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Private polling in elections and voter welfare ☆

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Abstract

We study elections in which two candidates poll voters about their preferred policies before taking policy positions. In the essentially unique equilibrium, candidates who receive moderate signals adopt more extreme platforms than their information suggests, but candidates with more extreme signals may moderate their platforms. Policy convergence does not maximize voters' welfare. Although candidates' platforms diverge in equilibrium, they do not do so as much as voters would like. We find that the electorate always prefers less correlation in candidate signals, and thus private over public polling. Some noise in the polling technology raises voters' welfare.

JEL classification: P16; C72; D72

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1. Introduction

Since the seminal papers of Hotelling [18], Downs [12], and Black [5], spatial competition models have greatly advanced our understanding of elections and campaigning. The central prediction is the median voter theorem, possibly the most famous result in political economy: Given voters with single-peaked preferences over a unidimensional policy space and two office-motivated candidates who are perfectly informed about voter preferences, both candidates locate at the median voter's preferred policy in the unique equilibrium. In reality, however, candidates often differentiate their platforms. A host of researchers have documented empirically that candidates' platforms diverge significantly from the estimated median voter's preferred policy, and yet are not too extreme.[1]

Our analysis starts with the observation that, in practice, political candidates do not know voters' policy preferences with certainty when selecting platforms. Determining the median voter's location is a difficult task, especially in the context of a complex political debate. Accordingly, candidates devote substantial resources to gathering information about voters through private polling. Eisinger [13] finds that since the Roosevelt administration, private polls have been an integral part of the White House modus operandi. Medvic [23] finds that 46 percent of all spending on U.S. Congressional campaigns in 1990 and 1992 was devoted to the hiring of political consultants, primarily political pollsters. In addition, the major parties provide polling services to their candidates. Private polling information is jealously guarded by candidates and parties. Indeed, Nixon had polls routinely conducted, but did not disclose results even to the Republican National Committee; and F.D. Roosevelt described private polling as his "secret weapon" (Eisinger [13]).

We develop a model of elections in which candidates receive private polling information about voters' preferences. Each candidate then updates about the median voter's preferred policy and the likely platform of the opponent and commits to an electoral platform, with the candidate whose platform is closest to the actual median policy winning. In our model, the median policy is given by $\mu = \alpha + \beta$, where α is independently and uniformly distributed, and candidates receive signals about β, which is symmetrically distributed around the ex-ante median. One interpretation of this median policy decomposition is that voters are unwilling or unable to provide pollsters accurate summaries about all of their views, as is suggested by the empirical work of Gelman and King [14]. Another interpretation is that candidates learn about the position β initially preferred by the median voter, after which electoral preferences may shift by α during the electoral campaign.[2] Our construction is completely general with respect to correlation in polling signals, capturing both private and public polling.

[1] See, for example, the National Election Survey data estimating presidential candidates' platforms from 1964 to 1972 (Page [30], Chapters 3 and 4) and for the 1984 and 1988 races (Merrill and Grofman [24], pages 55–56). Budge et al. [6] compare estimates of the U.S. and British median voters based on survey data (such as the NES and British Election Survey) with estimates of candidates' platforms derived from speech and writing context analyses. They find clear evidence of divergence from the median policy, and no evidence of extremization. Poole and Rosenthal [32] obtain similar findings using roll call voting to estimate Congress-persons' platforms (pages 62–63).

[2] For example, after platforms have been selected, a weakening economy may change voters' views about increased fiscal spending; or terrorist attacks may alter voters' views about civil rights restrictions.

In a companion paper, Bernhardt, Duggan, and Squintani [3] (henceforth BDS) show that in any pure strategy equilibrium, after receiving a signal, a candidate locates at the median of the posterior distribution over the location of the median voter, where the posterior is *conditioned on both candidates receiving that same signal.* As a consequence, each candidate locates more extremely than his best estimate of the median given his private signal.[3] The substantive content of this characterization is seriously limited by the possibility that the pure strategy equilibrium may not exist. In BDS, we provide sufficient conditions for the pure strategy equilibrium to exist. In this paper we show that these conditions are also necessary under some regularity assumptions. These conditions are implausible unless there are few possible signals, or unless signals are so precise that the probability that the opponent receives the same signal (rather than just a near-by signal) exceeds one half.

This finding leads us to prove that, even when a pure strategy equilibrium does not exist, there always exists a unique mixed-strategy equilibrium in which the locations of the candidates follow a strong order with respect to their signals, defined later in the paper. We derive the closed-form solution of this equilibrium and generate several empirical predictions. First, we show that candidates with sufficiently moderate signals adopt their pure strategy equilibrium platforms, locating more extremely than their information suggests, while candidates who receive more and more extreme signals mix over policy positions, tempering their positions by more and more toward the ex-ante median policy. This result reflects that a politician whose pollster predicts greater shifts in the median anticipates that he is more likely to compete against an opponent with a more moderate signal, who will take a more moderate platform. The result is broadly consistent with the empirical evidence that candidates' platforms significantly diverge from the median voter's preferred policy, and yet are not too extreme.

We then turn to the effect of the statistical properties of the polling technology on equilibrium platforms. We show that an increase in the precision of the candidates' signals leads candidates to locate more extremely, in the sense of first order stochastic dominance. This finding is consistent with the concurrent trends of platform polarization (see the NES data as reported in Budge et al. [6]) and technological improvement in polling. The effect of increased signal correlation across candidates (which can be induced by public polling, for example) is ambiguous for candidates with extreme signals, but it unambiguously moderates their locations following moderate signals.

We then provide a thorough analysis of the welfare properties of private polling and equilibrium outcomes. Our analysis builds on the observation that in a model with office-motivated candidates who share symmetric information on the unknown median policy, à la Wittman [35] or Calvert [8], candidates' platforms converge to the median of the median policy distribution and do not offer voters with enough choice. If one were to introduce exogenously a small amount of dispersion in candidate platforms, then each candidate's individual platform would target the median less accurately. Collectively, however, the platform closest to the realized median would generally be more accurate than the median of the median policy distribution. Because candidates care only about winning, they do not internalize this externality. As a result, candidates do not provide enough platform dispersion from the standpoint of the electorate. We identify conditions under which this insight extends endogenously to the asymmetric information setting considered

[3] BDS consider a more general version of the model studied here, but the lack of structure imposed on the distribution of the median policy limits the analysis to a study of the existence and continuity properties of equilibria.

in this paper: Candidates' platforms diverge in equilibrium due to private polling, but not by as much as voters would like.

Our welfare analysis then proceeds to show that greater signal correlation makes voters worse off: Correlation reduces both the degree by which candidates "extremize" their platforms given their signals, as well as the probability that candidates receive different signals, choose distinct platforms, and thus provide more variety to the electorate. In contrast, the effect of signal precision on welfare is non-monotonic. Increased polling accuracy raises the probability that candidates correctly identify the median voter's preferred policy, raising the welfare from any one candidate's platform. However, increased polling accuracy also raises the probability that the candidates adopt similar platforms, reducing the choice that candidates give voters. The net effect is that up to some point, raising precision raises welfare, but too much precision has the opposite effect.

These final two results have implications for public policy. First, the electorate prefers private to public polling, because sharing information raises the correlation between candidates' information and adversely reduces platform diversity. This finding provides support for public polling bans that does not rest on claims that public polling may distort elections because of bandwagon effects or effects on voter participation. Second, because greater precision eventually reduces voter welfare, campaign spending caps that limit resources devoted to polling may raise voter welfare, even when campaign advertising is truly informative and beneficial to the electorate.

The set of papers considering aspects of elections with privately-informed candidates begins with Ledyard [20], who raised the issue of privately-informed candidates and considered examples exploring the effects of the order of candidate position-taking, public polls, and repeated elections. Chan [9] studies a three-signal variant of our model, showing that a pure strategy equilibrium exists when signals are almost uninformative, but more generally, his analysis does not consider the possibility that pure strategy equilibria fail to exist. Within his framework, he finds that signal precision can reduce voter's welfare. Ottaviani and Sørensen [29] numerically analyze a model of financial analysts who receive private signals of a firm's earnings and simultaneously announce forecasts, with rewards depending on relative forecast accuracy. The case of two analysts can be re-interpreted as a model of electoral competition with privately-informed candidates. They show that greater competition increases the strategic bias in forecasts: With more forecasters, forecasts grow more extreme.[4]

There is a growing literature on the strategic incentives of polled citizens in delivering their responses to pollsters. In Morgan and Stocken [27], citizens have private information of common value, but differ in their ideology. The policy maker implements a policy after polling citizens. They find that full truthful revelation is impossible as the poll size grows large, but full information aggregation can arise in equilibrium. Closer to our model, Meirowitz [25] studies a two-candidate Downsian election where prior to presenting platforms, candidates have access to polls. In contrast to our focus, he considers public polls, abstracting from private information. He finds that for most environments honest poll responses cannot occur in a perfect Bayesian equilibrium.

While we identify adverse effects of polling in that candidates do not differentiate sufficiently when polling is too precise, Taylor and Yildirim [34] and Goeree and Grosser [15] identify dif-

[4] More distantly related, Heidhues and Lagerlöf [17] study a setting where candidates know more than voters about the optimal policies for voters, and candidates can pre-commit to one of two exogenous policy alternatives. Martinelli [22] analyzes a related setting where platforms are endogenous and voters have private information. If voters' information is biased, equilibrium results in less than full convergence even if parties know the optimal policy.

ferent adverse effects of polling. Unlike us, they suppose that voting is costly, and consider only public polls. They show that public polls increase voter turnout but reduce expected welfare, because they coordinate the minoritarian group to participate to the election at higher rates than the majoritarian group so as to induce a "toss-up" election.

Other models that generate platform separation feature policy-motivated candidates (Wittman [35], Calvert [8]), platform-motivated candidates (Callander and Wilkie [7] and Kartik and McAfee [19]), heterogeneity in candidate valence (Aragones and Palfrey [1], Groseclose [16]), the threat of entry by a third candidate (Palfrey [31]), or citizen-candidate models where candidates cannot commit to positions (Osborne and Slivinski [28], Besley and Coate [4]). Models that reverse the informational environment so that voters see candidate attributes with noise that is affected by campaign advertising include Austen Smith [2], Coate [10,11] and Prat [33].

2. The electoral framework

Two political candidates, A and B, simultaneously choose policy platforms on the real line, where we use x to denote candidate A's platform and y to denote B's. There is a unique median voter, whose preferred policy position is given by μ. Candidate A wins the election if his platform is closer to the median voter's preferred position than candidate B's, i.e., if $|x-\mu|<|y-\mu|$, and A loses if he locates further away. If $|x-\mu|=|y-\mu|$, then the election is decided by a fair coin toss, so that A wins with probability one half.

Candidates do not observe μ, but they receive information about voters' preferred policies from private polls. Polling generates signals about the date 1 location of the median voter, given by β. Then candidates choose platforms. Finally, the election is at date 2. Between dates 1 and 2, the median voter's preferred platform may shift, so that the median voter's final preferred position is $\mu=\alpha+\beta$.[5] For simplicity, we assume that β is a discrete random variable with support on $-R,\dots,-1,0,1,\dots,R$, where $E[\beta]$ is normalized to zero, and α is independently and uniformly distributed on $[-a,a]$, with $a>2R$. This assumption captures the idea that the unresolved uncertainty in the median voter's position at the time of polling can sometimes outweigh policy preferences elicited through polling. Technically, α ensures that the distribution of μ is uniform on the relevant range (see below).

Polling provides candidates with private real-valued signals i and j about β, drawn from the finite set $-K,\dots,-1,0,1,\dots,K$. Signals are drawn prior to the shift in the median voter's position, and are therefore independent of α. We use $P(i,j,b)$ to denote the joint prior probability of candidate A's signal i and candidate B's signal j and realization $\beta=b$. Analogously, we use $P(i,j)$ to denote the marginal probability of signals i and j, $P(i)$ to represent the marginal probability of signal i; and $P(b)$ to denote the marginal probability of b. We focus on a symmetric environment in which $P(b)=P(-b)>0$, and $P(i,j|b)=P(-i,-j|-b)>0$. Given a subset I of signals, we denote the distribution of the median μ conditional on one candidate receiving signal i and the other receiving a signal in the set I by $F_{i,I}$, and let $f_{i,I}$ represent the associated density. We denote the median of $F_{i,I}$ by $m_{i,I}$, and note that $m_{i,I}$ equals the conditional expectation of b, $E[b|i,I]=\sum_b bP(b|i,I)$. We denote the conditional distributions given one signal i and two signals i,j by F_i and $F_{i,j}$, and the associated medians by m_i and $m_{i,j}$. We assume that P and F are both symmetric with $P(i,j)=P(j,i)$ and $F_{i,j}=F_{j,i}$ for all signals i and j. This implies that the candidates have access to equally informative polling technologies.

[5] An alternative interpretation is that α captures an additional source of error inherent in the polling process, e.g., α may represent a component of policy preferences about which voters are unable or unwilling to divulge information.

Because we associate larger signals with larger realizations of β, we assume that $m_{i,I} < m_{j,I}$ whenever $i < j$. We further assume that signals are "self-reinforcing", so that $i > 0$ implies $m_{i,i} > m_i$ and $m_{i,i} > m_{i,I\setminus\{i\}}$, where $m_{i,I\setminus\{i\}}$ denotes the median conditional on the other candidate receiving a signal other than i.[6] In our symmetric model $m_{i,j} = -m_{-i,-j}$ for all i, j. Hence, the signal $i = 0$ is "uninformative", i.e., the median following signal $i = 0$ equals the ex-ante median, $m_{00} = m_0 = E[\beta] = 0$. We denote the vector of medians $(m_{i,i})_{i\in I}$ by M. Our assumptions on the distributions of α and β simplify equilibrium calculations as they imply that the distribution $F_{i,I}$ is linear over the relevant interval $[-R, R]$:

$$F_{i,I}(z) = \frac{a - m_{i,I} + z}{2a}, \quad \text{for all } z \in [-R, R].$$

We believe that the qualitative properties of our equilibrium analysis do not hinge on this structure.

We next highlight the structural assumptions on the conditional distributions of a candidate's signal given the opponent's signal realization that we use in our analysis. This structure is mild. Assumption (A1) is a stochastic dominance restriction on the conditional distributions of signals. It says that when one candidate receives a higher signal, the other candidate is also more likely to receive a higher signal. Assumption (A2) says the higher is a candidate's signal, the more likely it is to exceed his opponent's. Assumption (A3) says that a candidate is most likely to receive signal k when the other candidate also receives k. Our characterization in Theorem 3 imposes no other structure on candidates' signals, for example, capturing conditionally-independent and perfectly correlated signals as special cases.

(A1) For all signals i, k with $k < i$,

$$\sum_{j:\ j<\ell} P(j|k) \geqslant \sum_{j:\ j<\ell} P(j|i), \quad \text{for all signals } \ell.$$

(A2) For all signals i, k with $k < i$,

$$\sum_{j:\ j<i} P(j|i) \geqslant \sum_{j:\ j<k} P(j|k) \quad \text{and} \quad \sum_{j:\ j\leqslant i} P(j|i) \geqslant \sum_{j:\ j\leqslant k} P(j|k).$$

(A3) For all signals i and k, $P(k|k) \geqslant P(k|i)$.

To illustrate our assumptions, we introduce the following example.

Example 1. Let there be $2K + 1$ ex-ante equally-likely values of $b \in \{-K, \dots, 0, \dots, K\}$, and $2K + 1$ possible signals, $i, j \in \{-K, \dots, 0, \dots, K\}$. With probability $q < 1$, the candidates receive the same signal, and with probability $1 - q$ they receive conditionally-independent signals. A signal is correct with probability $p \geqslant \frac{1}{2K+1}$, and with equal probability $\frac{1-p}{2K}$ any of the other signals is drawn. Thus, p captures the accuracy of the polling signals, and q is a measure of the correlation between the candidates' signals. Calculations reveal that for any i,

[6] The assumption that signals are self-reinforcing is not used in the equilibrium analysis. Rather, it is employed in the interpretation of equilibrium platforms. As we will show, when a player endowed with a signal i plays a pure-strategy, he adopts the platform $m_{i,i}$ in equilibrium. Because signals are self-reinforcing, this implies that the player adopts a more extreme platform than his signal i suggests.

$$P(i|i) = q + (1-q)\left(2K\left(\frac{1-p}{2K}\right)^2 + p^2\right),$$
$$P(j|i) = (1-q)\left(\frac{1-p}{2K}\right)\left((2K-1)\frac{1-p}{2K} + 2p\right), \quad \text{for } j \neq i,$$

and it is straightforward to check that assumptions (A1), (A2), and (A3) hold. Solving for the median conditional only on a candidate's own signal i and for the median conditional on both candidates receiving the same signal i yields

$$m_i = i\left(p - \frac{1-p}{2K}\right) \quad \text{and} \quad m_{i,i} = i\,\frac{(qp + (1-q)p^2) - (q\frac{1-p}{2K} + (1-q)(\frac{1-p}{2K})^2)}{q + (1-q)(2K(\frac{1-p}{2K})^2 + p^2)},$$

and due to symmetry, $m_0 = m_{0,0} = 0$. Since $m_i < m_{i,i}$ for all $i > 0$, signals are self-reinforcing.

Having described the polling technology, we now turn to describe electoral outcomes. If candidate A locates to the left of B, i.e., $x < y$, then candidate A wins when $\mu < (x+y)/2$. Conversely, if $x > y$, then A wins when $\mu > (x+y)/2$. The probability that B wins is just one minus the probability that A wins. Thus, the probability that candidate A wins when A adopts platform x following signal i and B adopts platform y following signal j is

$$\pi_A(x, y|i, j) = \begin{cases} F_{i,j}(\frac{x+y}{2}) & \text{if } x < y, \\ 1 - F_{i,j}(\frac{x+y}{2}) & \text{if } y < x, \\ \frac{1}{2} & \text{if } x = y. \end{cases}$$

We define a Bayesian game between the candidates in which pure strategies for candidates A and B are vectors $X = (x_i)$ and $Y = (y_j)$, respectively, and the solution concept is Bayesian equilibrium. The equilibrium is *symmetric* if $X = Y$. Given pure strategies X and Y, candidate A's interim expected payoff conditional on signal i and A's ex-ante expected payoff are

$$\Pi_A(X, Y|i) = \sum_j P(j|i)\pi_A(x_i, y_j|i, j) \quad \text{and} \quad \Pi_A(X, Y) = \sum_i P(i)\Pi_A(X, Y|i),$$

respectively. The ex-ante game is a two-player, constant-sum game: For all X and Y, $\Pi_A(X, Y) + \Pi_B(X, Y) = 1$. Because the game is constant sum, equilibria are interchangeable in the sense that if (X, Y) and (X', Y') are equilibria, then so are (X, Y') and (X', Y).

These concepts extend to mixed strategies, where candidate behavior is described by cumulative distribution functions over platforms. Such strategies capture the possibility that a candidate's position following a signal may not be precisely predicted by his opponent. We let G_i represent the distribution over platforms adopted by a candidate after receiving signal i. A mixed strategy is then a vector $G = (G_i)$ of cumulative distribution functions. A mixed strategy equilibrium is a pair (G, H) of mixed strategies such that candidate A's strategy G maximizes her expected payoff given each signal i, and similarly for candidate B's strategy H.

When pure strategy equilibria fail to exist, we search for mixed-strategy equilibria that satisfy a simple monotonicity condition. Specifically, we assume that following any given signal, if candidates do not locate deterministically, then they locate according to a distribution with connected support, where these supports are non-overlapping and ordered according to their signals. In the following definition, $\underline{x}_i$ and $\bar{x}_i$ denote the lower and upper bounds of a candidate's mixed strategy distribution following signal i. When these bounds coincide, i.e., $\underline{x}_i = \bar{x}_i$, the candidate locates deterministically.[7]

Definition 1. A mixed strategy G is *ordered* if (a) for all signals i, $\mathrm{Supp}(G_i) = [\underline{x}_i, \bar{x}_i]$, and (b) for all signals i and j with $i < j$, $\bar{x}_i \leqslant \underline{x}_j$.

This concludes the description of our model. We next derive the unique ordered mixed strategy equilibrium and determine its empirical and welfare properties.

3. Equilibrium analysis

BDS show that when a pure strategy equilibrium exists, it is unique and take a simple form: After receiving a signal, each candidate locates at the median of the distribution of μ conditional on *both* candidates receiving that signal.

Theorem 1 *(BDS). If (X, Y) is a pure strategy equilibrium, then $x_i = y_i = m_{i,i}$ for all signals i.*

Intuitively, if the candidates were to locate symmetrically at a platform $x_i = y_i$ away from $m_{i,i}$ following signal i, then either candidate, say A, could exploit this by moving slightly toward $m_{i,i}$. For instance, suppose that $x_i > m_{i,i}$. Conditional on B receiving signal realization i, A's expected payoff for playing x_i is $1/2$. By slightly moving toward $m_{i,i}$ his expected payoff increases discontinuously to almost $F_{i,i}(x_i)$. This quantity exceeds $1/2$ because $F_{i,i}(m_{i,i}) = 1/2$, and $x_i > m_{i,i}$. If instead B receives a signal other than i, A's payoff would vary continuously with A's location. Averaging expected payoffs across opponent's signals, we obtain that a slight deviation from x_i toward $m_{i,i}$ would raise A's payoff, which is impossible in equilibrium. When, instead $x_i = m_{i,i}$, then because $F_{i,i}(x_i) = F_{i,i}(m_{i,i}) = 1/2$, a slight deviation from x_i marginally reduces the probability of winning the election below $1/2$, if the opponent has the same signal i, and changes the probability of winning continuously if the opponent has any other signal.[8]

This result is reminiscent of Milgrom's [26] findings on common-value second-price auctions. There, competition drives the equilibrium bid of type θ to the expected value of the good conditional on the top two types being equal to θ, because a winning bidder ties with an opponent with the same type. Here, because candidates maximize the probability of winning, the relevant statistic is the median rather than the mean. In contrast to the auction setting, however, we will show that a pure-strategy equilibrium need not exist, and that in the (essentially unique) ordered mixed strategy equilibrium, extreme-signal candidates moderate their platform relatively to their private information. In auction terminology, they bid more conservatively than in Milgrom's equilibrium.

Theorem 1 has strong implications due to the self-reinforcing nature of signals: A corollary is that candidates take policy positions that are extreme relative to the median of μ given only their own information. This result runs counter to the standard platform convergence result,

[7] While stronger than a monotonicity condition that would only require that strategies be ordered in first-order stochastic dominance, the requirement that mixed strategies be ordered in the support ensures that there is a unique equilibrium. We conjecture that this structure is unnecessary, but have no proof when there are more than five signals.

[8] To conclude that there are no asymmetric pure-strategy equilibria, note that if (X, Y), with $x_i \neq m_{i,i}$ for some i, is a pure strategy equilibrium, then symmetry implies that (Y, X) is an equilibrium, and interchangeability implies that (X, X) is an equilibrium, contradicting the result that $x_i = y_i = m_{i,i}$ in any symmetric equilibrium.

as it highlights a tendency toward polarization in elections with private polling. But while the pure-strategy equilibrium characterization yields strong implications, its substantive content is obviously limited to settings where the pure strategy equilibrium exists.

In BDS, we introduce sufficient conditions for pure-strategy equilibrium existence. Our next result highlights the possibility that the pure strategy equilibrium does not exist by showing that the sufficient conditions introduced in BDS are also necessary: The pure strategy equilibrium exists *only* if each signal i is a median of the probability distribution $P(\cdot|i)$. Formally, for every signal i, we must have

$$\sum_{j:\ j\leqslant i} P(j|i) \geqslant \sum_{j:\ j>i} P(j|i) \quad \text{and} \quad \sum_{j:\ j<i} P(j|i) \leqslant \sum_{j:\ j\geqslant i} P(j|i). \tag{1}$$

To see why condition (1) is necessary, suppose it is violated. Specifically, suppose that when a candidate's signal is i, the probability her opponent's signal j is strictly smaller than i exceeds the probability that j is weakly larger than i. If the candidate takes a platform smaller than the equilibrium prescription $m_{i,i}$, then he increases the probability of winning the election when the opponent receives a signal $j < i$, and decreases the probability of winning when $j \geqslant i$. Because the probability that $j < i$ exceeds the probability that $j \geqslant i$, the candidate gains from this deviation from the posited equilibrium. To facilitate the simple expression of our result, we frame our pure-strategy equilibrium non-existence result using a simplifying condition on the conditional medians. Specifically, we assume that $m_{i,j}$ is the average of the two conditional medians $m_{i,i}$ and $m_{j,j}$. This assumption implies that, when a candidate has a signal i and is considering the possibility that the opponent's signal is j, he ties the election by following the equilibrium strategy $m_{i,i}$, given that the opponent plays the equilibrium strategy $m_{j,j}$. As a result, whether he has an incentive to take a platform that is less than the equilibrium platform $m_{i,i}$ depends only on whether the probability $P(j|i)$ of opponent types $j < i$ is greater or less than the probability $P(j|i)$ of opponent types $j \geqslant i$; and vice versa for deviations greater than $m_{i,i}$.

Theorem 2. *Suppose that $m_{i,j} = [m_{i,i} + m_{j,j}]/2$ for all signals i and j. A pure strategy equilibrium exists only if condition* (1) *is satisfied for all signals i.*

While satisfied when signals are perfectly correlated ex-ante, condition (1) is likely to be restrictive. It is particularly restrictive for the most extreme signals, for which $P(i|i) \geqslant 1/2$ is implied. The significance of this non-existence result rests on the observation that in elections with finely-detailed polling and hence many possible signals, condition (1) is not plausibly satisfied, unless signals are so precise that the probability that, conditional on receiving the most extreme signal, the opponent receives the same signal (not just a nearby one) is larger than one half. In light of this negative result, we turn to the analysis of equilibria of the electoral game in which candidates adopt mixed strategies.

We begin by characterizing the maximal set C of signals i that satisfy the inequalities in (1). We show that C is a non-empty set of *moderate* signals, symmetric around the signal zero. Conditional on such moderate signals, candidates expect only a small shift in the median μ away from the ex-ante median $E[\mu] = 0$. We let $\bar{c} = \max C$.

Lemma 1. *Under* (A1)*, the set C is non-empty. Adding* (A2)*, C is a set of moderate signals*: *for any signal i, we have $i \in C$ if and only if $-\bar{c} \leqslant i \leqslant \bar{c}$.*

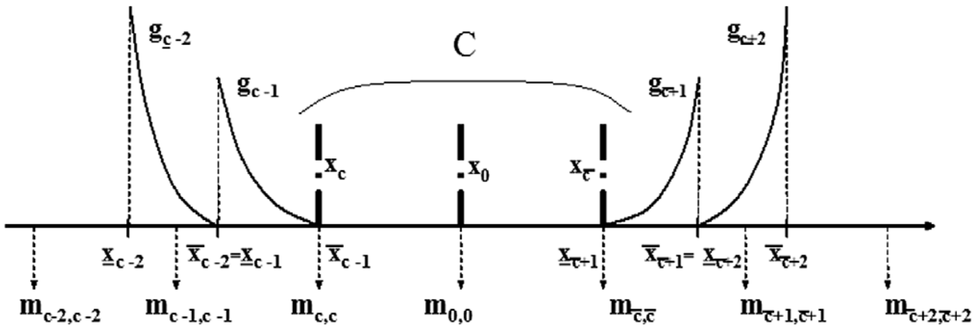

Fig. 1. Equilibrium diagram.

We now calculate the unique ordered mixed-strategy equilibrium. We prove in Theorem 3 that candidates who receive moderate signals in the set C play the pure-strategy $m_{i,i}$, and hence extremize their platforms, due to the reinforcing nature of signals. In contrast, candidates with extreme signals, say $i > \bar{c}$, adopt convex, increasing mixed strategy densities that place all probability mass on locations that are more moderate than $m_{i,i}$. Further, the supports of the distributions following extreme signals are adjacent. A symmetric characterization holds for signals $i < -\bar{c}$. Fig. 1 depicts the ordered equilibrium.

The intuition for these results is that following an extreme signal $i > \bar{c}$, a candidate believes that with high probability, the opponent's signal is less than i. In order to compete against a candidate who likely holds a smaller signal, he moderates his platform relative to $m_{i,i}$. Theorem 1 indicates that such moderation means that he turns to a mixed strategy. In sum, when a pollster predicts a dramatic shift in voter's preferences, we predict that a candidate will not follow such extreme recommendations, and instead will moderate his platforms and make his platform less easily predictable by his opponent. The joint effect of extremization following moderate signals in C, and moderation following extreme signals outside C, substantiates our claim that private polling induces outcomes that diverge from the median, but yet do not exhibit extreme polarization.

Theorem 3. *Under* (A1)–(A3)*, there exists a unique ordered mixed strategy equilibrium. For all $i \in C$, candidates locate at $m_{i,i}$, and for all $i > \bar{c}$, candidates mix according to an increasing, convex density,*

$$g_i(x) = \frac{\Phi_i}{2}\sqrt{\frac{m_{i,i} - \underline{x}_i}{(m_{i,i} - x)^3}} > 0 \tag{2}$$

on the interval $[\underline{x}_i, \bar{x}_i]$ with $\bar{x}_i < m_{i,i}$. Here,

$$\Phi_i = \frac{\sum_{j:\, j<i} P(j|i) - 1/2}{P(i|i)} > 0 \quad \text{and} \quad \underline{x}_{i+1} = m_{i,i}\left[1 - \left(\frac{\Phi_i}{\Phi_i + 1}\right)^2\right] + \underline{x}_i\left(\frac{\Phi_i}{\Phi_i + 1}\right)^2, \tag{3}$$

with $\underline{x}_{\bar{c}+1} = m_{\bar{c},\bar{c}}$. These supports are adjacent, in the sense that $\bar{x}_{i-1} = \underline{x}_i$ for all $i > \bar{c}$. The equilibrium is symmetric across signals i around the signal zero.

The proof in Appendix A proceeds sequentially. We first work under assumptions (A1) and (A2) only. We prove that an equilibrium mixed strategy cannot put a probability atom following signal i on any platform other than $m_{i,i}$. Further, because a candidate must be indifferent over all positions in the support of his mixed strategy, the second-order condition must hold as an equality over any non-degenerate interval in the support, taking the simple form

$$3g_i(x)f_{i,i}(x) + g_i'(x)\big(2F_{i,i}(x) - 1\big) = 0.$$

We solve this system of ordinary differential equations for a mixed-strategy equilibrium up to the initial condition $g_i(\underline{x}_i)$. We use this characterization to show that for signals $i \in C$, the candidate necessarily places probability one on the conditional median $m_{i,i}$, and that for signals $i \notin C$, the candidate mixes according to a continuous distribution G_i. We then show that there can be no gaps between the supports of the candidates' mixed strategies. This result provides the initial conditions $g_i(\underline{x}_i)$ to conclude the calculation of g_i. Adding (A3) to conditions (A1) and (A2), we show that each candidate's payoffs are single-peaked in x_i around the support of his mixed strategy—candidates play best responses with probability one—and hence the densities g_i induce the unique mixed-strategy equilibrium.[9]

Our explicit solution for the equilibrium allow us to calculate a number of statistics of potential empirical interest. Owing to equilibrium symmetry across signals i, we henceforth focus on signals $i > \bar{c}$ whenever studying extreme signals $i \notin C$. For any such signal, the cumulative distribution function with which candidates mix on the interval $[\underline{x}_i, \bar{x}_i]$ is given by

$$G_i(x) = \Phi_i\left[\sqrt{\frac{m_{i,i} - \underline{x}_i}{m_{i,i} - x}} - 1\right]. \tag{4}$$

Further, the expected platform of a candidate with signal $i > \bar{c}$ is a weighted average of $\underline{x}_i$ and $m_{i,i}$,

$$E[x_i] = \frac{\Phi_i}{\Phi_i + 1}\underline{x}_i + \frac{1}{\Phi_i + 1}m_{i,i}. \tag{5}$$

We conclude this section by documenting a continuity property of pure-strategy equilibrium. Note that $\Phi_i \downarrow 0$ is equivalent to $\sum_{j:\ j \geqslant i} P(j|i) \uparrow 1/2$, so that condition (1) for candidates to adopt their pure strategy location of $m_{i,i}$ becomes close to being satisfied. The closed-form solution for the equilibrium mixed strategy in Theorem 3 reveals that as Φ_i goes to zero, candidates place almost all probability close to their pure strategy location of $m_{i,i}$, providing robustness of the pure strategy equilibrium (in a probabilistic sense) with respect to small deviations from the inequalities in (1).

In the next section, we use these equilibrium statistics to derive implications of signal precision and correlation on equilibrium platform extremization.

4. Positive implications

In this section, we exploit the closed-form solution for G_i to develop a qualitative understanding of how candidates use their private information when choosing platforms. We first examine how the propensity of candidates to extremize their location varies with their signals. We then determine how the statistical properties of the candidates' polling technology—the correlation and precision of their signals—affect platform choices. By linking these properties to different types of elections, we obtain a number of novel empirical predictions.

[9] In the proof of the theorem, (A2) and (A3) are used only to address signals $i \notin C$. It follows that if (1) holds for all signals, then (A1) delivers existence of the pure strategy equilibrium.

Signal realization. We first derive a key characterization result: Candidates with more moderate signals expect to locate more extremely relative to their information than do candidates with more extreme signals. Specifically, we identify simple statistical conditions under which candidates with extreme signals $i > \bar{c}$ expect to locate further away from $m_{i,i}$ as i increases. Intuitively, as the shift in voter's preferences predicted by the pollster becomes more extreme, we predict that the candidates will become less likely to follow the pollster's recommendations. This result reinforces a key positive insight: Polling yields platform divergence, due to the self-reinforcing nature of signals, but not platform polarization.

First note that plausibly the coefficients Φ_i in the characterization of Theorem 3 increase in i for $i > \bar{c}$. In fact, under (A2), $\sum_{j:\ j \leqslant i} P(j|i)$ rises with i: Increasing a candidate's signal raises the probability that his signal is at least as high as his opponent's. Hence, as a candidate's signal grows more extreme, Φ_i rises as long as the conditional probability $P(i|i)$ that the other candidate receives the same signal does not rise too sharply with i.

When Φ_i is increasing in $i > \bar{c}$, it follows immediately from (4) that $G_i(m_{i,i} - x)$ rises with $i > \bar{c}$. Hence, to prove that candidates with more extreme signals expect to locate more moderately relative to their information, we just need to show that $m_{i,i} - \underline{x}_i$ is strictly increasing in i. As we show in the proof of the next proposition, a gross sufficient condition for this is that the distance between successive conditional medians, $m_{i+1,i+1} - m_{i,i}$, not fall too quickly with i for $i \geqslant \bar{c}$.

Proposition 1. *Suppose that for all signals $i > \bar{c}$, $\Phi_{i+1} \geqslant \Phi_i$. Then, there exists $\delta > 0$ such that if $m_{j+1,j+1} - m_{j,j} > m_{j,j} - m_{j-1,j-1} - \delta$ for all $j \geqslant \bar{c}$, then $m_{i,i} - E[x_i]$ is strictly increasing in i for $i \geqslant \bar{c}$.*

Precision. In Example 1, signal precision is represented by the probability p that the signal is correct. To describe the general implications of an increase in signal precision, we introduce simple restrictions that are consistent with the implications of increasing signal precision in Example 1.[10] Consider any signal $i \geqslant 0$. Under reasonable structural assumptions, increasing signal precision implies that when candidate A receives signal i, candidate B is also more likely to receive this signal, so that $P(i|i)$ increases. Similarly, increasing precision should cause $\sum_{j:\ j<i} P(j|i)$ to fall. Finally, increasing precision plausibly raises $m_{i,i}$ when $i > 0$, as higher precision increases the "informational content" of signals. These properties hold in Example 1 when we increase p.

The next result shows that increasing precision causes candidates to locate more extremely in equilibrium, in the sense of first order stochastic dominance. This result delivers the insight that candidates may have stronger tendencies to polarize platforms in higher-profile elections, such as presidential races, where candidates have more resources to devote to private polling, which, in turn, yield more precise polling outcomes. Empirically, this prediction is consistent with the concurrent trends of technological improvement in polling and platform polarization (see the NES data as reported in Budge et al. [6]).

Proposition 2. *Suppose that for all signals $i > 0$, both $P(i|i)$ and $m_{i,i}$ increase and $\sum_{j:\ j<i} P(j|i)$ decreases. Then candidates locate more extremely in equilibrium: For all signals $i > 0$, $G_i(x)$ decreases for all x, strictly so for all x in the support of G_i in the initial specification of the model.*

[10] A treatment and discussion of different concepts of signal precision can be found in Lehman [21].

The proof follows from inspection of Eq. (4) in Theorem 3: The cumulative distribution function G_i decreases for all x, because the increase in precision decreases Φ_i and at the same time it increases $m_{i,i}$.

Proposition 2 uses an "absolute" measure of extremization, but it does not reveal how precision affects the likelihood that a candidate chooses a platform more extreme than his estimate of the median voter's position. It shows that raising precision increases the probability of locating more extremely, but raising precision also raises m_i. This leads us to perform a numerical analysis of Example 1, with $K = 1$, and hence three possible signal realizations. We find that a candidate is always likely to extremize his location in the sense that he locates more extremely than his forecast, given his signal, of the median voter's position. In particular, the probability a candidate chooses a platform more moderate than his information suggests, i.e., $x_1 \in [0, m_1]$, is bounded from above by $\frac{1}{2}(\sqrt{2} - 1) \approx 0.207$, achieving this bound when signals are uninformative, i.e., $p \downarrow \frac{1}{3}$, and signals are uncorrelated, i.e., $q = 0$. Further, raising the signal accuracy p always raises the probability that a candidate locates more extremely than his information suggests.

Correlation. To address the impact of correlation, we generalize Example 1 as follows. Assume that, as in Example 1, with probability q both candidates receive the same signal drawn from $P(\cdot|b)$ for each realization of b, and with probability $1 - q$ candidates receive conditionally-independent signals drawn from the same $P(\cdot|b)$ distributions. The next result summarizes the effects of increased signal correlation, as measured by the coincidence probability q, on the equilibrium strategies of the candidates.

Proposition 3. *Suppose that conditional on the realization b, candidates receive the same signal with probability q and conditionally-independent signals with probability $1 - q$. Raising the signal correlation q causes candidates to locate more moderately following moderate signals $i \in C$. For extreme signals i outside the set C, increasing correlation generates countervailing effects. Specifically, raising q lowers Φ_i for all signals $i > \bar{c}$, and it lowers $m_{i,i}$ for all positive signals*:

$$\frac{d\Phi_i}{dq} < 0 \quad \text{for } i > \bar{c} \quad \text{and} \quad \frac{dm_{i,i}}{dq} < 0 \quad \text{for } i > 0.$$

Further, the set C is weakly increasing in q.

Combining Proposition 3 with Theorem 3 yields direct characterizations. Every moderate signal i belonging to the set C remains in C when correlation increases, and candidate locations following such signals become more moderate. This result is intuitive: A moderate signal i induces a candidate to play $m_{i,i}$, the median conditional on both signals being i, and when signals become more correlated, $m_{i,i}$ shifts inward toward m_i, the median conditional on only one signal being equal to i. For extreme signals $i > \bar{c}$, a decrease in Φ_i leads candidates to take more extreme platforms, but the decrease in conditional medians $m_{i,i}$ following positive signals shifts the lower bounds of supports, $\underline{x}_i$, in toward the ex-ante median $m_{0,0}$. Thus, increasing correlation generates countervailing effects on candidate location following extreme signals. This highlights an essential difference between an increase in correlation and an increase in precision: Both decrease Φ_i, but increased correlation shifts conditional medians in toward $m_{0,0}$ producing a moderating effect.

To gain insight into which effect dominates, we return to Example 1, with three signal realizations. There, the pure strategy equilibrium exists for $P(1|1) \geqslant 0.5$, and increasing q reduces $m_{1,1}$, causing candidates to moderate their positions. However, for $P(1|1) < 0.5$, the equilibrium is in mixed strategies, and using the expression (5), we can show that $\frac{dE[x_1]}{dq} > 0$. That is, raising correlation causes the expected platform to become more extreme when the equilibrium is in mixed strategies, but to become less extreme when the equilibrium is in pure strategies.

5. Voter welfare

We now investigate the properties of socially optimal platforms and the consequences of equilibrium platform choices for voter welfare. To evaluate the optimality of equilibrium strategies, we adopt the standpoint of a social planner who does not have an informational advantage over the candidates in the electoral game, and whose role is simply to choose strategies for the two candidates to maximize voter welfare. To maintain consistency with equilibrium choices, we restrict the social planner to monotone strategies that are symmetric around zero. That is, the social planner selects the same strategy X for the two candidates, where $x_i > x_j$ for signals $i > j$ and $x_i = x_{-i}$ for all i.[11]

We first show that given simple statistical conditions on the polling technology, candidates locate too moderately for the electorate after observing their pollsters' signals: Voters prefer that candidates take even more extreme positions than they do in equilibrium. We then investigate how the statistical properties of the polling technology affect voter welfare. In particular, we show that increased correlation in candidate signals reduces voter welfare; and we illustrate how the optimal amount of noise in the polling technology from the perspective of voters is affected by the correlation in signals and the amount of uncertainty about the median voter's location.

We consider a distribution of voter preferences that has median equal to zero and that is subject to a stochastic shift, μ. Hence, we focus on aggregate preference uncertainty. Voter v's preferred policy, θ_v, is defined relative to the median voter's preferred policy μ: $\theta_v = \mu + \delta_v$, where δ_v represents the position of v's ideal point relative to μ, and a change in μ simply shifts the distribution of voter ideal points. We assume quadratic utilities, so that a voter with ideal point θ receives utility $u(\theta, z) = -(\theta - z)^2$ from policy outcome z, and we assume that each voter votes for the candidate whose platform is closest to his preferred policy. We let $W_{\delta_v}(X)$ represent the utility that voter v expects to receive if candidates use strategy X.

The next result implies that all voters have identical preferences over candidate strategies that are symmetric around zero, and that their ex-ante utility W_{δ_v} is strictly concave in X.

Lemma 2. *For all strategies X symmetric around zero, voter v's expected utility from X is a fixed amount δ_v^2 less than the expected utility of the median voter*:

$$W_{\delta_v}(X) = -\delta_v^2 + W_0(X).$$

Further, the welfare function $W_0(X)$ is a concave function of X.

Lemma 2 implies that without loss of generality we can focus on the median voter's welfare:

[11] This restriction makes it more difficult to find welfare-improving platforms for candidates, and hence strengthens our negative results on social inefficiency of equilibrium.

$$W_0(X) = -\sum_b \sum_i P(b,i,i) \int_{-a}^{a} \frac{(\alpha + b - x_i)^2}{2a} d\alpha$$
$$- \sum_b \sum_i \sum_{j:\ j>i} P(b,i,j) \left[\int_{-a}^{[x_i+x_j]/2-b} \frac{(\alpha + b - x_i)^2}{2a} d\alpha \right.$$
$$\left. + \int_{[x_i+x_j]/2-b}^{a} \frac{(\alpha + b - x_j)^2}{2a} d\alpha \right]$$
$$- \sum_b \sum_i \sum_{j:\ j<i} P(b,i,j) \left[\int_{-a}^{[x_i+x_j]/2-b} \frac{(\alpha + b - x_j)^2}{2a} d\alpha \right.$$
$$\left. + \int_{[x_i+x_j]/2-b}^{a} \frac{(\alpha + b - x_i)^2}{2a} d\alpha \right]. \tag{6}$$

In what follows, we therefore drop the subscript on $W(\cdot)$. Strict concavity of the welfare function $W(\cdot)$ implies that it has a unique maximizer, which we denote by X^*, that can be characterized using first-order conditions. By our symmetry assumptions, X^* is symmetric around zero, with $x_0^* = m_{0,0}$. Thus, we only need to consider the social optimality of candidate locations following signal realizations $i \neq 0$.

We now show that under simple statistical conditions on the polling technology, voters want candidates to take even more extreme positions than they do in a pure strategy equilibrium. That is, if voters could choose the amount by which candidates biased their location away from the ex-ante median following a signal realization, then they would *raise* the bias. This welfare result holds despite the thrust of Theorem 1 that pure-strategy equilibrium platforms are already extreme relative to expected location of the median voter conditional on pollsters' signals.

Underlying our welfare result is the fact that "extremizing" platforms raises expected platform separation. Voters value platform separation because they can always choose the candidate closer to the realized median policy. Because pollsters' signals are only imperfect estimates of the realized median policy, sufficient platform separation increases the chance that one of the two candidates' platforms is closer to the realized median policy. But candidates are not willing to provide sufficient platform separation because each candidate only cares only about winning the election, i.e., each candidate only cares about his location being the closest one to the realized median. Hence, to maximize voters' expected welfare, platforms should be more extreme than those chosen by candidates. Posed differently, in equilibrium, candidates do not internalize the positive externality on voters' welfare of extremizing their platforms, thereby providing voters with more choice.

Formalizing this reasoning, we investigate the three-signal setting and show that candidates do not locate as extremely as voters would like. In such environments, we show in Appendix A that the derivative of W with respect to x_1 takes the following form (see Eq. (23)):

$$\frac{\partial W}{\partial x_1}(M) = \frac{1}{a} \sum_{j:\ j<1} P(1,j) \left[(a + m_{j,1} - m_{1,1})^2 + \sigma_{j,1}^2 - \left(\frac{m_{j,j} - m_{1,1}}{2} \right)^2 \right] > 0,$$

where $\sigma_{j,k}^2$ is the variance of β conditional on signals j and k. To see the inequality, note that the term corresponding to $j=-1$ is positive. Because $a > m_{1,1} - m_{-1,-1}$, the term corresponding to $j=0$ is also positive. It follows that social welfare would be raised by a marginal increase in the candidates' platform following signal $i=1$. Concavity and symmetry around zero then imply that *candidates locate more moderately following signals* $i=1$ *and* $i=-1$ *than is optimal* in the pure-strategy equilibrium when it exists, i.e., $m_{1,1} < x_1^*$ and $x_{-1}^* < m_{-1,-1}$. More generally, given the characterization in Theorem 3, this extends immediately to mixed strategy equilibria. Intuitively, the voters gain from the increased separation in candidates' platforms when candidates receive different signals more than they lose in the increased bias of each candidate's platform relative to the estimate of the median voter's location, conditional on his signal. The following result summarizes these observations.

Proposition 4. *Consider a coarse three-signal setting. When the pure-strategy equilibrium exists, candidates locate too moderately relative to the social optimum following signals* $i \neq 0$. *Adding* (A1)–(A3)*, candidates locate too moderately relative to the social optimum with probability one following a signal* $i \neq 0$ *in the ordered mixed strategy equilibrium.*

Extending this result to arbitrary numbers of signal realizations requires more structure. We introduce the following two assumptions.

(A4) For all signals $i > 0$,

$$\sum_{j:\ j<i} P(j|i) > \sum_{j:\ j>i} P(j|i).$$

(A5) For all signals $i > 0$, $j > 0$, $j \neq i$,

$$\frac{P(j|i)}{P(-j|i)} > \frac{3m_{i,i} + m_{j,j}}{|m_{i,i} - m_{j,j}|}.$$

The condition in (A4) is not unreasonably restrictive, as there are $2i$ more signals to the left of i than there are to the right.[12] The inequality in (A5) says that if one candidate receives a positive signal i, then the probability the other candidate receives a positive signal j is sufficiently greater than the probability of receiving the more distant signal $-j$. Condition (A5) is satisfied if candidate signals are sufficiently correlated or sufficiently precise, as in high-profile elections in which candidates have substantial resources to devote to accurate polling. Conditions (A4) and (A5) are always (vacuously) satisfied in a three-signal model.

Armed with (A4) and (A5), we have the following result.

Theorem 4. *Under* (A4)*, when a is sufficiently large, we have* $\frac{\partial W}{\partial x_i}(M) > 0$ *for all signals* $i > 0$. *Adding* (A5)*, the conditional medians are more moderate than the socially optimal platforms*: $m_{i,i} < x_i^*$ *for all signals* $i > 0$. *An analogous result holds for signals* $i < 0$. *Hence, with* (A1)–(A3)*, candidates locate in equilibrium too moderately relative to the social optimum following every signal* $i \neq 0$.

[12] In fact, with symmetry of $P(\cdot|0)$ around the zero signal, (A4) would be implied by (A2) if the latter condition were strengthened so that the comparisons of conditional probabilities held with strict inequality.

As we show in Appendix A, under (A4), the partial derivative of welfare with respect to any signal $i > 0$ at the vector of medians M is strictly positive for all signals i, $\frac{\partial W}{\partial x_i}(M) > 0$, as long as a is sufficiently large.[13] Hence, voter welfare is increased by extremizing platforms slightly past their equilibrium locations, because for all signals i, the associated pure-strategy equilibrium location x_i coincides with the conditional median $m_{i,i}$; and under (A1)–(A3), $m_{i,i}$ exceeds the upper bound $\bar{x}_i$ of the support of the mixed strategy distribution G_i.

But the local result in the first part of Theorem 4 does not ensure that the *optimal* locations are more extreme than equilibrium platforms for all signal realizations. When the partial derivative of $W(\cdot)$ with respect to the location x_i following some signal j decreases sufficiently fast in x_i, it may be that the social planner extremizes the optimal location x_i^* so far past $m_{i,i}$ that he needs to "compensate" by moderating x_j^*. To rule out this pathological possibility, we introduce assumption (A5). We then use the quadratic form of the partial derivative of $W(\cdot)$ with respect to x_i, when viewed as a function of candidate positions following other signals. Assumption (A5) ensures that the cross-partial derivatives of $W(\cdot)$ are well-behaved even under the "worst case scenario" in which $x_j = m_{i,i}$ for positive signals $j < i$ and $x_j = m_{j,j}$ for signals $j > i$. We conclude that under reasonable statistical conditions on the polling technology, if the voters could choose the amount by which candidates biased their location away from the ex-ante median following a signal realization, they would choose to *increase* the bias.

We now turn to address how welfare is affected by the statistical properties of the polling technology. As in the analysis of Section 4, we address the impact of correlation by assuming that conditional on the realization b, candidates receive the same signal with probability q, and conditionally-independent signals with probability $1 - q$.

Theorem 5, below, shows that an increase in signal correlation reduces welfare when candidates select platforms $x_i = m_{i,i}$ for all signals i, as in the unique pure strategy equilibrium. Intuitively, an increment in signal correlation q increases the chance that candidates choose the same platform in equilibrium, and hence do not provide the voters with any platform separation. Voters' welfare is thus reduced, as the electorate values platform separation. This result bears the unexpected implication that voters may prefer private polling by candidates to public polling. Hence, we provide novel support for provisions that ban public polling—our argument does not rest on claims that public polling may distort elections because of bandwagon effects or distortions in voters' participation.

Theorem 5. *Suppose that conditional on the realization b, candidates receive the same signal with probability q, and conditionally-independent signals with probability* $1 - q$. *If* $\frac{\partial W}{\partial x_i}(M) > 0$ *for all signals i* > 0*, then an increase in signal correlation q reduces voter welfare*: $\frac{dW}{dq}(M) < 0$.

This result is proved as follows. From Proposition 3, we know that an increase in correlation shifts the conditional medians inward, decreasing candidate separation. Under our regularity assumptions, this indirect effect of increasing correlation reduces voter welfare. The proof then consists of showing that the direct effect of increasing correlation also reduces voter welfare.

While increased correlation monotonically reduces voter welfare, the effect of increasing signal precision on voter welfare is more subtle. It is immediate that perfectly precise polling dominates uninformative polling. We now prove that the impact of signal precision on welfare

[13] Even when a is not large, we have $\frac{\partial W}{\partial x_i}(M) > 0$ for any positive signal i under further conditions that we omit and make available upon request to the authors.

is not monotonic: While increasing precision up to some level raises welfare, eventually further increases in precision lower welfare. This is because as signals become very precise, there is a high chance that candidates receive signals close to each other. As a result, candidates play close platforms with high probability, and reduce welfare by not providing voters with enough platform separation. Because polling precision should be monotonic in the amount of resources devoted to polling, our result provides novel support for campaign spending caps. Spending caps that limit resources devoted to polling may raise voter welfare, even when campaign advertising is truly informative and otherwise beneficial to the electorate.

To formalize this reasoning, we show that welfare is raised if rather than having perfectly precise polling signals, signals are slightly noisy. We say that signals are *perfectly precise* if for all b and for signals $i = j = b$, $P(i, j|b) = P(i|b) = 1$.[14] When signals are perfectly precise, both candidates receive the "correct" signal and choose the same equilibrium platform, which equals the realization of β. We now prove that a small reduction in precision makes all voters better off, as it allows candidates to receive different signals and hence differentiate their platforms, thereby providing voters more choice. In particular, we suppose that candidates receive conditionally-independent signals, and that the probability a candidate's signal is correct is only $1 - \epsilon$. Inspection of the proof reveals that the result extends as long as when candidates receive the wrong signal, their signals are not perfectly correlated. Thus, we prove that welfare increases if we introduce signal noise such that the probability that both candidates receive "incorrect" signals is infinitesimal relative to the probability that one candidate receives the "correct" signal and one receives the "incorrect" one.

Theorem 6. *Suppose that candidates receive conditionally-independent signals. If signals are perfectly precise, i.e., for all b and for signals $i = j = b$, $P(i, j|b) = P(i|b) = 1$, then a sufficiently small reduction in signal precision increases social welfare in equilibrium.*

While Theorem 6 reveals that voters are better off if polling is not too accurate, it does not shed light on the optimal amount of noise to introduce to the polling technology from the perspective of voters, and how that optimal noise varies with the specifics of the economic environment. To glean such insights, we consider a binary variant of Example 1 in which there are two equally-likely values of $b \in \{-1, 1\}$ and two possible signals, $i \in \{-1, 1\}$. As in Example 1, signals are correct with probability p, and conditional on the realization b, candidates receive the same signal with probability q and conditionally-independent signals with probability $1-q$. Increasing the signal quality p raises the probability that both candidates receive the "correct" signal, but it also increases the probability that candidates receive the same signal, and hence do not offer voters variety. Fig. 2 presents level sets of voter welfare W as functions of (p, a) when $q = 0$, and level sets of W in (p, q) for $a = 2$.

Fig. 2 illustrates two important qualitative features. First, the value of precisely targeting β decreases with a. This reflects the fact that as a rises, there is more uncertainty about the median voter's preferred platform, which raises the value of increased platform choice. As a result, the optimal precision decreases in a: Voters want less accurate polls, so that candidates are less likely to receive the same signal and thus provide greater choice. Second, the optimal signal precision rises with the signal correlation q. Intuitively, when signal correlation is increased, the candidates

[14] Accordingly, we assume that $R \leqslant K$, so each realization of β may be associated with a unique element of K.

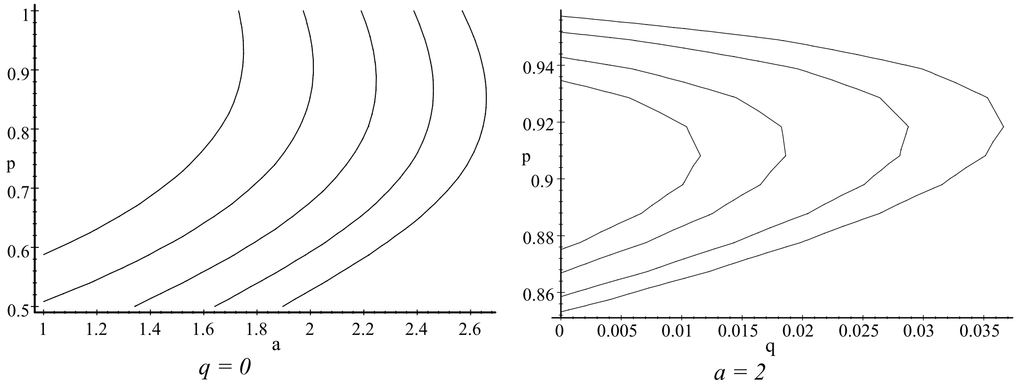

Fig. 2. Welfare level curves.

are more likely to receive the same signal, raising the value of targeting the median voter more accurately.

6. Conclusion

This paper shows how private polling radically alters the nature of the strategies of office-motivated candidates, overturning the apparently robust result of platform convergence. Specifically, in any pure strategy equilibrium, candidates' platforms over-emphasize their private information: Candidates locate at the median given that *both* receive the same signal. When candidates are not sufficiently likely to receive the same signal, equilibrium is characterized by mixed strategies. In the mixed strategy equilibrium, candidates who receive moderate signals adopt more extreme platforms than their information suggests, while candidates who receive more extreme signals moderate their platforms relative to their pollsters' advice.

We show that some platform differentiation always increases voters' welfare. Although candidates differentiate their platforms in equilibrium, voters would prefer that candidates extremize their positions by even more. From the perspective of voters, this paper finds that there is an optimal amount of noise in the polling technology. That is, the marginal social value of better information for candidates about voters becomes negative, once polls are sufficiently accurate. This suggests a rationale for campaign spending limits—such limits reduce expenditures on polling, thereby reducing the precision of candidates' signals and possibly raising voter welfare. So too, this suggests that voters may want to give dishonest answers to political pollsters in order to add noise to their polling technology. Finally, the electorate prefers private to public polling, because the increased signal correlation due to public polling reduces the diversity of platforms that candidates provide voters.

Our analysis suggests fruitful directions for future research. First, because the strategic value of better information is always positive for candidates, it is conceptually straightforward to endogenize the choice of costly polling technologies by candidates. Second, it would be worthwhile to determine how outcomes are affected when candidates have ideological preferences, and to endogenize contributions by ideologically-motivated lobbies to fund polling by candidates. Finally, as Ledyard [20] observes, it would be useful to uncover how equilibrium outcomes are affected when candidates sequentially choose platforms, so the second candidate can see where the first locates, and hence can unravel the latter's signal, before locating.

Appendix A

Proof of Theorem 2. The result is proved with minor modifications of the proof of Theorem 2 in BDS, hence the proof is omitted and made available upon request. □

Proof of Lemma 1. First, let k be the highest signal such that $\sum_{j:\ j>k} P(j|k) > \frac{1}{2}$. (If such a k does not exist, then the minimum signal $-K$ satisfies the inequalities in (1), and we can move to the second part of the argument.) Clearly, k is less than the maximum signal: $k < K$. We claim that signal $k+1$ satisfies (1). First, note that

$$\sum_{j:\ j\leqslant k} P(j|k+1) \leqslant \sum_{j:\ j\leqslant k} P(j|k) \leqslant \frac{1}{2}, \tag{7}$$

where the first inequality above follows from (A1) and the second follows from the definition of k. Then (7) implies

$$\sum_{j:\ j<k+1} P(j|k+1) \leqslant \sum_{j:\ j\geqslant k+1} P(j|k+1).$$

Finally, from $k+1 > k$ and the assumption that k is the highest signal such that $\sum_{j:\ j>k} P(j|k) > \frac{1}{2}$, we have $\sum_{j:\ j>k+1} P(j|k+1) \leqslant \frac{1}{2}$, i.e.,

$$\sum_{j:\ j>k+1} P(j|k+1) \leqslant \sum_{j:\ j\leqslant k+1} P(j|k+1).$$

Therefore, $k+1$ satisfies (1), and $C \neq \emptyset$. Second, we establish that, adding (A2), C is connected. Let ℓ be the lowest signal such that $\sum_{j:\ j<\ell} P(j|\ell) > \frac{1}{2}$. Repeating the argument above, we see that $\ell-1$ satisfies (1). By (A1), $k+1 \leqslant \ell-1$. Take any i such that $k+1 < i < \ell-1$. If $i \notin C$, then we may assume without loss of generality that $\sum_{j:\ j>i} P(j|i) > \frac{1}{2}$. Then the second part of (A2) implies that $\sum_{j:\ j>k+1} P(j|k+1) > \frac{1}{2}$, a contradiction. Finally, consider i such that $i < k+1$ or $i > \ell-1$, and without loss of generality assume the former. If $\sum_{j:\ j>i} P(j|i) \leqslant \frac{1}{2}$, then $i < k$. Then the second part of (A2) implies that $\sum_{j:\ j>k} P(j|k) \leqslant \frac{1}{2}$, a contradiction. Hence, $C = \{j \mid k+1 \leqslant j \leqslant \ell-1\}$ is connected. □

Proof of Theorem 3. We proceed in two parts. We first characterize the unique ordered equilibrium under assumptions (A1) and (A2). Then we prove existence under the additional assumption (A3).

Part 1. Under (A1) and (A2), if there is an ordered equilibrium, then it is unique and has the symmetric form specified in Theorem 3.

Proof. Let (G, H) be an ordered equilibrium. We first assume the equilibrium is symmetric, so that $G = H$. Then x is a differentiable point of a candidate's expected payoff, conditional on signal i, if and only if the following holds: for all signals j, if G_j puts positive probability on x, then $x = m_{i,j}$. At every point of differentiability $x \in \mathrm{Supp}(G_i)$, the derivative of the candidate's expected payoff function at x, conditional on signal i, with respect to x_i is:

$$\sum_{j:\, m_{j,j}<x} P(j|i)\left[-f_{i,j}\left(\frac{x+m_{j,j}}{2}\right)\frac{(G_j(m_{j,j})-G_j(m_{j,j})^-)}{2}\right]$$
$$+\sum_{j:\, x<m_{j,j}} P(j|i)\left[f_{i,j}\left(\frac{x+m_{j,j}}{2}\right)\frac{(G_j(m_{j,j})-G_j(m_{j,j})^-)}{2}\right]$$
$$+\sum_{j} P(j|i)\left[\int_{-\infty}^{x} -f_{i,j}\left(\frac{x+z}{2}\right)\frac{g_j(z)}{2}\,dz+\big(1-F_{i,j}(x)\big)g_j(x)\right.$$
$$\left.+\int_{x}^{\infty} f_{i,j}\left(\frac{x+z}{2}\right)\frac{g_j(z)}{2}\,dz-F_{i,j}(x)g_j(x)\right], \tag{8}$$

where g_i is the density of G_i, wherever it is defined. BDS show generally that the supports of equilibrium mixed strategies are bounded by the left- and right-most conditional medians.

Our premise of non-overlapping supports, and the fact that $f_{i,j}$ is constant at $\frac{1}{2a}$ over the relevant range under our assumptions, allows us to simplify (8) greatly. If g_k is defined at $x \in [\underline{x}_k, \bar{x}_k]$, then the derivative of the candidate's expected payoff, conditional on signal i, with respect to x_i is

$$\frac{1}{4a}\left[-\sum_{j:\, j<k} P(j|i)+\sum_{j:\, j>k} P(j|i)+P(k|i)\big[1-2G_k(x)+4(m_{i,k}-x)g_k(x)\big]\right].$$

If $x \in (\bar{x}_{k-1}, \underline{x}_k)$, then the interior term in brackets simplifies, and the derivative becomes

$$\frac{1}{4a}\left[-\sum_{j:\, j<k} P(j|i)+\sum_{j:\, j\geqslant k} P(j|i)\right]. \tag{9}$$

Clearly, the candidate must be indifferent among all locations in any interval in the support of G_i, and the first order condition must hold at all points of differentiability on such an interval.

Step 1. *For all k and all z, if G_k puts positive probability on z, then $z = m_{k,k}$.* Condition (C4*) of [3] is satisfied in our model, and this step therefore follows from their Theorem 7.

Step 2. *If G_i is continuous in an interval $[\underline{x}_i, \hat{x})$, then given $g_i(\underline{x}_i)$, the density g_i on $[\underline{x}_i, \hat{x})$ is characterized by the candidates' second-order condition.* Since the candidate's expected payoff is constant over the interval, it must in particular be linear over this interval, so the second-order condition must be satisfied with equality. Differentiating (8), we have

$$P(i|i)\big[-3g_i(x)+4(m_{i,i}-x)g_i'(x)\big]=0,$$

for all $x \in (\underline{x}_i, \hat{x})$. Since the candidate chooses the platform $\underline{x}_i$ with zero probability, we include it in the interval as well, yielding a differential equation in g_i that is easily solved. We find that

$$g_i(x)=g_i(\underline{x}_i)\left(\frac{m_{i,i}-\underline{x}_i}{m_{i,i}-x}\right)^{\frac{3}{2}} \tag{10}$$

for all $x \in [\underline{x}_i, \hat{x}]$, with associated distribution

$$G_i(x)=g_i(\underline{x}_i)(m_{i,i}-\underline{x}_i)^{3/2}\left(\frac{2}{\sqrt{m_{i,i}-\underline{x}_i}}-\frac{2}{\sqrt{m_{i,i}-x}}\right). \tag{11}$$

Thus, the second-order condition pins down the density g_i and distribution G_i on $[\underline{x}_i, \hat{x})$ up to the initial condition $g_i(\underline{x}_i)$.

Step 3. *For all $i \in C$, G_i is the point mass on $m_{i,i}$.* Suppose not, so $\underline{x}_i < \bar{x}_i$. As the argument is symmetric, suppose without loss of generality that $\underline{x}_i < m_{i,i}$. By Step 1, G_i is continuous on $[\underline{x}_i, m_{i,i})$. The first-order condition can be written as

$$g_i(x) = \frac{\sum_{j:\, j<i} P(j|i) + P(i|i)(1 - 2G_i(x)) - \sum_{j:\, i<j} P(j|i)}{4P(i|i)(m_{i,i} - x)}. \tag{12}$$

Note that $m_{i,i} \notin [\underline{x}_i, \bar{x}_i]$, for otherwise by (11) would imply that G_i takes values greater than one in a neighborhood of $m_{i,i}$. Therefore, $\bar{x}_i < m_{i,i}$, and G_i is continuous on the entire interval $[\underline{x}_i, \bar{x}_i]$. Substituting $x = \bar{x}_i$ in (12) and using $i \in C$, we have

$$g_i(x) = \frac{\sum_{j:\, j<i} P(j|i) - \sum_{j:\, i\leqslant j} P(j|i)}{4P(i|i)(m_{i,i} - x)} < 0,$$

a contradiction.

Step 4. *For all $i > \bar{c}$, G_i is continuous.* First, suppose that $\underline{x}_i = \bar{x}_i$; then from Step 1, G_i puts probability one on $m_{i,i}$. We claim that for all $j < i$, we have $\bar{x}_j < m_{i,i}$. If $j \in C$, then this follows from Step 3 and $m_{j,j} < m_{i,i}$. If $j > \bar{c}$ and $\bar{x}_j = m_{i,i}$, then Eq. (12) implies g_j is negative in a neighborhood of $\bar{x}_j$, a contradiction, establishing the claim. Consider any $z < m_{i,i}$ such that $\bar{x}_j < z$ for all $j < i$. The derivative of the candidate's expected payoff, conditional on i, at z is given by (9), which is negative, as $i > \bar{c}$. Therefore, a sufficiently small move from $m_{i,i}$ to $z < m_{i,i}$ raises the candidate's expected payoff, a contradiction. Therefore, $\underline{x}_i < \bar{x}_i$. As in Step 3, we must have $m_{i,i} \notin [\underline{x}_i, \bar{x}_i]$, and then G_i is continuous.

Step 5. *For all $i > \bar{c}$, supports are adjacent, in the sense that $\bar{x}_{i-1} = \underline{x}_i$.* Suppose $\bar{x}_{i-1} < \underline{x}_i$. As in Step 4, the derivative in the interval $[\bar{x}_{i-1}, \underline{x}_i]$ is given by (9), which is negative. As above, a sufficiently small move from $\underline{x}_i$ to $z < \underline{x}_i$ raises the candidate's expected payoff, a contradiction.

Step 6. *For all $i > \bar{c}$, the distribution G_i and its increasing, convex density g_i are given by* (2) *and* (4); *the upper bound of the support of G_i is strictly less than $m_{i,i}$; the lower bounds of the supports are as in* (3); *and the expected value of x_i is given by* (5). The parameters $\underline{x}_i$ and $g_i(\underline{x}_i)$ are determined by the first-order condition, which yields

$$g_i(\underline{x}_i) = \frac{\sum_{j:\, j<i} P(j|i) - \sum_{j:\, j\geqslant i} P(j|i)}{4P(i|i)(m_{i,i} - \underline{x}_i)} = \frac{\Phi_i}{2} \frac{1}{(m_{i,i} - \underline{x}_i)},$$

when evaluated at $\underline{x}_i$. Substituting for $g_i(\underline{x}_i)$ in (10) and (11) yields the following expressions for the density g_i and distribution G_i on the support $[\underline{x}_i, \bar{x}_i]$:

$$g_i(x) = \frac{\Phi_i}{2}\left[\frac{(m_{i,i} - \underline{x}_i)^{\frac{1}{2}}}{(m_{i,i} - x)^{\frac{3}{2}}}\right] \quad \text{and} \quad G_i(x) = \Phi_i\left[\sqrt{\frac{m_{i,i} - \underline{x}_i}{m_{i,i} - x}} - 1\right].$$

The condition $G_i(x) = 1$ determines the upper bound $\bar{x}^i$ of the support, which by Step 5 coincides with $\underline{x}_{i+1}$. Note that for $\underline{x}_i < m_{i,i}$, a solution to $G_i(x) = 1$ does indeed exist for all $i > \bar{c}$, since $\frac{2}{\sqrt{m_{i,i} - x}}$ goes to infinity as x increases to $m_{i,i}$. By Steps 3 and 5, $\underline{x}_{\bar{c}+1} = m_{\bar{c},\bar{c}}$. Therefore, solving $G_i(\underline{x}_{i+1}) = 1$, the lower bounds are pinned down recursively by difference equation (3), with the initial condition $\underline{x}_{\bar{c}+1} = m_{\bar{c},\bar{c}}$. These observations, with an induction argument starting with $\underline{x}_{\bar{c}+1} = m_{\bar{c},\bar{c}}$, yield $\bar{x}_i < m_{i,i}$ for all signals i. The expectation (5) is derived simply by integrating. That g_i is increasing and convex is apparent from the functional form of the density.

Finally, we argue that there are no non-symmetric ordered equilibria (G, H), where $G \neq H$. Given any ordered equilibrium (G, H), because the electoral game is symmetric and zero-sum, the strategy pair (H, G) is also an equilibrium. By interchangeability, (G, G) is a symmetric equilibrium, so G is uniquely characterized above, as is H by an analogous argument. □

Part 2. Under (A1)–(A3), if one candidate adopts the strategy specified in Theorem 3, then the other candidate's payoff following each signal i is maximized by all $x_i \in [\underline{x}_i, \bar{x}_i]$.

Proof. Assume each candidate uses the mixed strategy G, defined in Theorem 3, and consider any signal $i > \bar{c}$. By construction, the candidate's expected payoff conditional on receiving signal i is constant on $[\underline{x}_i, \bar{x}_i]$. We show that the candidate's expected payoff falls as we move x_i further to the left of $\underline{x}_i$ or further to the right of $\bar{x}_i$. First, take k such that $k < i$ and $k \notin C$. We must show that

$$\frac{-\sum_{j:\, j<k} P(j|i) + \sum_{j:\, j>k} P(j|i)}{P(k|i)} + \left[1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x)\right] \geqslant 0. \tag{13}$$

By construction, the candidate's expected payoff following signal k is constant over his support, so at $x \in (\underline{x}_k, \bar{x}_k)$ we have

$$\frac{[\sum_{j:\, j>k} P(j|k) - \sum_{j:\, j<k} P(j|k)]}{P(k|k)} + \left[1 - 2G_k(x) + 4(m_{k,k} - x)g_k(x)\right] = 0. \tag{14}$$

By (A1) and (A3), we have

$$\frac{\sum_{j:\, j>k} P(j|i) - \sum_{j:\, j<k} P(j|i)}{P(k|i)} \geqslant \frac{\sum_{j:\, j>k} P(j|k) - \sum_{j:\, j<k} P(j|k)}{P(k|k)}.$$

Since $m_{i,k} > m_{k,k}$, we have

$$1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x) > 1 - 2G_k(x) + 4(m_{k,k} - x)g_k(x),$$

which implies that the left-hand side of (13) exceeds the left-hand side of (14), as required.

Now take $k \in C$. Note that the candidate's expected payoff conditional on signal i is discontinuous at $m_{k,k}$: Letting $\Pi(x_i|G, i)$ denote the expected payoff conditional on i from locating at x_i when the other candidate uses the mixed strategy G, we have

$$\lim_{w \downarrow m_{k,k}} \Pi_A(w|G, i) - \Pi_A(m_{k,k}|G, i) = P(k|i)\left[1 - F_{i,k}(m_{k,k}) - \frac{1}{2}\right] \geqslant 0,$$

where the inequality follows from $m_{k,k} < m_{i,k}$. Similarly,

$$\Pi_A(m_{k,k}|G, i) - \lim_{w \uparrow m_{k,k}} \Pi_A(w|G, i) \geqslant 0,$$

so the candidate's payoff function is non-decreasing at $m_{k,k}$. Over the interval $(\bar{x}_{k-1}, m_{k,k})$, the derivative of the candidate's payoff function conditional on signal i is proportional to

$$\sum_{j:\ j\geqslant k} P(j|i) - \sum_{j:\ j<k} P(j|i) \geqslant \sum_{j:\ j\geqslant k} P(j|k) - \sum_{j:\ j<k} P(j|k) \geqslant 0,$$

where the first inequality follows from (A1) and the second from the definition of $k \in C$.

Now take $k > i$. We must show that

$$-\sum_{j:\ j<k} P(j|i) + \sum_{j:\ j>k} P(j|i) + P(k|i)\big[1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x)\big] \tag{15}$$

is non-positive. By (A1), we have

$$-\sum_{j:\ j<k} P(j|k) + \sum_{j:\ j>k} P(j|k) \geqslant -\sum_{j:\ j<k} P(j|i) + \sum_{j:\ j>k} P(j|i). \tag{16}$$

Because (14) holds at $x \in (\underline{x}_k, \bar{x}_k)$ and $k > \bar{c}$, the left-hand side above is negative, which implies that $1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x)$ is positive. By (A3) and $m_{k,k} > m_{i,k}$, we have

$$P(k|k)\big[1 - 2G_k(x) + 4(m_{k,k} - x)g_k(x)\big] > P(k|i)\big[1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x)\big]. \tag{17}$$

Together, (14), (16), and (17) imply that (15) is negative, as required. □

Proof of Proposition 1. To do this, we use the difference equation (3) describing the relationship between $\underline{x}_{i+1}$ and $\underline{x}_i$ to solve for

$$m_{i+1,i+1} - \underline{x}_{i+1} = m_{i+1,i+1} - m_{i,i} + \left(\frac{\Phi_{i+1}}{\Phi_{i+1}+1}\right)^2 (m_{i,i} - \underline{x}_i), \quad \text{for } i \geqslant \bar{c}.$$

Note that $\underline{x}_{\bar{c}} = m_{\bar{c},\bar{c}} = \underline{x}_{\bar{c}+1}$, so that $(m_{\bar{c}+1,\bar{c}+1} - \underline{x}_{\bar{c}+1}) - (m_{\bar{c},\bar{c}} - \underline{x}_{\bar{c}}) = m_{\bar{c}+1,\bar{c}+1} - m_{\bar{c},\bar{c}} > 0$. Continuing inductively,

$$\begin{aligned}
&(m_{i+1,i+1} - \underline{x}_{i+1}) - (m_{i,i} - \underline{x}_i)\\
&\quad = [m_{i+1,i+1} - m_{i,i}] - [m_{i,i} - m_{i-1,i-1}] + \left(\frac{\Phi_{i+1}}{\Phi_{i+1}+1}\right)^2 (m_{i,i} - \underline{x}_i)\\
&\qquad - \left(\frac{\Phi_i}{\Phi_i+1}\right)^2 (m_{i-1,i-1} - \underline{x}_{i-1})\\
&\quad \geqslant [m_{i+1,i+1} - m_{i,i}] - [m_{i,i} - m_{i-1,i-1}]\\
&\qquad + \left(\frac{\Phi_{i+1}}{\Phi_{i+1}+1}\right)^2 \big[(m_{i,i} - \underline{x}_i) - (m_{i-1,i-1} - \underline{x}_{i-1})\big]\\
&\quad > [m_{i+1,i+1} - m_{i,i}] - [m_{i,i} - m_{i-1,i-1}],
\end{aligned}$$

where the first inequality follows because $\Phi_{i+1} \geqslant \Phi_i$, and the second inequality follows from the induction hypothesis. We have shown that when $[m_{i+1,i+1} - m_{i,i}]$ is constant, $m_{i,i} - \underline{x}_i$ is strictly increasing, with slack, delivering the result. □

Proof of Proposition 2. If a model is subject to a transformation consistent with an increase in signal precision, then the zero signal remains an element of C after the transformation. In fact, every signal that belongs to C before the transformation remains in the central set, say C', after the transformation. Given any signal $i > 0$ such that $i \in C$, the shift in equilibrium platform is therefore just $\Delta m_{i,i} > 0$, as required. Given any signal $i > 0$ such that $i \in C' \setminus C$, the equilibrium mixed strategy before the transformation is described in Theorem 3. This mixed strategy puts probability one on platforms less than $m_{i,i}$, and the equilibrium platform after the transformation puts probability one on the new conditional median, say $m'_{i,i}$. Since $m'_{i,i} > m_{i,i}$, the claim of the proposition is fulfilled. Let $\bar{c}'$ be the greatest element of the new set C'. It is evident that for signals $i > \bar{c}'$, the transformation decreases Φ_i. Moreover, the foregoing implies that the lower bound of the candidates' support following signal $\bar{c}' + 1$ after the transformation, denoted $\underline{x}'_{\bar{c}'+1}$, weakly exceeds the lower bound before the transformation, which is $\underline{x}_{\bar{c}'+1}$. Therefore, expression (4) in Theorem 3 implies that the transformation leads to a stochastic improvement in the candidates' equilibrium distribution following signal $\bar{c} + 1$. This further increases the lower bound $\bar{x}_{\bar{c}'+2}$ for signal $\bar{c}' + 2$ and leads to a stochastic improvement following $\bar{c}' + 2$. An induction argument based on these observations yields the proposition. □

Proof of Proposition 3. To see that $\frac{d\Phi_i}{dq} < 0$ for $i > \bar{c}$, write Φ_i as $\Phi_i = \frac{([\sum_{j:\ j<i} P(j,i)] - \frac{P(i)}{2})}{P(i,i)}$. Note that

$$P(i,i) = \sum_b \bigl[qP(i|b) + (1-q)P(i|b)^2\bigr]P(b),$$

$$\frac{dP(i,i)}{dq} = \sum_b \bigl[1 - P(i|b)\bigr]P(i|b)P(b),$$

$$P(j,i) = \sum_b \bigl[(1-q)P(i|b)P(j|b)\bigr]P(b) \quad \text{for } j \neq i,$$

$$\frac{dP(j,i)}{dq} = -\sum_b P(i|b)P(j|b)P(b) \quad \text{for } j \neq i.$$

Therefore, using the fact that $P(i)$ is independent of q, we have

$$\begin{aligned}
\frac{\Phi_i}{dq} &\propto P(i,i)\biggl[-\sum_b \sum_{j:\ j<i} P(i|b)P(j|b)P(b)\biggr] \\
&\quad - \biggl[\biggl[\sum_{j:\ j<i} P(j,i)\biggr] - \frac{P(i)}{2}\biggr]\biggl[\sum_b \bigl[1 - P(i|b)\bigr]P(i|b)P(b)\biggr] \\
&< 0,
\end{aligned}$$

where the inequality follows from $i > \bar{c}$, which implies $\sum_{j:\ j<i} P(j|i) > \frac{1}{2}$. To see that $\frac{dm_{i,i}}{dq} < 0$, note that

$$m_{i,i}(q) = E[b|i,i,q] = \sum_b b\biggl[\frac{qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b)}{\sum_{b'}[qP(i|b')P(b') + (1-q)P(i|b')P(i|b')P(b')]}\biggr].$$

Therefore, for all $i > 0$, we have

$$\begin{aligned}
\frac{dm_{i,i}}{dq}(q) &\propto \sum_b b\bigl[P(i|b)P(b) - P(i|b)P(i|b)P(b)\bigr] \\
&\quad \times \sum_{b'} \bigl[qP\bigl(i|b'\bigr)P\bigl(b'\bigr) + (1-q)P\bigl(i|b'\bigr)P\bigl(i|b'\bigr)P\bigl(b'\bigr)\bigr] \\
&\quad - \sum_{b'} \bigl[P\bigl(i|b'\bigr)P\bigl(b'\bigr) - P\bigl(i|b'\bigr)P\bigl(i|b'\bigr)P\bigl(b'\bigr)\bigr]
\end{aligned}$$

$$
\begin{aligned}
&\times \sum_b b\big[qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b)\big] \\
&\propto \frac{\sum_b b[P(i|b)P(b) - P(i|b)P(i|b)P(b)]}{\sum_b [P(i|b)P(b) - P(i|b)P(i|b)P(b)]} \\
&\quad - \frac{\sum_b b[qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b)]}{\sum_b [qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b)]} \\
&= \frac{\sum_b b[1 - P(i|b)]P(i|b)P(b)}{\sum_b [1 - P(i|b)]P(i|b)P(b)} \\
&\quad - \frac{\sum_b b[qP(i|b) + (1-q)P(i|b)P(i|b)]P(b)}{\sum_b [qP(i|b) + (1-q)P(i|b)P(i|b)]P(b)} \\
&= m_{i,I\setminus\{i\}} - m_{i,i} \\
&< 0,
\end{aligned}
$$

as required. That C is weakly increasing in q follows from the fact that $\frac{dP(i,j)}{dq} < 0$ for all $j \neq i$. □

Proof of Lemma 2. The first part of the proof establishes that a voter v's welfare is just δ_v^2 less than the welfare of the median voter ($\delta_v = 0$). The second part of the proof establishes that the welfare function $W_0(\cdot)$ is strictly concave.

First, to calculate the welfare of a voter with relative ideal point δ_v, note that for any b, if both candidates receive a signal i, then they both locate at x_i, yielding voter δ_v expected utility of $(1/a)\int_{-a}^{a} u(\alpha + b + \delta_v, x_i)\,d\alpha$. If candidates receive different signals i and j, with $i < j$, one locates at x_i, while the other locates at x_j, and the candidate closest to the median voter wins the election. That is, the candidate at x_i wins if $\mu < [x_i + x_j]/2$, or equivalently if $\alpha < [x_i + x_j]/2 - b$; and the candidate at x_j wins if $\mu > [x_i + x_j]/2$, or equivalently if $\alpha > [x_i + x_j]/2 - b$. Thus, voter δ_v's expected utility is

$$
\frac{1}{2a}\int_{-a}^{[x_i+x_j]/2-b} u(\alpha + b + \delta_v, x_i)\,d\alpha + \frac{1}{2a}\int_{[x_i+x_j]/2-b}^{a} u(\alpha + b + \delta_v, x_j)\,d\alpha. \tag{18}
$$

Therefore,

$$
\begin{aligned}
W_{\delta_v}(X) &= \sum_b \sum_i P(b,i,i)\frac{1}{2a}\int_{-a}^{a} u(\alpha + b + \delta_v, x_i)\,d\alpha \\
&\quad + \sum_b \sum_i \sum_{j:\ j>i} P(b,i,j)\Bigg[\frac{1}{2a}\int_{-a}^{[x_i+x_j]/2-b} u(\alpha + b + \delta_v, x_i)\,d\alpha \\
&\quad + \frac{1}{2a}\int_{[x_i+x_j]/2-b}^{a} u(\alpha + b + \delta_v, x_j)\,d\alpha\Bigg]
\end{aligned}
$$

$$+\sum_b\sum_i\sum_{j:\,j<i}P(b,i,j)\left[\frac{1}{2a}\int_{-a}^{[x_i+x_j]/2-b}u(\alpha+b+\delta_v,x_j)\,d\alpha\right.$$
$$\left.+\frac{1}{2a}\int_{[x_i+x_j]/2-b}^{a}u(\alpha+b+\delta_v,x_i)\,d\alpha\right]. \tag{19}$$

For any b and any pair i, j with $i < j$, we aggregate the term (18) with the corresponding term for $b' = -b$, $i' = -i$, and $j' = -j$, so that $j' < i'$, so as to obtain:

$$-\frac{1}{2a}\int_{-a}^{[x_i+x_j]/2-b}-(\alpha+b+\delta_v-x_i)^2\,d\alpha-\frac{1}{2a}\int_{[x_i+x_j]/2-b}^{a}(\alpha+b+\delta_v-x_j)^2\,d\alpha$$
$$-\frac{1}{2a}\int_{[-x_i-x_j]/2+b}^{a}-(\alpha-b+\delta_v+x_i)^2\,d\alpha-\frac{1}{2a}\int_{-a}^{[-x_i-x_j]/2+b}(\alpha-b+\delta_v+x_j)^2\,d\alpha$$
$$=-\delta_v^2-2\delta_v(b-x_i)\left[a+\frac{x_i+x_j}{2}-b\right]\frac{1}{2a}-2\delta_v(b-x_j)\left[a-\frac{x_i+x_j}{2}+b\right]\frac{1}{2a}$$
$$-\delta_v^2-2\delta_v(-b+x_i)\left[a-\left[-\frac{x_i+x_j}{2}+b\right]\right]\frac{1}{2a}$$
$$-2\delta_v(-b+x_j)\left[-\frac{x_i+x_j}{2}+b+a\right]\frac{1}{2a}$$
$$-\frac{1}{2a}\int_{-a}^{[x_i+x_j]/2-b}(\alpha+b-x_i)^2\,d\alpha-\frac{1}{2a}\int_{[x_i+x_j]/2-b}^{a}(\alpha+b-x_j)^2\,d\alpha$$
$$-\frac{1}{2a}\int_{[-x_i-x_j]/2+b}^{a}-(\alpha-b+x_i)^2\,d\alpha-\frac{1}{2a}\int_{-a}^{[-x_i-x_j]/2+b}(\alpha-b+x_j)^2\,d\alpha$$
$$=-\delta_v^2-\frac{1}{2a}\int_{[-x_i-x_j]/2+b}^{a}u(\alpha-b,-x_i)\,d\alpha-\frac{1}{2a}\int_{-a}^{[-x_i-x_j]/2+b}u(\alpha-b,-x_j)\,d\alpha$$
$$-\delta_v^2-\frac{1}{2a}\int_{-a}^{[x_i+x_j]/2-b}u(\alpha+b,x_i)\,d\alpha-\frac{1}{2a}\int_{[x_i+x_j]/2-b}^{a}u(\alpha+b,x_j)\,d\alpha,$$

where we used $\int_{-a}^{a}\alpha\,d\alpha=0$. The case for pairs $i = j$ follows from analogous manipulations. This establishes that the voter's welfare is just δ_v^2 less than the welfare of the median voter ($\delta_v = 0$).

Second, we differentiate the expression (19) for voter welfare with $\delta_v = 0$ to obtain:

$$\begin{aligned}\frac{\partial W_0}{\partial x_i}(X) = \sum_b \Bigg[& P(i,i,b)\frac{1}{a}\int_{-a}^{a}(b+\alpha-x_i)\,d\alpha \\ & + 2\sum_{j:\ j<i} P(i,j,b)\frac{1}{a}\int_{\frac{x_j+x_i}{2}-b}^{a}(b+\alpha-x_i)\,d\alpha \\ & + 2\sum_{j:\ j>i} P(i,j,b)\frac{1}{a}\int_{-a}^{\frac{x_i+x_j}{2}-b}(b+\alpha-x_i)\,d\alpha\Bigg],\end{aligned}$$

where we note that in (19), each signal pair (k,ℓ) with $k<\ell$ appears once in the sum over i and $j>i$ and once in the sum over i and $j<i$. After simplifying, the above expression becomes

$$\begin{aligned}\frac{\partial W_0}{\partial x_i}(X) = {} & \frac{1}{2a}\sum_b P(i,i,b)\big[(b+a-x_i)^2-(b-a-x_i)^2\big] \\ & + \frac{1}{a}\sum_{j:\ j<i}\sum_b P(i,j,b)\left[(b+a-x_i)^2-\left(\frac{x_j-x_i}{2}\right)^2\right] \\ & + \frac{1}{a}\sum_{j:\ j>i}\sum_b P(i,j,b)\left[\left(\frac{x_j-x_i}{2}\right)^2-(b-a-x_i)^2\right].\end{aligned}$$

Viewing the summands above as quadratic functions of the random variable b, we use mean-variance analysis and the fact that $E[b|j,i]=m_{j,i}$ to derive the expression

$$\begin{aligned}\frac{\partial W_0}{\partial x_i}(X) = {} & \frac{1}{2a}P(i,i)\big[(m_{i,i}+a-x_i)^2-(m_{i,i}-a-x_i)^2\big] \\ & + \frac{1}{a}\sum_{j:\ j<i} P(i,j)\left[(m_{j,i}+a-x_i)^2-\left(\frac{x_j-x_i}{2}\right)^2+\sigma_{j,i}^2\right] \\ & + \frac{1}{a}\sum_{j:\ j>i} P(i,j)\left[\left(\frac{x_j-x_i}{2}\right)^2-(m_{j,i}-a-x_i)^2-\sigma_{j,i}^2\right], \end{aligned} \tag{20}$$

where $\sigma_{j,i}^2$ is the variance of b conditional on signals j and i.

Taking one further derivative, the cross partial with respect to x_i and x_j with $j \neq i$ is then

$$\frac{\partial^2 W_0}{\partial x_i \partial x_j}(X) = P(i,j)\frac{|x_i-x_j|}{2a}, \tag{21}$$

where we use monotonicity of strategies in signal. The second partial with respect to x_i is

$$\begin{aligned}\frac{\partial^2 W_0}{\partial^2 x_i}(X) = {} & \frac{1}{2a}P(i,i)\big[-2(a+m_{i,i}-x_i)+2(m_{i,i}-x_i-a)\big] \\ & + \frac{1}{a}\sum_{j:\ j<i} P(i,j)\left[-2(a+m_{i,j}-x_i)+\frac{x_j-x_i}{2}\right]\end{aligned}$$

$$
\begin{aligned}
&- \frac{1}{a} \sum_{j:\ j>i} P(i,j)\left[-2(m_{i,j} - x_i - a) + \frac{x_j - x_i}{2}\right] \\
&= \frac{P(i)}{a}\left[-2a - 2\sum_{j:\ j<i} P(j|i)(m_{i,j} - x_i) + 2\sum_{j:\ j>i} P(j|i)(m_{i,j} - x_i)\right] \\
&\quad - \sum_{j:\ j\neq i} P(i,j)\frac{|x_j - x_i|}{2a}.
\end{aligned}
$$

Therefore, we may decompose the Hessian of W_0 into two matrices, $H = D + E$, where E is a symmetric matrix such that

$$
e_{i,j} = P(i,j)\frac{|x_j - x_i|}{2a} \quad \text{for } i \neq j \quad \text{and} \quad e_{i,i} = -\sum_{j:\ j\neq i} P(i,j)\frac{|x_j - x_i|}{2a}, \tag{22}
$$

and D is a diagonal matrix such that

$$
d_{i,i} = \frac{P(i)}{a}\left[-2a - 2\sum_{j:\ j<i} P(j|i)(m_{i,j} - x_i) + 2\sum_{j:\ j>i} P(j|i)(m_{i,j} - x_i)\right].
$$

Because x_i is bounded above by R, it follows that

$$
d_{i,i} \leqslant \frac{P(i)}{a}[-2a + 2R + 2R] < 0,
$$

where the inequality follows from $a > b_N - b_1 = 2R$. Thus, D is negative definite. We now argue that E is negative semi-definite. Let $t \in \Re^{2K+1}$ be an arbitrary vector, and note that

$$
\begin{aligned}
\sum_{i,j} t_i t_j e_{ij} &= \sum_i t_i^2 e_{ii} + \sum_i \sum_{j:\ j\neq i} t_i t_j e_{ij} \\
&= \sum_i t_i^2 \left(\sum_{j:\ j\neq i} -e_{ij}\right) + \sum_i \sum_{j:\ j\neq i} t_i t_j e_{ij} \\
&= \sum_{\{i,j\}:\ i\neq j} \left(-t_i^2 e_{ij} - t_j^2 e_{ji}\right) + \sum_{\{i,j\}:\ i\neq j} (t_i t_j e_{ij} + t_j t_i e_{ji}) \\
&= \sum_{\{i,j\}:\ i\neq j} e_{ij}\left(-t_i^2 + 2t_i t_j - t_j^2\right) \\
&= \sum_{\{i,j\}:\ i\neq j} -e_{ij}(t_i - t_j)^2 \\
&\leqslant 0,
\end{aligned}
$$

where the second equality follows from condition (22), the fourth equality follows from symmetry of E, and the final inequality uses $e_{ij} \geqslant 0$ when $i \neq j$. Therefore, E is negative semi-definite, and $H = D + E$ is negative definite. We conclude that W_0 is strictly concave. □

Proof of Theorem 4. We first note that substituting $m_{j,j}$ for x_j for all j, in expression (20), the partial derivative of welfare at the vector of medians is

$$\frac{\partial W}{\partial x_i}(M) = \frac{1}{a}\sum_{j:\ j<i} P(i,j)\left[\big(a-(m_{i,i}-m_{j,i})\big)^2+\sigma_{j,i}^2-\left(\frac{m_{i,i}-m_{j,j}}{2}\right)^2\right]$$
$$-\frac{1}{a}\sum_{j:\ j>i} P(i,j)\left[\big(a-(m_{j,i}-m_{i,i})\big)^2+\sigma_{j,i}^2-\left(\frac{m_{j,j}-m_{i,i}}{2}\right)^2\right]. \tag{23}$$

Hence, when a is sufficiently large, $\frac{\partial W}{\partial x_i}(M) > 0$ for all i, under (A4).

Recall that $x_i^* = -x_{-i}^*$ for each signal i, and in particular we have $x_0^* = 0$. Define the mapping $\phi : \Re^K \to \Re^{2K+1}$ by

$$\phi(x_1,\dots,x_K) = (-x_K,\dots,-x_1,0,x_1,\dots,x_K),$$

and define the symmetrized welfare function $\hat{W} : \Re^K \to \Re$ by

$$\hat{W}(x_1,\dots,x_K) = W\big(\phi(x_1,\dots,x_K)\big).$$

For simplicity, we denote K-tuples by $\hat{X} = (x_1,\dots,x_K)$; in particular, the K-tuple of conditional medians is $\hat{M} = (m_{1,1},\dots,m_{K,K})$. By Lemma 2 and linearity of ϕ, $\hat{W}$ is strictly concave. Furthermore, it has a unique maximizer, $\hat{X}^*$, and by the above argument it follows that $X^* = \phi(\hat{X}^*)$. Note that, by the chain rule, for all signals $i > 0$ and all $\hat{X}$, we have

$$\frac{\partial \hat{W}}{\partial x_i}(\hat{X}) = \frac{\partial W}{\partial x_{-i}}\big(\phi(\hat{X})\big)\frac{\partial \phi_{-i}}{\partial x_i}(\hat{X}) + \frac{\partial W}{\partial x_i}\big(\phi(\hat{X})\big)\frac{\partial \phi_i}{\partial x_i}(\hat{X}) = 2\frac{\partial W}{\partial x_i}\big(\phi(\hat{X})\big).$$

Furthermore, for all $j \neq i$, we have

$$\frac{\partial^2 \hat{W}}{\partial x_i \partial x_j}(\hat{X}) = 2\left[\frac{\partial^2 W}{\partial x_i \partial x_j}\big(\phi(\hat{X})\big) - \frac{\partial^2 \hat{W}}{\partial x_i \partial x_{-j}}\big(\phi(\hat{X})\big)\right]$$
$$= \frac{1}{a}\big[P(i,j)|x_i - x_j| - P(i,-j)(x_i + x_j)\big], \tag{24}$$

where the second equality uses (21). Consider the welfare maximization problem with the additional constraint that candidates locate at or above the conditional medians corresponding to all signals:

$$\begin{aligned}&\max_{\hat{X}}\ \hat{W}(\hat{X})\\ &\text{s.t. } x_i \geqslant m_{i,i} \quad \text{for all } i > 0.\end{aligned} \tag{25}$$

Because the domain of this problem is convex, it has a unique solution, say $\hat{X}'$. We seek to show that at this solution, the constraints in (25) are slack, i.e., $x_i' > m_{i,i}$ for all signals $i > 0$. Then concavity of W implies $x_i^* = x_i' > m_{i,i}$ for all $i > 0$, as required. To this end, suppose $x_i' = m_{i,i}$ for some $i > 0$, where without loss of generality we take i to be the lowest such signal. Therefore, we must have

$$\frac{\partial \hat{W}}{\partial x_i}\big(\hat{X}'\big) \leqslant 0. \tag{26}$$

Consider any $\hat{X}$ such that $x_j = x_j'$ for all $j \leqslant i$ and $x_j \geqslant m_{j,j}$ for all $j > i$. Given signal $j > i$, from (24), we see that

$$\frac{\partial^2 \hat{W}}{\partial x_i \partial x_j}(\hat{X}) > 0 \tag{27}$$

if and only if

$$\frac{P(i,j)}{P(i,-j)} > \frac{m_{i,i}+x_j}{x_j-m_{i,i}}.$$

Differentiating the right-hand side of the latter inequality, we see that it is decreasing in x_j, and so it is maximized over $x_j \geqslant m_{j,j}$ at $x_j = m_{j,j}$. Thus, (27) follows from assumption (A5). Now define $\hat{X}''$ so that $x''_j = x'_j$ for all $j \leqslant i$ and $x''_j = m_{j,j}$ for all $j > i$, in effect just decreasing candidate positions following signals $j > i$ to their conditional medians. Then (26) and (27) imply

$$\frac{\partial \hat{W}}{\partial x_i}\big(\hat{X}''\big) \leqslant 0. \tag{28}$$

This contradicts $\frac{\partial \hat{W}}{\partial x_i}(\hat{M}) > 0$ immediately if $i = 1$, so assume $i > 1$, and note that $x''_j > m_{j,j}$ for all signals $j < i$. Let $I \subseteq \{1,\dots,i-1\}$ be any subset of signals less than i, and define $\hat{X}^I$ so that

$$x^I_j = \begin{cases} m_{i,i} & \text{if } j < i \text{ and } j \in I,\\ m_{j,j} & \text{else.}\end{cases}$$

That is, for the subset of signals in I, we move candidate positions up to $m_{i,i}$; after all other signal realizations, we position candidates at their conditional medians. Note that $\hat{X}''$ is a convex combination of such vectors:

$$\hat{X}'' \in \operatorname{conv}\big\{\hat{X}^I \;\big|\; I \subseteq \{1,\dots,i-1\}\big\}.$$

Note also that $\hat{X}^{\emptyset} = \hat{M}$, and that $\frac{\partial \hat{W}}{\partial x_i}(\hat{X}^{\emptyset}) > 0$. More generally, we have

$$\begin{aligned}
&\frac{\partial \hat{W}}{\partial x_i}\big(\hat{X}^I\big) - \frac{\partial \hat{W}}{\partial x_i}\big(\hat{X}^{\emptyset}\big)\\
&\quad = 2\sum_{j\in I}\frac{1}{a}\left[P(-j,i)\left[\left(\frac{m_{j,j}+m_{i,i}}{2}\right)^2 - m_{i,i}^2\right] + P(j,i)\left(\frac{m_{i,i}-m_{j,j}}{2}\right)^2\right]\\
&\quad = 2\sum_{j\in I}\frac{1}{a}\left[P(-j,i)\left(\frac{m_{j,j}-m_{i,i}}{2}\right)\left(\frac{3m_{i,i}+m_{j,j}}{2}\right) + P(j,i)\left(\frac{m_{i,i}-m_{j,j}}{2}\right)^2\right]\\
&\quad \geqslant 0,
\end{aligned}$$

where the inequality follows from assumption (A5). Therefore, $\frac{\partial \hat{W}}{\partial x_i}(\hat{X}^I) > 0$ for all subsets I. The expression (20) shows that the partial derivative of $\hat{W}$ with respect to x_i is a concave (weighted) quadratic function of positions $(x_1,\dots,x_{i-1})$, and therefore the set

$$Z = \left\{(x_1,\dots,x_{i-1},m_{i,i},\dots,m_{K,K}) \;\middle|\; \frac{\partial \hat{W}}{\partial x_i}(x_1,\dots,x_{i-1},m_{i,i},\dots,m_{K,K}) > 0\right\}$$

is convex. In particular, since $\hat{X}^I \in Z$ for all I, and since $\hat{X}''$ is a convex combination of the $\hat{X}^I$, we have $\hat{X}'' \in Z$, i.e., $\frac{\partial \hat{W}}{\partial x_i}(\hat{X}'') > 0$, contradicting (28). We conclude that $x'_i > m_{i,i}$ for all signals $i > 0$, as required. □

Proof of Theorem 5. We write $W(X;q)$ to bring out the dependence of welfare on correlation given locations X, and we write $m_{i,i}(q)$ to bring out the dependence of the conditional median on correlation. The effect of an increase in correlation q may be decomposed as follows:

$$\begin{aligned}\frac{dW}{dq}(M;q) &= \frac{\partial W}{\partial q}(M;q) + \sum_i \frac{\partial W}{\partial x_i}(M;q)\frac{dm_{i,i}}{dq}(q)\\ &= \frac{\partial W}{\partial q}(M;q) + \sum_{i=1}^{K}\left[\frac{\partial W}{\partial x_i}(M;q)\frac{dm_{i,i}}{dq}(q) + \frac{\partial W}{\partial x_{-i}}(M;q)\frac{dm_{-i,-i}}{dq}(q)\right]\\ &= \frac{\partial W}{\partial q}(M;q) + 2\sum_{i=1}^{K}\frac{\partial W}{\partial x_i}(M;q)\frac{dm_{i,i}}{dq}(q),\end{aligned}$$

where the third equality follows from symmetry about zero. By hypothesis, $\frac{\partial W}{\partial x_i}(M;q) > 0$ for all signals $i > 0$, and Proposition 3 delivers $\frac{dm_{i,i}}{dq}(q) < 0$ for all $i > 0$. To see that $\frac{\partial W}{\partial q}(M;q) < 0$, note that

$$\begin{aligned}W(M;q) = &-\sum_b\sum_i\big[qP(i|b) + (1-q)P(i|b)P(i|b)\big]P(b)\int_{-a}^{a}\frac{(\alpha+b-m_{ii})^2}{2a}\,d\alpha\\ &-\sum_b\sum_i\sum_{j>i}(1-q)P(i|b)P(j|b)P(b)\left[\int_{-a}^{[m_{ii}+m_{jj}]/2-b}\frac{(\alpha+b-m_{ii})^2}{2a}\,d\alpha\right.\\ &\left.+\int_{[m_{ii}+m_{jj}]/2-b}^{a}\frac{(\alpha+b-m_{jj})^2}{2a}\,d\alpha\right]\\ &-\sum_b\sum_i\sum_{j<i}(1-q)P(i|b)P(j|b)P(b)\left[\int_{-a}^{[m_{ii}+m_{jj}]/2-b}\frac{(\alpha+b-m_{jj})^2}{2a}\,d\alpha\right.\\ &\left.+\int_{[m_{ii}+m_{jj}]/2-b}^{a}\frac{(\alpha+b-m_{ii})^2}{2a}\,d\alpha\right].\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial W}{\partial q}(M;q) = &-\sum_b\sum_i P(i|b)\big(1-P(i|b)\big)P(b)\int_{-a}^{a}\frac{(\alpha+b-m_{ii})^2}{2a}\,d\alpha\\ &+\sum_b\sum_i\sum_{j>i}P(i|b)P(j|b)P(b)\left[\int_{-a}^{[m_{ii}+m_{jj}]/2-b}\frac{(\alpha+b-m_{ii})^2}{2a}\,d\alpha\right.\\ &\left.+\int_{[m_{ii}+m_{jj}]/2-b}^{a}\frac{(\alpha+b-m_{jj})^2}{2a}\,d\alpha\right]\end{aligned}$$

$$+\sum_b\sum_i\sum_{j<i}P(i|b)P(j|b)P(b)\left[\int_{-a}^{[m_{ii}+m_{jj}]/2-b}\frac{(\alpha+b-m_{jj})^2}{2a}\,d\alpha + \int_{[m_{ii}+m_{jj}]/2-b}^{a}\frac{(\alpha+b-m_{ii})^2}{2a}\,d\alpha\right].$$

Given signals i and $j>i$, note that $\frac{m_{i,i}+m_{j,j}}{2}-b<\alpha$ implies $(\alpha+b-m_{j,j})^2>(\alpha+b-m_{i,i})^2$. Thus, we have

$$-\int_{-a}^{a}\frac{(\alpha+b-m_{ii})^2}{2a}\,d\alpha \leqslant -\int_{-a}^{[m_{ii}+m_{jj}]/2-b}\frac{(\alpha+b-m_{ii})^2}{2a}\,d\alpha - \int_{[m_{ii}+m_{jj}]/2-b}^{a}\frac{(\alpha+b-m_{jj})^2}{2a}\,d\alpha.$$

Similarly, given signals i and $j<i$, $\alpha<\frac{m_{i,i}+m_{j,j}}{2}-b$ implies $(\alpha+b-m_{j,j})^2<(\alpha+b-m_{i,i})^2$. Thus, we have

$$-\int_{-a}^{a}\frac{(\alpha+b-m_{ii})^2}{2a}\,d\alpha \leqslant -\int_{-a}^{[m_{ii}+m_{jj}]/2-b}\frac{(\alpha+b-m_{jj})^2}{2a}\,d\alpha - \int_{[m_{ii}+m_{jj}]/2-b}^{a}\frac{(\alpha+b-m_{ii})^2}{2a}\,d\alpha.$$

Finally, we conclude that $\frac{\partial W}{\partial q}(M;q)\leqslant 0$, which delivers the desired result. □

Proof of Theorem 6. In what follows, P^ϵ, $m^\epsilon_{i,i}$, and W^ϵ denote the components of our model when signals are conditionally independent with precision $1-\epsilon$. For $\epsilon>0$ sufficiently small, it is straightforward to show that all symmetric equilibria, and therefore all equilibria, are in pure strategies: For example, suppose $z^\epsilon=\inf\operatorname{supp}G^\epsilon_i<m^\epsilon_{i,i}$ in equilibrium for arbitrarily small ϵ; then platforms close to z^ϵ lose with probability close to $1-\epsilon$, but the probability of winning when locating at $m^\epsilon_{i,i}$ is close to $\frac{1-\epsilon}{2}$. Given that equilibria are in pure strategies, Theorem 1 implies that $x_i=m^\epsilon_{i,i}$ for all signals i. Further, for $b=i$, we have $m^\epsilon_{i,i}\to m_{i,i}=b$ as ϵ goes to zero. For signals i and j and realization b, define

$$W(i,j|b)=\frac{1}{2a}\int_{-a}^{[m_{i,i}+m_{j,j}]/2}(\alpha+b-m_{i,i})^2\,d\alpha-\frac{1}{2a}\int_{[m_{i,i}+m_{j,j}]/2}^{a}(\alpha+b-m_{j,j})^2\,d\alpha$$

for $i<j$, with a similar definition for $i>j$, and

$$W(i,i|b)=-\frac{1}{2a}\int_{-a}^{a}(\alpha+b-m_{i,i})^2\,d\alpha,$$

when the candidates receive the same signals. We use the same conventions for the notation $W^\epsilon(i, j|b)$, substituting $m^\epsilon_{i,i}$ and $m^\epsilon_{j,j}$ where appropriate. By assumption, we have

$$\lim_{\epsilon\to 0} \frac{\sum_{j\neq b} P^\epsilon(j, b|b)}{\epsilon} = 1 \quad \text{and} \quad \lim_{\epsilon\to 0} \frac{\sum_{i,j\neq b} P^\epsilon(i, j|b)}{\epsilon} = 0.$$

Then welfare conditional on realization b under perfect precision is $W(b) = W(b, b|b)$, and with slightly noisy signals it is

$$W^\epsilon(b) = P^\epsilon(b, b|b)W^\epsilon(b, b|b) + \sum_{j\neq b} P^\epsilon(j, b|b)W^\epsilon(j, b|b) + \sum_{i,j\neq b} P^\epsilon(i, j|b)W^\epsilon(i, j|b).$$

Therefore,

$$\begin{aligned}\frac{W^\epsilon(b) - W(b)}{\epsilon} \geqslant{}& \frac{P^\epsilon(b, b|b)}{\epsilon}\big[W^\epsilon(b, b|b) - W(b, b|b)\big] \\ &+ \frac{\sum_{j\neq b} P^\epsilon(j, b|b)}{\epsilon} \min_{j\neq b}\big[W^\epsilon(j, b|b) - W(b, b|b)\big] + D^\epsilon,\end{aligned}$$

where D^ϵ goes to zero with ϵ. Using the fact that $m_{b,b} = b$, note that

$$\begin{aligned}W^\epsilon(b, b|b) - W(b, b|b) &= -\frac{1}{2a}\int_{-a}^{a} \big(\alpha + b - m^\epsilon_{b,b}\big)^2 d\alpha + \frac{1}{2a}\int_{-a}^{a} \alpha^2\, d\alpha \\ &= -\frac{1}{2a}\int_{-a}^{a} \big[\big(\alpha + b - m^\epsilon_{b,b}\big)^2 - (\alpha + b - m_{b,b})^2\big]\, d\alpha \\ &= -\frac{1}{2a}\int_{-a}^{a} \big[-\big(m^\epsilon_{b,b} - m_{b,b}\big)\big(2b + 2\alpha - m^\epsilon_{b,b} - m_{b,b}\big)\big]\, d\alpha \\ &= \big(m^\epsilon_{b,b} - m_{b,b}\big)\big(2b - m^\epsilon_{b,b} - m_{b,b}\big).\end{aligned}$$

Further,

$$\begin{aligned}m_{b,b} - m^\epsilon_{b,b} &= b - \frac{\sum_{b'} b' P^\epsilon(b, b|b')P(b')}{\sum_{b''} P^\epsilon(b, b|b'')P(b'')} \\ &= b - \frac{bP^\epsilon(b, b, b) + \sum_{b'\neq b} b' P^\epsilon(b, b, b')}{P^\epsilon(b, b, b) + \sum_{b''\neq b} P^\epsilon(b, b, b'')}.\end{aligned}$$

Hence,

$$\frac{m_{b,b} - m^\epsilon_{b,b}}{\epsilon} \leqslant \frac{b}{\epsilon}\left[\frac{\sum_{b''\neq b} P^\epsilon(b, b, b'')}{P^\epsilon(b, b, b)}\right],$$

which goes to zero with ϵ by assumption. Finally, without loss of generality, let

$$\min_{j\neq b}\big[W^\epsilon(j, b|b) - W(b, b|b)\big]$$

be achieved at $k > b$ as ϵ goes to zero. Then

$$\min_{j\neq b}\big[W^{\epsilon}(j,b|b)-W(b,b|b)\big]$$
$$=\int_{-a}^{[m^{\epsilon}_{b,b}+m^{\epsilon}_{k,k}]/2}\big[\big(\alpha+b-m^{\epsilon}_{b,b}\big)^2-\alpha^2\big]\,d\alpha+\int_{[m^{\epsilon}_{k,k}+m^{\epsilon}_{b,b}]/2}^{a}\big[\big(\alpha+b-m^{\epsilon}_{k,k}\big)^2-\alpha^2\big]\,d\alpha$$

has positive limit $\int_{[k+b]/2}^{a}[(\alpha+b-k)^2-\alpha^2]\,d\alpha>0$. Therefore,

$$\lim_{\epsilon\to 0}\frac{W^{\epsilon}(b)-W(b)}{\epsilon}\geqslant\int_{[k+b]/2}^{a}\big[(\alpha+b-k)^2-\alpha^2\big]\,d\alpha>0.$$

Since this is true for all b, the result follows. □

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