Trade and Uncertainty

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Abstract

We offer a new explanation as to why international trade is so volatile in response to economic shocks. Our approach combines the uncertainty shock idea of Bloom (2009) with a model of international trade, extending the idea to the open economy. Firms import intermediate inputs from home or foreign suppliers, but with higher costs in the latter case. Due to fixed costs of ordering firms hold an inventory of intermediates. We show that in response to an uncertainty shock firms optimally adjust their inventory policy by cutting their orders of foreign intermediates disproportionately strongly. In the aggregate, this response leads to a bigger contraction in international trade flows than in domestic economic activity. We confront the model with newly-compiled monthly aggregate U.S. import data and industrial production data going back to 1962, and also with disaggregated data back to 1989. Our results suggest a tight link between uncertainty and the cyclical behavior of international trade.

Keywords: Uncertainty shock, trade collapse, inventory, real options, imports, intermediates

JEL Codes: E3, F1
1. Introduction

The recent global economic crisis saw a sharp decline in output. However, the accompanying decline in international trade volumes was even sharper, and almost twice as big. Globally, industrial production fell 12% and trade volumes fell 20% in the 12 months from April 2008, shocks of a magnitude not witnessed since the 1930s (Eichengreen and O’Rourke 2010). Standard models of international macroeconomics and international trade fail to account for the severity of the event now known as the Great Trade Collapse.

In this paper, we attempt to explain why international trade is so volatile in response to economic shocks, in the recent crisis as well as in prior episodes. On the theoretical side, we combine the uncertainty shock concept due to Bloom (2009) with a model of international trade. Bloom’s real options approach is motivated by high-profile events that trigger an increase in uncertainty about the future path of the economy, for example the 9/11 terrorist attacks or the collapse of Lehman Brothers. In the wake of such events, firms adopt a ‘wait-and-see’ approach, slowing down their hiring and investment activities. Bloom shows that bouts of heightened uncertainty can be modeled as second-moment shocks to demand or productivity and that these events typically lead to sharp recessions. Once the degree of uncertainty subsides, firms revert to their normal hiring and investment patterns, and the economy recovers.

We extend the uncertainty shock approach to the open economy. In contrast to Bloom’s (2009) closed-economy set-up, we develop a theoretical framework in which firms import intermediate inputs from foreign or domestic suppliers. This structure is motivated by the observation that a large fraction of international trade now consists of capital-intensive intermediate goods such as car parts and electronic components or capital investment goods, a feature of the global production system which has taken on increasing importance in recent decades.
In our model, due to fixed costs of ordering associated with transportation, firms hold an inventory of intermediate inputs, but the costs are larger for foreign inputs. Following the inventory model with time-varying uncertainty by Hassler (1996), we show that in response to a large uncertainty shock in business conditions, whether to productivity or the demand for final products, firms optimally adjust their inventory policy by cutting their orders of foreign intermediates more strongly than orders for domestic intermediates. In the aggregate, this differential response leads to a bigger contraction and subsequently a stronger recovery in international trade flows than in domestic trade. Thus, international trade exhibits more volatility than domestic economic activity. In a nutshell, uncertainty shocks magnify the response of international trade, given the differential cost structure.

Our model generates some additional testable predictions. First, the magnification effect is increased by larger fixed costs of ordering. Intuitively, the larger the fixed costs of ordering, the more reluctant firms are to order intermediate inputs from abroad if uncertainty rises. This is a testable hypothesis to the extent that fixed costs vary across domestic and foreign orders.

Second, the magnification effect is muted for industries characterized by high depreciation rates. Perishable goods are a case in point since they have extremely high depreciation rates. The fact that such goods have to be ordered frequently means that importers have little choice but to keep ordering them frequently even if uncertainty rises. Conversely, durable goods can be considered as the opposite case of very low depreciation rates. Our model predicts that for those goods we should expect the largest degree of magnification in response to uncertainty shocks. Intuitively, the option value of waiting is most easily realized by delaying orders for durable goods. We find strong evidence of this pattern in the data when we examine the cross-industry response of imports to elevated uncertainty.
In sum, our model leads to various predictions in a unified framework. In contrast to conventional static trade models such as the gravity equation, we focus on the dynamic response of international trade. In addition, we highlight second-moment shocks and thus move beyond the first-moment shocks traditionally employed in the literature, such as shocks to productivity or trade costs. Our approach is relevant for researchers and policymakers alike who seek to understand the recovery process in response to the Great Recession, and may also be relevant for understanding historical events like the Great Depression. It could also help account for the response of international trade in future economic crises.

On the empirical side, we confront the model with high-frequency monthly U.S. import and industrial production data, some of it new and hand-collected, going back to 1962. Our results suggest a tight link between uncertainty shocks as identified by Bloom (2009) and the cyclical behavior of international trade. That is, the behavior of trade can be well explained with standard uncertainty measures such as a VXO stock market volatility index. Bloom (2009) identifies 17 high-volatility episodes since the early 1960s such as the assassination of JFK, the 1970s oil price shocks, the Black Monday market crash of October 1987, the 1998 bailout of Long-Term Capital Management, 9/11, and the collapse of Lehman Brothers in September 2008. As Bloom (2009) shows, these high-volatility episodes are strongly correlated with alternative indicators of uncertainty.

In particular, we argue that the Great Trade Collapse of 2008/09 can to a large extent be explained by the exceptional degree of uncertainty triggered by subprime lending and rising further up to, and especially after, the collapse of Lehman Brothers. According to our empirical results, the unusually large decline in trade is thus a response to the unusually large increase in uncertainty at the time. Although it stands out quantitatively, we show that qualitatively the Great

\[\text{\footnotesize{\cite{Bloom2009}}\text{Similarly, Bloom, Bond and Van Reenen (2007) provide empirical evidence that fluctuations in uncertainty can lead to quantitatively large adjustments of firms’ investment behavior.}}\]
Trade Collapse is quite comparable to previous post-World War II contractions in international trade. In fact, our aim is to empirically account for trade recessions more generally, not only for the Great Trade Collapse. In addition, we confirm the cross-industry predictions coming from our theoretical model.

We are certainly not the first authors to consider general uncertainty and real option values in the context of international trade, but so far the literature has not examined the role of uncertainty shocks in an open-economy model of inventory investment. For example, Baldwin and Krugman (1989) adopt a real options approach to explain the hysteresis of trade in the face of large exchange rate swings but their model only features standard first-moment shocks. More recently, the role of uncertainty has attracted new interest in the context of trade policy and trade agreements (Handley 2012; Handley and Limão 2012; Limão and Maggi 2013). Similar to our approach, Taglioni and Zavacka (2012) empirically investigate the relationship between uncertainty and trade for a panel of countries with a focus on aggregate trade flows. But, as they do not provide a theoretical mechanism, they do not speak to variation across industries.

The trade collapse of 2008/09 has been documented by various authors (see Baldwin 2009 for a collection of approaches, and Bems, Johnson and Yi 2013 for a survey). Eaton, Kortum, Neiman, and Romalis (2011) develop a structural model of international trade in which the decline in trade can be related to first-moment shocks, namely a collapse in the demand for tradable goods, and an increase in trade frictions, both of which originate outside the model. They find that a collapse in demand explains the vast majority of declining trade. Our approach


\footnote{Leibovici and Waugh (2012) show that the increase in implied trade frictions can be rationalized by a model with time-to-ship frictions such that agents need to finance future imports upfront (similar to a cash-in-advance technology) and become less willing to import in the face of a negative income shock.}
is different in that we explicitly model the collapse in demand by considering second-moment uncertainty shocks. Firms react to the uncertainty by adopting a ‘wait-and-see’ approach, and we do not require an increase in trade frictions to account for the excess volatility of trade.

Our approach is consistent with the view that trade frictions did not materially change in the recent crisis. Evenett (2010) and Bown (2011) find that protectionism was contained during the Great Recession. This view is underlined by Bems, Johnson, and Yi (2013). More specifically, Kee, Neagu, and Nicita (2013) find that less than two percent of the Great Trade Collapse can be explained by a rise in tariffs and antidumping duties. Bown and Crowley (2013) find that compared to previous downturns, during the Great Recession governments notably refrained from imposing temporary trade barriers against partners who experienced economic difficulties.

Examining Belgian firm-level data during the 2008/09 recession, Behrens, Corcos, and Mion (2013) find that most changes in international trade across trading partners and products occurred at the intensive margin, while trade fell most for consumer durables and capital goods. Similarly, Bricongne, Fontagné, Gaulier, Taglioni, and Vicard (2012) confirm the overarching importance of the intensive margin for French firm-level export data. Levchenko, Lewis, and Tesar (2010) stress that sectors with goods used as intermediate inputs experienced substantially bigger drops in international trade. Likewise, Bems, Johnson, and Yi (2011) confirm the important role of trade in intermediate goods. These findings are consistent with our modeling approach.

Our model is cast in terms of real variables, and we do not model monetary effects and prices. This modeling strategy is supported by the empirical regularity documented by Gopinath, Itskhoki, and Neiman (2012) showing that prices of differentiated manufactured goods (both durables and nondurables) hardly changed during the trade collapse of 2008/09. They conclude that the sharp
decline in the value of international trade in differentiated goods was “almost entirely a quantity phenomenon.” We therefore focus on modeling real variables.\footnote{In contrast, prices of non-differentiated manufactures decreased considerably. In the empirical part of the paper, however, we most heavily rely on differentiated products.}

Amiti and Weinstein (2011) and Chor and Manova (2012) highlight the role of financial frictions and the drying up of trade credit. However, based on evidence from Italian manufacturing firms Guiso and Parigi (1999) show that the negative effect of uncertainty on investment cannot be explained by liquidity constraints. Paravisini, Rappoport, Schnabl, and Wolfenzon (2011) find that while Peruvian firms were affected by credit shocks, there was no significant difference between the effects on exports and domestic sales. We do not rely on credit frictions, but such effects may be complementary to our approach.

Engel and Wang (2011) point out the fact that the composition of international trade is tilted towards durable goods. Building a two-sector model in which only durable goods are traded, they can replicate the higher volatility of trade relative to general economic activity. Instead, we relate the excess volatility of trade to inventory adjustment in response to uncertainty shocks. As this mechanism in principle applies to any industry, compositional effects do not drive the volatility of international trade in our model.

Finally, our paper is related to Alessandria, Kaboski, and Midrigan (2010a; 2011) who rationalize the decline in international trade by changes in firms’ inventory behavior driven by a first-moment supply shock. In contrast, we focus on the role of increased uncertainty, modeled as a second-moment shock. Heightened uncertainty was arguably a defining feature of the Great Recession, and we employ an observable measure of it.\footnote{Yilmazkuday (2012) compares a number of competing explanations for the Great Trade Collapse in a unified framework. Consistent with our approach, he finds that a model with an inventory adjustment mechanism fits the data best.}

The paper is organized as follows. In section 2, to motivate our approach, we show that impulse responses to uncertainty shocks are stronger for U.S. imports
than U.S. industrial production at the aggregate level. In sections 3, 4 and 5 we outline our theoretical model, conduct comparative statics and provide theoretical simulation results. Section 6 presents the main part of our empirical evidence with disaggregated data. In section 7 we ask to what extent uncertainty shocks can empirically account for the recent Great Trade Collapse. Section 8 concludes.

2. Motivation: Uncertainty Shocks and International Trade

The world witnessed an unusually steep decline in international trade during the Great Recession of 2008/09, generally the steepest since the Great Depression. International trade plummeted by 30% or more in many cases. Some countries suffered particularly badly. For example, Japanese imports declined by about 40% from September 2008 to February 2009. In addition, the decline was remarkably synchronized across countries. Baldwin (2009, introductory chapter) notes that “all 104 nations on which the WTO reports data experienced a drop in both imports and exports during the second half of 2008 and the first half of 2009.” The synchronization hints at a common cause (Imbs 2010).

To motivate our approach, we first showcase the simplest possible evidence on the importance of uncertainty shocks for trade using aggregate data on real imports and industrial production (IP). We estimate a simple vector autoregression (VAR) with monthly data from 1962 through 2012, following the econometric specification in the seminal work of Bloom (2009) exactly, with the data used here and below as detailed in the appendix.

Figure ?? presents the VAR results for both imports and IP side by side. The impulse response functions (IRFs) are based on a one-period uncertainty shock where the Bloom uncertainty indicator increases by one unit (the indicator is an equity market index, VXO, and more details follow in the main empirical part of the paper). The bottom line is very clear from this figure. In response to
Figure 1: IRFs at aggregate level for uncertainty shocks.

Notes: Sample is 1962:1–2012:2. The quadvariate VAR ordering as in Bloom (2009) is: stock market, volatility, log employment, followed lastly by either log real imports or log IP. No rescaling of shocks. 95% confidence intervals shown. See text and appendix.

the uncertainty shock, both industrial production and imports decline. But the response of imports is considerably stronger, about 5 to 10 times as strong in its period of peak impact during year one. The response of imports is also highly statistically significant. At its peak the IRF is 3 or 4 standard errors below zero, whereas the IRF for IP is only just about 2 standard errors below zero, and only just surmounts the 95% confidence threshold.

While we will argue throughout the paper that uncertainty shocks can go a long way in explaining the behavior of international trade in recessions, we note that static gravity equations typically fail to explain the disproportionate decline in trade. They can only match the trade collapse if they incorporate increases
in bilateral trade frictions such as tariff hikes combined with a sufficiently large trade cost elasticity (Eaton, Kortum, Neiman, and Romalis 2011). However, most evidence indicates that trade policy barriers moved little during the recession (Evenett 2010; Bown 2011; Kee, Neagu, and Nicita 2013), while freight rates actually declined for most modes of shipping, given the slackening of trade flows and surplus capacity. In the absence of rising trade costs, it is similarly difficult to relate the excessive responsiveness of trade to ‘back-and-forth trade’ or ‘vertical specialization’ (Bems, Johnson, and Yi 2011). For example, if demand for final goods drops by 10%, then in the standard framework demand for intermediates typically also drops by 10% throughout the supply chain.

We therefore turn to a different model of trade with dynamic effects arising from uncertainty shocks to account for outcomes like the Great Trade Collapse.

3. A Model of Trade with Uncertainty Shocks

We build on Hassler’s (1996) setting of investment under uncertainty to construct a model of trade in intermediate goods. Following the seminal contribution by Bloom (2009) we then introduce second-moment uncertainty shocks.

Hassler’s (1996) model starts from the well-established premise that uncertainty has an adverse effect on investment. In our set-up we model ‘investment’ as firms’ investing in intermediate goods. Due to fixed costs of ordering firms build up an inventory of intermediate goods that they run down over time and replenish at regular intervals. The intermediate goods can be either ordered domestically or imported from abroad. Thus, we turn the model into an open economy.

In addition, firms face uncertainty over ‘business conditions’ (in Bloom’s terminology), which means they experience unexpected fluctuations in productivity and demand. What’s more, the degree of uncertainty varies over time. Firms might therefore enjoy periods of calm when business conditions are relatively
stable, or they might have to weather ‘uncertainty shocks’ that lead to a volatile business environment characterized by large fluctuations. Overall, this formulation allows us to model the link between production, international trade and shifting degrees of uncertainty. Hassler’s (1996) key innovation is to formally model how changes in uncertainty influence investment. His model therefore serves as a natural starting point for our analysis of uncertainty shocks.

3.1. Production and Demand

As in Bloom (2009), each firm has a Cobb-Douglas production function

\[ F(A, K, L) = AK^aL^{1-a}, \]  

where \( A \) is productivity, \( L \) is domestic labor and \( K \) is an intermediate production factor that depreciates at rate \( \delta \). Each firm faces isoelastic demand \( Q \) with elasticity \( \varepsilon \)

\[ Q = BP^{-\varepsilon}, \]  

where \( B \) is a demand shifter. As we focus on the firm’s short-run behavior, we assume that the firm takes the wage rate and the price of the intermediate production factor as given.\(^7\) We thus adopt a partial equilibrium approach to keep the model tractable.

3.2. Inventory and Trade

The input factor \( K \) is an intermediate input factor (or a composite of such inputs). As the firm has to pay fixed costs of ordering per shipment \( f \), it stores the intermediate factor as inventory and follows an \( s, S \) inventory policy. Scarf (1959)

\(^6\)Both \( K \) and \( L \) are stock variables.

\(^7\)The prices of differentiated manufactured goods in international trade were essentially unchanged during the trade collapse of 2008/09, as documented by Gopinath, Itskhoki, and Neiman (2012). Their evidence further motivates our assumption of a given input price.
shows that in the presence of such fixed costs of ordering, an \( s, S \) policy is an optimal solution to the dynamic inventory problem. We assume that the intermediate factor is either ordered from abroad or sourced domestically, leading to imports or domestic trade flows, respectively. The corresponding fixed costs are \( f_F \) and \( f_D \) with \( f_F \geq f_D > 0 \), where \( F \) stands for foreign and \( D \) for domestic.

Given the intermediate input price and the wage rate, it follows that the firm employs a constant ratio of intermediates and labor regardless of productivity fluctuations. That is, the Cobb-Douglas production function implies that the firm’s use of the intermediate factor \( K \) is proportional to output \( Q \). Similar to Hassler (1996) we assume that the firm has a target level of intermediates to be held as inventory, denoted by \( M^* \), which is proportional to both \( K \) and \( Q \). Thus, we can write

\[
m^* = c + q,
\]

where \( c \) is a constant, \( m^* \equiv \ln(M^*) \) denotes the logarithmic inventory target and \( q \equiv \ln(Q) \) denotes logarithmic output. Grossman and Laroque (1990) show that such a target level can be rationalized as the optimal solution to a consumption problem in the presence of adjustment costs.

In our context the target level can be similarly motivated if it is costly for the firm to adjust its level of production up or down.

We follow Hassler (1996) in modeling the dynamic inventory problem. In particular, we assume a quadratic loss function that penalizes deviations \( z \) from the target \( m^* \) as \( \frac{1}{2}z^2 \) with \( z \equiv m - m^* \). Clearly, in the absence of ordering costs

\[8\] Guided by empirical evidence we do not model firms’ switching from a foreign to a domestic supplier, or vice versa. Thus, the source of the intermediate good is exogenous for the firm. However, we provide comparative statics on \( f_F \) and \( f_D \). See sections 4.3 and 5 for details.

\[9\] In their model consumers have to decide how much of a durable good they should hold given that they face fluctuations in their wealth. Adjustment is costly due to transaction costs. The optimal amount of the durable good turns out to be proportional to their wealth.

\[10\] The loss associated with a negative deviation could be seen as the firm’s desire to avoid a stockout, while the loss associated with a positive deviation could be interpreted as the firm’s desire to avoid excessive storage costs.
the firm would continuously set \( m \) equal to the target \( m^* \). However, since we assume positive ordering costs \( (f > 0) \), the firm faces a trade-off of balancing the fixed costs on the one hand and the costs of deviating from the target on the other. Changes in inventory are brought about whenever the firm pays the fixed costs \( f \) to adjust \( m \) (and costs are higher when the input is sourced from abroad).

We solve for the optimal solution to this inventory problem subject to a stochastic process for output \( q \). The optimal control solution can be characterized as follows: when the deviation of inventory \( z \) reaches a lower trigger point \( s \), the firm orders the amount \( \phi \) so that the inventory rises to a return point of deviation \( S = s + \phi \). Formally, we can state the problem as

\[
\min_{\{I_t, z_t\}} \left\{ E_0 \int_0^\infty e^{-rt} \left( \frac{1}{2} z_t^2 + I_t f \right) dt \right\}
\]

subject to

\[
\begin{align*}
z_0 &= x; \\
z_{t+dt} &= \begin{cases} 
\text{free} & \text{if } m_t \text{ is adjusted}, \\
z_t - \delta dt - dq & \text{otherwise}; 
\end{cases} \\
I_t dt &= \begin{cases} 
1 & \text{if } m_t \text{ is adjusted}, \\
0 & \text{otherwise}. 
\end{cases}
\end{align*}
\]

\( I_t \) is a dummy variable that takes on the value 1 whenever the firm adjusts \( m_t \) by paying \( f, r > 0 \) is a constant discount rate, and \( \delta > 0 \) is the depreciation rate for the intermediate so that \( dK_t/K = \delta dt \). Note that the intermediate input only depreciates if used in production, not if it is merely in storage as inventory.\(^{11}\)

\(^{11}\)In our trade and production data at the 4-digit industry level, examples of the intermediate factor \( K \) include ‘electrical equipment’, ‘engines, turbines, and power transmission equipment’, ‘communications equipment’ and ‘railroad rolling stock.’ We can consider the firm described in our model as ordering a mix of such products. We therefore think of a situation where inventories are essentially a factor of production, for instance spare parts for when machines break down (Ramey 1989).
3.3. Business Conditions with Time-Varying Uncertainty

Due to market clearing output can move because of shifts in productivity $A$ in equation (??) or demand shifts $B$ in equation (??). Like Bloom (2009), we refer to the combination of supply and demand shifters as business conditions. Specifically, we assume that output $q$ follows a stochastic marked point process that is known to the firm. With an instantaneous probability $\lambda/2$ per unit of time and $\lambda > 0$, $q$ shifts up or down by the amount $\varepsilon$:

$$q_{t+dt} = \begin{cases} 
q_t + \varepsilon & \text{with probability } (\lambda/2)dt, \\
q_t & \text{with probability } 1 - \lambda dt, \\
q_t - \varepsilon & \text{with probability } (\lambda/2)dt.
\end{cases}$$

The shock $\varepsilon$ can be interpreted as a sudden change in business conditions. Through the proportionality between output and the target level of inventory embedded in equation (??), a shift in $q$ leads to an updated target inventory level $m^*$. Following Hassler (1996) we assume that $\varepsilon$ is sufficiently large such that it becomes optimal for the firm to adjust $m^*$ sufficiently to lead to a negative deviation $z$ that reaches below the lower trigger point $s$. As a result the firm restocks $m$. Vice versa, a negative shock reduces $m^*$ sufficiently such that $z$ reaches above the upper trigger point and the firm destocks $m$.

Thus, to keep our model tractable we allow the firm to both restock and destock depending on the direction of the shock.

The arrival rate of shocks $\lambda$ is the measure of uncertainty and thus a key parameter of interest. We interpret changes in $\lambda$ as changes in the degree of uncertainty. Note that $\lambda$ determines the frequency of shocks, not the size of

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12Hassler (1996, section 4) reports that relaxing the large shock assumption, while rendering the model more difficult to solve, appears to yield no qualitatively different results.

13To keep the exposition concise we do not explicitly describe the upper trigger here and focus on the lower trigger point $s$ and the return point $S$. But it is straightforward to characterize the upper trigger point. See Hassler (1996) for details.
shocks. This feature is consistent with $\lambda$ determining the second moment of shocks, not their first moment. More specifically, as the simplest possible set-up we follow Hassler (1996) by allowing uncertainty $\lambda_\omega$ to switch stochastically between two states $\omega \in \{0, 1\}$: a state of low uncertainty $\lambda_0$ and a state of high uncertainty $\lambda_1$ with $\lambda_0 < \lambda_1$. The transition of the uncertainty states follows a Markov process

$$\omega_{t+dt} = \begin{cases} \omega_t & \text{with probability } 1 - \gamma_\omega dt, \\ \bar{\omega}_t & \text{with probability } \gamma_\omega dt, \end{cases}$$

(6)

where $\bar{\omega}_t = 1$ if $\omega_t = 0$, and vice versa. The probability of switching the uncertainty state in the next instant $dt$ is therefore $\gamma_\omega dt$, with the expected duration until the next switch given by $\gamma_\omega^{-1}$.

Below we will choose parameter values for $\lambda_0$, $\lambda_1$, $\gamma_0$ and $\gamma_1$ that are consistent with uncertainty fluctuations as observed over the past few decades.\textsuperscript{14} The firm knows the parameters of the stochastic process described by (??) and (??) and takes them into account when solving its optimization problem (??)\textsuperscript{15}

3.4. Solving the Inventory Problem

The Bellman equation for the inventory problem is

$$V(z_t, \omega_t) = \frac{1}{2} z_t^2 dt + (1 - r dt) E_t V(z_{t+dt}, \omega_{t+dt}).$$

(7)

The cost function $V(z_t, \omega_t)$ at time $t$ in state $\omega_t$ thus depends on the instantaneous loss element from the minimand (??), $z_t^2 dt/2$, as well as the discounted expected

\textsuperscript{14}Overall, the stochastic process for uncertainty is consistent with Bloom’s (2009). In his setting uncertainty also switches between two states (low and high uncertainty) with given transition probabilities. But he models uncertainty as the time variation of the volatility of a geometric random walk.

\textsuperscript{15}When we simulate the model in section 5, we consider a large number of firms that are identical apart from receiving idiosyncratic shocks. Those firms do not behave strategically, and there are no self-fulfilling bouts of uncertainty.
cost at time \( t + dt \). The second term can be further broken down as follows:

\[
E_t V(z_{t+dt}, \omega_{t+dt}) = V(z_{t}, \omega_{t}) + \lambda_{\omega} dt \{ V(S_{\omega}, \omega_{t}) + f - V(z_{t}, \omega_{t}) \} + \gamma_{\omega} dt \{ V(z_{t}, \bar{\omega}) - V(z_{t}, \omega_{t}) \},
\]

where \( V_z \) denotes the derivative of \( V \) with respect to \( z \). The expected cost at time \( t + dt \) thus takes into account the cost of depreciation over time through the term involving \( \delta \). It also captures the probability \( \lambda_{\omega} dt \) of a shock hitting the firm’s business conditions (in which case the firm would pay the ordering costs \( f \) to return to point \( S_{\omega} \)), as well as the probability \( \gamma_{\omega} dt \) that the uncertainty state switches from \( \omega_t \) to \( \bar{\omega}_t \).

Equations (8) and (9) form a system of two differential equations for the two possible states \( \omega_t \) and \( \bar{\omega}_t \). Following Hassler (1996) we show in the technical appendix how standard stochastic calculus techniques lead to a solution for the system. We have to use numerical methods to obtain values for the four main endogenous variables of interest: the bounds \( s_0 \) and \( S_0 \) for the state of low uncertainty \( \lambda_0 \), and the bounds \( s_1 \) and \( S_1 \) for the state of high uncertainty \( \lambda_1 \). It turns out that in either state, the cost function \( V \) reaches its lowest level at the respective return point \( S_{\omega} \). This point represents the level of inventory the firm ideally wants to hold given the expected outlook for business conditions and given it has just paid the fixed costs \( f \) for adjusting its inventory.\footnote{It would not be optimal for the firm to return to a point at which the cost function is above its minimum. The intuition is that in that case, the firm would on average spend less time in the lowest range of possible cost values.}

As in Hassler (1996), the following condition can be derived from the Bellman equation:\footnote{For details of the derivation see Hassler (1996, appendix 2).}

\[
\frac{1}{2} \left( s_{\omega}^2 - S_{\omega}^2 \right) = (r + \lambda_{\omega}) f + \gamma_{\omega} \{ f - (V(s_{\omega}, \bar{\omega}) - V(S_{\omega}, \bar{\omega})) \} > 0. \quad (9)
\]
This expression can be shown to be strictly positive since $(r + \lambda_\omega) f > 0$ and 
\[ \gamma_\omega \left\{ f - (V(s_\omega, \overline{w}_t) - V(S_\omega, \overline{w}_t)) \right\} \geq 0. \]
This last non-negativity result holds because the smallest value of $V$ can always be reached by paying the fixed costs $f$ and stocking up to $S_\omega$. That is, for any $z_t$, the cost value $V(z_t, \overline{w}_t)$ can never exceed the minimum value $V(S_\omega, \overline{w}_t)$ plus $f$. It therefore also follows that $V(s_\omega, \overline{w}_t)$ can never exceed $V(S_\omega, \overline{w}_t) + f$, i.e., $V(s_\omega, \overline{w}_t) \leq V(S_\omega, \overline{w}_t) + f$.

Recall that the lower trigger point $s_\omega$ is expressed as a deviation from the target level $m^*$. We therefore have $s_\omega < 0$. Conversely, the return point $S_\omega$ is always positive, $S_\omega > 0$. The fact that expression (??) is positive implies $|s_\omega| > S_\omega$, i.e., the lower trigger point is further from the target than the return point. Why does this asymmetry arise? Intuitively, in the absence of uncertainty the firm would stock as much inventory as to be at the target value on average. That is, its inventory would be below and above the target exactly half of the time, with the lower trigger point and return point equally distant from the target. However, in the presence of uncertainty this symmetry is no longer optimal. There is now a positive probability that output $q$ gets hit by a shock according to equation (??). Whenever a shock hits, the firm adjusts its inventory to the return point $S_\omega$\(^{18}\). If the return point were the same distance from the target as the lower trigger point, the firm’s inventory would on average be above target. To avoid this imbalance the firm chooses a return point that is relatively close to the target\(^{19}\).

### 4. Time-Varying Uncertainty and Inventory Behavior

The main purpose of this section is to explore how the firm endogenously changes its $s, S$ bounds in response to increased uncertainty. Our key result is that the firm lowers the bounds in response to increased uncertainty. In addition, we are

\(^{18}\)Recall from section 3.3 that for tractability the shock $\varepsilon$ is assumed to be sufficiently large.

\(^{19}\)Although we will fill in more details in section 4, we can refer interested readers to Figure ?? where we illustrate the difference between no uncertainty (cases 1 and 2) and positive uncertainty (cases 3a and 3b).
interested in comparative statics for the depreciation rate $\delta$ and the fixed cost of ordering $f$. As explained in the preceding section, the model cannot be solved analytically. Instead, we use numerical methods.

4.1. Parameterizing the Model

We choose the same parameter values for the interest rate and rate of depreciation as Bloom (2009), i.e., $r = 0.065$ and $\delta = 0.1$ per year. The interest rate value corresponds to the long-run average for the U.S. firm-level discount rate. Based on data for the U.S. manufacturing sector from 1960 to 1988, Nadiri and Prucha (1996) estimate depreciation rates of 0.059 for physical capital and 0.12 for R&D capital. As reported in their paper, those are somewhat lower than estimates by other authors. We therefore take $\delta = 0.1$ as a reasonable baseline value.

For the stochastic uncertainty process described by equations (??) and (??) we choose parameter values that are consistent with Bloom’s (2009) data on stock market volatility. In his Table II he reports that an uncertainty shock has an average half-life of two months. This information can be expressed in terms of the transition probabilities in equation (??) with the help of a standard process of exponential decay for a quantity $D_t$:

$$D_t = D_0 \exp(-gt).$$

Setting $t$ equal to 2/12 years yields a rate of decay $g = 4.1588$ for $D_t$ to halve. The decaying quantity $D_t$ in that process can be thought of as the number of discrete elements in a certain set. We can then compute the average length of time that an element remains in the set. This is the mean lifetime of the decaying quantity, and it is simply given by $g^{-1}$. It corresponds to the expected duration of the high-uncertainty state, $\gamma_1^{-1}$, so that $\gamma_1 = g = 4.1588$. Thus, the average duration of the high-uncertainty period follows as $4.1588^{-1} = 0.2404$ years.
Bloom (2009) furthermore reports 17 uncertainty shocks in 46 years. Hence, an uncertainty shock arrives on average every $46/17 = 2.7059$ years. Given the duration of high-uncertainty periods from above, this implies an average duration of low-uncertainty periods of $2.7059 - 0.2404 = 2.4655$ years. It follows $\gamma_0 = 2.4655^{-1} = 0.4056$.

The uncertainty term $\lambda \, dt$ in the marked point process (?) indicates the probability that output is hit in the next instant by a supply or demand shock that is sufficiently large to shift the target level of inventory. Thus, the expected length of time until the next shock is $\lambda^{-1}$. It is difficult to come up with an empirical counterpart of the frequency of such shocks since they are unobserved. For the baseline level of uncertainty we set $\lambda_0 = 1$, which implies that the target level of inventory is adjusted on average once a year. This value can therefore be interpreted as an annual review of inventory policy.

However, we note that our results are not very sensitive to the $\lambda_0$ value. In our baseline specification we follow Bloom (2009, Table II) by doubling the standard deviation of business conditions in the high-uncertainty state. This corresponds to $\lambda_1 = 4\lambda_0 = 4^{20}$ In the comparative statics below we also experiment with other values for $\lambda_1$. An uncertainty shock is defined as a sudden shift from $\lambda_0$ to $\lambda_1$.

Finally, we need to find an appropriate value for the fixed costs of ordering, $f_F$ and $f_D$. Based on data for a U.S. steel manufacturer, Alessandria, Kaboski, and Midrigan (2010b) report that “domestic goods are purchased every 85 days, while foreign goods are purchased every 150 days.” To match the behavior of foreign import flows we set $f_F$ to ensure that the interval between orders is on average 150 days in the low-uncertainty state. In the model the interval between orders corresponds to the normalized bandwidth, $(S_0 - s_0)/\delta$. In the case of $f_F$ we set it equal to 150 days, or $150/365$ years.

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20 For a given $\lambda$, the conditional variance of process (?) is proportional to $\lambda$ so that the standard deviation is proportional to the square root of $\lambda$. Thus, we quadruple $\lambda_0$ to double the standard deviation.

21 In the model the interval between orders corresponds to the normalized bandwidth, $(S_0 - s_0)/\delta$. In the case of $f_F$ we set it equal to 150 days, or $150/365$ years.
imply \( f_D = 0.00001057 \). These fixed costs differ by a large amount (by a factor of about 5.5), and that difference might seem implausibly large. However, we show in the simulation section that quantitatively, we can obtain large declines in trade flows with values for \( f_F \) that are not as high as in our baseline specification. That is, we are able to obtain a large decline in trade flows for a ratio of \( f_F / f_D \) that is lower than implied by the above values.

4.2. A Rise in Uncertainty

Given the above parameter values we solve the model numerically. Figure ?? illustrates the change in \( s, S \) bounds in response to rising uncertainty. The vertical scale indicates the percentage deviation from the target \( m^* \). Note that there are two sets of \( s, S \) bounds: one set for the low-uncertainty state 0, and the other for the high-uncertainty state 1. The level of low uncertainty is fixed at \( \lambda_0 = 1 \) but the level of high uncertainty \( \lambda_1 \) varies on the horizontal axis (as our baseline value we will use \( \lambda_1 = 4 \) in later sections). At \( \lambda_0 = \lambda_1 = 1 \) the bounds for the two states by construction coincide. As the \( s, S \) bounds are endogenous, all of them in principle shift in response to rising values of \( \lambda_1 \). But clearly, the bounds for the low-uncertainty state are essentially not affected by rising values of \( \lambda_1 \).

Two observations stand out. First, the lower trigger point always deviates further from the target than the return point. This is true for both states of uncertainty, i.e., \(|s_0| > S_0\) and \(|s_1| > S_1\). As explained in the context of equation (??), in the presence of uncertainty a symmetric band around the target, i.e., \(|s_\omega| = S_0\), would not be optimal. The reason is that with uncertainty, there is a positive probability of the firm’s output getting hit by a shock, leading the firm to adjust its inventory to the return point. Thus, the higher the shock probability, the more frequently the firm would adjust its inventory above target. To counteract this tendency it is optimal for the firm to set the return point closer to the target compared to how close the lower trigger point is to the target.
Second, the bounds for the high-uncertainty state decrease with the extent of uncertainty, i.e., $\partial S_1 / \partial \lambda_1 < 0$ and $\partial s_1 / \partial \lambda_1 < 0$. The intuition for the drop in the return point $S_1$ is the same as above – increasing uncertainty means more frequent adjustment so that $S_1$ needs to be lowered to avoid excessive inventory holdings. The intuition for the drop in the lower trigger point $s_1$ reflects the rising option value of waiting. Suppose the firm is facing low inventory and decides to pay the fixed costs of ordering $f$ to stock up. If the firm gets hit by a shock in the next instant, it would have to pay $f$ again. The firm could have therefore saved one round of fixed costs by waiting. Waiting longer corresponds to a lower value of $s_1$. This logic follows immediately from the literature on uncertainty and the option value of waiting (McDonald and Siegel 1986; Dixit 1989; Pindyck 1991).

Figure 2 shows that the decline in the lower trigger point $s_1$ compared to $s_0$ can be quite substantial for high degrees of uncertainty. It plots the percentage...
Figure 3: How uncertainty decreases the lower trigger point (compared to the low-uncertainty state).

Figure 3 summarizes the main qualitative results in a compact way. Case 1 depicts the (hypothetical) situation where both fixed costs $f$ and uncertainty $\lambda$ are negligible. Due to the very low fixed costs the bandwidth (i.e., the height of the box) is tiny, and due to the lack of uncertainty the $s_1$ and $S_1$ bounds are essentially symmetric around the target level $m^*$. In case 2 the fixed costs become larger,
which pushes both $s_1$ and $S_1$ further away from the target but in a symmetric way. Cases 3a and 3b correspond to the situation we consider in this paper with non-negligible degrees of uncertainty as in Figures ??–??. The uncertainty in case 3a induces two effects compared to case 2. First, both $s_1$ and $S_1$ shift down so that they are no longer symmetric around the target. Second, the bandwidth increases further (see Figure ??). A shift to even more uncertainty (case 3b) reinforces these two effects.

4.3. Comparative Statics

4.3.1 Varying the Fixed Costs of Ordering

We assume fixed costs of ordering to be lower when the intermediate input is ordered domestically, i.e., $f_D < f_F$. Figure ?? shows the effect of using the value $f_D$ from above that corresponds to an average interval of 85 days between domestic orders compared to the baseline value $f_F$ in Figure ?? that corresponds to 151 days.

Figure 4: How uncertainty increases the $s, S$ bandwidth (compared to the low-uncertainty state).
days. Lower fixed costs imply more frequent ordering and therefore allow the firm to keep its inventory closer to the target level. This means that for any given level of uncertainty, the optimal lower trigger point does not deviate as far from the target compared to the high fixed cost scenario.

### 4.3.2 Varying the Depreciation Rate

Some types of imports are inherently difficult to store as inventory, for instance food products and other perishable goods. We model this inherent difference in storability with a higher rate of depreciation of $\delta = 0.2$ compared to the baseline value of $\delta = 0.1$. In general, the larger the depreciation rate, the smaller the decreases in the lower trigger point and the return point in response to heightened uncertainty. Intuitively, with a larger depreciation rate the firm orders more frequently. The value of waiting is therefore diminished. Figure ?? graphs the percentage decline in the lower trigger point $s_1$ relative to $s_0$ for both the baseline
Figure 6: The effect of a lower fixed costs of ordering on the decrease in the lower trigger point.

depreciation rate (as in Figure ??) and the higher value.

Figure 7: The effect of a higher depreciation rate on the decrease in the lower trigger point.
5. Simulating an Uncertainty Shock

So far we have described the behavior of a single firm. We now simulate 50,000 firms that receive shocks according to the stochastic uncertainty process in equations (??) and (??). These shocks are idiosyncratic for each firm but drawn from the same distribution. The firms are identical in all other respects. We use the same parameter values as in section 4.1.

We should add that we do not model an extensive margin response, i.e., firms neither enter nor exit over the simulation period. This approach seems reasonable given that most of the changes in the value of international trade during the trade collapse of 2008/09 happened at the intensive margin (see Behrens, Corcos, and Mion 2013; Bricongne et al. 2012). Allowing for extensive margin responses would be an important avenue for future research. We conjecture that the extensive margin would amplify uncertainty shocks. Firms would likely exit in the face of higher uncertainty and enter once the recovery takes hold, thus reinforcing the effects of higher uncertainty.

5.1. A Permanent Uncertainty Shock

We simulate an uncertainty shock by permanently shifting the economy from low uncertainty $\lambda_0$ to high uncertainty $\lambda_1$. A key result from section 4.2 is that firms lower their $s, S$ bounds in response to increased uncertainty. This shift leads to a strong downward adjustment of intermediate input inventories and thus a strong decline in imports. Figure ?? plots simulated imports of intermediate goods in the economy (normalized to 1 for the average value). Given our parameterization imports decrease by about 25% in response to the shock. The decrease happens quickly, in about one month, followed by a quick recovery and in fact an overshoot (we comment on the overshoot below). As in Bloom (2009), this pattern of sharp

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22 Neither do we model firms’ switching from a foreign to a domestic supplier, or vice versa.
contraction and recovery is typical for uncertainty shocks.

While the trade collapse and recovery happen quickly in the simulation, this process takes longer in the data. For instance, during the Great Recession German imports peaked in the second quarter of 2008, rapidly declined by 32% and only returned to their previous level by the third quarter of 2011.23 Such persistence could be introduced into our simulation by staggering firms’ responses. Currently, all firms perceive uncertainty in exactly the same way and thus synchronize their reactions. It might be more realistic to introduce some degree of heterogeneity by allowing firms to react at slightly different times. In particular, firms might have different assessments as to the time when uncertainty has faded and business conditions have normalized. This would tend to stretch out the recovery of trade. Moreover, delivery lags could be introduced that vary across industries. We

23 Most high-income countries experienced similar patterns. U.S. and Japanese imports declined by 38% and 40% over that period, respectively (source: IMF, Direction of Trade Statistics).

Figure 8: Simulating the response of aggregate imports to an uncertainty shock.
abstracted from such extensions here in order to keep the model tractable.

We stress that the short-run dynamics in Figure ?? are purely driven by a second-moment shock (we discuss first-moment shocks in section 5.2). The extent of orders and imports in the long run is ultimately determined by the depreciation rate $\delta$ as intermediates depreciate over time.\footnote{As firms are equally likely to receive positive or negative shocks, the effects of restocking and destocking cancel in the aggregate.} Average aggregate orders in the long run (i.e., once the economy settles into a new steady state) are the same as before the uncertainty shock hits.

In contrast to imports, the short-run dynamics of output are not characterized by systematic fluctuations in our model. As a result of the stochastic process (??), output is driven by idiosyncratic mean-zero shocks that wash out in the aggregate. As we explain in more detail in section 5.2, if output shifted due to first-moment shocks, then demand for intermediates and thus imports would move one-for-one due to the Cobb-Douglas production function (??) and fixed input prices.\footnote{As discussed in the context of equation (??), the use of inputs is proportional to output.} For example, a ten percent decline in demand would translate into a ten percent decline in imported intermediate goods. Our framework can therefore best be interpreted as explaining the excess volatility of trade flows that arises in addition to any first moment movements, or as explaining the magnified response of trade flows relative to output.

\subsection{The Volatility Effect}

The reaction of aggregate imports can be thought of in terms of two effects, depicted in Figure ?? . The blue line (at the bottom) represents a ‘pure’ uncertainty effect, and the red line (at the top) is a volatility effect. The black line (in the middle) is the total effect as in Figure ??.

The decomposition is computed as follows. The uncertainty effect only captures the shifting down of the $s, S$ bounds (i.e., we use the lower bounds whilst
holding the degree of volatility fixed at $\lambda_0$). Once the uncertainty shock hits, firms decrease their lower trigger point such that they initially take longer to run down their inventory. This leads to a sharp drop in orders of imported intermediate inputs. Once firms approach the new lower trigger point, they start restocking. This leads to the sharp recovery in orders.

The volatility effect is an overshoot caused by the increased probability of firms receiving a shock (i.e., we switch to $\lambda_1$ from $\lambda_0$ whilst holding the $s, S$ bounds fixed).\footnote{This effect is analogous to Bloom’s ‘volatility overshoot’ (see Bloom 2009, section 4.4).} Recall that a shock $\varepsilon$ moves output symmetrically in either direction with equal probability. Suppose that all firms were exactly at the return point ($z = S$). Then the size of negative orders (induced by $z$ being pushed above the upper trigger point) and the size of positive orders (induced by $z$ being pushed below the lower trigger point) would be the same. Switching to $\lambda_1$ increases the frequency

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9.png}
\caption{Simulating the response of aggregate imports to an uncertainty shock: the total effect (baseline), the ‘pure’ uncertainty effect and the volatility effect.}
\end{figure}
of orders, but given that negative and positive orders are of the same size and of equal probability, there would be no net effect on aggregate orders. However, most firms are in fact below the return point \((z < S)\), which means that they have not stocked up in a while. Positive orders are therefore larger than negative orders, and increasing the frequency leads to a rise in aggregate orders.\(^{27}\) We stress that the volatility effect plays an important role in this simulation because we assume a permanent increase in uncertainty. It would matter less for a temporary increase.

Note that the total (baseline) effect reaches beyond the volatility effect in Figure ?? about one-and-a-half months into the period of heightened uncertainty. This happens due to the interaction of the volatility and uncertainty effects. While the volatility effect implies more frequent ordering and thus larger aggregate orders, it is reinforced by the increase in the bandwidth \((S - s)\), which entails larger restocking orders all else being equal. With a larger bandwidth in place as a result of the uncertainty effect, the volatility effect is in fact stronger compared to the scenario with no interaction as indicated by the red line.

In Figure ?? we illustrate the inventory position of the average firm. Specifically, we plot the average deviation of imported intermediates from the target level. In the steady state before the uncertainty shock hits, this deviation is essentially zero as firms on average hold precisely the amount of inventory that minimizes their loss function. After the shock has hit, their average inventories initially decline sharply as firms decrease their lower trigger point. This is driven by the uncertainty effect described above. But at the same time, the higher volatility means that firms are more likely to restock, implying a rising average deviation over time. Although this volatility effect sets in immediately, it is initially dominated

\[^{27}\text{One implication is that the upper trigger point does not matter for the size of orders. The reason is that the shock size is such that once a shock hits, there is always adjustment (i.e., the upper trigger point is always breached given a shock in that direction). Therefore, only the return point and the lower trigger point matter for the size of orders as they mark the range of inventory that the average firm holds. Since depreciation can only decrease but never increase inventory, the average firm’s inventory can never be above the return point.}\]
by the uncertainty effect. In Figure ?? firms’ inventories eventually start rising after just under a month into the period of heightened uncertainty.

5.1.2 Comparative Statics

In Figure ?? we plot the total effect of an uncertainty shock for three different values of fixed costs $f_F$. The black line corresponds to our baseline value of $f_F = 0.00005846$. The remaining two lines in grey correspond to smaller values of $f_F$. Their values are $f_F = 0.00004846$ for the dark grey line and $f_F = 0.00003846$ for the light grey line. Although the latter value is about a third smaller than the baseline value, imports still drop by over 20 percent (compared to 25 percent in the baseline scenario).

The insight is that although the trade collapse becomes less severe with smaller fixed cost values (as predicted by the theory), quantitatively the collapse is not so
Figure 11: Simulating aggregate imports with different values of fixed costs of ordering.

Sensitive to fixed costs above a certain threshold. In case of the light grey line in Figure ??, the foreign fixed cost value is only 3.6 times as large as the domestic fixed cost value ($f_D = 0.00001057$ in section 4.1, so $f_F/f_D = 3.64$). In contrast, Alessandria, Kaboski, and Midrigan (2010a) use a ratio of $f_F/f_D = 6.54$, a large difference in frictions. The reason that smaller and arguably more plausible values of $f_F$ suffice is as follows. The decline of the lower trigger point in response to an uncertainty shock (as depicted in Figure ??) is increasing but concave in $f_F$. Thus, increases in $f_F$ have a strong marginal impact when $f_F$ is low. Once $f_F$ is high, increases have a weak impact on the lower trigger point. For instance, the impact associated with the baseline value of $f_F$ makes up more than two-thirds (72 percent) of the impact associated with doubling $f_F$.

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28In their benchmark case, Alessandria, Kaboski, and Midrigan (2010a, Table 4) choose values for fixed costs of ordering that correspond to 23.88 percent of mean revenues for foreign orders and 3.65 percent of mean revenues for domestic orders.

29Given the parameterization in section 4.1, the baseline value of $f_F = 0.00005846$ is associated
In Figure 12 we plot the effect of an uncertainty shock for three different values of the depreciation rate $\delta$. The black line is for our baseline value of $\delta = 0.10$. The dark grey line corresponds to $\delta = 0.125$ and the light grey line is for $\delta = 0.15$. As predicted by the theory, higher rates of depreciation tend to entail a smaller adjustment of $s$, $S$ bands so that the decline in imports is not as pronounced.

5.2. A First-Moment Shock

In section 5.1 we only considered a second-moment shock. Whereas aggregate imports display distinct short-run dynamics, aggregate output is flat because positive and negative shocks at the firm level are of equal probability and thus exactly offset each other. The increase in volatility due to the second-moment

\[ \frac{27.7}{38.4} = 0.72. \]
Figure 13: Simulating the response of aggregate imports to a negative ten percent first-moment shock.

The idiosyncratic shocks that lead to inventory adjustment at the firm level wash out in the steadystate.\(^{30}\) In our model a negative ten percent productivity shock does not change this result.

We now consider a first-moment shock. Due to the Cobb-Douglas production function (??) and due to the assumption of fixed input prices, it follows that the optimal ratios of the production factors to output, $K/Q$ and $L/Q$, do not vary over time in our model. At the firm level, a shock to output as in the stochastic process (??), which could be driven by a supply or demand shock or their combination, therefore translates into a one-for-one movement of inputs and inventories. For instance, if a firm experiences a negative ten percent productivity shock (or demand shock), this translates into a ten percent decline in $K$ and thus a ten percent decline in imports (assuming the firm sources $K$ from abroad).

In the aggregate, it is ultimately the rate of depreciation that drives inventory behavior and thus imports in the steady state.\(^{30}\) In our model a negative ten percent...
Figure 14: U.S. real imports, IP, total factor productivity, and real GDP from 2006:Q1 to 2011:Q4.

Source: Quarterly data. TFP from the Federal Reserve Bank of San Francisco, utilization adjusted. IP is from the OECD, quarterly. Imports and GDP are from the Bureau of Economic Analysis. Log scale with units set such that the fourth quarter of 2007 is 0.

percent permanent productivity shock therefore corresponds to a ten percent cut in the depreciation rate. This is the shock we consider in Figure ?? We leave the degree of uncertainty and the $s, S$ bands unchanged at their baseline levels (i.e., as in the low-uncertainty state). The drop in demand leads to a gradual decline in aggregate imports by around ten percent in total.

We should note that in the context of our sample, we find no evidence of a large, negative U.S. productivity shock which might account for the observed trade collapse in this manner. As the dotted line in Figure ?? shows, during the Great Trade Collapse total factor productivity (TFP) in fact increased. Thus, a TFP-based explanation seems unlikely to account for the direction, let alone the severity of the Great Trade Collapse, and this (and the lack of plausible first-moment shocks to trade frictions) motivates our focus on second-moment shocks.
Alessandria, Kaboski, and Midrigan (2010a) also develop an $s, S$ inventory model with a band of inaction as in our model. However, they only consider first-moment shocks (in particular a negative supply shock) but no second moment shocks. In contrast to Figure ??, their model nevertheless generates a decline in imports that exceeds the decline in output or sales. The reason is that they treat the intermediate input as a flow variable that needs to be replaced fully every period, and that the firm has a desired inventory-to-sales ratio above 1. Once sales take a hit, a multiplier effect kicks in such that imports are reduced more than one-for-one because firms run down their high levels of inventory. In our model, however, the imported input factor is not fully absorbed in the production process. It only depreciates by rate $\delta$, which is less than 100 percent in our parameterization. We do not impose a desired inventory-to-sales ratio above 1. Instead, we generate a disproportionate decline in imports through an endogenous adjustment of $s, S$ inventory bands caused by a second-moment shock.

6. **Empirical Evidence**

To explore the relationship between uncertainty, production, and international trade we run vector autoregressions (VARs) with U.S. data. In particular, we follow the seminal work of Bloom (2009) in running a VAR to generate an impulse response function (IRF) relating the reactions of key model quantities, in this case not only industrial production but also imports, to the underlying impulses which take the form of shocks to uncertainty.

We contend that, as with the application to production, the payoffs to an uncertainty-based approach can be substantial again in the new setting we propose for modeling trade volatility. Recall that in the view of Bloom (2009, p. 627):

More generally, the framework in this paper also provides one response to the “where are the negative productivity shocks?” critique
of real business cycle theories. In particular, since second-moment shocks generate large falls in output, employment, and productivity growth, it provides an alternative mechanism to first-moment shocks for generating recessions.

The same might then be said of theories of the trade collapse that rely on negative productivity shocks, or other first-moment shocks. So by the same token, the framework in our paper provides one response to the “where are the increases in trade frictions?” objection that is often cited when standard static models are unable to otherwise explain the amplified nature of trade collapses in recessions, relative to declines in output.

Our theoretical model, and empirical evidence, can thus be seamlessly integrated with the Bloom (2009) view of uncertainty-driven recessions, whilst matching a crucial and recurrent feature of international economic experience: the highly magnified volatility of trade, which has been a focus of inquiry since at least the 1930s, and which, since the onset of the Great Recession has flared again as an object of curiosity and worry to scholars and policymakers alike.

6.1. Three Testable Hypotheses

To look ahead and quickly sum up the bottom line, our empirical results expose several new and important stylized facts, all of which are consistent with, and thus can motivate our previously described theoretical framework. Specifically we focus on testing three empirical propositions that would be implied by our theory.

- First, trade volumes do respond to uncertainty shocks, the effects are quantitatively and statistically significant, and are robust to different samples and methods. In addition, trade volumes respond much more to uncertainty shocks than does the volume of industrial production; that is to say, there is something fundamentally different about the dynamics of traded goods
supplied via the import channel, as compared to supply originating from domestic industrial production.

- Second, we confirm that these findings are true not just at the aggregate level, but also at the disaggregated level, indicating that the amplified dynamic response of traded goods is not just a sectoral composition effect.

- Third, we find that the dynamic response of traded goods to uncertainty shocks is greatest in durable goods sectors as compared to nondurable goods sectors, consistent with the theoretical model where a decrease in the depreciation parameter (interpreted as a decrease in perishability) leads to a larger response.

The following parts of this section are structured as follows. The first part briefly spells out the empirical VAR methods we employ based on Bloom (2009). The second part spells out the data we have at our disposal, some of it newly collected, to examine the differences between trade and industrial production in this framework. The subsequent parts discuss our findings on the first three testable hypotheses noted above, and we discuss the corroborative evidence in the next section, before concluding.

6.2. Computing the Responses to an Uncertainty Shock

In typical business cycle empirical work, researchers are often interested in the response of key variables, most of all output, to various shocks, most often a shock to the level of technology or productivity. The analysis of such first-moment shocks has long been a centerpiece of the macroeconomic VAR literature. The innovation of Bloom (2009) was to construct, simulate and empirically estimate a model where the key shock of interest is a second-moment shock, which is conceived of as an ‘uncertainty shock’ of a specific form. This shock amounts to
an increase in the variance, but not the mean, of a composite ‘business condition’ disturbance in the model, which can be flexibly interpreted as a demand or supply shock. For empirical purposes when the model is estimated using data on the postwar U.S., Bloom proposes that changes in the market price of the VXO index, a daily options-based implied stock market volatility for a 30-day horizon, be used as a proxy for the uncertainty shock, with realized volatility used when the VXO is not available. A plot of this series, scaled to an annualized form, and extended to 2012, is shown in Figure ??.

31 As Bloom (2009, Figure 1) notes: “Pre-1986 the VXO index is unavailable, so actual monthly returns volatilities are calculated as the monthly standard deviation of the daily S&P500 index normalized to the same mean and variance as the VXO index when they overlap from 1986 onward. Actual and VXO are correlated at 0.874 over this period. The asterisks indicate that for scaling purposes the monthly VXO was capped at 50. Uncapped values for the Black Monday peak are 58.2 and for the credit crunch peak are 64.4. LTCM is Long Term Capital Management.”
Following Bloom (2009) we evaluate the impact of uncertainty shocks using VARs on monthly data from 1962 (the same as in Bloom) to February 2012 (going beyond Bloom’s end date of June 2008). Bloom’s full set of variables, in VAR estimation order are as follows: log(S&P500 stock market index), stock-market volatility indicator, Federal Funds Rate, log(average hourly earnings), log(consumer price index), hours, log(employment), and log(industrial production). For simplicity, for the main results presented in this section, all VARs of this form are estimated using a quad-variate VAR (log stock-market levels, the volatility indicator, log employment, and lastly the industrial production or trade indicator).

6.3. Data

Many of our key variables are taken from the exact same sources as Bloom (2009): log industrial production in manufacturing (Federal Reserve Board of Governors, seasonally adjusted), employment in manufacturing (BLS, seasonally adjusted), a monthly stock-market volatility indicator as above, and the log of the S&P500 stock-market index. All variables are HP detrended using a filter value of $\lambda = 129,600$. We follow these definitions exactly as in Bloom, and full details are provided in the appendix. Collection of these data was updated to February 2012.

However, in some key respects, our data requirement are much larger. For starters, we are interested in assessing the response of trade, so we needed to collect monthly import volume data. In addition, we are interested in computing disaggregated responses of trade and industrial production (IP) in different sectors.

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32 For comparability, we follow exactly the same definitions here and we thank Nicholas Bloom for providing us with an updated series extended to 2012.

33 In terms of VAR variable ordering and variable definitions we follow Bloom (2009) exactly for comparability. As Bloom notes: “This ordering is based on the assumptions that shocks instantaneously influence the stock market (levels and volatility), then prices (wages, the consumer price index (CPI), and interest rates), and finally quantities (hours, employment, and output). Including the stock-market levels as the first variable in the VAR ensures the impact of stock-market levels is already controlled for when looking at the impact of volatility shocks.”
in the aftermath of uncertainty shocks, in an attempt to gauge whether some of
the key predictions of our theory are sustained. Thus, we needed to assemble new
monthly trade data (aggregate and disaggregate) as well as new disaggregated
monthly IP data to complement the Bloom data.

We briefly explain the provenance of these newly collected data, all of which
will also be HP filtered for use in the VARs as above.

• U.S. aggregated monthly real import volume. These data run from 1962:1
to 2012:2. After 1989, total imports for general consumption were obtained
from the USITC dataweb, where the data can be downloaded online. From
1968 to 1988 data were collected by hand from FT900 reports, where the
imports series are only available from 1968 as F.A.S. (free alongside ship)
at foreign port of export, general imports, seasonally unadjusted; the series
then change to C.I.F. (cost, insurance and freight) value available beginning
in 1974, and the definition changes to customs value in 1982. Prior to 1968
we use NBER series 07028, a series that is called “total imports, free and
dutiable” or else “imports for consumption and other”; for the 1962 to
1967 window this NBER series is a good match, as it is sourced from the
same FT900 reports as our hand-compiled series. The entire series was then
deflated by the monthly CPI.

• U.S. disaggregated monthly real imports. These data only run from 1989:1 to
2012:2. In each month total imports for general consumption disaggregated
at the 4-digit NAICS level were obtained from the USITC dataweb, where
the data can be downloaded online. All series were then deflated by the
monthly CPI. In this way 108 sector-level monthly real import series were
compiled.

• U.S. disaggregated monthly industrial production. These data only run from
1972:1 to 2012:2 at a useful level of granularity. Although aggregate IP data
are provided by the Fed going back to 1919, the sectorally disaggregated IP data only start in 1939 for 7 large sectors, with ever finer data becoming available in 1947 (24 sectors), 1954 (39 sectors) and 1967 (58 sectors). However, it is in 1972 that IP data are available using the 4-digit NAICS classification which permits sector-by-sector compatibility with the import data above. Starting in 1972 we use the Fed G.17 reports to compile sector-level IP indices, which affords data on 98 sectors at the start, expanding to 99 in 1986.

6.4. Results 1: IRFs at Aggregate Level for Trade versus IP

We begin with the simplest possible evidence on the importance of uncertainty shocks for the dynamics of trade flows, using aggregate data on real imports and industrial production.

Following Bloom (2009) exactly, a baseline quad-variate VAR is estimated for both series, which are placed last in the ordering. Ordering is stock market, volatility, log employment, followed lastly by either log real imports or log IP. Data differ from Bloom in that we have updated all series through February 2012 so as to include the response to the 2008 financial crisis. However, our results are not sensitive to this extension of the sample. The presentation also differs from Bloom in that we do not rescale the IRFs at this stage, since we are only interested in the comparative responses of internationally traded and domestically produced goods.

In Figure ?? we already presented the VAR results for both imports and IP side by side. The impulse response functions (IRFs) are based on a one-period uncertainty shock where the Bloom uncertainty indicator (i.e., VXO or its proxy) increases by one unit. The VXO, later superseded by the VIX, is the Chicago Board Options Exchange Market Volatility Index, a measure of the implied volatility of S&P 500 equity index options. It is a proxy for the market’s expectation of stock market volatility over the subsequent 30-day period.
The bottom line is very clear from this figure. In response to the uncertainty shock, both industrial production and imports decline. But the response of imports is considerably stronger, about 5 to 10 times as strong in its period of peak impact during year one. The response of imports is also highly statistically significant. At its peak the IRF is 3 or 4 standard errors below zero, whereas the IRF for IP is only just about 2 standard errors below zero, and only just surmounts the 95% confidence threshold.

These results offer prima facie confirmation of the mechanisms suggested in our theoretical model. Indeed to the extent that the Bloom (2009) result for IP has proven novel, robust, and influential, one might argue that our finding of a import response to uncertainty that is almost an order of magnitude larger is also notable, especially since it opens an obvious route towards finding an explanation for the amplification effects seen during the recent trade collapse, a puzzle where, as we have seen, no fully convincing theoretical explanation has yet been given.

However, to make that claim more solid, we must convince the reader that the theoretical mechanisms we propose are indeed at work. To do that, we delve more deeply into the dynamics of disaggregated trade and IP in the wake of uncertainty shocks. In the following we demonstrate that, taking into account cross-sectoral variations in perishability/durability and also in the intensity of downstream intermediate use, the empirical evidence closely matches our model’s predictions. We find that imports of any good are, in general, more responsive to uncertainty shocks than domestic IP, whether in broad sectors (like End Use categories), or at a much more disaggregated level (e.g., NAICS 4-digit sectors). However, the aggregate results seen above will be shown to mask substantial sectoral heterogeneity in response to uncertainty shocks. At the end we will be able to weight the responses, for both imports and IP, and compute a simulated response to the 2008 uncertainty shock aggregated across sectors. We will show that this response closely matches the observed data, with import volume falling
about twice as much as a basket of industrial production.

6.5. Results 2: IRFs with Coarse Disaggregation

Proceeding to a coarse level of disaggregation we now investigate IRFs for uncertainty shocks when trade and IP data are divided into either End Use categories (a Bureau of Economic Analysis classification) or into Market Groups (a Fed classification). The purpose is to see whether the aggregate result holds up at the sectoral level, and, if there is any departure, to see if there is any systematic variation that is yet consistent with our model’s more detailed predictions for heterogeneous goods.

Figure ?? shows IRFs for real imports disaggregated into 6 BEA 1-digit End Use categories. The response to an uncertainty shock varies considerably across these sectors, but in a manner consistent with predictions from theory. There is essentially no response for the most perishable, or least durable, types of goods in End Use category 0. These goods include foods, feeds and beverages. This response matches up with cases in our model when the depreciation parameter is set very high. In this case the response to uncertainty shocks diminishes towards zero. Responses are also weak for other nondurable consumer goods (End Use 4) and the residual category of imports not elsewhere specified (End Use 5).

In contrast, some sectors show a very large response to an uncertainty shocks, notably End Use categories 1, 2, and 3, which include industrial inputs, capital goods, and autos. These are all sectors characterized by either high durability and/or high downstream intermediate use. Again, our theory predicts that it is precisely these sectors that will experience the largest amplitude response to an uncertainty shock.

It is not possible to compare these IRFs to the corresponding response of domestic IP using the same End Use classification, since we cannot obtain IP disaggregated by End Use code. However, we can obtain both imports and IP
**Figure 16:** Import IRFs by End Use category for uncertainty shocks.

Source: Sample is 1989:1–2012:2. Imports by End Use 1-digit from USITC dataweb, deflated by CPI; all other data as in Bloom (2009), updated. Uncertainty shocks for quadivariate VARs as in Bloom (2009). Ordering is stock market, volatility, log employment, followed lastly by log real imports. Data updated through February 2012. No rescaling of shocks. 95% confidence intervals shown. See text and appendix.
the import and IP responses directly. To facilitate this, all responses are shown on
the same scale.

In panel (a) the results for imports are compatible with those above based on
the End Use categories. Here, under the Fed Market Groups the largest amplitude
responses to an uncertainty shock are seen for materials, business equipment and
consumer durables. The responses are between a 1 and 2 percent drop at peak.
The weakest response is for consumer nondurables, which shows about a 0.5
percent drop at peak, although this is barely statistically significant at the 95%
level.

By contrast, in panel (b) the results for IP are very muted indeed. Confidence
intervals are tighter, so these responses do in all cases breach the 95% confidence
interval within a range of steps. However, the magnitude of the response is
qualitatively different from imports. The consumer durables response is just
below 1 percent at peak for IP, whereas it was almost twice as large, near 1.5
percent, for imports. Materials and business equipment fall at peak by about 0.25
percent for IP, but fell about four times as much in the case of imports. Consumer
nondurables in IP are barely perturbed at all.

6.6. Results 3: IRFs with Finer Disaggregation

Our next set of results aims to study dynamic responses to uncertainty shocks
at an even finer level of disaggregation, whilst still allowing for comparability
between import and IP responses.

For these purposes we move to the 3- or 4-digit NAICS level of classification,
again sourcing the data from USITC dataweb and the Fed G.17 releases at a
monthly frequency starting in 1989. The overlap between these two sources allows
us to work with 51 individual sectors, as seen in Figure ??.

A similar pattern emerges here, consistent with previous results, whereby the
Figure 17: Import and IP IRFs by Fed Market Group for uncertainty shocks.

(a) Real imports

(b) Industrial production

Notes: Sample is 1989:1–2012:2. Imports via concordance from USITC dataweb, deflated by CPI; IP from Fed G.17; all other data as in Bloom (2009), updated. Uncertainty shocks for quadvariate VARs as in Bloom (2009). Ordering is stock market, volatility, log employment, followed lastly by either log real imports or log IP. Data updated through February 2012. No rescaling of shocks. 95% confidence intervals shown. See text.
responsiveness in any sector tends to be higher for real imports (CPI deflated) than for industrial production. There are some exceptions but these are generally to be found in only a few sectors. The bars in Figure ?? are ordered from top to bottom starting with largest negative real import response measured by the average sectoral IRF over months 1–12.

Some of the sectors are also obviously quite peculiar. One is basically a non-manufacturing sector, and not very tradable — namely logging (NAICS 1133, which is resource intensive and not highly traded apart from imports from Canada). This does fit the general pattern of imports being more volatile than domestic output, but it may reflect downstream use in the heavily procyclical construction industry (we discuss downstream use in the next section). Another oddity is tobacco manufacturing (NAICS 3122) where the response goes heavily against the prevailing pattern, with tobacco imports rising sharply after an uncertainty shock, and domestic supply basically flat. Still, this response is consistent with clinical studies showing that the use of tobacco may rise, and the ability of people to quit smoking may fall, in stressful periods of hard economic times.

Less unusual cases where the negative response of IP exceeds real imports are: Audio and video equipment manufacturing (NAICS 3343); Household and institutional furniture and kitchen cabinet manufacturing (3371); Industrial machinery manufacturing (3332); Ventilation, heating, air-conditioning, and commercial refrigeration equipment manufacturing (3334); Leather and allied product manufacturing (316); Apparel manufacturing (315); Nonmetallic mineral mining and quarrying (2123); Metalworking machinery manufacturing (3335). Still, out of 51 sectors, these are a minority. But generally, and especially for the high-response sectors where responses are significantly different from zero, the real import bar is larger and more negative than the IP bar. The essence of this pattern is revealed in Figure ?? which presents a scatter of the average real import one-year IRF on the vertical axis against the average IP one-year IRF on the horizontal axis. There
Figure 18: Import and IP IRFs compared in months 1–12.

**Notes:** Average IRF for months 1–12. Sample is 1989:1–2012:2. Imports from USITC data web, deflated by CPI; IP from Fed G.17; all other data as in Bloom (2009), updated. Uncertainty shocks for quadvariate VARs as in Bloom (2009). Ordering is stock market, volatility, log employment, followed lastly by either log real imports or log IP. Data updated through February 2012. No rescaling of shocks. 95% confidence intervals shown. See text and appendix.
**Figure 19:** Import and IP IRFs compared in months 1–12.

Notes: Average IRF for months 1–12. Sample is 1989:1–2012:2. Imports from USITC dataweb, deflated by CPI; IP from Fed G.17; all other data as in Bloom (2009), updated. Uncertainty shocks for quadvariate VARs as in Bloom (2009). Ordering is stock market, volatility, log employment, followed lastly by either log real imports or log IP. Data updated through February 2012. No rescaling of shocks. 95% confidence intervals shown. See text and appendix. The correlation of the two variables is 0.2544 (with a significance level of \( p = .0716 \)). However, the correlation falls to 0.1467 and is not statistically significant (\( p = .3092 \)) when tobacco (3122) is excluded.

is only a very weak correlation between these responses (0.25), and when the outlier tobacco sector is excluded the correlation essentially vanishes (it falls to 0.15, but is not statistically significant).

What is more striking in the figure, however, is the general asymmetry relative to the 45 degree line. Most points lie in the lower-left quadrant where both real imports and IP react negatively to an uncertainty. In that quadrant, we do find points above the diagonal, where the IP response is more negative than the real import response – but generally these deviations from the diagonal are small. In contrast, several sectors fall well below the diagonal by a significant margin, indicating a much sharper negative response of real imports compared to IP.
7. **Can the Great Trade Collapse Be Explained?**

Can our model, which takes second-moment uncertainty shocks as its main driver, provide a plausible account of the Great Trade Collapse of 2008/09? We conclude by using a simulation exercise to argue that it could.

The four months following the collapse of Lehman Brothers were characterized by particularly strong increases in uncertainty as measured by the volatility index VXO in September to December 2008, with elevated volatility persisting into the first quarter of 2009. To simulate this shock we choose to feed into the model a series of exogenous volatility shocks which generate a path of volatility similar to that observed. That is, we assume that the dynamics are driven primarily by an exogenous shock to the system from the volatility index and the subsequent endogenous response of the variables in the system.

We observe that the own-response of volatility to itself in the orthogonalized impulse response is about 3. We begin with a simulated VXO level of 20, and
then subject the process to monthly shocks of $+20, +5, +5, +5, +5, +5, +5$ starting in September 2008; the first shock takes the simulated VXO up to just over 80, and the additional shocks keep the simulated VXO very elevated for several months before the decay commences. In actuality, the VXO rises from its pre-crisis mean of about 20 to almost 90 in the last quarter of 2008, a shift of +70, and thus the simulated impulses matches the actual path of VXO quite well, as shown in Figure ??.

Given these VXO shocks, the model-implied and the actual observed responses of IP and real imports are shown in Figure ???. As can be seen, the model is capable of explaining a large fraction of the actual observed IP response, especially up to 6 months out. It is also capable of explaining most of the real import response over a similar horizon. Overall, the simulations show that our model can on average explain over three-quarters of the imports collapse out to the 12-month horizon.
But we can see that, especially in early to mid-2009, some additional factors must have been at work that are not captured by the uncertainty shock. Nonetheless, overall the comparison of the model IRFs with actual data shows that the evidence is consistent with a large fraction of the Great Trade Collapse being explicable in terms of second-moment uncertainty shocks, rather than the more conventional first-moment explanations seen in the literature to date.

8. Conclusion

Following the seminal work of Bloom (2009), we introduce second-moment uncertainty shocks into a dynamic, open-economy model. Firms import intermediate inputs and due to fixed costs of ordering store them according to an optimal $s, S$ inventory policy. We show that elevated uncertainty leads firms to shift down their $s, S$ bands. This induces a sharp contraction of international trade flows followed by a swift recovery. In contrast, output remains unaffected unless conventional first-moment shocks are introduced. Uncertainty shocks can therefore explain why trade is more volatile than domestic economic activity.

Our results offer an explanation for the Great Trade Collapse of 2008/09 and previous trade slowdowns in a way that differs from the conventional static trade models or dynamic inventory models seen before. We argue that imports and industrial production can be modeled as reacting to uncertainty shocks in theory and in practice. Such second-moment shocks are needed since the required first-moment shocks are either absent on the impulse side or insufficient on the propagation side (for plausible parameters) to explain the events witnessed. We also show that there is substantial heterogeneity in responses at the sectoral level, both for imports and industrial production, in a way consistent with the model.


This appendix shows how the solution to the system of differential equations implied by equations (??) and (??) can be found. We closely follow Hassler (1996) and refer to his appendix for further details.

We plug the expression for \(E_t V(z_{t+\Delta t}, \omega_{t+\Delta t})\) from equation (??) into equation (??). We then set \(\Delta t^2 = 0\) and divide by \(\Delta t\) to arrive at the following system of differential equations:

\[
rV(z_t, \omega_t) = \frac{1}{2} z_t^2 - \delta V_z(z_t, \omega_t) + \lambda \omega \{ V(S_\omega, \omega_t) + f - V(z_t, \omega_t) \} + \gamma \omega \{ V(z_t, \bar{\omega}_t) - V(z_t, \omega_t) \}.
\]

The set of solutions to this system is given by

\[
V(z_t, 0) = \alpha_0 + \beta_0 z_t + c_1 \exp(\rho_1 z_t) + c_2 \exp(\rho_2 z_t) + \phi_0 + \frac{1}{\Delta} \{ \lambda_1 \gamma_0 V(S_1, 1) + \lambda_0 \psi_1 V(S_0, 0) \}
\]

for the state of low uncertainty, and

\[
V(z_t, 1) = \alpha_1 + \beta_1 z_t + v_1 c_1 \exp(\rho_1 z_t) + v_2 c_2 \exp(\rho_2 z_t) + \phi_1 + \frac{1}{\Delta} \{ \lambda_1 \psi_0 V(S_1, 1) + \lambda_0 \gamma_1 V(S_0, 0) \}
\]

for the state of high uncertainty, where \(c_1\) and \(c_2\) are the integration constants. The parameters \(\psi_0, \psi_1, \Delta, \alpha_0, \alpha_1, \beta_0, \beta_1, \phi_0\) and \(\phi_1\) are given by

\[
\psi_\omega = r + \lambda_\omega + \gamma_\omega,
\]

\[
\Delta = \psi_0 \psi_1 - \gamma_0 \gamma_1,
\]

\[
\alpha_\omega = \frac{1}{\Delta} (r + \lambda_\omega + \gamma_\omega) ,
\]

\[
\beta_\omega = -\frac{\delta}{\Delta} (\psi_\omega \alpha_\omega + \gamma_\omega \alpha_\omega) ,
\]

\[
\phi_\omega = \frac{1}{\Delta} (\psi_\omega (\lambda_\omega f - \delta \beta_\omega) + \gamma_\omega (\lambda_\omega f - \delta \beta_\omega)) ,
\]
where $\bar{w} = 1$ if $\omega = 0$, and vice versa. $[v_i, 1]'$ is the eigenvector that corresponds to the eigenvalue $\rho_i$ of the matrix

$$\begin{bmatrix}
\frac{1}{\delta} & -(r + \lambda_1 + \gamma_1) & \gamma_1 \\
\gamma_0 & -(r + \lambda_0 + \gamma_0)
\end{bmatrix}$$

for $i = 1, 2$. Expressions for $V(S_0, 0)$ and $V(S_1, 1)$ can be obtained by setting $V(z_t, 0) = V(S_0, 0)$ and $V(z_t, 0) = V(S_1, 1)$ in equations (??) and (??), respectively, and then solving the two resulting equations.

Six key equations describe the solution. They are two value-matching conditions positing for each state of uncertainty that the value of the cost function at the return point must be equal to the value at the lower trigger point less the fixed ordering costs $f$:

$$V(S_0, 0) = V(s_0, 0) - f,$$
$$V(S_1, 1) = V(s_1, 1) - f.$$ 

The remaining four equations are smooth-pasting conditions:

$$V_z(S_0, 0) = 0,$$
$$V_z(s_0, 0) = 0,$$
$$V_z(S_1, 1) = 0,$$
$$V_z(s_1, 1) = 0.$$ 

These six conditions determine the six key parameters: the return points $S_0$ and $S_1$, the lower trigger points $s_0$ and $s_1$ as well as the two integration constants $c_1$ and $c_2$. Numerical methods have to be used to find them.
Empirical Appendix

Sources for Data Used in Empirical Analysis

We follow Bloom (2009, p. 630) and estimate the empirical responses of model quantities to uncertainty shocks using a VAR approach. Bloom estimates a range of VARs on monthly data from June 1962 to June 2008. In his basic 4-variable system the variables in Cholesky estimation order are log(S&P500 stock market index), the stock-market volatility indicator, log(employment), and log(industrial production). This ordering is based on the assumption that shocks instantaneously influence the stock market (levels and volatility), and only later quantities (hours, employment, and output). Including the stock-market levels as the first variable in the VAR ensures that the impact of stock-market levels is already controlled for when looking at the impact of volatility shocks. All variables are Hodrick-Prescott (HP) detrended ($\lambda = 129,600$) in the VAR estimations, and the same procedure is followed here.

To this empirical framework we make three additions: we extend all the data through December 2012, we add a time series for real imports at the aggregate level, and we add time series for real imports and industrial production at the disaggregated 4-digit NAICS level. The sources are as follows.


Stock-market volatility indicator: June 1962 to June 2008 from Bloom (2009). “Pre-1986 the VXO index is unavailable, so actual monthly returns volatilities are calculated as the monthly standard deviation of the daily S&P500 index normalized to the same mean and variance as the VXO index when they overlap from 1986 onward. Actual and VXO are correlated at 0.874 over this period. [...] Monthly VXO was capped at 50. Uncapped values for the Black Monday peak are 58.2 and for the credit crunch peak are 64.4. LTCM is Long Term Capital Management.” For comparability, we follow exactly the same definitions here and we thank Nicholas Bloom for providing us with an updated series extended to 2012.


Real Imports, Aggregated: These data run from January 1962 to February 2012. After 1989, total imports for general consumption were obtained from the USITC dataweb, where the data can be downloaded online. From 1968 to 1988 data were collected by hand from FT900 reports, where the imports series are only available from 1968 as F.A.S. at foreign port of export, general imports, seasonally unadjusted; the series then change to C.I.F. value available beginning in 1974, and the definition changes to customs value in 1982. Prior to 1968 we use NBER series 07028, a series that is called “total imports, free
and dutiable” or else “imports for consumption and other”; for the 1962 to 1967 window this NBER series is a good match, as it is sourced from the same FT900 reports as our hand-compiled series. To obtain real values we deflate by the U.S. series for Consumer Price Index for All Urban Consumers: All Items, Not Seasonally Adjusted (CPIAUCNS), constructed by the U.S. Department of Labor, Bureau of Labor Statistics, and taken from FRED http://research.stlouisfed.org/fred2/.

**Industrial Production, Disaggregated:** These data only run from January 1972 to February 2012 at a useful level of granularity. Although aggregate IP data are provided by the Fed going back to February 1919, the sectorally disaggregated IP data only start in 1939 for 7 large sectors, with ever finer data becoming available in 1947 (24 sectors), 1954 (39 sectors) and 1967 (58 sectors). However, it is in 1972 that IP data are available using the 4-digit NAICS classification which will permit sector-by-sector compatibility with the import data above. Starting in 1972 we use the Fed G.17 reports to compile sector-level IP indices, which affords data on 98 sectors at the start, expanding to 99 in 1986. Monthly values with data by NAICS 4-digit group and by Fed Market Group. Mapped into End Use categories using a concordance with 2010 gross value added weights also from the G.17 report.

**Real Imports, Disaggregated:** These data only run from January 1989 to February 2012. In each month total imports for general consumption disaggregated at the 4-digit NAICS level were obtained from the USITC dataweb, where the data can be downloaded online. All series were then deflated by the monthly CPI. In this way 108 sector-level monthly real import series were compiled. Mapped into Fed Market Group categories using a concordance. To obtain real values we deflate by the U.S. CPI as above.

**List of NAICS 4-Digit Codes**

| 1111  | Oilseeds and Grains               | 3149 | Other Textile Products      |
| 1112  | Vegetables and Melons             | 3151 | Knit Apparel                 |
| 1113  | Fruits and Tree Nuts              | 3152 | Apparel                      |
| 1114  | Mushrooms, Nursery and Related Products | 3159 | Apparel Accessories         |
| 1119  | Other Agricultural Products       | 3161 | Leather and Hide Tanning     |
| 1121  | Cattle                           | 3162 | Footwear                     |
| 1122  | Swine                            | 3169 | Other Leather Products       |
| 1123  | Poultry and Eggs                 | 3211 | Sawmill and Wood Products    |
| 1124  | Sheep, Goats and Fine Animal Hair | 3212 | Veneer, Plywood, and Engineered Wood Products |
| 1125  | Farmed Fish and Related Products  | 3219 | Other Wood Products          |
| 1129  | Other Animals                    | 3221 | Pulp, Paper, and Paperboard Mill Products |
| 1132  | Forestry Products                | 3222 | Converted Paper Products     |
| 1133  | Timber and Logs                  | 3231 | Printed Matter and Related Product, NESOI |
| 1141  | Fish, Fresh, Chilled or Frozen and Other Marine Products | 3241 | Petroleum and Coal Products |
| 2111  | Oil and Gas                      | 3251 | Basic Chemicals              |
| 2121  | Coal and Petroleum Gases         | 3252 | Resin, Synthetic Rubber, & Artificial & Synthetic Fibers & Filament |
| 2122  | Metal Ores                       | 3253 | Pesticides, Fertilizers and Other Agricultural Chemicals |
| 2123  | Nonmetallic Minerals             | 3254 | Pharmaceuticals and Medicines |
| 3111  | Animal Foods                     | 3255 | Paints, Coatings, and Adhesives |
| 3112  | Grain and Oiled Milling Products  | 3256 | Soaps, Cleaning Compounds, and Toilet Preparations |
| 3113  | Sugar and Confectionery Products  | 3259 | Other Chemical Products and Preparations |
| 3119  | Foods, NESOI                     | 3261 | Plastics Products            |
| 3121  | Beverages                        | 3262 | Rubber Products              |
| 3122  | Tobacco Products                 | 3271 | Clay and Refractory Products |
| 3131  | Fibers, Yarns, and Threads       | 3272 | Glass and Glass Products     |
| 3132  | Fabrics                          | 3273 | Cement and Concrete Products |
| 3133  | Finished and Coated Textile Fabrics| 3274 | Lime and Gypsum Products     |
| 3141  | Textile Furnishings              | 3279 | Other Nonmetallic Mineral Products |
|       |                                  | 3311 | Iron and Steel and Ferroalloy |
|       |                                  | 3312 | Steel Products From Purchased Steel |
|       |                                  | 3313 | Aluminum and Aluminum and Processing |
|       |                                  | 3314 | Nonferrous Metal (Except Aluminum) and Processing |
|       |                                  | 3315 | Foundries                    |

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<td>Published Printed Music and Music Manuscripts</td>
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<tr>
<td>9100</td>
<td>Waste and Scrap</td>
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<td>Used or Second-Hand Merchandise</td>
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<td>9800</td>
<td>Goods Returned to Canada (Exports Only); U.S. Goods Returned and Reimported Items (Imports Only)</td>
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