

Trade Costs and the Open Macroeconomy*

Dennis Novy

University of Warwick, Department of Economics, Coventry CV4 7AL, United Kingdom

d.novy@warwick.ac.uk

Abstract

Trade costs are known to be a major obstacle to international economic integration. Following the approach of New Open Economy Macroeconomics, this paper explores the effects of international trade costs in a micro-founded general equilibrium model that allows for different degrees of exchange rate pass-through. Trade costs are shown to create an endogenous home bias in consumption and the model performs well in matching empirical trade shares for OECD countries. In addition, trade costs reduce cross-country output and consumption correlations and they magnify exchange rate volatility. Trade costs turn a monetary expansion into a beggar-thy-neighbor policy.

Keywords: Trade Frictions; Pass-Through; Exchange Rates; Consumption Correlations; International Trade

JEL classification: F12; F31; F41

I. Introduction

Trade costs have long been known as a major obstacle to international economic integration. In a recent survey, James Anderson and Eric van Wincoop (2004) show that empirical trade costs are large even when formal barriers to trade do not exist. They argue that the tariff equivalent of representative international trade costs is around 74 percent. This paper puts trade costs in the spotlight by integrating them into a rigorous micro-founded general equilibrium model.

Following the approach of New Open Economy Macroeconomics, I introduce iceberg trade costs into a two-country open economy model with nominal price rigidities. Although I allow for asymmetric country sizes, due to logarithmic utility the complexity of the model is kept at a manageable level so that analytical solutions can be derived for the asymmetric equilibrium. The combination of trade costs and asymmetric country sizes allows me to yield

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trade shares that match empirically observed trade shares for OECD countries.

The paper shows that even moderate trade costs can substantially interfere with international linkages. For example, they easily reduce the cross-country correlations of consumption by more than half compared to a frictionless world. Those lower levels match empirically observed consumption correlations. Intuitively, by raising the price of imported goods trade costs render domestic goods more attractive to consumers. As a consequence, spending is predominantly kept within the domestic country and consumption is tilted towards domestic goods, creating an endogenous home bias in consumption. This *containment effect* of trade costs tends to isolate two countries from each other and makes them behave more like closed economies. Shocks hitting one country therefore have a reduced bearing on the other, weakening current account movements and the international correlations of consumption and output. However, as a result of weaker international linkages, nominal and real exchange rates respond more strongly to shocks. This helps to explain the high exchange rate volatility observed in the data.

An additional feature of the model is to allow for varying degrees of exchange rate pass-through. This extension, adopted from Caroline Betts and Michael Devereux (2000), is particularly suitable in the context of trade costs. As Andrew Atkeson and Ariel Burstein (2008) argue, lack of pass-through implies that prices for the same good can differ across countries. Such deviations from the law of one price require market segmentation. This can be justified by trade costs.

Betts and Devereux (2000) show that in a world with zero trade costs, lack of pass-through dramatically reduces cross-country consumption correlations. In contrast, I find that in the presence of trade of costs, the behavior of key economic variables does not heavily depend on the degree of pass-through. For example, for plausible trade cost values cross-country consumption correlations are rather similar in the two opposite cases of local currency pricing (i.e. zero pass-through) and producer currency pricing (i.e. full pass-through). Likewise, if trade costs are present the distinction between local and producer currency pricing is no longer as sharp for welfare considerations. Regardless of the degree of pass-through trade costs turn a monetary expansion into a beggar-thy-neighbor policy.

The model falls into the New Open Economy Macroeconomics literature that has evolved from Maurice Obstfeld and Kenneth Rogoff's (1995) seminal contribution. It represents a micro-founded two-country general equilibrium framework with monopolistic competition and one-period price stickiness. The key contribution of my paper is to combine this set-up with iceberg trade costs as the central modelling device. As a consequence of trade costs, many conclusions from Obstfeld and Rogoff's (1995) paper no longer hold. For example consumption is no longer highly correlated across countries and positive monetary shocks no

longer lead to symmetric welfare gains in both countries. In addition, I extend the model to productivity shocks. The results are qualitatively similar to monetary shocks in that trade costs dampen international linkages and increase exchange rate volatility.¹

Paul Krugman (1980) is the first author to introduce iceberg trade costs into a monopolistic competition framework but my model is more closely related to the paper by Obstfeld and Rogoff (2000). They have given trade costs new impetus by pointing out their potential to elucidate major puzzles of international macroeconomics like the consumption correlations puzzle. But Obstfeld and Rogoff (2000) only use a small open endowment economy model, as do Paul Bergin and Reuven Glick (2006) who introduce heterogeneous iceberg trade costs and endogenous tradability. Allan Brunner and Kanda Naknoi (2003) integrate trade costs into a more rigorous two-country general equilibrium model with production but they only allow for full pass-through and do not consider welfare implications.

The inclusion of trade costs yields results that are in many respects similar to the ones obtained by David Backus and Gregor Smith (1993) and Harald Hau (2000) in their models with nontradable goods. Hau (2000) also finds that consumption becomes less correlated across countries and that both nominal and real exchange rates are more volatile. I incorporate nontradable goods into my model but I find that in contrast to trade costs, nontradable goods would clearly overpredict the extent of international trade.² In order to match empirical trade shares, an implausibly large share of over 90 percent of goods would have to be nontradable. The reason is that nontradability simply reduces the range of goods available to consumers without changing relative prices. But trade costs drive a wedge between the prices of domestic and imported goods and, provided that demand is elastic, generate nonlinear shifts in expenditure patterns towards domestic goods.

Charles Engel (1999) and V. V. Chari, Patrick Kehoe and Ellen McGrattan (2002) provide an additional reason for choosing trade costs over nontradable goods. They show that the relative price of nontradable goods accounts for virtually none of U.S. and European real exchange rate movements. Instead, the real exchange rate appears to be driven almost exclusively by the relative price of tradable goods.³

Francis Warnock (2003) shows that the assumption of a home bias in preferences can generate results that are qualitatively similar to nontradable goods. However, empirical micro-evidence by Carolyn Evans (2007) shows that national preferences are negligible in

¹Most papers in this literature only consider nominal shocks, for example Svensson and van Wijnbergen (1989), Hau (2000) and Warnock (2003).

²This result is not as apparent in Hau's (2000) paper because he assumes countries to be symmetric.

³Naknoi (2008) presents a Ricardian trade model to explain the empirical observation that under fixed exchange rate regimes the nontraded real exchange rate is relatively more important. I abstract from fixed exchange rates.

explaining international trade flows relative to transportation costs and tariffs. Likewise, Helpman (1999) argues that there is no clear evidence of home bias in preferences. I therefore adopt trade costs to generate an *endogenous* home bias in consumption.

John Fender and Chong Yip (2000) consider a unilateral tariff but not symmetric trade costs. Ravn and Mazzenga (2004) evaluate the effect of transportation costs in a real business cycle approach with symmetric countries. In addition, they assume a home bias in preferences. But since transportation costs and home bias work in the same direction, it is difficult to establish the relative contribution of each assumption in explaining empirical trade shares and cross-country correlations. Another difference is that by construction, their flexible-price environment precludes the analysis of varying degrees of exchange rate pass-through.

The paper is structured as follows. Section II introduces trade costs into a New Open Economy Macroeconomics model with sticky prices. Section III describes its flexible-price equilibrium, establishing the endogenous home bias in consumption and matching the model with empirical trade shares. Section IV discusses the effects of monetary and productivity shocks on exchange rate volatility and international output and consumption correlations. Section V conducts a welfare analysis. Section VI concludes.

II. A Model with Trade Costs

The model follows the New Open Economy Macroeconomics literature and is based on the setting with varying degrees of exchange rate pass-through in Betts and Devereux (2000). As a new ingredient there exist exogenous iceberg trade costs τ , where τ represents the fraction of goods that melts away during the trading process with $0 \leq \tau < 1$. If $\tau = 0$ we have the special case of frictionless trade that is customary in the literature. In the extreme case of $\tau \rightarrow 1$, trade between the two countries breaks down and they become closed economies.

Households choose among a continuum $[0, 1]$ of differentiated, nondurable and tradable goods which are produced by monopolistic firms. The sizes of the Home and Foreign countries are n and $1 - n$ with $0 < n < 1$. As in Betts and Devereux (2000), it is assumed that s with $0 \leq s \leq 1$ is the fraction of firms in each country that engage in local currency pricing (LCP). The remaining firms engage in producer currency pricing (PCP).

Households

Households derive utility from consumption C_t and also from real money balances M_t/P_t due to a transactionary motive but they dislike work effort h_t . In Home country notation

utility is given by

$$U_t = \sum_{v=t}^{\infty} \beta^{v-t} \left(\ln C_v + \gamma \ln \left(\frac{M_v}{P_v} \right) + \eta \ln(1 - h_v) \right) \quad (1)$$

with the composite consumption index defined as

$$C_t \equiv \left(\int_0^1 c_{it}^{\left(\frac{\rho-1}{\rho}\right)} di \right)^{\frac{\rho}{\rho-1}} \quad (2)$$

where ρ is the elasticity of substitution with $\rho > 1$.⁴ c_{it} is consumption of good i at time t . β is the subjective discount factor with $0 < \beta < 1$. The parameters β , γ , η and ρ are positive and identical across countries. All above variables (C_t and h_t etc.) are Home per-capita variables. Since all households within one country are identical by construction, the corresponding Home aggregate quantities are nC_t and nh_t etc. Note that unlike in Warnock (2003) there is no home bias in preferences.

The Home consumption-based price index is defined as the minimum expenditure subject to $C_t = 1$ and can be derived as⁵

$$P_t = \left[\int_0^n p_{it}^{1-\rho} di + \int_n^{n+(1-n)s} \left(\frac{1}{1-\tau} p_{it}^* \right)^{1-\rho} di + \int_{n+(1-n)s}^1 \left(\frac{1}{1-\tau} e_t q_{it}^* \right)^{1-\rho} di \right]^{\frac{1}{1-\rho}} \quad (3)$$

The Foreign price index is given by

$$P_t^* = \left[\int_0^{ns} \left(\frac{1}{1-\tau} q_{it} \right)^{1-\rho} di + \int_{ns}^n \left(\frac{1}{1-\tau} \frac{1}{e_t} p_{it} \right)^{1-\rho} di + \int_n^1 q_{it}^{*1-\rho} di \right]^{\frac{1}{1-\rho}} \quad (4)$$

where prices p represent goods prices denominated in Home currency and prices q represent goods prices denominated in Foreign currency. In general, asterisks indicate Foreign country variables but in the context of goods prices an asterisk means that a price is *set* by a Foreign firm. Thus, all p_{it}^* are set by Foreign firms in Home currency and all q_{it}^* are set by Foreign firms in Foreign currency.

The goods in the range $[0, n]$ are produced by Home firms and the goods in the range $[n, 1]$ are produced by Foreign firms. In the Home price index (3), Foreign firms price in local

⁴As in Obstfeld and Rogoff (1995) and Betts and Devereux (2000), real money balances in the utility function could alternatively be specified as $\frac{\gamma}{1-\epsilon} \left(\frac{M_v}{P_v} \right)^{1-\epsilon}$ with $\epsilon > 0$. This specification would yield additional features such as exchange rate overshooting but those features are not essential to understand the role of trade costs. Results are available from the author on request.

⁵The derivations of this section are outlined in Appendix A.

currency for the goods in the range $[n, n + (1 - n)s]$, i.e. they set the corresponding prices p_{it}^* in Home currency. The range $[n + (1 - n)s, 1]$ represents the goods produced by Foreign firms that set prices q_{it}^* in producer (i.e. Foreign) currency. These are converted into Home currency through multiplying by the nominal exchange rate e_t , which is defined as the Home price of Foreign currency.

Note that the factor $\frac{1}{1-\tau}$ is included in the range $[n, 1]$ of Home price index (3) as well as in the range $[0, n]$ of Foreign price index (4). The reason is that all prices p_{it} , p_{it}^* , q_{it} , q_{it}^* are f.o.b. (free on board) unit prices that are charged at the factory gate. If a Foreign good is shipped to the Home country, only the fraction $(1 - \tau)$ arrives. The Home consumer must therefore buy $\frac{1}{1-\tau}$ units in the Foreign country so that one full unit arrives in the Home country. Hence, from a Home consumer's perspective $\frac{1}{1-\tau}p_{it}^*$ is the c.i.f. (cost, insurance and freight) unit price of a Foreign good priced in local currency, and $\frac{1}{1-\tau}e_tq_{it}^*$ is the c.i.f. unit price of a Foreign good priced in producer currency. One can think of this f.o.b./c.i.f. relationship as firms' charging an additional markup for shipping the purchased goods over to the destination country.⁶

Asset markets are complete domestically such that each household owns an equal share of an initial stock of domestic currency and an equal share of all domestic firms. There is no bond denominated in Foreign currency but there is free and costless trade in a Home currency nominal discount bond. F_t represents the Home holdings of the bond maturing in period $t + 1$ and d_t is its price. The Home budget constraint at time t in per-capita terms is thus given by

$$P_t C_t + M_t + d_t F_t = W_t h_t + \pi_t + M_{t-1} + Z_t + F_{t-1} \quad (5)$$

where W_t is the nominal wage rate and π_t are Home firms' profits. If the Home government generates revenue from printing money, it gives out nominal lump-sum transfers Z_t to its citizens such that

$$Z_t = M_t - M_{t-1} \quad (6)$$

The Home consumption demand function can be derived as

$$c_{it} = \left(\frac{\xi_{it}}{P_t} \right)^{-\rho} C_t \quad (7)$$

⁶However, the fraction τ of goods gets lost in the trading process so that firms do not receive the additional markup. The model can be extended such that trade costs are rebated to consumers but this does not change the qualitative results. Details are available from the author on request.

where

$$\xi_{it} = \begin{cases} p_{it} & \text{for } 0 \leq i \leq n \\ \frac{1}{1-\tau} p_{it}^* & \text{for } n \leq i \leq n + (1-n)s \\ \frac{1}{1-\tau} e_t q_{it}^* & \text{for } n + (1-n)s \leq i \leq 1 \end{cases} \quad (8)$$

analogous to the three terms in price index (3).

Firms

Each firm faces the same linear production technology

$$y_t = h_t \quad (9)$$

where y_t denotes Home per-capita output and h_t is Home per-capita labor input. The i subscript is dropped as all firms face the same production technology. Home output can be divided into output destined for the domestic country, denoted by x_t , and output destined for the Foreign country, denoted by z_t

$$y_t = x_t + z_t \quad (10)$$

Labor markets in each country are perfectly competitive so that the internationally immobile workers are wage-takers. The Home per-capita profit function for any $s \in [0, 1]$ is then given by

$$\pi_t = s(p_t x_t + e_t q_t z_t) + (1-s)(p_t x_t + p_t z_t) - W_t y_t \quad (11)$$

Note that (11) is expressed in f.o.b. terms and that z_t is the amount of Home output that is *shipped* to Foreign. Due to trade costs only the fraction $(1-\tau)$ of z_t arrives and is *consumed* in Foreign. The first term on the right-hand side of (11) reflects the revenue of firms that engage in local currency pricing, charging the Foreign currency price q_t to Foreign consumers. The second term is the revenue from firms that engage in producer currency pricing, charging the Home currency price p_t . The last term of (11) constitutes the costs of production.

Appendix A shows that profit maximization leads to the standard markups for Home firms

$$p_t = e_t q_t = \frac{\rho}{\rho-1} W_t \quad (12)$$

and for Foreign firms

$$q_t^* = p_t^*/e_t = \frac{\rho}{\rho-1} W_t^* \quad (13)$$

Equations (12) and (13) imply that in f.o.b. terms there is no price discrimination across countries under flexible prices. Firms receive the same revenue per unit, no matter whether

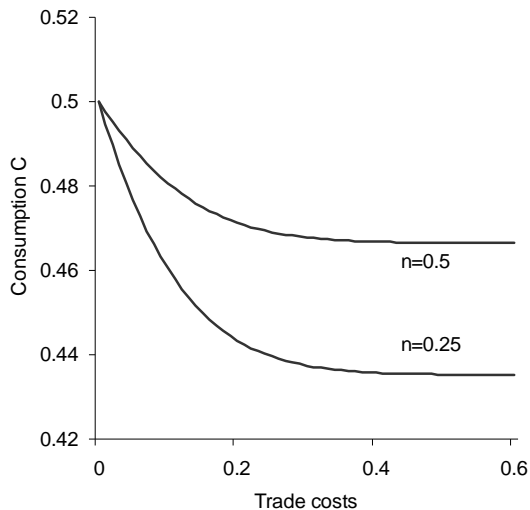


Figure 1: Trade costs reduce consumption in a nonlinear way.

they sell their products to Home or Foreign consumers. Note that equations (12) and (13) only hold when firms are able to set their prices freely, not when prices are sticky.

III. The Flexible-Price Equilibrium

The question of interest in this section is how trade costs affect the flexible-price equilibrium compared to a perfect, frictionless world. As usual in the New Open Economy Macroeconomics literature, it is assumed that firms set prices after the exchange rate and wages are known and that initially there are no bond holdings and no lump-sum transfers so that $F = F^* = Z = Z^* = 0$. The time index t is dropped to denote initial equilibrium values.

An Endogenous Home Bias in Consumption

By comparing individual goods prices in (8) one can easily see that trade costs drive up the price of imported goods and thus render domestic goods more attractive. As a result, consumers spend a bigger fraction of their income on domestic goods. This buffering feature of trade costs will be referred to as the *containment effect* of trade costs, meaning that spending tends to be retained within the domestic country. Trade costs therefore lead to an *endogenous home bias* in consumption in each country.

The home bias arises although the preference specification in (2) is symmetric such that consumers equally desire all goods, regardless of where they are produced. Of course, abandoning the symmetry by introducing an exogenous home bias in preferences, for example as in Warnock (2003), would be an alternative way of explaining the home bias. However,

Carolyn Evans (2007) finds empirically that the only significant reason for the tendency of consumers to purchase domestic goods are locational factors arising due to geographic distance and legal regulations – but not consumer preferences.⁷ Her findings are therefore consistent with the preference specification in (2) and the model’s iceberg trade costs but not with Warnock’s (2003) assumption of home bias in preferences. My approach is therefore to use trade costs as a way to rationalize and endogenize home bias.

As shown in Appendix B, equilibrium labor supply is not affected by trade costs because of the logarithmic utility specification in (1). However, trade costs reduce consumption, real profits and real wages and hence, they make individuals worse off.⁸ Figure 1 illustrates the reduction in consumption with a numerical example.⁹ Two characteristic features of trade costs stand out. First, trade costs operate in a nonlinear fashion. Moderate values, say, $\tau = 0.1$, already have a sizeable impact. Second, trade costs have a more detrimental impact on small countries, as can be seen in the case of $n = 0.25$. Intuitively, since all the goods produced in the world are equally desired by consumers, smaller countries are more open economies and therefore more exposed to trade costs.

Trade Shares

In a frictionless world the model would predict that a country’s expenditure share on domestic goods equals its share of world output. The remainder would be spent on imported goods. Empirically, however, countries tend to spend a much larger fraction of their incomes on domestic goods and the fraction spent on imported goods is smaller. Figure 2 shows that the model performs reasonably well in predicting empirical trade shares for the OECD countries.¹⁰ Country size, measured as the share of world GDP, is plotted against the share of GDP spent on imported goods. The data are taken from the CIA World Factbook for the year 2007 (PPP-adjusted). The graph also contains the European Union aggregate, which is the data point with the highest GDP share (22 percent) and the lowest trade share (10

⁷Evans (2007) compares prices and quantities of imported goods produced by American firms for domestic sale with those of the same goods produced by foreign affiliates of these American firms for local sale. Her data set encompasses seven industries, ranging from transportation equipment to food products, across nine OECD countries over the period 1985-1994. She finds that the ad-valorem tariff equivalent of producing domestically and shipping abroad ranges between 51 and 105 percent across industries, which considerably reduces the attractiveness of the foreign goods for local consumers. Establishing and selling from a local affiliate, however, does not lead to any negative effect on sales of these foreign products when compared to sales of local goods. For example, French consumers do not intrinsically prefer French to American beer, only if it is cheaper.

⁸Formally, $\partial U/\partial \tau < 0$ and $\partial U^*/\partial \tau < 0$. For given money supply trade costs also decrease equilibrium real money balances.

⁹ $\rho = 11$

¹⁰The analytical expression for the trade share is given in Appendix C.2 after equation (??). The Foreign country is treated as the rest of the world.

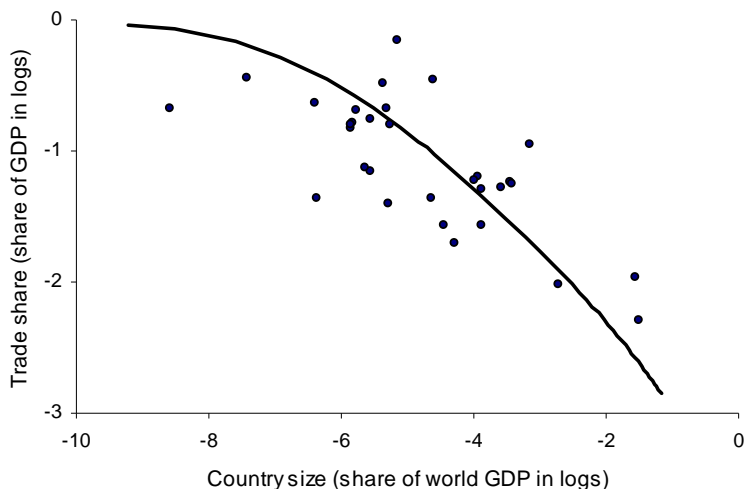


Figure 2: Trade shares and country sizes are inversely related. The solid line represents the model's prediction. Each data point refers to an OECD country (data for 2007).

percent).

The key parameters underlying Figure 2 are chosen as $\tau = 0.37$ and $\rho = 8$. These values are based on the seminal paper by Eaton and Kortum (2002). They run a gravity regression with data for the year 1990 and, assuming an elasticity of $\rho = 8$ they estimate a range of 58-78 percent for trade costs expressed as a tariff equivalent. Given that trade costs are likely lower in 2007, I choose the lower end of that range, corresponding to $\tau = 0.37$ when converted into the iceberg form.¹¹ The choice of $\rho = 8$ lies in the middle of the range typically suggested in the trade literature. In their survey on trade costs, Anderson and van Wincoop (2004) conclude that the elasticity of substitution is usually estimated between 5 and 10.

Figure 2 is robust to alternative values. Anderson and van Wincoop (2004, Table 7) present various combinations of trade costs and substitution elasticities that prominent papers in the trade literature have found. Using U.S.-Canadian trade data for the year 1993, Anderson and van Wincoop (2003) estimate $\tau = 0.47$ under the assumption of $\rho = 5$ and $\tau = 0.26$ under the assumption of $\rho = 10$. Both combinations yield roughly the same trade shares as plotted in Figure 2. All these trade cost values are substantial because they capture a wide range of trade barriers such as transportation costs, tariffs, informational costs, bureaucratic hurdles etc. Even in the European Union where tariffs were abolished in the 1960s, significant trade barriers remain in the form of technical barriers to trade, for example packaging and labelling requirements (see Chen and Novy, 2008). In contrast, Ravn and

¹¹The tariff equivalent of iceberg trade costs is given by $0.58 = 1/(1 - \tau) - 1$, implying $\tau = 0.37$.

Mazzenga (2004) focus on transportation costs. They infer the magnitude of transportation costs from c.i.f./f.o.b. ratios, using a tariff equivalent value of 20 percent ($\tau = 0.17$). That number can be interpreted as a lower bound.

Nontradable Goods

As Hau (2000) shows, the assumption of nontradable goods also leads to a home bias in consumption. To check whether nontradable goods are a viable alternative to trade costs for generating realistic trade shares, I incorporate them into a version of the model with zero trade costs. Like Hau (2000) I assume that in each country only an exogenous fraction ω of firms produces tradable goods.¹² Appendix B.1 provides the details. In contrast to Hau (2000) and Warnock (2003) I solve for a steady state in which the two countries can have different sizes. This allows me to calculate the fraction of tradable goods that corresponds to empirical trade shares and country sizes.

The general conclusion is that nontradable goods fail to generate realistic trade shares. For example, the UK produces about three percent of world GDP ($n = 0.03$) and has a trade share of around 29 percent of GDP. This trade share would correspond to $\omega = 0.04$, i.e. only four percent of all goods in the economy are tradable. This share clearly seems too low in the face of the UK's large engagement in international trade. Hau (2000) generates much more favorable values, for example $\omega = 0.48$ for the average of OECD countries between 1973-1993, but only because he constrains countries to be symmetric ($n = 0.5$). Once trade shares are matched with actual country sizes the implied values of ω become implausibly low.

Why do empirical trade shares correspond to reasonable values of trade costs (see Figure 2) but not to reasonable values of ω ? The intuition is that trade costs change relative prices by driving a wedge between domestic and foreign goods. Given that demand is elastic ($\rho > 1$) even small amounts of trade costs can have a large impact on the allocation of expenditure. However, nontradable goods do not change relative prices. They merely restrict the range of goods available to consumers.

IV. Sticky Prices and Shocks

This section examines how key economic variables respond to shocks when prices are sticky for one period and when trade costs impede the international exchange of goods. It is assumed that all prices (p_t, p_t^*, q_t, q_t^*) are preset every period and that firms choose prices to be optimal in the absence of shocks. They therefore preset the prices of the initial flexible-

¹²It turns out that the parameter ω corresponds to $(1 - \tau)^{\rho-1}$ in the model with trade costs. This correspondence goes through for the log-linearized model in Sections IV and V.

price equilibrium. For a sufficiently small shock in period t , firms have an incentive to meet the post-shock market demand since they are monopolistic competitors and still make profits. As there is no capital in the model, prices and all other variables reach their new long-run equilibrium in $t + 1$, one period after the shock hits the economy. Log-linear approximations are taken around the pre-shock flexible-price equilibrium of Section III. For any variable X let $\widehat{X}_{t+k} \equiv (X_{t+k} - X)/X$ be the percentage deviation from the initial equilibrium at time $t + k$ for $k = 0, 1$.

There are monetary shocks and productivity shocks.¹³ As in Obstfeld and Rogoff (1995), productivity shocks are modelled as shocks to the disutility of labor in utility function (1). A fall in the parameter η can be interpreted as a positive productivity shock in the sense that less labor is required to produce a given amount of output. The Foreign country draws the productivity shock $\widehat{\eta}_t^*$ which may be different from $\widehat{\eta}_t$.¹⁴ Monetary shocks are modelled as shocks to M and M^* . It is assumed that all shocks are permanent ($\widehat{\eta}_t = \widehat{\eta}_{t+1}$, $\widehat{\eta}_t^* = \widehat{\eta}_{t+1}^*$, $\widehat{M}_t = \widehat{M}_{t+1}$ and $\widehat{M}_t^* = \widehat{M}_{t+1}^*$).¹⁵ Tables 1a and 1b summarize their effects on key variables. The next subsection discusses the real exchange rate. The subsequent subsection concentrates on the nominal exchange rate, consumption, output and the current account. Full analytical derivations are given in Appendix C.

The Real Exchange Rate

The short-run responses of the price indices to an exchange rate movement in period t can be obtained by log-linearizing (3) and (4). Under local currency pricing ($s = 1$) the price indices are not affected by nominal exchange rate movements since all prices are fixed irrespective of trade costs ($\widehat{P}_t = \widehat{P}_t^* = 0$). A nominal exchange rate depreciation therefore directly translates into a real exchange rate depreciation ($\widehat{\psi}_t = \widehat{e}_t$).

But when at least some prices are sticky in producer currency ($0 \leq s < 1$), a depreciation of the Home currency increases the Home price level and decreases the Foreign price level. Trade costs weaken the effect that exchange rate movements have on price indices.¹⁶ Intuitively, trade costs act like buffers that shift consumption towards domestic goods through their containment effect and thus decrease the weight of imported goods in the price index. In the limit as the countries become closed economies ($\tau \rightarrow 1$), the price indices are

¹³The model can be easily extended to government spending shocks as in Betts and Devereux (2000). Results are available from the author on request.

¹⁴For simplicity η and η^* are the same in the initial equilibrium.

¹⁵As short-run output is demand-determined, purely temporary productivity shocks at time t would be entirely absorbed by short-run leisure so that no other variables would need to adjust, see Obstfeld and Rogoff (1995, p. 653).

¹⁶Formally, $\left| \partial \widehat{P}_t / \partial \widehat{e}_t \right| / \partial \tau < 0$ and $\left| \partial \widehat{P}_t^* / \partial \widehat{e}_t \right| / \partial \tau < 0$.

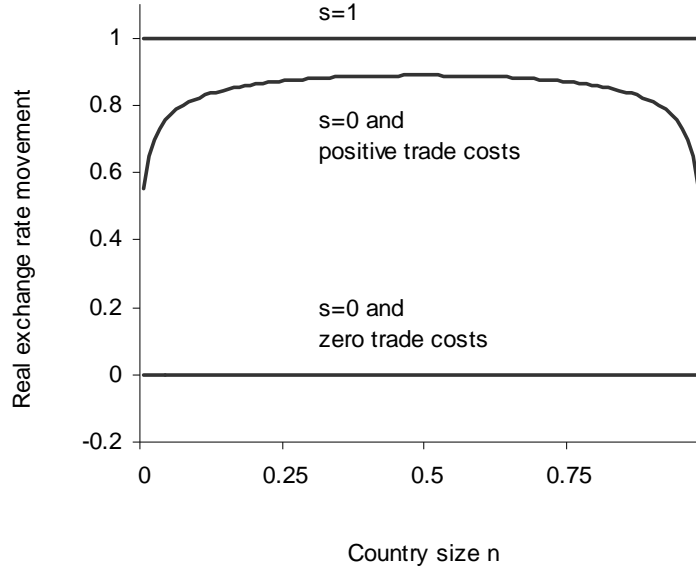


Figure 3: Trade costs increase the real exchange rate movement under producer currency pricing ($s = 0$).

completely insulated from exchange rate movements.

This weakening effect of trade costs has consequences for the real exchange rate movement which can be expressed as

$$\widehat{\psi}_t = \widehat{e}_t + \widehat{P}_t^* - \widehat{P}_t = (s + \chi(1 - s)) \widehat{e}_t \quad (14)$$

where χ is a function of trade costs and exogenous parameters only. χ has the property that $\chi = 0$ for $\tau = 0$ and that it monotonically increases in τ such that $0 < \chi < 1$ for $0 < \tau < 1$.¹⁷ Consider the case of producer currency pricing ($s = 0$) and a depreciation ($\widehat{e}_t > 0$). In the absence of trade costs ($\tau = \chi = 0$) the price index movements are exactly offset by the nominal exchange rate so that the real exchange rate does not move at all ($\widehat{\psi}_t = 0$). But in the presence of trade costs the price index movements are weakened and the real exchange rate is no longer fixed ($\widehat{\psi}_t > 0$). The real exchange rate movement is stronger for higher trade costs with $\widehat{\psi}_t = \widehat{e}_t$ in the limit as $\tau \rightarrow 1$.¹⁸

Figure 3 illustrates this behavior with the numerical example from Section III.¹⁹ In the

¹⁷See Appendix C.1 for details.

¹⁸Formally, for $0 \leq s < 1$, $\left| \partial \widehat{\psi}_t / \partial \widehat{e}_t \right| / \partial \tau > 0$ and $\lim_{\tau \rightarrow 1} \left| \partial \widehat{\psi}_t / \partial \widehat{e}_t \right| = 1$ since $\lim_{\tau \rightarrow 1} \chi = 1$. This finding also implies that for some degree of producer currency pricing and a given series of nominal exchange rate movements, trade costs render the real exchange rate more volatile (see below for simulation results).

¹⁹ $\tau = 0.37$ and $\rho = 8$. Figure 3 and all subsequent figures are drawn for one-percent shocks.

presence of trade costs real exchange rate movements approach the ones under local currency pricing. This effect is so strong that even under full exchange rate pass-through, trade costs of a reasonable magnitude come close to a situation as if all prices were fixed in local currency.

The Nominal Exchange Rate, Consumption, Output and the Current Account

One can express the nominal exchange rate movement in period t in terms of exogenous shocks and parameters as

$$\widehat{e}_t = \frac{a_1(\widehat{M}_t - \widehat{M}_t^*) + a_2(\widehat{\eta}_t - \widehat{\eta}_t^*)}{a_1s + a_3(1-s)} \quad (15)$$

where

$$\begin{aligned} a_1 &= 1 + \frac{\sigma\beta}{1-\beta} - \chi \left(1 + \frac{\sigma\beta}{\rho(1-\beta)} \right) > 0 \\ a_2 &= \frac{(\sigma-1)\beta}{1-\beta} > 0 \\ a_3 &= (\rho-1)(1-\chi^2) + a_1 > 0 \\ \sigma &\equiv \frac{\rho-1+\rho\eta}{\rho-1+\eta} > 1 \end{aligned}$$

with $1 < \sigma < \rho$. $\chi \geq 0$ depends on trade costs τ and country size n (see Appendix C). Since a_1 , a_2 and a_3 are all greater than zero, a positive Home monetary shock ($\widehat{M}_t > 0$) leads to a depreciation of the Home currency ($\widehat{e}_t > 0$), whereas a positive Home productivity shock ($\widehat{\eta}_t < 0$) leads to an appreciation.

Table 1a summarizes the responses of key variables to a positive Home monetary shock. The results can be understood with the help of two ‘switching’ effects. Under local currency pricing ($s = 1$) relative prices and thus relative demand are fixed, but measured in domestic currency Home firms generate higher revenue because of the exchange rate depreciation. This will be referred to as the *income-switching effect*. As a result of the containment effect this additional income is predominantly spent on domestic goods, leading to a higher increase in Home output and a lower increase in Foreign output ($\widehat{h}_t > \widehat{h}_t^*$). But in the absence of trade costs the additional income would be spent evenly across the two countries ($\widehat{h}_t = \widehat{h}_t^*$). Apart from output trade costs do not affect the reaction of other variables to the monetary shock. In particular, as can be seen from (15) the nominal exchange rate does not behave differently because the factor a_1 cancels.

More generally, it can be said that trade costs dampen international linkages. Take the case of producer currency pricing ($s = 0$). Price indices are no longer fixed and the familiar *expenditure-switching effect* comes into play. But as trade costs reduce the movement of price

Table 1a **A monetary shock and
the impact of trade costs ($\widehat{M} > 0$)**

	LCP ($s = 1$)		PCP ($s = 0$)	
	Direction	Impact of τ	Direction	Impact of τ
Short run				
\widehat{e}_t	+	=	+	>
\widehat{P}_t	0	=	+	<
\widehat{P}_t^*	0	=	-	<
\widehat{C}_t	+	=	+	>
\widehat{C}_t^*	0	=	+	<
\widehat{h}_t	+	>	+	<
\widehat{h}_t^*	+	<	-	<
Current account				
dF_t	0	=	+	<
Long run				
\widehat{C}_{t+1}	0	=	+	<
\widehat{C}_{t+1}^*	0	=	-	<
\widehat{h}_{t+1}	0	=	-	<
\widehat{h}_{t+1}^*	0	=	+	<

+ up, 0 unchanged, - down, > reinforced, = neutral, < attenuated.

Table 1b **A productivity shock and
the impact of trade costs ($\widehat{\eta} < 0$)**

	LCP ($s = 1$)		PCP ($s = 0$)	
	Direction	Impact of τ	Direction	Impact of τ
Short run				
\widehat{e}_t	-	>	-	>
\widehat{P}_t	0	=	-	<
\widehat{P}_t^*	0	=	+	<
\widehat{C}_t	0	=	+	<
\widehat{C}_t^*	0	=	-	<
\widehat{h}_t	0	=	-	<
\widehat{h}_t^*	0	=	+	<
Current account				
dF_t	-	<	-	<
Long run				
\widehat{C}_{t+1}	+	>	+	>
\widehat{C}_{t+1}^*	+	<	+	<
\widehat{h}_{t+1}	+	<	+	<
\widehat{h}_{t+1}^*	-	<	-	<

+ up, 0 unchanged, - down, > reinforced, = neutral, < attenuated.

indices, the nominal exchange rate must depreciate more strongly than it would without trade costs.²⁰ Trade costs dampen the increase in output of Home goods and they dampen the decrease in output of Foreign goods. They obstruct the positive spillover of the monetary stimulus such that Home consumption increases more strongly than Foreign consumption. The current account response is therefore also dampened, toning down the long-run responses of consumption and output.²¹ Those reactions are qualitatively similar to the behavior of variables in the presence of nontradable goods (Hau, 2000) and home bias in preferences (Warnock, 2003).

Trade costs also dampen international linkages in the face of productivity shocks. Table 1b illustrates that the long-run response to a positive Home productivity shock is qualitatively the same for local and producer currency pricing. Due to the containment effect of trade costs the benefits of the Home productivity improvement are tilted towards Home consumers in the sense that Home long-run consumption increases more strongly, whereas Foreign long-run consumption increases less strongly. In the short run, no variables adjust under local currency pricing apart from the nominal exchange rate because relative prices do not change and the productivity shock is entirely absorbed by leisure.¹⁵ Under producer currency pricing the expenditure-switching effect leads to an increase in Home consumption and a decrease in Foreign consumption but both those movements are toned down by trade costs.

Finally, trade costs have the general feature that compared to a frictionless world they lead to quantitatively more similar responses of variables under local and producer currency pricing. Put differently, one could say that the quantitative responses of variables under $s = 1$ and $s = 0$ converge with increasing trade costs. This is true for both monetary and productivity shocks.²² Take Figure 3 as an example. For most country sizes the real exchange rate responses under $s = 1$ and $s = 0$ are quantitatively rather similar with trade costs, whereas they are markedly different without trade costs.²³ This pattern holds up for all variables in Tables 1a and 1b.

Simulation Results

Two conclusions can be drawn from the results above. First, trade costs make both nominal

²⁰Formally, $\partial (a_1/a_3) / \partial \tau > 0$ is required. This is generally the case unless trade costs are very low (roughly below 2 percent) and one country is overwhelmingly big (roughly over 98 percent of world size).

²¹Devereux (2000) provides a detailed discussion of the impact on the current account.

²²Formally, $\partial \left| \hat{X}_{s=1} - \hat{X}_{s=0} \right| / \partial \tau < 0$. This is generally the case unless trade costs are unreasonably low (roughly below 2 percent) and one country is overwhelmingly big (roughly over 98 percent of world size).

²³Note that trade costs can at most boost the real exchange movement up to the value under zero exchange rate pass-through ($s = 1$) but not further. This is also true for the nominal exchange rate movement.

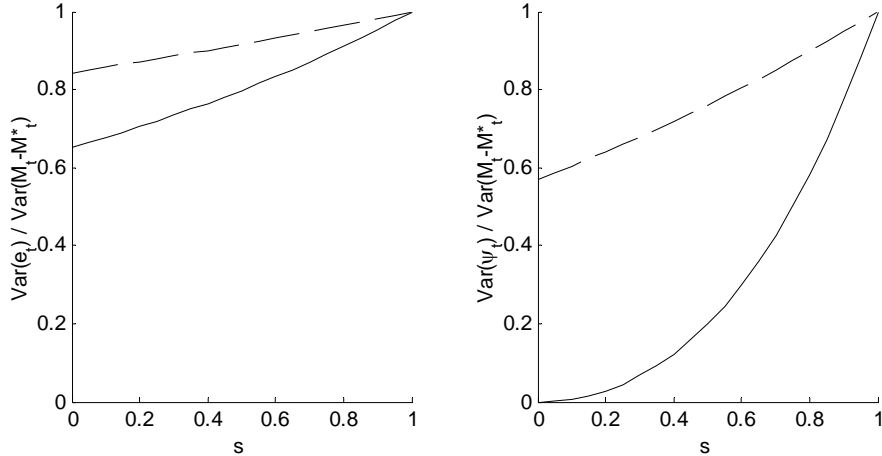


Figure 4: Trade costs render the nominal (left) and real (right) exchange rates more volatile ($\tau = 0$ for solid lines, $\tau > 0$ for dashed lines).

and real exchange rates more volatile. Second, trade costs dampen international linkages and reduce cross-country consumption and output correlations. These two conclusions are illustrated by simulation results in Figures 4 and 5. Each simulated observation is constructed from 100 replications over a draw of 100 periods for uncorrelated shocks to M_t and M_t^* or $\hat{\eta}_t$ and $\hat{\eta}_t^*$. As discussed in Section III, trade costs and the elasticity of substitution are chosen as $\tau = 0.37$ and $\rho = 8$ based on Eaton and Kortum (2002). The country size is set to $n = 0.05$ which falls in the range of large OECD countries such as Japan, Germany or the UK. As in Betts and Devereux (2000) β is chosen to be 0.94, implying a real interest rate of about 6 percent, roughly the average long-run real return on stocks. The steady-state value of η is chosen as 10/11 to match a consumption-constant elasticity of labor supply near unity. This value is low by the standards of the real business cycle literature but higher than most microeconomic estimates.²⁴

Figure 4 plots the volatility of the nominal and real exchange rates against the extent of local currency pricing s . Volatility is measured as the relative variance of exchange rate movements and monetary shocks, $Var(\hat{e}_t)/Var(\widehat{M}_t - \widehat{M}_t^*)$ and $Var(\hat{\psi}_t)/Var(\widehat{M}_t - \widehat{M}_t^*)$. Under producer currency pricing ($s = 0$) trade costs increase the volatility of the nominal exchange rate by 29 percent (from 0.65 to 0.84). The volatility of the real exchange rate jumps up from zero to 0.57. Thus, the differences in volatility between producer and local currency pricing tend to diminish when trade costs are present.²⁵

²⁴The elasticity is given by $(\rho - 1) / (\rho\eta) = 0.9625$.

²⁵This result holds up for productivity shocks. For $s = 0$ the volatility of the nominal exchange rate, measured as $Var(\hat{e}_t)/Var(\hat{\eta}_t - \hat{\eta}_t^*)$, increases by 75 percent from 0.12 to 0.21. Real exchange rate volatility, measured as $Var(\hat{\psi}_t)/Var(\hat{\eta}_t - \hat{\eta}_t^*)$, jumps up from zero to 0.14. These volatilities are hardly sensitive to

Given the difficulties measuring monetary and productivity shocks, it is not easy to compare Figure 4 to empirically observed exchange rate volatilities. Instead, I follow Chari, Kehoe and McGrattan (2002) in computing the standard deviation of the exchange rate relative to the standard deviation of output. For quarterly U.S. exchange rate data with respect to a number of European countries between 1973-2000 they find ratios of 4.67 for the nominal exchange rate and 4.36 for the real exchange rate. For $s = 1$ and positive trade costs my model yields a ratio of 1.53 for both the nominal and real exchange rates in response to monetary shocks. For zero trade costs the ratio would be lower at 1.27.²⁶

Although trade costs move exchange rate volatility in the right direction, the numbers clearly fall short of the empirically observed volatilities. As Chari, Kehoe and McGrattan (2002) explain, they can only successfully match the empirical ratios by assuming a large degree of risk aversion equal to $\vartheta = 5$. They show that the degree of risk aversion is almost one-for-one related to the ratio of standard deviations. But since my model is based on logarithmic utility, implying a degree of risk aversion equal to $\vartheta = 1$, it is by construction not well suited to match the high empirical volatility of exchange rates. Nevertheless, trade costs increase exchange rate volatility and therefore seem a promising tool for generating higher volatility for more moderate levels of risk aversion (i.e. $\vartheta < 5$).²⁷

Empirically, output is more strongly correlated across countries than consumption. For example, based on quarterly data for 14 OECD countries from 1970-2000 Ravn and Mazzenga (2004) report cross-country correlations of 0.33 for output and 0.22 for consumption. However, the literature on international business cycles has struggled to explain the low cross-correlation of consumption known as the “consumption correlations puzzle” (for a discussion see Obstfeld and Rogoff, 2000).

Figure 5 visualizes the cross-country correlations of consumption growth and output growth, $Corr(\hat{C}, \hat{C}^*)$ and $Corr(\hat{h}, \hat{h}^*)$, as predicted by the model. The top panels depict correlations in response to monetary shocks and the bottom panels depict correlations in response to productivity shocks. As a general pattern trade costs push consumption and output correlations towards zero.

Trade costs match the empirical consumption correlations rather well. For $s = 0$ the

alternative country sizes, for instance $n = 0.25$.

²⁶For $s = 0$ the model yields significantly lower ratios (0.37 for the nominal and 0.30 for the real exchange rate) but those numbers match the fact that nominal exchange rates are more volatile. The ratios are similar in the case of productivity shocks (0.31 and 0.26). A larger country size, for instance $n = 0.25$, slightly increases these ratios but they are still below 1.

²⁷Ravn and Mazzenga (2004, Table 5) predict a standard deviation of the real exchange rate below 1, compared to an empirical value of 4.03 in quarterly OECD data from 1970-2000. In contrast to Chari, Kehoe and McGrattan (2002), their model is based on flexible prices and therefore cannot capture the fact that in the short run real exchange rates are predominantly driven by nominal exchange rate movements (see Rogoff, 1996).

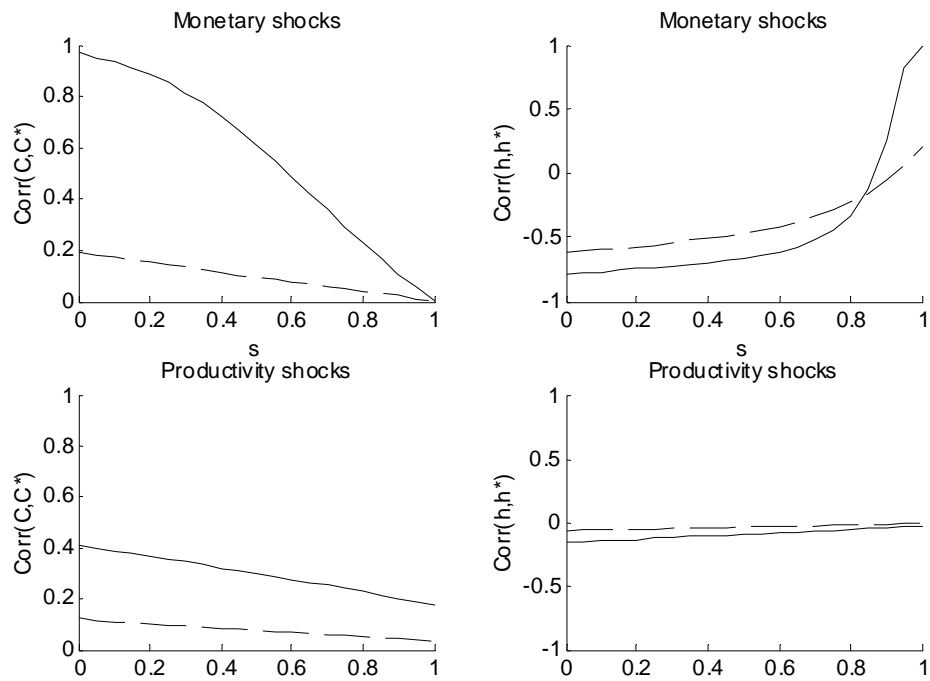


Figure 5: Trade costs push international consumption and output correlations towards zero ($\tau = 0$ for solid lines, $\tau > 0$ for dashed lines, monetary shocks in the top panels, productivity shocks in the bottom panels).

correlation is 0.19 for monetary shocks and 0.12 for productivity shocks, compared to 0.97 and 0.41 in a frictionless world. Thus, trade costs substantially reduce cross-country correlations, in particular in the case of full pass-through. The correlation of 0.12 for productivity shocks can be most readily compared to the value of 0.49 that Ravn and Mazzenga (2004, Table 6) obtain when they assume that technology shocks are uncorrelated and have no cross-country spillovers. Their baseline specification with more persistent technology shocks yields an even higher correlation of 0.86.²⁸ Backus, Kehoe and Kydland (1992), who use the same persistent technology shock process as Ravn and Mazzenga (2004), are not able to generate low cross-country correlations either even if they introduce transportation costs into their flexible price model. It therefore seems that trade costs in combination with sticky prices are more effective at reducing cross-country consumption correlations.

In contrast, the model does not generate realistic output correlations. Although trade costs move them in the right direction, output correlations tend to be negative and therefore do not match the positive correlation observed empirically. Similarly, Ravn and Mazzenga (2004) report output correlations that are close to zero and in some cases negative. But as Chari, Kehoe and McGrattan (2002) show, a business cycle model with investment and capital is more successful in generating realistic output correlations when combined with monetary shocks and sticky prices.

V. Trade Costs and Welfare

How do trade costs affect the welfare properties of the model? To address this issue, I adopt Obstfeld and Rogoff's (1995) methodology and decompose the utility function (1) into $U_t = U_t^R + U_t^M$ where U_t^R consists of the consumption and labor terms and U_t^M of real money balances. As Obstfeld and Rogoff (1995) argue, unless real money balances are assigned an implausibly large weight γ in (1), U_t^R dominates the overall welfare effect of a shock and U_t^M can be neglected. Taking log-linear approximations and noting that $\widehat{C}_{t+1} = \widehat{C}_{t+1+k}$, $\widehat{h}_{t+1} = \widehat{h}_{t+1+k}$ as well as $\widehat{\eta}_{t+1} = \widehat{\eta}_{t+1+k}$ for $k = 1, 2, \dots$ yields

$$dU_t^R = \left[\widehat{C}_t - \left(\frac{\rho - 1}{\rho} \right) \widehat{h}_t + \eta \ln(1 - h) \widehat{\eta}_t \right] + \frac{\beta}{(1 - \beta)} \left[\widehat{C}_{t+1} - \left(\frac{\rho - 1}{\rho} \right) \widehat{h}_{t+1} + \eta \ln(1 - h) \widehat{\eta}_{t+1} \right]$$

The notation for the Foreign country is analogous. Table 2 summarizes the welfare effects.

The welfare effect of an increase in Home productivity ($\widehat{\eta} < 0$) is particularly easy to understand. It unambiguously raises welfare in both countries. But due to the containment

²⁸Moreover, symmetric country sizes ($n = 0.5$), as implicitly assumed by Ravn and Mazzenga (2004), yield the lowest possible consumption correlation. For example, the correlation of 0.12 with $n = 0.05$ as mentioned above compares to a correlation of 0.05 in the case of country symmetry.

Table 2 The impact of trade costs on welfare

	LCP ($s = 1$)		PCP ($s = 0$)	
	Direction	Impact of τ	Direction	Impact of τ
$\widehat{M} > 0$				
dU_t^R	+	<	+	>
dU_t^{*R}	-	<	+ becomes -	
$\widehat{\eta} < 0$				
dU_t^R	+	>	+	>
dU_t^{*R}	+	<	+	<

+ up, - down, > reinforced, < attenuated.

effect trade costs reinforce the welfare gain for Home households and dampen the welfare gain for Foreign households. Given that with zero trade costs the Home welfare gain exceeds the Foreign gain ($dU_t^R > dU_t^{*R}$), this result implies that trade costs increase the welfare gap.

The welfare effect of a Home monetary expansion ($\widehat{M} > 0$) depends on the degree of exchange rate pass-through. As Table 1a illustrates, under local currency pricing ($s = 1$) trade costs only affect the response of short-run labor effort. They increase the labor effort of Home households, thus making them worse off compared to a frictionless world. The opposite is true for Foreign households. As depicted in the top panel of Figure 6 (drawn for $n = 0.05$), trade costs therefore reduce the welfare gap between Home and Foreign. In the limit when the two countries become closed economies ($\tau \rightarrow 1$) Foreign households are not at all affected.

As the bottom panel of Figure 6 illustrates, the welfare response under producer currency pricing ($s = 0$) is hump-shaped with the welfare gap peaking at intermediate trade cost values around $\tau = 0.2$. Table 1a shows that trade costs affect both the short-run and the long-run responses of consumption and labor effort. Trade costs reinforce the increase in short-run consumption \widehat{C}_t and dampen the increase in short-run labor effort \widehat{h}_t so that in the short run, trade costs make Home households better off compared to a frictionless world. This short-run effect can be explained by the expenditure-switching effect that favors the Home country and that is reinforced by trade costs. In contrast, trade costs dampen the increase in long-run consumption \widehat{C}_{t+1} as well as the fall in long-run labor effort \widehat{h}_{t+1} so that in the long run, trade costs make Home households worse off. It turns out that the positive short-run effect dominates the negative long-run effect up to an intermediate value of roughly $\tau = 0.2$.

Thus, trade costs initially increase the welfare gap under producer currency pricing but they reduce it under local currency pricing. The middle panel of Figure 6 plots the welfare reaction for an intermediate degree of pass-through ($s = 0.5$). Now the welfare gap is fairly

stable up to $\tau = 0.2$ and only shrinks for higher trade costs.

In summary, the welfare results stand in sharp contrast to Obstfeld and Rogoff's (1995) finding that monetary shocks entail positive and symmetric international welfare spillovers. Their finding refers to the special case of zero trade costs and full pass-through. Figure 6 shows that this special case is not robust to trade costs and lower degrees of pass-through.

VI. Conclusion

The focus of this paper is to investigate the implications of international trade costs for key macroeconomic variables. This is achieved by integrating iceberg trade costs into a micro-founded two-country general equilibrium model with asymmetric country sizes based on the New Open Economy Macroeconomics literature. The inclusion of trade costs is motivated by Anderson and van Wincoop's (2004) recent survey showing that empirical trade costs are widespread and large with a tariff equivalent of representative international trade costs of around 74 percent.

Even moderate trade costs tend to have a substantial impact on the behavior of key economic variables. By increasing the price of foreign goods, trade costs tilt consumption towards domestic goods and create an endogenous home bias in consumption. This feature of trade costs is crucial in matching empirical trade shares for OECD countries. In contrast, nontradable goods would overpredict trade shares.

By impeding international trade flows trade costs tend to isolate countries from each other and therefore weaken international linkages, leading to smaller current account movements as well as lower international output and consumption correlations. However, the weakened international linkages force nominal and real exchange rates to move more strongly in response to shocks. Trade costs therefore increase exchange rate volatility.

Finally, the model is designed to allow for varying degrees of exchange rate pass-through. It turns out that trade costs render the distinction between producer and local currency pricing less relevant. For example, simulation results show that cross-country consumption correlations are rather similar under complete and incomplete pass-through. In the same way, trade costs turn a monetary expansion into a beggar-thy-neighbor policy regardless of the degree of exchange rate pass-through.

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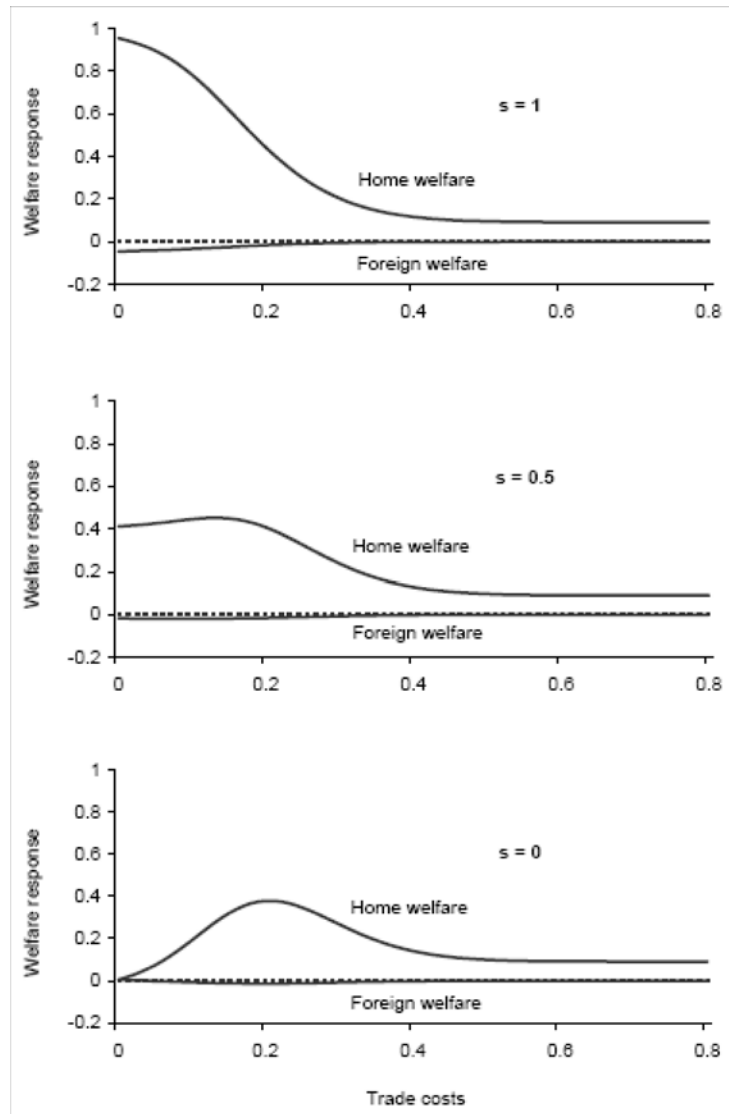


Figure 6: Trade costs and welfare for varying degrees of exchange rate pass-through.

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Appendix A: Households and Firms

Appendix A outlines the derivations of the expressions in Section II. Demand function (7) is derived by maximizing C_t in (2) subject to the expenditure K_t given by

$$K_t = \int_0^n p_{it} c_{it} \, di + \int_n^{n+(1-n)s} \frac{1}{1-\tau} p_{it}^* c_{it} \, di + \int_{n+(1-n)s}^1 \frac{1}{1-\tau} e_t q_{it}^* c_{it} \, di$$

This also results in price index (3). Maximizing utility (1) subject to the two-period intertemporal budget constraint constructed from (5)

$$P_{t+1} C_{t+1} + M_{t+1} + d_{t+1} F_{t+1} = W_{t+1} h_{t+1} + \pi_{t+1} + \frac{d_t - 1}{d_t} M_t + Z_{t+1} + \frac{1}{d_t} [W_t h_t + \pi_t + M_{t-1} + Z_t + F_{t-1} - P_t C_t]$$

yields the optimality condition for labor supply

$$\frac{\eta}{1-h_t} = \frac{W_t}{P_t C_t} \tag{A1}$$

and the money demand function

$$\frac{M_t}{P_t} = \frac{\gamma C_t}{1-d_t} \tag{A2}$$

as well as the intertemporal consumption stream

$$d_t P_{t+1} C_{t+1} = \beta P_t C_t \tag{A3}$$

The corresponding equations for the Foreign country are analogous.

Let us now turn to profit maximization. The profit function (11) can be rewritten as

$$\pi_t = (p_t - W_t) (x_t + (1 - s) z_t^{PCP}) + (e_t q - W_t) s z_t^{LCP}$$

From (7) insert the demand functions

$$x_t = \left(\frac{p_t}{P_t} \right)^{-\rho} n C_t \quad (\text{A4})$$

$$z_t^{LCP} = \frac{1}{1 - \tau} \left(\frac{\frac{1}{1 - \tau} q_t}{P_t^*} \right)^{-\rho} (1 - n) C_t^* \quad (\text{A5})$$

$$z_t^{PCP} = \frac{1}{1 - \tau} \left(\frac{\frac{1}{1 - \tau} p_t}{P_t^*} \right)^{-\rho} (1 - n) C_t^* \quad (\text{A6})$$

Then maximize with respect to p_t and q_t to yield (12). (13) can be derived analogously.

Appendix B: The Flexible-Price Equilibrium

Appendix B gives the derivations and analytical results of Section III. In order to derive the relative wage $W/(eW^*)$ consistent with (12) and (13) I adopt a ‘guess and verify’ solution strategy by imposing that the relative wage in equilibrium equals an unknown parameter α , i.e. $W/(eW^*) = \alpha$. The ultimate aim is to solve for α . Plug α into the price indices (3) and (4), also using the markups (12) and (13). This results in the real wages

$$\frac{W}{P} = \frac{\rho - 1}{\rho} \theta^{\frac{1}{\rho - 1}} \quad (\text{A7})$$

$$\frac{W^*}{P^*} = \frac{\rho - 1}{\rho} \theta^*{}^{\frac{1}{\rho - 1}} \quad (\text{A8})$$

where

$$\theta = n + (1 - n)(1 - \tau)^{\rho - 1} \alpha^{\rho - 1} \quad (\text{A9})$$

$$\theta^* = (1 - n) + n(1 - \tau)^{\rho - 1} \alpha^{1 - \rho} \quad (\text{A10})$$

I derive a solution for α below.

In equilibrium the per-capita supply of labor and thus per-capita output is the same in both countries

$$h = h^* = y = y^* = \frac{\rho - 1}{\rho - 1 + \rho\eta} \quad (\text{A11})$$

(A11) can be derived by combining (5), (9), (10), (11), (12) and (A1), noting that in the initial equilibrium $M_{t-1} = M_t$ and $Z = Z^* = F = F^* = 0$. The equilibrium real wages are

given in (A7) and (A8). Combining (9)-(12), (A7) and (A11) yields Home real profits

$$\frac{\pi}{P} = \frac{h}{\rho} \theta^{\frac{1}{\rho-1}} \quad (\text{A12})$$

Foreign real profits can be derived similarly as

$$\frac{\pi^*}{P^*} = \frac{h^*}{\rho} \theta^{*\frac{1}{\rho-1}} \quad (\text{A13})$$

By inserting (A7) and (A11) into (A1) Home consumption can be derived as

$$C = h \theta^{\frac{1}{\rho-1}} \quad (\text{A14})$$

or

$$y = h = C \theta^{\frac{-1}{\rho-1}} \quad (\text{A15})$$

Similarly, Foreign consumption follows as

$$C^* = h^* \theta^{*\frac{1}{\rho-1}} \quad (\text{A16})$$

such that

$$C^* = \left(\frac{\theta^*}{\theta} \right)^{\frac{1}{\rho-1}} C \quad (\text{A17})$$

To solve for α take (10) and insert the demand functions (A4)-(A6). Use $W/(eW^*) = \alpha$, (A7), (A8) and (A17) to obtain

$$y = C \theta^{\frac{-\rho}{\rho-1}} [n + (1-n)(1-\tau)^{\rho-1} \alpha^{\rho-1} (\theta/\theta^*) \alpha^{-2\rho+1}] \quad (\text{A18})$$

Equation (A18) equals equation (A15) only if the term in the brackets of (A18) equals θ . Comparing the brackets with the expression for θ in (A9), it therefore must be $(\theta/\theta^*) \alpha^{-2\rho+1} = 1$ such that

$$\alpha = (\theta/\theta^*)^{\frac{1}{2\rho-1}} \quad (\text{A19})$$

It follows

$$\theta = n + (1-n)(1-\tau)^{\rho-1} \left(\frac{\theta}{\theta^*} \right)^{\frac{\rho-1}{2\rho-1}} > n \quad (\text{A20})$$

$$\theta^* = (1-n) + n(1-\tau)^{\rho-1} \left(\frac{\theta^*}{\theta} \right)^{\frac{\rho-1}{2\rho-1}} > (1-n) \quad (\text{A21})$$

θ and θ^* consist of exogenous parameters only. θ and θ^* cannot be solved analytically but by repeated substitution they converge to one unique value for all admissible parameters. For $\tau = 0$ it follows $\theta = \theta^* = 1$, and for $0 < \tau < 1$ it follows $n < \theta < 1$ and $(1 - n) < \theta^* < 1$.

The nominal exchange rate is obtained by plugging (A7) and (A8) into $W/(eW^*) = \alpha$ and then using (A19) as well as (A2) and its Foreign equivalent

$$e = \frac{M}{M^*} \frac{C^*}{C} \left(\frac{\theta}{\theta^*} \right)^{\frac{\rho}{(2\rho-1)(\rho-1)}} \quad (\text{A22})$$

The real exchange rate can be expressed as

$$\psi \equiv \frac{eP^*}{P} = \left(\frac{\theta}{\theta^*} \right)^{\frac{\rho}{(2\rho-1)(\rho-1)}} \quad (\text{A23})$$

Note that real wages in (A7) and (A8), real profits in (A12) and (A13) and consumption in (A14) and (A16) are reduced by trade costs since $\partial\theta/\partial\tau < 0$ and $\partial\theta^*/\partial\tau < 0$. Relative quantities can be expressed as

$$\frac{C}{C^*} = \frac{\pi/P}{\pi^*/P^*} = \frac{W/P}{W^*/P^*} = \left(\frac{\theta}{\theta^*} \right)^{\frac{1}{\rho-1}} \quad (\text{A24})$$

If $\tau > 0$, (A20) and (A21) imply $\theta = \theta^*$ if the two countries are symmetric ($n = 1 - n = 0.5$), whereas $\theta > \theta^*$ is implied if the Home country is bigger ($n > 1 - n$), and vice versa. Thus, in the presence of trade costs a smaller country faces relatively lower consumption, a lower real wage and lower real profits.

Nontradable Goods

I assume that trade costs are zero ($\tau = 0$) and that only the fraction $1 \geq \omega > 0$ of firms in each country produces tradable goods. The fraction $(1 - \omega)$ is not available to foreign consumers. The Home per-capita expenditure function can then be written as

$$PC = \int_0^{\omega n} p_i^T c_i \, di + \int_{\omega n}^n p_i^{NT} c_i \, di + \int_n^{n+\omega(1-n)} e q_i^{*T} c_i \, di$$

where T and NT denote tradable and nontradable. The model can be solved as outlined above. Profit maximization leads to $p^T = p^{NT} = p$ and $q^{*T} = q^{*NT} = q^*$ where p and q^* are given in (12) and (13). Equations (A7), (A8), (A11)-(A17) and (A22)-(A24) go through but

θ and θ^* are replaced by

$$\tilde{\theta} = n + (1 - n)\omega \left(\frac{\tilde{\theta}}{\tilde{\theta}^*} \right)^{\frac{\rho-1}{2\rho-1}} > n \quad (\text{A25})$$

$$\tilde{\theta}^* = (1 - n) + n\omega \left(\frac{\tilde{\theta}^*}{\tilde{\theta}} \right)^{\frac{\rho-1}{2\rho-1}} > (1 - n) \quad (\text{A26})$$

Comparing (A25) and (A26) to (A20) and (A21) one can see that ω corresponds to $(1 - \tau)^{\rho-1}$ in the model with trade costs. The derivations of Appendix C go through when θ and θ^* are replaced by $\tilde{\theta}$ and $\tilde{\theta}^*$, in particular for the trade shares given after equation (A36).

Appendix C: Sticky Prices and Shocks

Appendix C outlines the derivations of the expressions discussed in Section IV.

The Real Exchange Rate

The short-run movements of the price indices are given by

$$\hat{P}_t = (1 - s) \left(1 - \frac{n}{\theta} \right) \hat{e}_t \quad (\text{A27})$$

$$\hat{P}_t^* = -(1 - s) \left(1 - \frac{1 - n}{\theta^*} \right) \hat{e}_t \quad (\text{A28})$$

For a given nominal depreciation of the Home currency the behavior of the price indices can be summarized as

$$\begin{aligned} \hat{P}_t = \hat{P}_t^* = \hat{P}_{t,\tau=0} = \hat{P}_{t,\tau=0}^* = 0 & \quad \text{if } s = 1 \\ -\hat{e}_t < \hat{P}_{t,\tau=0}^* < \hat{P}_t^* < 0 < \hat{P}_t < \hat{P}_{t,\tau=0} < \hat{e}_t & \quad \text{if } 0 \leq s < 1 \end{aligned}$$

The parameter χ in (14) is defined as

$$\chi \equiv \frac{n}{\theta} + \frac{1 - n}{\theta^*} - 1$$

with $\chi = 0$ for $\tau = 0$ and $0 < \chi < 1$ for $0 < \tau < 1$.

The Nominal Exchange Rate, Consumption, Output and the Current Account

The derivation of the exchange rate movement (15) is sketched first. Noting that $F_{t-1} = 0$,

combine (5), (6) and (11) to get the overall Home budget constraint

$$P_t C_t + d_t F_t = p_t x_t + s e_t q_t z_t^{LCP} + (1-s) p_t z_t^{PCP} \quad (\text{A29})$$

where

$$x_t = \left(\frac{p_t}{P_t} \right)^{-\rho} n C_t \quad (\text{A30})$$

$$z_t^{LCP} = \frac{1}{1-\tau} \left(\frac{\frac{1}{1-\tau} q_t}{P_t^*} \right)^{-\rho} (1-n) C_t^* \quad (\text{A31})$$

$$z_t^{PCP} = \frac{1}{1-\tau} \left(\frac{\frac{1}{1-\tau} p_t}{P_t^*} \right)^{-\rho} (1-n) C_t^* \quad (\text{A32})$$

represent goods market clearing conditions. Analogously, for the Foreign country

$$P_t^* C_t^* + \frac{d_t}{e_t} F_t^* = q_t^* x_t^* + s \frac{p_t^*}{e_t} z_t^{*LCP} + (1-s) q_t^* z_t^{*PCP} \quad (\text{A33})$$

where F_t^* represents Foreign holdings of the bond maturing in period $t+1$ and

$$x_t^* = \left(\frac{q_t^*}{P_t^*} \right)^{-\rho} (1-n) C_t^* \quad (\text{A34})$$

$$z_t^{*LCP} = \frac{1}{1-\tau} \left(\frac{\frac{1}{1-\tau} p_t^*}{P_t^*} \right)^{-\rho} n C_t^* \quad (\text{A35})$$

$$z_t^{*PCP} = \frac{1}{1-\tau} \left(\frac{\frac{1}{1-\tau} e_t q_t^*}{P_t^*} \right)^{-\rho} n C_t^* \quad (\text{A36})$$

Take log-linear approximations and combine these equations. Note that by using (A30)-(A32), (A34)-(A36), $z^{LCP} = z^{PCP} = z$ and $z^{*LCP} = z^{*PCP} = z^*$ as well as (A23) and (A24) one finds expressions for the domestic expenditure shares ($x/y = \frac{n}{\theta}$ in Home and $x^*/y^* = \frac{1-n}{\theta^*}$ in Foreign) as well as for the import expenditure shares or trade shares ($z/y = 1 - \frac{n}{\theta}$ in Home and $z^*/y^* = 1 - \frac{1-n}{\theta^*}$ in Foreign). Subtract the approximation of (A33) from the approximation of (A29), using $d F_t^* = -\frac{n}{1-n} d F_t$, $d = \beta$, $CP = py = \alpha e C^* P^* = \alpha e q^* y^*$ where $\alpha = (\theta/\theta^*)^{\frac{1}{2\rho-1}}$ from above. Also use (A27) and (A28). The resulting equation is

$$\hat{e}_t = \frac{(1-\chi) \left(\hat{C}_t - \hat{C}_t^* \right) + \frac{(1-n+\alpha n)\beta d F_t}{(1-n)PC}}{(1-\chi)s + (1-\chi)(\rho-1+\chi\rho)(1-s)} \quad (\text{A37})$$

Approximations of long-run market clearing and optimality conditions are required in order to express the current account term $d F_t$ in (A37) as dependent on exogenous variables. The following equations need to be log-linearized and combined

$$P_{t+1}C_{t+1} + d_{t+1}F_{t+1} = p_{t+1}y_{t+1} + F_t \quad (\text{A38})$$

$$P_{t+1}^*C_{t+1}^* + \frac{d_{t+1}}{e_{t+1}}F_{t+1}^* = q_{t+1}^*y_{t+1}^* + \frac{F_t^*}{e_t} \quad (\text{A39})$$

$$x_{t+1} = \left(\frac{p_{t+1}}{P_{t+1}} \right)^{-\rho} nC_{t+1} \quad (\text{A40})$$

$$z_{t+1} = \frac{1}{1-\tau} \left(\frac{\frac{1}{1-\tau} \frac{p_{t+1}}{e_{t+1}}}{P_{t+1}^*} \right)^{-\rho} (1-n)C_{t+1}^* \quad (\text{A41})$$

$$\frac{1}{1-h_{t+1}} = \frac{\rho-1}{\rho\eta_{t+1}} \frac{p_{t+1}}{P_{t+1}C_{t+1}} \quad (\text{A42})$$

$$x_{t+1}^* = \left(\frac{q_{t+1}^*}{P_{t+1}^*} \right)^{-\rho} (1-n)C_{t+1}^* \quad (\text{A43})$$

$$z_{t+1}^* = \frac{1}{1-\tau} \left(\frac{\frac{1}{1-\tau} e_{t+1} q_{t+1}^*}{P_{t+1}^*} \right)^{-\rho} nC_{t+1}^* \quad (\text{A44})$$

$$\frac{1}{1-h_{t+1}^*} = \frac{\rho-1}{\rho\eta_{t+1}^*} \frac{q_{t+1}^*}{P_{t+1}^*C_{t+1}^*} \quad (\text{A45})$$

Note that (12) and (13) also hold at time $t+1$. Also note that $d F_t = d F_{t+1}$ and $d F_t^* = d F_{t+1}^*$. (A42) and (A45) are the households' optimal labor supply decisions combined with the long-run versions of markups (12) and (13). As a result one yields

$$\frac{(1-n+\alpha n) d F_t}{(1-n)PC} = \frac{\sigma(\rho-\chi)}{\rho(1-\beta)} \left(\widehat{C}_{t+1} - \widehat{C}_{t+1}^* \right) + \frac{\sigma-1}{1-\beta} \left(\widehat{\eta}_{t+1} - \widehat{\eta}_{t+1}^* \right) \quad (\text{A46})$$

Now make use of the intertemporal Euler equation (A3) and its Foreign equivalent in order to derive

$$\left(\widehat{C}_{t+1} - \widehat{C}_{t+1}^* \right) = \left(\widehat{C}_t - \widehat{C}_t^* \right) - (\chi(1-s) + s) \widehat{e}_t \quad (\text{A47})$$

noting $d_t^* = d_t(e_{t+1}/e_t)$ and $\widehat{P}_{t+1} - \widehat{P}_{t+1}^* = \widehat{e}_{t+1}$ from (A23) and also using (A27) and (A28). Combine this result with the money market clearing condition (A2) and its Foreign equivalent to yield

$$(1-s)(1-\chi)\widehat{e}_t = \left(\widehat{M}_t - \widehat{M}_t^* \right) - \left(\widehat{C}_t - \widehat{C}_t^* \right) - \frac{\beta}{1-\beta} (\widehat{e}_t - \widehat{e}_{t+1}) \quad (\text{A48})$$

From (A22) note that

$$\widehat{e}_{t+1} = \left(\widehat{M}_{t+1} - \widehat{M}_{t+1}^* \right) - \left(\widehat{C}_{t+1} - \widehat{C}_{t+1}^* \right) \quad (\text{A49})$$

and recall the assumption that monetary and productivity shocks are permanent ($\widehat{M}_t = \widehat{M}_{t+1}$, $\widehat{M}_t^* = \widehat{M}_{t+1}^*$, $\widehat{\eta}_t = \widehat{\eta}_{t+1}$ and $\widehat{\eta}_t^* = \widehat{\eta}_{t+1}^*$). Combine (A37) and (A46)-(A49) to obtain (15).

The current account term dF_t in (A46) can be expressed as a function of exogenous variables by using (A37), (A46) and (A47)

$$\frac{(1-n+\alpha n)dF_t}{(1-n)PC} = \frac{\frac{1}{1-\beta}}{\frac{\sigma\beta}{1-\beta} + \frac{\rho-\rho\chi}{\rho-\chi}} \left[\sigma(\rho-1)(1-\chi^2)(1-s)\widehat{e}_t + \frac{(\sigma-1)(\rho-\rho\chi)}{\rho-\chi} (\widehat{\eta}_t - \widehat{\eta}_t^*) \right] \quad (\text{A50})$$

where \widehat{e}_t is given in (15) as a function of exogenous variables.

In addition to (A46), \widehat{C}_{t+1} and \widehat{C}_{t+1}^* as the long-run consumption movements can also be expressed separately with the help of the log-linear approximations of (A38)-(A45) as

$$\begin{aligned} \widehat{C}_{t+1} &= \frac{\rho}{(\rho-1)(\rho-\chi)} \left[\left\{ \rho - \frac{1-n}{\theta^*} - \frac{\alpha n}{1-n} \left(1 - \frac{n}{\theta} \right) \right\} \frac{1}{\sigma} \frac{(1-\beta)dF_t}{PC} - \frac{\sigma-1}{\sigma} \left\{ \left(\rho - \frac{1-n}{\theta^*} \right) \widehat{\eta}_t + \left(1 - \frac{n}{\theta} \right) \widehat{\eta}_t^* \right\} \right] \\ \widehat{C}_{t+1}^* &= \frac{\rho}{(\rho-1)(\rho-\chi)} \left[\left\{ 1 - \frac{1-n}{\theta^*} - \frac{\alpha n}{1-n} \left(\rho - \frac{n}{\theta} \right) \right\} \frac{1}{\sigma} \frac{(1-\beta)dF_t}{PC} - \frac{\sigma-1}{\sigma} \left\{ \left(1 - \frac{1-n}{\theta^*} \right) \widehat{\eta}_t + \left(\rho - \frac{n}{\theta} \right) \widehat{\eta}_t^* \right\} \right] \end{aligned}$$

The long-run output movements also follow from the log-linearizations of (A38)-(A45) as

$$\begin{aligned} \widehat{h}_{t+1} &= \frac{\rho\eta}{\rho-1+\rho\eta} \left(-\frac{(1-\beta)dF_t}{PC} - \widehat{\eta}_t \right) \\ \widehat{h}_{t+1}^* &= \frac{\rho\eta}{\rho-1+\rho\eta} \left(\frac{\alpha n}{1-n} \frac{(1-\beta)dF_t}{PC} - \widehat{\eta}_t^* \right) \end{aligned}$$

The short-run consumption movements can be derived by log-linearizing and combining (A2) and its Foreign equivalent, both for periods t and $t+1$, as well as (A3) and its Foreign equivalent, resulting in

$$\widehat{C}_t^w = \widehat{M}_t^w - \widehat{P}_t^w$$

where $\widehat{X}_t^w \equiv n\widehat{X}_t + (1-n)\widehat{X}_t^*$. \widehat{P}_t^w can be replaced by exogenous variables via (15), (A27) and (A28). Then \widehat{C}_t and \widehat{C}_t^* can be computed as

$$\widehat{C}_t = \widehat{C}_t^w + (1-n)(\widehat{C}_t - \widehat{C}_t^*) \quad (\text{A51})$$

$$\widehat{C}_t^* = \widehat{C}_t^w - n(\widehat{C}_t - \widehat{C}_t^*) \quad (\text{A52})$$

where $(\widehat{C}_t - \widehat{C}_t^*)$ follows from combining (15), (A46), (A47) and (A50).

Finally, the short-run output movements \widehat{h}_t and \widehat{h}_t^* can be derived as follows. Note that

$$\begin{aligned} h_t &= x_t + sz_t^{LCP} + (1-s)z_t^{PCP} \\ h_t^* &= x_t^* + sz_t^{*LCP} + (1-s)z_t^{*PCP} \end{aligned}$$

Log-linearize these two equations using (A30)-(A32) and (A34)-(A36). The results are

$$\begin{aligned} \widehat{h}_t &= \frac{n}{\theta} \left(\rho \widehat{P}_t + \widehat{C}_t \right) + \left(1 - \frac{n}{\theta} \right) \left(\rho \widehat{P}_t^* + \widehat{C}_t^* \right) + (1-s)\rho \left(1 - \frac{n}{\theta} \right) \widehat{e}_t \\ \widehat{h}_t^* &= \frac{1-n}{\theta^*} \left(\rho \widehat{P}_t^* + \widehat{C}_t^* \right) + \left(1 - \frac{1-n}{\theta^*} \right) \left(\rho \widehat{P}_t + \widehat{C}_t \right) - (1-s)\rho \left(1 - \frac{1-n}{\theta^*} \right) \widehat{e}_t \end{aligned}$$

where \widehat{P}_t , \widehat{P}_t^* , \widehat{C}_t , \widehat{C}_t^* and \widehat{e}_t are given in (A27), (A28), (A51), (A52) and (15).