Week 9:
Asymmetric Information and Moral Hazard

Dr Daniel Sgroi

Reading: Snyder and Nicholson, Chapter 18.

With thanks to Peter J. Hammond.

Asymmetric Information

Many transactions in economics involve two parties:
one buyer and one seller.

There is asymmetric information in case
one party to the transaction has relevant information
that is unavailable to the other.

The more informed party than take advantage of the less informed.

In ancient Rome the problem was already recognized
by the orator Cicero, who cites an earlier Babylonian discussion.

Insider dealing on stock markets has been outlawed,
even if those laws are not always easy to enforce.

Efficient transactions can be seriously impeded
when one side has better information.

The Value of Contracts

Well designed contracts may help to mitigate
some of the inefficiencies associated with asymmetric information.

But rarely do contracts completely eliminate the inefficiencies.

In a standard economic model, one party
who proposes the contract is called the principal.

Another party called the agent decides
whether or not to accept the contract
and then how to perform under its terms.

Typically the agent has superior information.

Two Leading Models

There are two leading models of asymmetric information.

1. In a hidden action or moral hazard model,
   the agent's choice of action affects the principal,
   but the principal does not observe the actions directly.
2. In a hidden type or adverse selection model,
   the agent's "type" remains as private information
   until after the contract has been signed.

First, Second, and Third Best

With complete information, the principal could simply propose
a first-best contract to maximize her profit,
subject to a participation constraint requiring the agent
to be no worse off agreeing to the contract
than he would be by not agreeing to the contract.

This is like the take it or leave it offer
that "solves" the ultimatum game.

With incomplete information, however,
when the principal is less well informed than the agent,
this limits the principal's opportunities to appropriate all the gains.

The principal can only discover the agent's extra information
by providing incentives for the agent to reveal it.

The principal can do no better than a second-best contract
which respects the relevant incentive constraints.

Adding further constraints leads to the third best, fourth best, etc.

Hidden Actions

The principal would like the agent
to take an action that maximizes their joint benefit.

But, the principal may be unable to observe the agent's action.

If so, the agent will typically prefer to "shirk" or take it easy.

Measured Performance

Contracts can mitigate shirking
by tying compensation to observable outcomes.

For example, sales staff get paid commissions,
workers get paid “piecework” rates related to perceived output,
and bankers’ bonuses should relate
to the long-run profits or losses they generate.

Often, the principal is anyway more concerned
with outcomes than with actions,
so she may as well devise a contract
which pays the agent according to measured performance.

Risk for the Agent

There is a problem with paying the agent by results.

The outcome may depend in part
on random factors beyond the agent's control.

So tying the agent's compensation to outcomes
exposes the agent to risk.

A risk averse agent will then require a risk premium to be paid
before he will accept the contract.

The participation constraint becomes more severe,
and limits how much benefit the principal can extract
from the contractual relationship.

The Owner–Manager Relationship

Consider a firm with one representative owner
who plans to hire the expertise of one manager.

The owner offers a contract to the manager.

The manager decides whether or not to accept the contract,
and also, if he accepts,
what level of effort $e \geq 0$ to put into running the firm.

Increasing e adds to the firm's gross profit
but is personally costly to the manager.

The Owner's Preferences

Suppose the firm's gross profit π_g is equal to $e + \epsilon$,
where ϵ is a random variable representing demand, cost,
and other economic factors beyond the agent's control.

Like an econometrician, we assume
that ϵ is normally distributed with mean 0 and variance σ^2.

Let s be the manager's salary.
Then the firm owner's net profit π_n equals $\pi_g - s$.

We assume the owner owns
a very well diversified portfolio of many firms.
This allows us to treat the owner as risk neutral,
with preferences represented by the expected value of net profit,
which is $\mathbb{E}\pi_n = \mathbb{E}(\pi_g - s) = \mathbb{E}(e + \epsilon - s) = e - \mathbb{E}s$.

The Manager's Preferences

Assume the manager is risk averse
with a constant rate of absolute risk aversion equal to $\alpha > 0$.

Also, let $c(e)$ be the manager's cost of exerting effort e,
where $c'(e) > 0$ and $c''(e) > 0$.

Specifically, assume that
the manager's von Neumann–Morgenstern utility function
takes the form $u(s, e) = v(s - c(e))$ where $v(x) = -e^{-\alpha x}$.

We will pay attention to special cases when the manager's salary
is a linear function $s = a + b\pi_g = a + b(e + \epsilon)$
of the firm's gross profit π_g,
or of the manager's effort e and the random shock ϵ.

This gives s a normal distribution with mean $\mathbb{E}s = a + be$
and variance $\operatorname{var} s = \mathbb{E}(s - a - be)^2 = \mathbb{E}(b\epsilon)^2 = b^2\mathbb{E}\epsilon^2 = b^2\sigma^2$.

As shown in the appendix, the manager's expected utility is

$$\mathbb{E}u = v(\mathbb{E}s - \tfrac{1}{2}\alpha \operatorname{var} s - c(e)) = -\exp\{-\alpha[\mathbb{E}s - \tfrac{1}{2}\alpha \operatorname{var} s - c(e)]\}.$$

The Manager's Participation Constraint

With complete information, the owner can simply instruct the manager to exert an optimal level of effort e^*.

Then the owner contracts to pay the manager a fixed salary s^*
if his observed level of effort is e^*,
but 0 if he is observed to shirk by exerting less effort.

For the manager to accept this contract, his expected utility

$$\mathbb{E}u = -\exp[-\alpha(\mathbb{E}s - \tfrac{1}{2}\alpha \operatorname{var} s - c(e^*))] = -\exp[-\alpha(s^* - c(e^*))]$$

should satisfy the participation constraint

$$\mathbb{E}u = -\exp[-\alpha(s^* - c(e^*))] \geq \bar{u},$$

where $\bar{u}$ is the manager's "reservation utility"
from the best alternative use of his time.
This inequality is equivalent to the constraint $s^* - c(e^*) \geq \bar{s}$
for a "reservation income" $\bar{s}$ satisfying $\bar{u} = -\exp(-\alpha\bar{s})$.

First-Best Contract

The owner's optimal contract is described by the pair (e^*, s^*) which maximizes expected net profit $\mathbb{E}\pi_n = e - s$ subject to the agent's participation constraint $s - c(e) \geq \bar{s}$.

The participation constraint implies that $s \geq c(e) + \bar{s}$ and so $\mathbb{E}\pi_n = e - s \leq e - c(e) - \bar{s}$ with equality only if the owner pays the lowest possible salary $s = c(e) + \bar{s}$, and then chooses e^* to maximize $\mathbb{E}\pi_n = e - c(e) - \bar{s}$.

This requires choosing e^* to satisfy the first-order condition $c'(e^*) = 1$.

Or, if $c'(0) \geq 1$, the optimum is the corner solution $e^* = 0$, where the manager is not hired at all.

In the special quadratic case $c = \gamma e + \frac{1}{2}\delta e^2$ with $\gamma \geq 0$, $\delta > 0$, the optimum e^* satisfies $\gamma + \delta e^* = 1$ when $e^* > 0$. Hence $e^* = \max\{0, (1-\gamma)/\delta\}$.

Linear Contracts

If the owner cannot observe the manager's effort, the contract cannot specify a payment s conditioned on e.

Nevertheless, the owner may still induce positive effort if the manager's salary is an increasing function of gross profit.

For example, suppose the owner contracts to pay an amount $s(\pi_g) = a + b\pi_g$ where a is a fixed salary and b indicates the power of the incentive scheme.

This relationship can be viewed as a three-stage game where:

1. first, the owner determines the payment scheme by choosing the constants a and b;
2. second, the manager decides whether or not to accept the contract;
3. third, provided he has accepted the contract, the manager decides how much effort e to exert.

The Manager's Optimal Effort

We use backward induction to find
a subgame perfect equilibrium of this three-stage game.

In the last stage, given the linear payment scheme $s(\pi_g) = a + b\pi_g$ and given that the manager has decided to accept the contract, his optimal effort $e^*(a, b)$ maximizes expected utility

$$\mathbb{E}u = -\exp\{-\alpha[\mathbb{E}s - \tfrac{1}{2}\alpha \operatorname{var} s - c(e)]\}$$

where s is a normally distributed with mean $\mathbb{E}s = a + be$ and variance $\operatorname{var} s = \mathbb{E}(s - a - be)^2 = b^2\mathbb{E}e^2 = b^2\sigma^2$.

So the manager's expected utility is

$$\mathbb{E}u = -\exp\{-\alpha[a + be - \tfrac{1}{2}\alpha b^2\sigma^2 - c(e)]\}$$

which is an increasing transformation of $a + be - \frac{1}{2}\alpha b^2\sigma^2 - c(e)$.

So optimal effort e^* satisfies the first-order condition $b = c'(e^*)$.
In particular, e^* is independent of a.

A Little Comparative Statics

In the special case when $c = \gamma e + \frac{1}{2}\delta e^2$,
the optimum e^* satisfies $\gamma + \delta e^* = b$ when $e^* > 0$.

Hence $e^* = \max\{0, (b - \gamma)/\delta\}$,
which increases with b once $b > \gamma$.

When $b > \gamma$, note that

$$c(e^*) = \frac{\gamma(b - \gamma)}{\delta} + \frac{(b - \gamma)^2}{2\delta} = \frac{(b - \gamma)(2\gamma + \beta - \gamma)}{2\delta} = \frac{b^2 - \gamma^2}{2\delta}.$$

Generally, differentiating the first-order condition $b = c'(e^*)$
w.r.t. b implies that $1 = c''(e^*)\dfrac{de^*}{db}$ or $\dfrac{de^*}{db} = \dfrac{1}{c''(e^*)} > 0$.

So the manager exerts more effort
as his share b of additional profit increases.

Does the Manager Accept the Contract?

If the manager accepts the contract,
and then chooses the optimal effort e^*, his expected utility is

$$\mathbb{E}u = -\exp\{-\alpha[a + be^* - \tfrac{1}{2}\alpha b^2\sigma^2 - c(e^*)]\}.$$

The participation constraint is $\mathbb{E}u \geq \bar{u}$,
which is equivalent to $a + be^* - \frac{1}{2}\alpha b^2\sigma^2 - c(e^*) \geq \bar{s}$.

This holds iff $a \geq \bar{s} + \frac{1}{2}\alpha b^2\sigma^2 + c(e^*) - be^*$.

In the quadratic case $c(e) = \gamma e + \frac{1}{2}\delta e^2$ with $b > \gamma$,
when $e^* = (b - \gamma)/\delta$ and $c(e^*) = (b^2 - \gamma^2)/2\delta$,
this requires that $a \geq \bar{s} + \frac{1}{2}\alpha b^2\sigma^2 - (b - \gamma)^2/2\delta$.

Generally, the participation constraint requires the fixed salary a
to be high enough, given the owner's choice of b
and the manager's induced choice of e^*.

Finding the Second-Best Contract

The manager's expected net profit can be written as

$$\mathbb{E}\pi_n = e^*(b) - a - be^*(b) = (1-b)e^*(b) - a$$

where we use the notation $e^*(b)$ to indicate how b influences the manager's choice of effort.

The manager will select the lowest fixed salary a satisfying the participation constraint $a \geq \bar{s} + \frac{1}{2}\alpha b^2\sigma^2 + c(e^*) - be^*$. So, as a function of b, the manager's expected net profit is $\mathbb{E}\pi_n = e^*(b) - a - be^*(b)$ or, after substituting for the optimal choice of a,

$$\mathbb{E}\pi_n = (1-b)e^*(b) - [\bar{s} + \tfrac{1}{2}\alpha b^2\sigma^2 + c(e^*(b)) - be^*(b)]$$

Cancelling the terms $-be^*(b) - (-be^*(b))$ reduces this to

$$\mathbb{E}\pi_n = e^*(b) - \bar{s} - \tfrac{1}{2}\alpha b^2\sigma^2 - c(e^*(b)).$$

First- versus Second-Best Effort

Maximizing $\mathbb{E}\pi_n = e^*(b) - \bar{s} - \frac{1}{2}\alpha b^2\sigma^2 - c(e^*(b))$ w.r.t. b gives the first-order condition $0 = \dfrac{de^*}{db} - \alpha b\sigma^2 - c'(e^*(b))\dfrac{de^*}{db}$.

But we saw earlier that $b = c'(e^*(b))$ and $\dfrac{de^*}{db} = \dfrac{1}{c''(e^*(b))}$, implying that $(1-b)/c''(e^*(b)) = \alpha b\sigma^2$ and so

$$b = c'(e^*(b)) = \frac{1}{1+\alpha\sigma^2 c''(e^*(b))} < 1.$$

Because the owner cannot observe e directly,
the manager is risk-averse, and output is risky,
the manager receives only a share $b < 1$ of incremental profit.

This dilutes the manager's incentives, and implies
that second-best effort $e^*(b)$, which solves $b = c'(e^*(b))$,
is less than first-best effort $e^*(1)$, which solves $1 = c'(e^*(1))$.

Quadratic Costs

Consider the quadratic case $c = \gamma e + \frac{1}{2}\delta e^2$ with $\gamma \geq 0$, $\delta > 0$.

Then $c''(e) = \gamma \geq 0$, independent of e, so $b = \frac{1}{1+\alpha\sigma^2\delta}$,
and $a = \bar{s} + \frac{1}{2}\alpha b^2\sigma^2 - (b-\gamma)^2/2\delta$ for this value of b.

The above results are valid if $\gamma \leq \frac{1}{1+\alpha\sigma^2\delta}$,
implying that the owner's optimal contract satisfies $b \geq \gamma$.

Otherwise, if $\gamma \leq \frac{1}{1+\alpha\sigma^2\delta}$,
the manager will put in no effort for the optimal linear contract,
so it is best not to hire the manager anyway.

Note that γ is the marginal cost of effort e
at the corner where $e = 0$.

Moral Hazard in Insurance

An insuree's unobservable effort
can reduce the probability of an insurance claim.

A person with full insurance
has a reduced incentive to undertake precautions.

This tends to increase the likelihood of an insured loss.

The adverse incentive effect of insurance coverage
on an individual's precautions, which may then change
the likelihood or the size of losses, or both,
is known as moral hazard.

Mathematical Model

Suppose a risk-averse individual has
a von Neumann–Morgenstern utility function of wealth
given by $u(W)$ where $u'(W) > 0$ and $u''(W) < 0$ for all $W \geq 0$.

The individual faces the possibility of a loss ℓ
that will reduce his initial wealth W_0.

The probability of loss is assumed to be a function $\pi(e)$
of the amount e that the individual spends on preventive measures.

Assume that $\pi'(e) < 0$ and $\pi''(e) > 0$,
so more preventative expenditure e
increases the probability of $1 - \pi(e)$ of avoiding a loss,
but at a declining rate.

Self Insurance

Without any insurance, or with only "self insurance", the individual has wealth:

1. $W_1 = W_0 - e$ in state 1, when there is no loss;
2. $W_2 = W_0 - e - \ell$ in state 2, when there is a loss of ℓ.

The individual's expected utility without insurance coverage is

$$\mathbb{E}u(W) = [1 - \pi(e)]u(W_0 - e) + \pi(e)u(W_0 - e - \ell).$$

The first-order condition for an optimal choice of e is

$$\begin{aligned} 0 = -\pi'(e)[u(W_0 - e) - u(W_0 - e - \ell)] \\ - [1 - \pi(e)]u'(W_0 - e) - \pi(e)u'(W_0 - e - \ell). \end{aligned}$$

Let $U^0 = u(W^0)$ and $W^0 = u^{-1}(U^0)$ respectively denote the level of expected utility and corresponding certainty equivalent wealth level achieved with this optimal level of precautionary expenditure e.

An Insurance Policy

Suppose an insurance company offers a contract whereby, in exchange for paying a premium p, the individual will receive a compensation payment of x in case a loss occurs.

An individual who takes this coverage has wealth:

1. $W_1 = W_0 - e - p$ in state 1, when there is no loss;
2. $W_2 = W_0 - e - p - \ell + x$ in state 2, when there is a loss of ℓ, but an insurance payout of x.

The individual's expected utility with this coverage is

$$\mathbb{E}u(W) = [1 - \pi(e)]u(W_1) + \pi(e)u(W_2)$$

First-Best Insurance Contract

In the first-best case with complete information,
the insurance company can monitor precautionary expenditure e.

The company should set the terms (x, p, e) of its contract,
including the precautions it requires the individual to undertake,
in order to maximize its expected profit $p - \pi(e)x$
subject to the participation constraint (or PC) requiring that
the expected utility $[1 - \pi(e)]u(W_1) + \pi(e)u(W_2)$ with insurance
cannot be less than
the expected utility $U^0 = u(W^0)$ without insurance.

Formally, the first-best contract (x^F, p^F, e^F) maximizes $p - \pi(e)x$
subject to $[1 - \pi(e)]u(W_1) + \pi(e)u(W_2) \geq U^0 = u(W^0)$
where $W_1 = W_0 - e - p$ and $W_2 = W_0 - e - p - \ell + x$.

Applying Jensen's Inequality

Consider any contract (x, p, e) satisfying constraint (PC)

$$[1-\pi(e)]u(W_1)+\pi(e)u(W_2) \geq U^0 = u(W^0)$$

where $W_1 = W_0 - e - p$ and $W_2 = W_0 - e - p - \ell + x$.

Let $\bar{W} := [1-\pi(e)]W_1 + \pi(e)W_2 = W_0 - e - p + \pi(e)(x-\ell)$ denote the consumer's expected wealth. Then Jensen's inequality (see appendix) and the constraint (PC) together imply that

$$u(\bar{W}) \geq [1-\pi(e)]u(W_1)+\pi(e)u(W_2) \geq U^0 = u(W^0).$$

Moreover the first inequality here must be strict except in the degenerate case when either $\pi(e)=0$, or $\pi(e)=1$, or $W_1 = W_2 = \bar{W}$.

The last possibility is the full insurance case when $x = \ell$, so the loss ℓ is compensated in full.

Hence $\bar{W} \geq W^0$, and in fact $\bar{W} > W^0$ except in the degenerate case.

Full Insurance

Rearranging the equality $\bar{W} = W_0 - e - p + \pi(e)(x - \ell)$ implies that $p - \pi(e)x = W_0 - e - \pi(e)\ell - \bar{W}$.

Because of Jensen's inequality and the constraint (PC), one has $\bar{W} \geq W^0$ and so $p - \pi(e)x \leq W_0 - e - \pi(e)\ell - W^0$, with strict inequality except in the degenerate case.

Since the objective is to maximize $p - \pi(e)x$, provided that the first-best effort e^F satisfies $0 < \pi(e^F) < 1$, the first-best contract (x^F, p^F, e^F) involves making $\bar{W} = W^0$ and so $p - \pi(e)x = W_0 - e - \pi(e)\ell - W^0$.

This can be done through full insurance with $x^F = \ell$, resulting in the degenerate case.

First-Best Effort

To find first-best effort e^F, choose (p, e) to maximize expected profit $p - \pi(e)\ell$ from a full insurance contract subject to constraint (PC), which simplifies to $p \leq W_0 - e - W^0$.

Hence e^F is chosen by the insurer to maximize the net profit $W_0 - e - W^0 - \pi(e)\ell$ it can earn from an optimal choice of p.

In particular, the first-order condition $-\pi'(e^F)\ell = 1$ for a maximum w.r.t. e equates:

- the marginal reduction $-\pi'(e^F)\ell$ in expected loss due to increased precautionary expenditure
- the marginal cost of that expenditure, which is 1.

Hence e^F is the socially efficient level of precaution.

Full Insurance Encourages Carelessness

Assume the insurance company cannot monitor the effort e at all.

Instead, the insured individual is free to choose whatever e it likes.

Consider a contract with premium p
and payment x in case of a claimed loss.

In case of full insurance when $x = \ell$,
expected utility simplifies to $u(W_0 - e - p)$.

We assume a negative e,
accepting money to increase the probability of an accident,
is ruled out by the criminal law.
So $u(W_0 - e - p)$ maximized by setting precautionary expenditure
to its minimum level $e = 0$.

The insurance company will have to charge $p = \pi(0)\ell$
in order to avoid a loss.

The individual's expected utility will be $u(W_0 - \pi(0)\ell)$,
without any risk, but paying a high premium.

Finding a Second-Best Contract . . .

Now suppose there is less than full insurance,
so the contract involves a premium p
and a payout $x < \ell$ in case of a loss.

The individual will choose $e^*(p, x)$
in order to maximize expected utility with this coverage, which is

$$\mathbb{E}u(W) = [1 - \pi(e)]u(W_0 - e - p) + \pi(e)u(W_0 - e - p - \ell + x)$$

This is an incentive constraint affecting the choice of contract.

The second-best contract will typically not involve full insurance.

Exposing the individual to some risk induces some precaution.

. . . is Non-Trivial

The second-best contract is described by a pair (p, x) chosen to maximize expected profit $p - \pi(e^*(p, x))x$ subject to the participation constraint

$$[1 - \pi(e^*(p, x))]u(W_1(p, x)) + \pi(e^*(p, x))u(W_2(p, x)) \geq \bar{U},$$

where $W_1(p, x) = W_0 - e^*(p, x) - p$
and $W_2(p, x) = W_0 - e^*(p, x) - p - \ell + x$.

A non-trivial optimization problem!

Normal Distribution

Your probability and statistics textbook should present the formula

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2\right]$$

for the density function of a normally or Gaussian distributed random variable with mean $\mu = \mathbb{E}x$
and variance $\sigma^2 = \operatorname{var} x = \mathbb{E}(x-\mu)^2$
— i.e., an $N(\mu, \sigma^2)$ random variable. In particular,

$$\int_{-\infty}^{\infty} f(x)dx = 1; \; \int_{-\infty}^{\infty} x\, f(x)dx = \mu; \; \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \sigma^2.$$

See also Section 9.7 in the 3rd edition of Sydsæter and Hammond *Essential Mathematics of Economic Analysis*.

Expected CARA Utility

Consider now the expected utility $\mathbb{E}u(x)$ of a random variable x with an $N(\mu, \sigma^2)$ distribution, when $u(x)$ is a constant absolute risk aversion (or CARA) utility function of the form $u(x) \equiv -e^{-\alpha x}$. In fact,

$$\mathbb{E}u = \int_{-\infty}^{\infty} -e^{-\alpha x} f(x) dx = -\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{\left[-\alpha x - \frac{1}{2\sigma^2}(x-\mu)^2\right]} dx.$$

After completing the square, this can be written as

$$\begin{aligned}
\mathbb{E}u &= -\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}[(x-\mu+\alpha\sigma^2)^2 + 2\mu\alpha\sigma^2 - \alpha^2\sigma^4]} dx \\
&= -e^{\left(-\alpha\mu + \frac{1}{2}\alpha^2\sigma^2\right)} \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu+\alpha\sigma^2)^2} dx \\
&= -e^{-\alpha\left(\mu - \frac{1}{2}\alpha\sigma^2\right)} \ = \ u(\mu - \tfrac{1}{2}\alpha\sigma^2) \ = \ u(\mathbb{E}x - \tfrac{1}{2}\alpha \operatorname{var} x)
\end{aligned}$$

because the last integrand is an $N(\mu - \alpha\sigma^2, \sigma^2)$ probability density function, whose integral must equal 1.

Jensen's Inequality

Suppose that u is any twice differential function
satisfying $u''(W) < 0$ for all $W \geq 0$.

Suppose that $\tilde{W}$ is a random variable for which
the expectations $\bar{W} := \mathbb{E}\tilde{W}$ and $\mathbb{E}u(\tilde{W})$ both exist.
We disregard the degenerate case when $\tilde{W} = \bar{W}$ with probability 1.

Consider the function $\theta \mapsto g(\theta)$ of one real variable
defined by $g(\theta) = \mathbb{E}u(\bar{W} + \theta(\tilde{W} - \bar{W}))$.
Then g is twice differentiable w.r.t. θ,
with first derivative $g'(\theta) = \mathbb{E}u'(\bar{W} + \theta(\tilde{W} - \bar{W})) \cdot (\tilde{W} - \bar{W})$
and second derivative $g''(\theta) = \mathbb{E}u''(\bar{W} + \theta(\tilde{W} - \bar{W})) \cdot (\tilde{W} - \bar{W})^2$.

Now $u''(W) < 0$ implies that $g''(\theta) < 0$ for all θ.
Also $g'(0) = \mathbb{E}u'(\bar{W}) \cdot (\tilde{W} - \bar{W}) = u'(\bar{W}) \cdot \mathbb{E}(\tilde{W} - \bar{W}) = 0$
because $\mathbb{E}\tilde{W} = \bar{W}$, and so $g(\theta) < g(0)$ for all $\theta \neq 0$.

In particular, putting $\theta = 1$ implies Jensen's inequality stating
that $\mathbb{E}u(\tilde{W}) \leq u(\mathbb{E}\tilde{W})$, with equality only in the degenerate case.