function. The log-likelihood function is estimated using the Kalman filter. The optimizer for the mode computation in the $KS2$ model uses a Monte-Carlo based routine (`mode_compute=6` in `DYNARE`). For the RBC and $KS1$ specifications we use Christopher Sims's `csminwel` to reach an acceptance rate closer to the usual standard for multivariate normal distribution (23%). Parameter estimates are virtually equal when using the Monte-Carlo based routine. The posterior kernel is simulated using the Random Walk Metropolis-Hastings sampling-like method. Each estimation consists of 40,000 draws and two chains. The unit root in $KS2$ is handled with the diffuse Kalman filter (Adjemian et al., 2011).

Table 4 shows the key results of the paper through model comparison. It reports the logarithm of the marginal data densities using the corresponding posterior model probabilities under the assumption that each model has equal prior probability. These were computed using the modified harmonic mean estimator proposed by Geweke (1999).

Table 4 suggests that among the three models, RBC, $KS1$, and $KS2$, the posterior model probability is 100% in favor of the $KS1$ over the other two specifications. Appendix A.A2 shows that these results are robust to more general preferences as CRRA.

If the Great Depression in the U.S is better represented in data by the $KS1$ specification, where investor's beliefs are exogenous and determined the unemployment rate, then a natural policy prescription is to focus efforts either in restore investors confidence or to compensate for the reduction in private investment with public investment.

Table 4—: Model Comparison

	RBC	**KS1**	**KS2**
Log marginal data density	-217.070968	-16.339786	-102.695149
Posterior model probability (%)	0	100	0

Note: Table 4 shows the results of the model comparison. The marginal density of the data conditional on the model is approximated using the Modified Harmonic Mean Estimator developed in Geweke (1999). We assume equal probability for each specification.

VII. Observed Policy

This section analyses the impulse response function of the model most favoured by data: the Keynesian Search model with exogenous investment ($KS1$).[17]

Figure 2 plots the Bayesian impulse response functions for the real interest rate, output, consumption, labour, and the stock market value in response to a *pessimistic* belief shock to investment expenditure. The x-axis measures periods in years. The y-axis shows the percentage deviation from the steady state. The shaded region represents 90% credible intervals.

A pessimistic belief shock is very much in line with the behaviour of the investment share to GDP plotted in Figure 1 (green dashed line) and is consistent with the notion of undermined confidence or depressed "animal spirits" in the aftermath of the Great Depression.

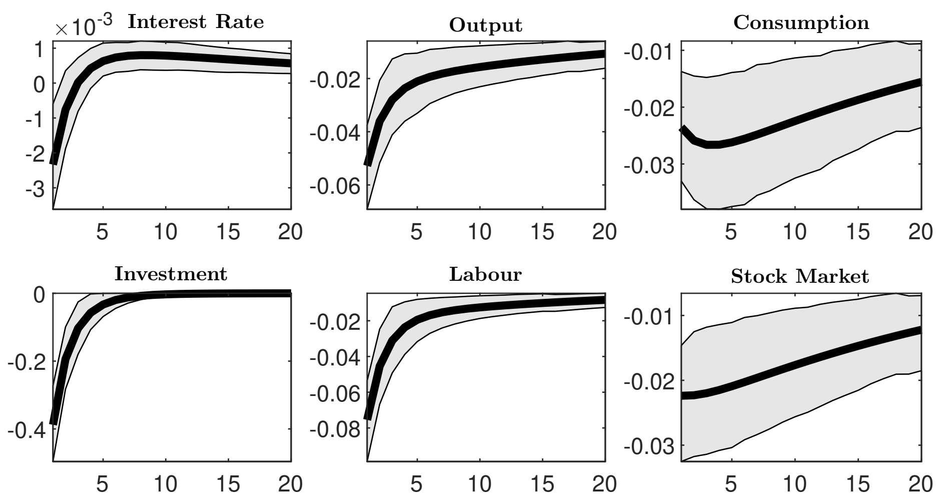
A negative shock to investment will hurt output on impact and reduce the real interest rate. Undermined investor confidence translates into lower production

[17] For completeness, we develop this same exercise for $KS2$ in Appendix A.A1.

and therefore lower labour demand that persists for roughly a decade according to the estimated parameters.

Because investment is exogenous, the reduction in the real interest will not translate into more capital accumulation. In fact, the reduction in capital accumulation has long-lasting effects on production and consumption, the latter taking more than 15 years in reaching its steady-state level. The stock market instead, defined as the present discounted value of future returns to capital, is negatively hit on impact, and similarly to the rest of the variables, takes some additional years to recover even when the confidence is restored, or in other words, until the shock dissipates.

Figure 2. : Bayesian Impulse Response Functions to Investment

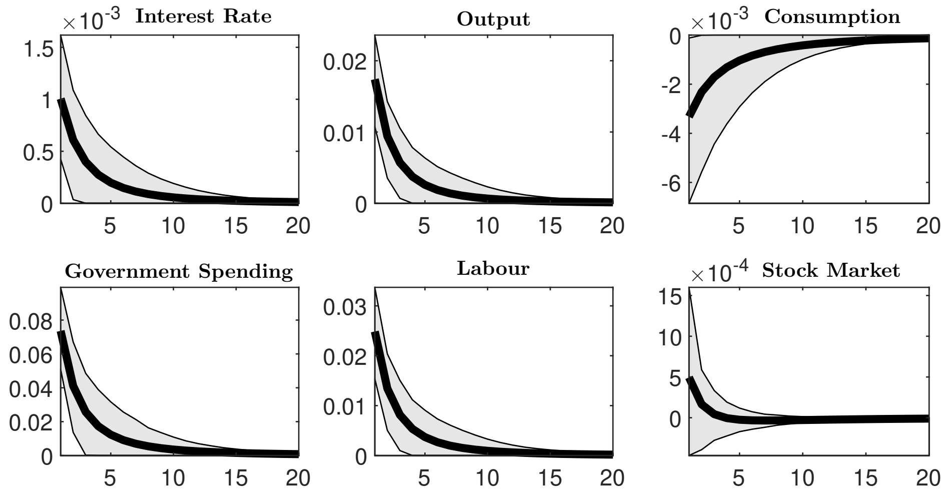


Note: Figure 2 plots the Bayesian impulse response functions for the real interest rate, output, consumption, labour, and the stock market value to a *negative* investment shock. The x-axis measures periods in years. The y-axis shows the percentage deviation from steady-state. The shaded region represents 90% credible intervals.

This comparative static exercise is very much in line with the persistence seen in macroeconomic data of the Great Depression in the U.S. Obviously, this model is not able to explain all the facts around this period of time, however, it provides a clear intuition about the persistent effects that low investor confidence or "animal spirits" can produce in the economy.

Figure 3 plots the Bayesian impulse response functions for the real interest rate, output, consumption, labour, and the stock market value in response to a positive shock to government spending.

Figure 3. : Bayesian Impulse Response Functions to Government Spending



Note: Figure 3 plots the Bayesian impulse response functions for the real interest rate, output, consumption, labour, and the stock market value to a *positive* government spending shock. The x-axis measures periods in years. The y-axis shows the percentage deviation from the steady state. The shaded region represents 90% credible intervals.

An increase in government spending makes output rise on impact. Higher aggregate demand requires more labour, which also increases at the time of the shock. In response, rational and forward-looking agents reduce consumption until the impulse of new spending vanishes. It is important to note that in this case, investment follows an exogenous and independent stochastic process. Therefore, the increase in the real interest will not affect the investment rate.

The stock market, defined as the present value of future capital returns, does not significantly increase on impact. The effect of government spending on employment quickly disappears as the one-time shock dissipates.

In terms of policy, this estimation exercise implies that restoring confidence in investors by replacing private investment with public investment has a more persistent effect than standard fiscal policy through government spending.

The conclusion is not to say that fiscal policy is useless, but that are other factors as important as fiscal policy that need to be considered when designing institutional arrays to deal with the business cycles.

VIII. Conclusions

Disastrous economic events such as the Great Depression and the Great Recession pose critical challenges to dynamic general equilibrium models with unique and bounded rational expectations in explaining high and persistent unemployment rates.

We provide an alternative explanation for the facts observed during the Great Depression in the U.S.: the Keynesian Search theory. This approach exploits both steady-state and dynamic indeterminacy to explain low-frequency movements in the unemployment rate. We solve these indeterminacies by specifying how agents form expectations about the future.

Using Bayesian estimation techniques, we find that the Keynesian Search specifications outperform the standard RBC model in our sample period. Among the two Keynesian Search models proposed, data favour the specification where aggregate demand is driven by an exogenous investment process.

This suggests that in terms of policy prescription an effective way to restore full employment is by replacing private investment with public investment.

This study provides evidence in favour of demand-side explanations for the facts seen during the Great Depression in the U.S.

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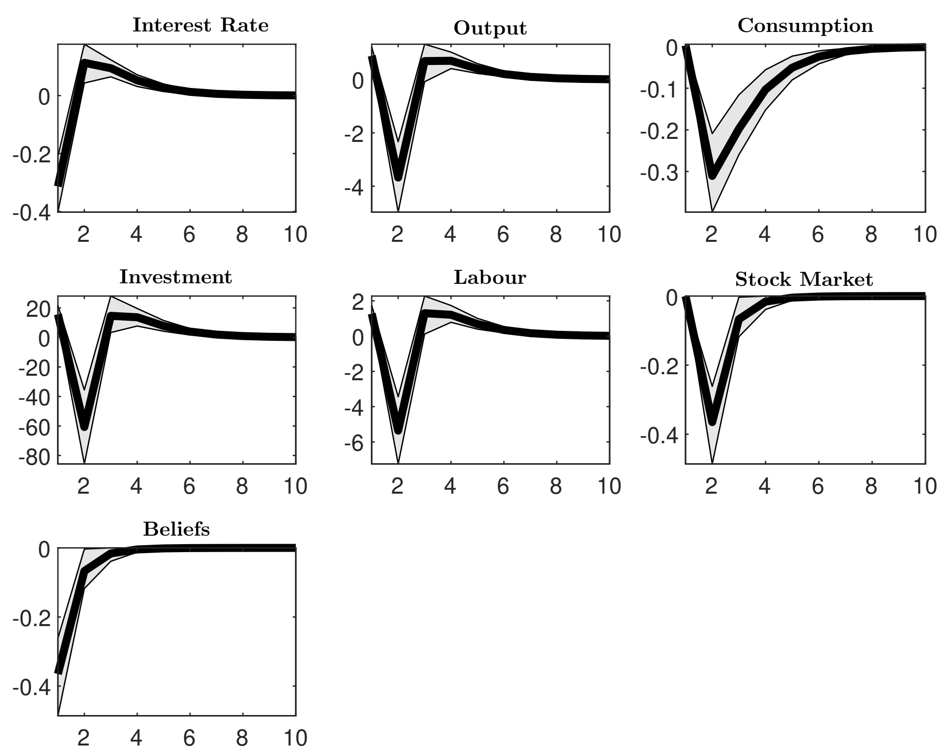
Appendix

A1. Observed policy for Keynesian Search 2

As in Section 5, we analyse the impulse response function of the Keynesian Search model where beliefs about the future value of the stock market are exogenous and stochastic (i.e, $KS2$). In line with the previous exercise, we model the recession as a sudden drop in consumer confidence that expresses itself as a reduction in the present value of future capital income. This replicates in some sense the idea of the "Black Friday" on October 25th, 1929, which is arguably the most devastating stock market crash in the history of the U.S.

Figure A1. : Bayesian Impulse Response Functions to a Belief Shock

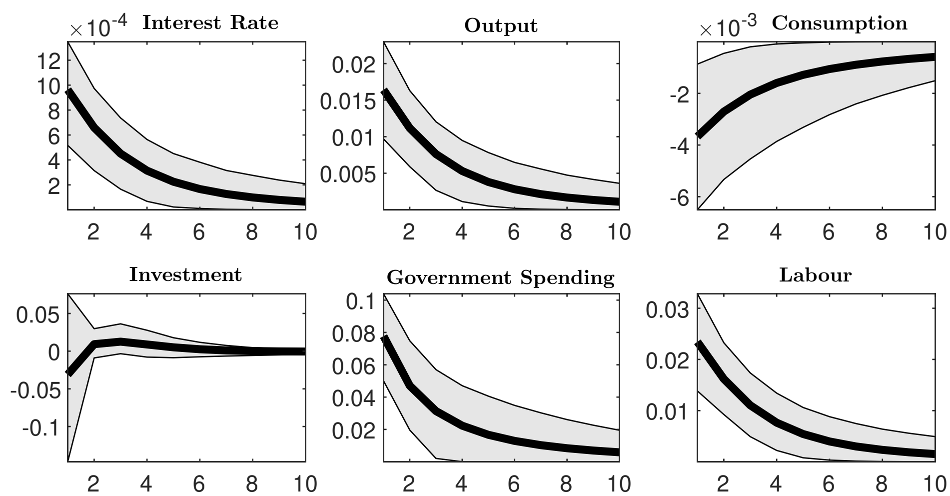


Note: Figure A1 plots the Bayesian impulse response functions for the real interest rate, output, consumption, labour, and the stock market value to a *negative* belief shock (zb) about the future value of the stock market. The x-axis measures periods in years. The y-axis shows the percentage deviation from the steady state. The shaded region represents 90% credible intervals.

Figure A1, shows that a temporary loss in consumer confidence has strong negative effects on consumption, investment, and output one period after the shock hits the economy. Because in this model aggregate demand determines the unemployment rate, the sharp reduction in output is mirrored by the unemployment rate.

However, different from the 1929 crash, the estimated parameters produce a very deep but short recession. After roughly four periods, most variables sharply recover, some of them overshoot the previous level, but then they converge back to their steady-state levels.

Figure A2. : Bayesian Impulse Response Functions to a Government Spending Shock



Note: Figure A2 plots the Bayesian impulse response functions for the real interest rate, output, consumption, labour, and the stock market value to a *positive* government spending shock. The x-axis measures periods in years. The y-axis shows the percentage deviation from the steady state. The shaded region represents 90% credible intervals.

In contrast, we can see in Figure A2 that government spending has more persistent effects on the real interest rate, output, and employment rate. These effects are still temporary, but more persistent than in the previous case. Is likely this lack of persistence that favours the Keynesian Search model where investment is exogenous rather than when beliefs about the future value of the stock market are.

If this were the model most favoured by data, the impulse response functions in figures A1 and A2 would suggest that a more effective way to restore full employment would be closer to fiscal policy rather than active management of the asset market during the Great Depression in the U.S.

A2. Results with CRRA preferences

The results from section 6 are robust to CRRA preferences. We replicate the previous exercise for this class of preferences without constraining the coefficient of risk aversion to be unity.

Table A1 summarizes the prior distribution of common parameters across models. This is the same as Table 1 but with the additional prior for the coefficient of risk aversion, φ.

Table A1—: Prior distributions (Common parameters across models)

						90% Interval	
Parameter	Range	Distribution	Mean	Mode	Std. dev.	Lower	Upper
λ	$[0,1)$	Beta	0.7000	0.7049	0.0500	0.6150	0.7795
δ	$[0,1)$	Beta	0.0300	0.0165	0.0200	0.0059	0.0686
β	$[0,1)$	Beta	0.9500	0.9578	0.0200	0.9133	0.9780
σ_γ	$\mathbb{R}^+$	Inv. Gamma	0.2000	0.1450	0.1000	0.1055	0.3729
σ_g	$\mathbb{R}^+$	Inv. Gamma	0.2000	0.1038	0.3000	0.0736	0.4742
ρ_γ	$[0,1)$	Beta	0.5000	0.5000	0.2000	0.1718	0.8282
ρ_g	$[0,1)$	Beta	0.5000	0.5000	0.2000	0.1718	0.8282
$log(\mu_\gamma)$	$\mathbb{R}$	Gaussian	0.0100	0.0100	0.2000	-0.3190	0.3389
$log(\mu_g)$	$\mathbb{R}$	Gaussian	-1.3110	-1.3110	0.2000	-1.6400	-0.9820
φ	$\mathbb{R}^+$	Gamma	3.5000	3.4286	0.5000	2.7201	4.3610

Note: Table A1 shows the prior distribution for parameters that are common across models. These parameters are described in equations (1) through (7). Table A1 replicates Table 1 but appends the prior distribution of the coefficient of risk aversion, φ.

Prior distribution of parameters that differ across models are the same as in Table 2, therefore are not reproduced here.

In the same way, Table A2 replicates the posterior estimates of Table 3 for the specification with CRRA preferences.

Table A3 confirms the key results of the paper. As before, it reports the logarithm of the marginal data densities using the corresponding posterior model probabilities under the assumption that each model has equal prior probability. These were computed using the modified harmonic mean estimator proposed by Geweke (1999).

Figure A3 suggests that among the three models, RBC, $KS1$, and $KS2$, the posterior model probability is 100% in favor of the $KS1$ over the other two specifications.

Table A2—: Posterior estimates with CRRA preferences

	RBC			**KS1**			**KS2**		
Parameter	Mean	90% CI		Mean	90% CI		Mean	90% CI	
λ	0.791	0.7478	0.8397	0.776	0.7244	0.823	0.535	0.4588	0.6143
δ	0.039	0.0265	0.0531	0.038	0.0275	0.0498	0.029	0.0130	0.0470
β	0.937	0.9044	0.9697	0.929	0.8992	0.9562	0.966	0.9363	0.9952
σ_γ	0.086	0.0614	0.1104	0.117	0.0761	0.1561	0.104	0.0708	0.1364
σ_g	0.102	0.0680	0.1379	0.075	0.0503	0.1006	0.074	0.0469	0.1002
ρ_γ	0.926	0.8726	0.9856	0.447	0.2106	0.6727	0.561	0.4070	0.7090
ρ_g	0.291	0.0491	0.5225	0.518	0.2220	0.8088	0.525	0.2105	0.8391
$log(\mu_\gamma)$	0.025	0.0035	0.0432	0.021	0.0073	0.0356	0.072	0.0442	0.0967
$log(\mu_g)$	-1.3038	-1.4010	-1.1854	-1.365	-1.4757	-1.2451	-1.408	-1.5005	-1.3102
ψ	0.178	0.1555	0.1967	-	-	-	-	-	-
σ_i	-	-	-	0.384	0.2691	0.4818	-	-	-
ρ_i	-	-	-	0.616	0.4704	0.7795	-	-	-
$log(\mu_i)$	-	-	-	-2.550	-2.7350	-2.3828	-	-	-
σ_b	-	-	-	-	-	-	0.250	0.1842	0.3222
ρ_b	-	-	-	-	-	-	0.850	0.7390	0.9611
$log(\mu_z)$	-	-	-	-	-	-	-0.940	-1.2793	-0.6226
φ	3.309	2.3996	4.0859	4.255	3.6591	4.8475	3.185	2.7080	3.7283

Note: Table A2 shows the posterior mean and the 90% credible intervals (Highest Posterior Density) for the model parameters in each specification when $\varphi \neq 1$, i.e, CRRA utility function. The log-likelihood function is estimated using the Kalman filter. The optimizer for the mode computation in the $KS2$ model uses a Monte-Carlo based routine (`mode_compute=6` in `DYNARE`). For the RBC and $KS1$ specifications we use Christopher Sims's `csminwel` to reach an acceptance rate closer to the usual standard for multivariate normal distribution (23%). Parameter estimates are virtually equal when using the Monte-Carlo based routine. The posterior kernel is simulated using the Random Walk Metropolis-Hastings sampling-like method. Each estimation consists of 40,000 draws and two chains. The unit root in $KS2$ is handled with the diffuse Kalman filter (Adjemian et al., 2011).

Table A3—: Model Comparison with CRRA Preferences

	RBC	**KS1**	**KS2**
Log marginal data density	-40.441925	-13.481287	-27.954074
Posterior model probability (%)	0	100	0

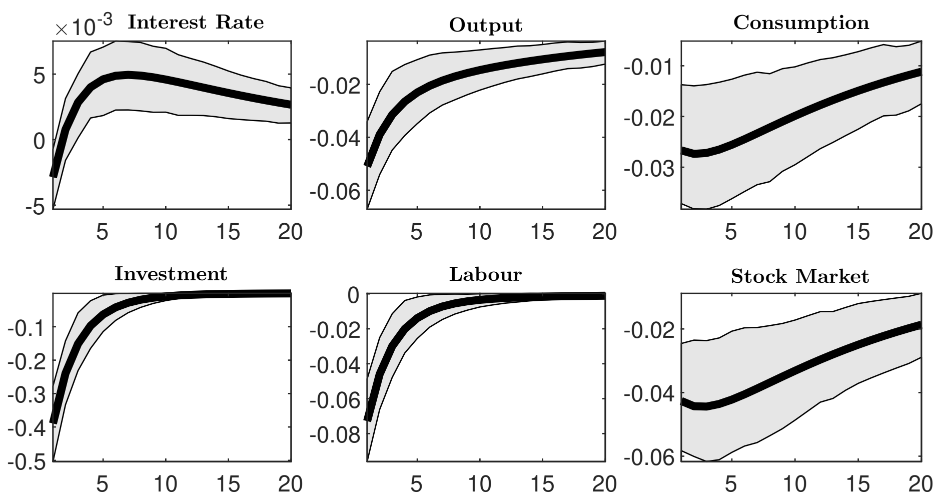
Note: Table A3 shows the results of the model comparison with CRRA preferences. The marginal density of the data conditional on the model is approximated using the Modified Harmonic Mean Estimator developed in Geweke (1999). We assume equal probability for each specification.

A3. Observed policy for Keynesian Search models

In this subsection, we replicate the previous exercise with the parameter estimation from the models with CRRA utility functions. The results are very similar to the previous section. However, in the case of KS2, the impulse response functions show smoother behaviour and higher persistence than in its logarithmic counterpart.

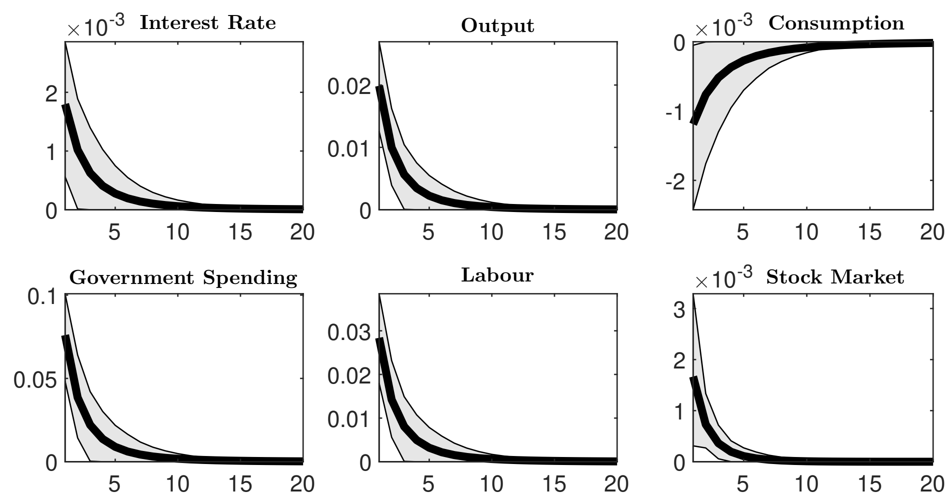
Keynesian Search 1 (*KS1*). —

Figure A3. : Bayesian Impulse Response Functions to Investment



Note: Figure A3 plots the Bayesian impulse response functions for the real interest rate, output, consumption, labour, and the stock market value to a *negative* investment shock. The x-axis measures periods in years. The y-axis shows the percentage deviation from the steady state. The shaded region represents 90% credible intervals.

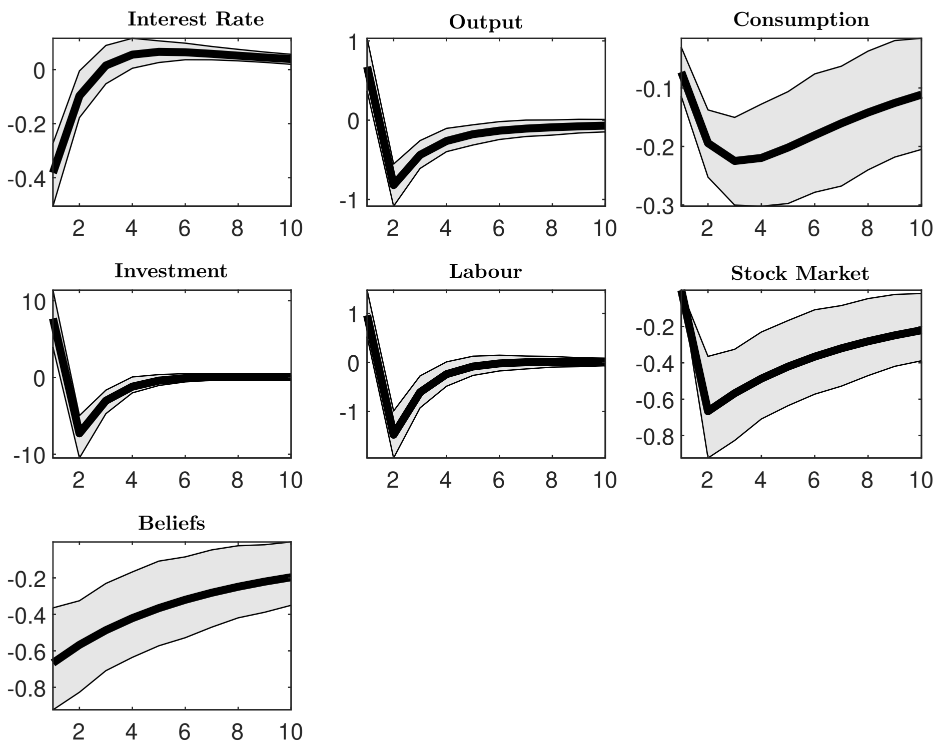
Figure A4. : Bayesian Impulse Response Functions to Government Spending



Note: Figure A4 plots the Bayesian impulse response functions for the real interest rate, output, consumption, labour, and the stock market value to a *positive* government spending shock. The x-axis measures periods in years. The y-axis shows the percentage deviation from the steady state. The shaded region represents 90% credible intervals.

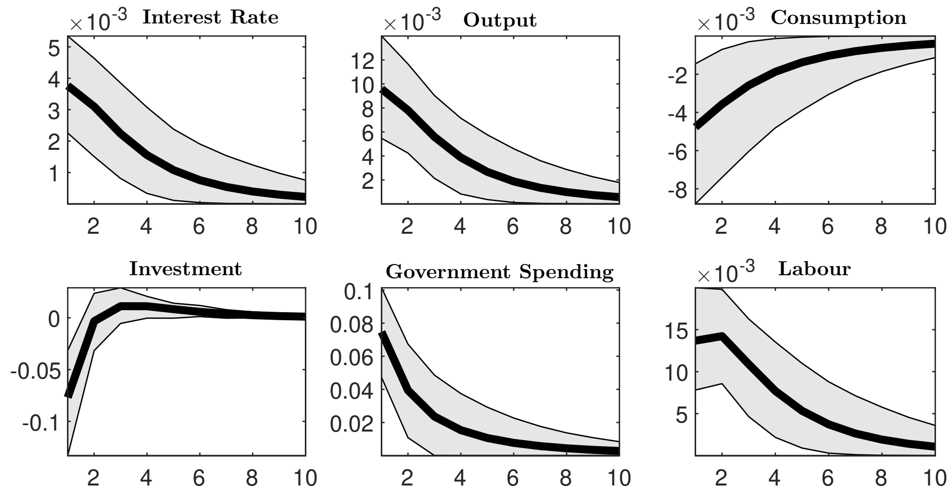
Keynesian Search 2 (*KS2*). —

Figure A5. : Bayesian Impulse Response Functions to a Belief Shock



Note: Figure A5 plots the Bayesian impulse response functions for the real interest rate, output, consumption, labour, and the stock market value to a *negative* belief shock (zb) about the future value of the stock market. The x-axis measures periods in years. The y-axis shows the percentage deviation from the steady state. The shaded region represents 90% credible intervals.

Figure A6. : Bayesian Impulse Response Functions to a Government Spending Shock



Note: Figure A6 plots the Bayesian impulse response functions for the real interest rate, output, consumption, labour, and the stock market value to a *positive* government spending shock. The x-axis measures periods in years. The y-axis shows the percentage deviation from the steady state. The shaded region represents 90% credible intervals.