The Optimal Choice of Pre-Launch Reviewer:

How Best to Transmit Information using Tests and Conditional Pricing

David Gill
Division of Economics
University of Southampton

Daniel Sgroi
Department of Economics
University of Warwick

November 2008

Motivation

- Public tests can be powerful in launching new products, standards, careers, ideas
    - Reviews
    - Accreditations
    - References
- Principal discovers whether she is "good" or "bad" type
    - Study the use of a public test by principal to try to convince agents to endorse her
    - Test is passed or failed
    - Information transmission problem
- Principal chooses toughness of test from a continuum of test types
    - Higher probability of passing a softer test
    - Greater impact from passing a tougher test
- And how pricing should respond to outcome of the public test
- Note: not studying problem of optimal test expertise

Outline

- A. Introduction
- B. Model
- C. Choice Problem
- D. Wanting to be Tested
- E. Extreme Tests are Best
- F. Choice of Extreme
- G. Importance of Price
- H. Conclusion

A. Introduction: Examples

- Beta-testers & magazine "previews" in software industry
- Job market candidates choosing referees
- Premieres at movie festivals
- Students choosing degree program to attempt
- Politicians "selling" a policy seeking support from think tanks
- Technology sponsors using standard setting organizations (SSOs)
    - “Technology sponsors attempt to build standards around their technologies by having them validated by SSOs that range from fully independent to largely captive special interest groups.” (Lerner & Tirole, AER 06)
- "Price" of endorsement can take different forms, e.g.
    - Standard market price
    - Salary
    - Degree of compromise offered on policy position

A. Introduction: Preview of Results

- Model not about signalling through *choice* of test; rather about signalling through *decision* of testing body
  - All equilibria are pooling
- Principal always chooses to be tested
  - Test complements the choice of price
- Choosing toughest or softest available is optimal
  - Ability to condition price on test result crucial in pushing choice to an extreme
- Toughest test possible where precision of agents' private information is low and prior not too positive
  - Launch innovative product or idea with bang on passing tough test
- Softest test possible where precision of private information is high
  - Well-know product or idea

A. Introduction: Related Literature 1/2

- Lerner & Tirole (AER 06) consider biased SSOs
- Sponsor attempts to receive certification from an SSO
  - Continuum of SSOs which vary in bias for or against sponsor
- Differences compared to our setup
  - Sponsor does not know her quality
  - SSO discovers sponsor's quality with certainty
  - Users receive no private information
  - Certification rule sensitive to any anticipated price response to decision
- Thus certifiers cannot counter bad private information
- And certification does not enable a rise in price
  - Sponsor happy to commit to price
  - Given certification rule adjusts if price set conditional on decision
- So no role for certifiers biased against the technology
- Choose SSO most biased in favor, subject to users adopting following certification

A. Introduction: Related Literature 2/2

- High initial prices in Taylor (REStud 99) and Bose et al. (RAND 07) play qualitatively similar role to tough reviewers
  - Bose et al.: if expensive good becomes successful conveys strong positive information to later buyers
  - Taylor: if expensive house not sold quickly, prospective buyers can attribute failure to sell to product being overpriced
- Payment structures to certification intermediaries
  - Lizzeri (RAND 99) and Albano & Lizzeri (IER 01)
  - Ask how intermediaries affect product quality
  - Disclosure may be incomplete, but no bias allowed
- Ottaviani & Prat (Econometrica 01) consider use of certifier
  - Again no bias; neither buyer nor seller fully informed
  - Public signal correlated with buyer's information reduces buyer's rents given 2nd-degree price discrimination
- Gill & Sgroi GEB 08: herding model without prices

B. Model: Timing

- Nature chooses unverifiable principal type $V \in \{0, 1\}$
  - "Good" type: $V = 1$
  - "Bad" type: $V = 0$
  - Prior $q \in (0, 1)$ that principal is good type
  - Type draw observed by principal but not agents
- N agents each receive a private signal about type of principal
- Principal decides whether to face public test
  - If tested, chooses type of test to face, which is public
  - Test is passed or failed
- Principal chooses price λ
  - Conditioned on chosen test type and test decision
- Agents simultaneously decide whether to endorse

B. Model: Test Technology

- Testing body receives one signal from $\{H, U, L\}$
- p_V^z is probability test receives signal z given principal of type V
  - $p_1^H > p_0^H$, so H is a high signal
  - $p_1^L < p_0^L$, so L is a low signal
  - $p_1^U = p_0^U$, so U is an uninformative signal
  - $p_V^z > 0$, so no signal is fully informative
  - When $p_1^H = p_0^L$, we call signal structure "symmetric"
- Testing body makes a decision $d \in \mathbb{D} = \{P, F\}$
- ϕ_P^z is probability test is passed on receiving signal z
- Principal chooses test type $\phi = \{\phi_P^H, \phi_P^U, \phi_P^L\}$
  - W.l.o.g. $\phi_P^H \geq \phi_P^L$, so P is good news and F bad news
  - Call tests with lower ϕ_P^U "tougher"
  - Call tests with higher ϕ_P^U "softer"
- Coarseness of binary report relative to trinary information is key

B. Model: Agents

- Each agent draws one signal from finite set $\mathbb{X} = \{0, 1, 2, ..., M\}$
  - Draws are i.i.d. conditional on V
  - And conditionally independent of test's signal
- p_V^m is probability agent receives signal m given principal of type V
  - $p_V^m > 0$, so no signal is fully informative
  - If $p_1^m = p_0^m$ for all m, agents receive no private information
- $\mu = \Pr[V = 1|m, \phi, d, \lambda]$ is an agent's posterior belief
- Agent endorses iff his $\mu \geq \lambda$
  - If agent endorses, $u = V - \lambda$
  - If doesn't endorse, $u = 0$
  - Endorse at indifference is w.l.o.g.
- $\mu^m = \Pr[V = 1|m]$ is an agent's posterior having observed only his private signal

C. Choice Problem: Introduction

- Principal aims to maximize expected revenue R
- Complicated choice problem
  - For every possible test and decision, $M+1$ possible prices
  - Anticipated price choices feed back into choice of test
  - And principal needs to worry about inferences from choice of test and price
- First, we rule out inference from choices per se
  - All equilibria are pooling
- Then we show that principal will always choose
  - $\phi_P^H = 1$, $\phi_P^L = 0$ and $\phi_P^U \in \{0,1\}$
  - Toughest or softest test available
- Finally, we compare the toughest and softest tests
  - Analytical results
  - And some numerical analysis

C. Choice Problem: Ruling out Separation 1/2

- Throughout we restrict principal to pure strategies
- Perfect Bayesian Equilibrium is the solution concept
- Separating equilibria are impossible
- At node where actions differed, action would fully reveal type
    - Bad principal could costlessly copy the choice of good principal
    - Would then be believed to be good for sure by all agents
    - And agents would ignore test
- *So in equilibrium choice of test and price uninformative*

C. Choice Problem: Ruling out Separation 2/2

- Equilibrium selection issue among pooling equilibria
  - Any pooled strategies form a PBE
- Conditional on pooling, let Ω be good type of principal's set of optimal strategies
- We restrict attention to pooling equilibria with strategies in Ω
  - *Good type of principal chooses test and pricing rule*
- Justified if, starting from a pooling equilibrium with strategies outside of Ω
  - Deviations to Ω weakly increase beliefs that principal is good type

C. Choice Problem: Revenue Function 1/2

- In equilibrium, an agent's posterior belief given by

$$\mu_d^m = \frac{\Pr\left[d|V=1,\phi\right]\mu^m}{\Pr\left[d|V=1,\phi\right]\mu^m + \Pr\left[d|V=0,\phi\right]\left(1-\mu^m\right)}$$

- Principal chooses price $\lambda \in \left\{\mu_d^0, \mu_d^1, ..., \mu_d^M\right\}$
- Setting $\lambda = \mu_d^m$ results in endorsements from agents with signals
  - $k : \mu^k \geq \mu^m$
  - Standard price-quantity trade-off
- If λ outside this set, could increase price a little with no loss of endorsements
- W.l.o.g. normalize number of agents to 1

C. Choice Problem: Revenue Function 2/2

- Given ϕ and d, optimal pricing results in expected revenue

$$\max_{m \in \mathbb{X}} \mu_d^m \sum_{k:\mu^k \geq \mu^m} p_1^k$$

- So given ϕ, but before test decision is known, expected revenue is

$$\begin{aligned} R &= \sum_{\mathbb{D}} \Pr\left[d|V=1,\phi\right] \max_{m \in \mathbb{X}} \mu_d^m \sum_{k:\mu^k \geq \mu^m} p_1^k \\ &= \sum_{\mathbb{D}} \max_{m \in \mathbb{X}} \Pr\left[d|V=1,\phi\right] \mu_d^m \sum_{k:\mu^k \geq \mu^m} p_1^k \end{aligned}$$

- Principal chooses ϕ to maximize this

D. Wanting to be Tested 1/2

Theorem 1:

- *The principal strictly prefers any test to not being tested at all*

- Let's compare any test $\overline{\phi}$ to not being tested
- In absence of a test

$$R = \max_{m \in \mathbb{X}} \mu^m \sum_{k:\mu^k \geq \mu^m} p_1^k$$

- Let $\widehat{m}$ be the maximizing m
  - Principal sets $\lambda = \mu^{\widehat{m}}$
  - To target agents with signals at least as strong as $\widehat{m}$

D. Wanting to be Tested 2/2

- Now suppose
  - Principal starts with belief $\mu^{\widehat{m}}$ about her type
  - After the test, principal must set $\lambda = \mu_d^{\widehat{m}}$
- Then test of no use as

$$\Pr\left[P|\overline{\phi}\right]\left(\mu_P^{\widehat{m}} - \mu^{\widehat{m}}\right) = \Pr\left[F|\overline{\phi}\right]\left(\mu^{\widehat{m}} - \mu_F^{\widehat{m}}\right)$$

- However
  - Principal knows she is a good type
  - And can choose how many agents to target conditional on d

E. Extreme Tests are Best: Test Toughness

Definition: (*For fixed* ϕ_P^H *and* ϕ_P^L)
(i) *Test "toughness" is decreasing in* ϕ_P^U
(ii) *Test "softness" is increasing in* ϕ_P^U

- A pass raises beliefs while a fail lowers them
  - $\mu_P^m > \mu^m > \mu_F^m$
- The softer the test
  - The smaller the positive impact of a pass: $\frac{\partial \mu_P^m}{\partial \phi_P^U} < 0$
  - And the bigger the negative impact of a fail: $\frac{\partial \mu_F^m}{\partial \phi_P^U} < 0$
  - But the greater the chance of passing
- The tougher the test
  - The bigger the positive impact of a pass
  - And the smaller the negative impact of a fail
  - But the smaller the chance of passing

E. Extreme Tests are Best: Convexity

- Remember

$$R = \sum_{\mathbb{D}} \max_{m \in \mathbb{X}} \Pr\left[d | V = 1, \phi\right] \mu_d^m \sum_{k:\mu^k \geq \mu^m} p_1^k$$

- Can prove that $\Pr\left[d | V = 1, \phi\right] \mu_d^m$ is strictly convex in ϕ_P^U
  - For any ϕ_P^H and ϕ_P^L
- Convex analysis then makes life easy
  - Maximum of strictly convex functions is strictly convex
  - As is sum of strictly convex functions
- Thus R is strictly convex in ϕ_P^U
  - *So maximized at $\phi_P^U = 1$ or $\phi_P^U = 0$, for any ϕ_P^H and ϕ_P^L*
- Always choose toughest or softest test
- Clearly, result extends to any finite set of toughness levels

E. Extreme Tests are Best: Intuition 1/2

- We've seen that all prices are decreasing in ϕ_P^U
  - Pass raises beliefs more the tougher the test
  - Fail not so damaging if test is tougher
- In fact, μ_P^m strictly convex in ϕ_P^U
  - Conditional on a pass, increasing toughness more powerful the tougher the test already is
- But probability of pass is linear in ϕ_P^U
- To maximize $\Pr\left[P|V=1,\phi\right]\mu_P^m$ principal chooses extreme ϕ_P^U
- Toughest test
  - Benefit from a steep increase in prices as $\phi_P^U \to 0$
  - While probability of pass falls linearly
- Softest test
  - Linear increase in probability of pass
  - While prices do not fall much as $\phi_P^U \to 1$

E. Extreme Tests are Best: Intuition 2/2

- $\Pr[F|V=1,\phi]\,\mu_F^m$ maximized at $\phi_P^U = 0$
  - Both $\Pr[F|V=1,\phi]$ and μ_F^m are decreasing in ϕ_P^U
- $\Pr[F|V=1,\phi]\,\mu_F^m$ also convex
- Summing over the 2 cases, principal always prefers extreme ϕ_P^U
- Chooses either extreme tough type
  - To maximize prices
  - At cost of lower probability of passing
- Or extreme soft type
  - To maximize probability of passing
  - At cost of lower prices

E. Extreme Tests are Best: High & Low Signals

- No comparable notion of toughness for ϕ_P^H and ϕ_P^L
- Higher ϕ_P^H
  - Raises beliefs more after a pass
  - And lowers them more after a fail ($\phi_F^H \downarrow$)
- Lower ϕ_P^L does same
- *For any* ϕ_P^U, *R maximized at* $\phi_P^H = 1$ *and* $\phi_P^L = 0$
  - Maximize μ_P^m and minimize μ_F^m
- Makes pass and fail signals as informative as possible
  - Principal knows her type to be good
  - So wants to transmit information as clearly as possible
  - If $\phi_P^H < 1$, so $\phi_F^H > 0$, some chance fail followed a high signal
  - If $\phi_P^L > 0$, some chance pass followed a low signal
- As $\phi_P^H \to \phi_P^L$, the test becomes uninformative

E. Extreme Tests are Best: Formal Results

Theorem 2:

- *Principal always selects* $\phi_P^H = 1$, $\phi_P^L = 0$ *and* $\phi_P^U \in \{0, 1\}$, *i.e., principal chooses a test which is as tough or as soft as possible on receiving an uninformative signal, and which always returns a pass on receiving a high signal and a fail on receiving a low signal*

Furthermore:

- *For any finite set of* ϕ_P^U *values such that* $\phi_{P1}^U > \phi_{P2}^U > ... > \phi_{PS}^U$, ϕ_{P1}^U *or* ϕ_{PS}^U *strictly preferred by principal to any intermediate test with* $\phi_{Ps}^U \notin \{\phi_{P1}^U, \phi_{PS}^U\}$

F. Choice of Extreme: Analytical Results

- Does principal prefer toughest ($\phi_P^U = 0$) or softest ($\phi_P^U = 1$) test?
  - When $\phi_P^H = 1$ and $\phi_P^L = 0$
- Suppose test's signals are symmetric
  - $p_1^H = p_0^L$

Theorem 3:

- *For sufficiently informative agent signals:*
  - *Principal strictly prefers softest test*
- *For sufficiently uninformative agent signals and* $q > \frac{1}{2}$*:*
  - *Principal strictly prefers softest test*
- *For sufficiently uninformative agent signals and* $q \leq \frac{1}{2}$*:*
  - *Principal strictly prefers toughest test*

F. Choice of Extreme: Intuition

- When agent signals are very uninformative
  - Principal expects agents to start with beliefs close to prior q
- So when q is high, limited scope to raise price using the test
  - Choose softest test, which is likely to be passed
  - (Even though failing a soft test is quite damaging)
- And when q is low, bigger scope to raise price using the test
  - Choose toughest test, which gives big upward impact if passed
  - Low initial agent beliefs encourages risk-taking
- When agent signals are very informative
  - Principal expects agents to start with good beliefs
  - Little upside from risking toughest test
  - So choose softest test

F. Choice of Extreme: Numerical Example 1/5

- For intermediate precision of agent information
  - Can't provide general description of test choice
  - Although a given principal can calculate optimal test
- Suppose $q = \frac{1}{2}$
- Agents signal set is binary
  - $p_1^1 = p_0^0 = p_A$
  - $p_0^1 = p_1^0 = 1 - p_A$
  - $p_A \in \left[\frac{1}{2}, 1\right)$ measures informativeness of agents' signals
- Test signals
  - $p_1^H = p_0^L = (p_T)^2$
  - $p_1^U = p_0^U = 2p_T(1 - p_T)$
  - $p_0^H = p_1^L = (1 - p_T)^2$
  - $p_T \in \left(\frac{1}{2}, 1\right)$ measures informativeness of test's signals
- Can think of test receiving two i.i.d. draws from binary signal set

F. Choice of Extreme: Numerical Example 2/5

Figure 2: $R(1) - R(0)$

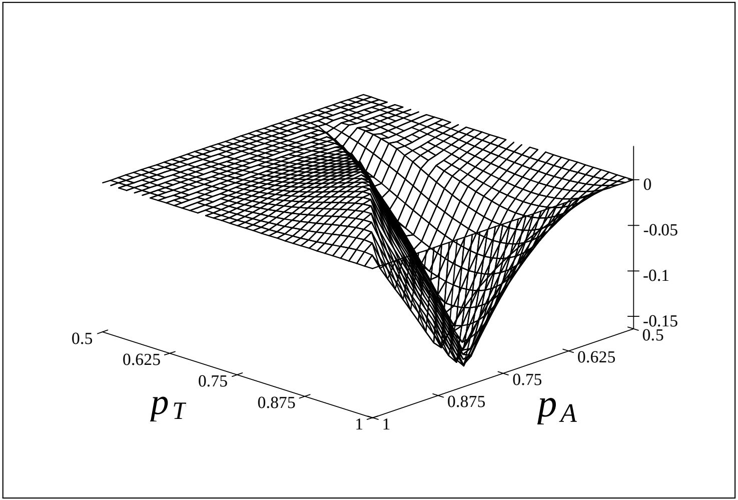
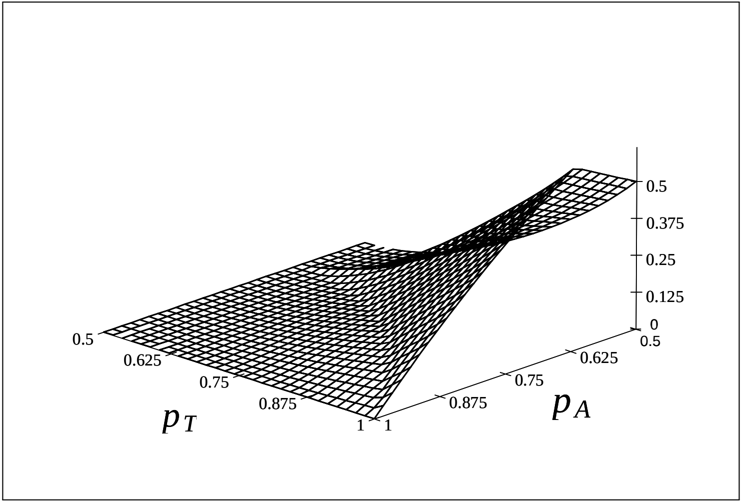


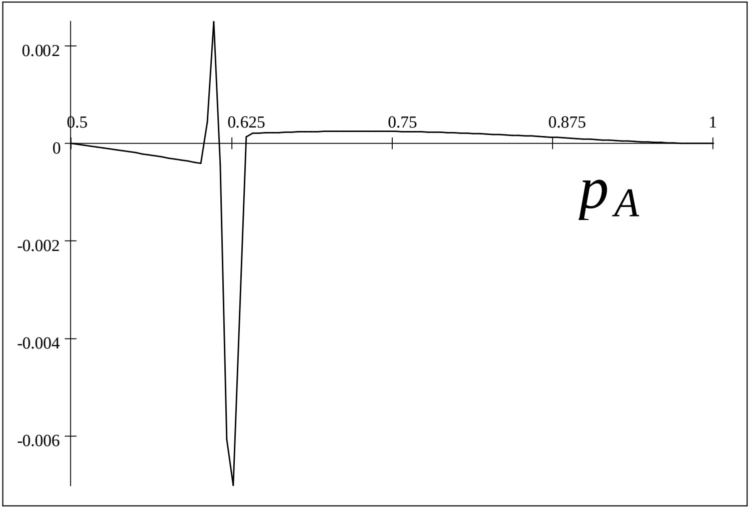
Figure 3: $\max\{R(1), R(0)\} - R(No\ Test)$



F. Choice of Extreme: Numerical Example 4/5

- Choice of test matters
  - Choosing correct extreme: up to 15% of max possible revenue
  - Optimal test vs. no test: up to 50% of max
- When toughest test is best
  - Choosing correct extreme matters more
  - Scope for prices to fall after failing a soft test
- As p_T rises
  - Importance of choosing to be tested goes up
- Over most of range of p_T
  - Toughest test best until p_A rises high enough
  - So Theorem 3 extends in natural way
  - Not true for very low p_T though

Figure 5: $R(1) - R(0)$ for $p_T = 0.55$



G. Importance of Price

- In Gill & Sgroi GEB 08
  - Principal seeks endorsement from sequence of agents
  - Potential for herding effects
- As length of sequence goes to 1
  - Becomes simplified analogue of this model
  - Except "price" is fixed
  - And information structure much less general
- P1: Any test with $\phi_P^U < \frac{1}{2}$ preferred to any other test with $\phi_P^U \geq \frac{1}{2}$
- P2: From continuum principal selects test with ϕ_P^U just below $\frac{1}{2}$
- Proof method is very different
  - Recursive method used to find R
- Despite differences, helps elucidate impact of prices
  - They drive optimal choice of test to an extreme

H. Conclusion

- Public tests
  - Can be important and effective way of transmitting information
  - But have received little attention in the literature
- Our principal always prefers to be tested
  - And choice of test can matter a lot
- With conditional pricing, choosing an extreme test is optimal
  - Toughest test where quality of information and prior are low
  - Softest test where quality of information is high
- In IO setting, integrate two key choices for firm launching a new product
  - Choice of initial price
  - Testing as a product marketing strategy
- Findings might explain existence and survival of
  - Reviewers with harsh styles, biases and critical approaches
  - And very easy tests or “yes-men”