Strategic Information Transmission in Networks *

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Abstract

This paper introduces a tractable model to study cheap talk in networks. Although the analysis is apparently very intricate, we provide sharp equilibrium and welfare characterisations, that we then use to address a number of theoretical economic questions. We first study how governments and private institutions can harness the power of social networks to promote social welfare and private goals. We show that more connected networks can decrease the ability of governments to use social networks to spread information in a community. We also show that the influential players in a community are those who diffuse information to many others who, in contrast, are poorly connected. Second, we advance the study of homophily, by finding that truthful communication across communities decreases as populations become larger. Finally, we consider the largely unexplored problem of information aggregation in intergovernmental organizations. We find that public meetings generally provide better incentives for information transmission than closed-door bilateral meetings.

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1 Introduction

This paper studies strategic communication in networks. Each player can send a message only to the players with whom he is linked in the network. The players' message may either be different for each linked player, or be common among them. Each player may exchange messages with few others, but when contemplating what to report, he must forecast how his messages will alter his counterparts' decisions, taking into account that they may also receive messages from players who are beyond his circle of contacts. Despite the intricacies of this problem, this paper provides a tractable model with sharp equilibrium and welfare characterizations. These characterizations allow us to formulate and solve a number of important theoretical economic questions.

First, we study how governments and private institutions can harness the power of social networks to promote social welfare and private goals. We show that more connected networks can decrease the ability of governments to use social networks to spread information in a community. We also show that influential players in a community are those who diffuse information to many others who, in contrast, are poorly connected. Second, we provide a new perspective to the study of homophily and segregation in communities, by studying equilibrium information transmission within and across groups with different preferences. Our main contribution here is to provide theoretical and testable predictions on how the size of a community affects the incentives to exchange information truthfully across communities. We finally study the information aggregation problem of different policy-makers whose policy choices effect each other. Our contribution is novel in the political economy literature on information aggregation, because we assume that individuals retain control of final actions, instead of presuming that choices are aggregated through voting.

The Basic Model and Results. Our model is a natural extension of the celebrated model of cheap talk by Crawford and Sobel (1982). There are n players, and a state of the world θ, which is unknown and uniformly distributed on the interval $[0, 1]$. Each player j

simultaneously chooses an action y_j, that influences the utility of all players. If θ were known, player i would like that each player's action were as close as possible to $\theta + b_i$, where b_i represents player i's bias relative to the common bliss point θ; specifically, player i's payoff is $-\sum_j (y_j - \theta - b_i)^2$.[1] Each player i is privately informed of a signal s_i, which takes a value of one with probability θ and a value of zero with complementary probability. Before players choose their actions, they may simultaneously report their signals to others. A player can send a message only to the set of players he is linked to– his communication neighbors. In our setup, every player's neighborhood is partitioned in sets of players, that we define as *audiences*. The player can differentiate his message only across but not within audiences. Hence, our model covers both the case of *private communication*, where every player can send a message privately to each player in his neighborhood, and, the case of *public communication*, where every player's message is common to all players in his neighborhood.

A communication strategy profile is described by a (directed) network in which each link represents a truthful message, termed *truthful network*. Our first result derives the equilibrium condition for truthful communication of player i with a group of receivers in audience J. This condition is expressed in terms of the distance between player i's bias and a weighted average of the receivers' biases and the number of truthful messages that each player in the audience J receives in equilibrium. The characterization identifies important equilibrium effects. First, each player's incentive to misreport a low signal in order to raise the action of lower bias neighbors is tempered by the loss incurred from the increase in actions of all higher bias neighbors who belong to the same audience. Second, the composition of these gains and losses depends on the number of players truthfully communicating in equilibrium with each player in the same audience. The reason for this is that the influence that player i's message has on player j's decision depends on player j's equilibrium information. Third, an increase in the

[1] The underlying assumption here is that players' interactions are of a global nature, but communication is local. If, for example, the communication network represents a particular organization structure, then it is appropriate to assume that each division of the organization cares about the global organization's performance. In other contexts, for example when the communication network represents an existing social network or R&D collaborations among firms, it is more appropriate to assume that also players' interactions are of a local nature. Our equilibrium characterization can be easily extended to these environments.

number of truthful messages that a player j in audience J is expected to receive has ambiguous effect on i's ability to truthfully communicate with the audience J. If communication from player i to player j is private there is a strong congestion effect: the willingness of player i to communicate truthfully with player j declines in the number of players communicating with j. However, if j is part of a larger audience and his preferences are distant from i's preferences, an increase in the number of truthful messages received by j decreases the influence that i has on j's decision. As a result, player i may be more willing to communicate truthfully with all players in audience J.

We then turn to welfare analysis. In our framework, an equilibrium maximizes the *ex-ante* utility of a player if and only if it maximizes the *ex-ante* utility of each one of the other players. We find that each player i's ex-ante payoff induced by a player j's choice is an increasing and concave function of the number of players who truthfully communicate with j. Hence, equilibria can be ranked in the *ex-ante* Pareto sense on the basis of the distribution of in-degree that they generate in their corresponding equilibrium networks. If the in-degree distribution of an equilibrium first order stochastic dominates the in-degree distribution of another equilibrium, the former is more efficient than the latter. Moreover, if the in-degree distribution of an equilibrium network is a mean preserving spread of the in-degree distribution of another equilibrium network, then the latter is more efficient than the former.

While derived in a simple quadratic-loss Beta-binomial model, our equilibrium and welfare results are based on general features of utility functions in the Crawford and Sobel (1982) framework, and on general features of statistical Bayesian models. Specifically, the key assumptions behind our results are (i) the assumptions that utility functions are single-peaked, strictly concave and ordered through a single-crossing condition, and (ii) the fact that the effect of a signal on the posterior update decreases with the precision of prior, i.e., in a multi-player communication model, that the effect of a player's truthfully reported signal decreases with the number of truthful messages received from other players.[2]

[2]However, our specific welfare results also depend on the *anonymity* assumption that each player i's payoff

Economic Questions. Our basic equilibrium and welfare characterizations allow us to formulate and solve a number of theoretical economic questions.

The analysis in our first setting studies how governments and private institutions can harness the power of social networks to promote social welfare and private goals. An external agent, with a bias $b > 0$, sends information to members of a community, each with a bias $b = 0$. After the advertising, each member of the network talks privately with each of his neighbours for T periods. We ask under which circumstances the external agent is able to influence network members, despite the conflicting preferences. We also ask what are the topological characteristics of the agents that the external player would like to target in order to maximise his influence.

This application of our model is in line with a few recent articles in economics who address somewhat related questions, see Ballester, Calvo-Armengol, and Zenou (2006), Campbell (2010), Chatterjee and Dutta (2010), Galeotti and Goyal (2010). The main departure of our work with the existing literature is our focus on the implication of strategic communication in determining the optimal targeting strategy. Our first insight is that an increase in connectivity of the network and/or an increase in the quality of information diffusion within the network may break the ability of governmental and private agents to use social networks in order to influence individuals' behaviour. The second insight is that key players in the network are those who diffuse information to many other agents, who, in turn, are poorly connected. Both insights are novel in the economic literature.

The second setting studies communication between two communities of players, where preferences are the same within groups, but different across groups. Considering private communication first, our results predict that individuals belonging to different communities are able to truthfully exchange information only if the two communities are sufficiently small. Furthermore, we predict that large groups of individuals influence the decisions of small groups by

depends on each player j's action through the same quadratic loss function.

credibly reporting information, while there is less truthful communication from small communities to large communities. The intuition for these results is that a member of a larger group has more access to information inside his own group and therefore he is less influenced by the messages he accesses from members of opposite communities. As a consequence, members of small groups have higher incentives to misreport information to large groups members, than vice-versa. This analysis offers a new perspective on a phenomenon that Lazersfeld and Merton (1954) termed *homophily*: the tendency of individuals to associate and exchange information with others who are similar to themselves.[3] A form of homophily is already present in the seminal work of Crawford and Sobel (1982): information transmission is more truthful if the principal and the agent have similar preferences. The added value of our analysis is that by studying cheap talk in a multi-player environment, we are able to provide insights on how the size of a community affects the incentives of truthful communication across communities.

We also consider whether private information transmission across communities can be improved by public communication, or by intermediate communication modes where each player must send the same message to an audience that does not necessarily includes all other players. We find that for large bias, private communication fares better than public communication, as it guarantees at least information transmission within communities. But intermediate communication performs even better than private communication, when each player's audience is balanced so as to include all fellow community members together with a minority of players from the other community. Public communication achieves full information aggregation for intermediate and low levels of bias. Hence, it outperforms private communication for intermediate levels of bias.[4]

In the final part of the paper, we consider the information aggregation problem when the bias difference across adjacent bias players is constant. We believe our model is able to provide

[3]Homophily has been documented across a wide range of characteristics, such as age, race, gender, religion and occupations, e.g., Moody (2001) and McPherson et al. (2001), whereas Currarini et al. (2009) provides a strategic foundation for these empirical patterns.

[4]Clearly, for small levels of bias, all communication modes induce full information aggregation.

useful insight for situations in which ideological policy-makers with private information of common value try to influence each other's decisions. Specifically, each politician would like that all decisions match his bliss point, which consists of the common state of the world plus an idiosyncratic ideological bias. Our analysis may be of relevance, for example, for a scenario where national leaders need to implement national environmental or economic policies with spillover effects on other countries. Most importantly, we propose a distinct contribution to the political economy literature on information aggregation in committees (see, e.g., Ottaviani and Sorensen (2001), Austen-Smith and Feddersen (2006), Visser and Swank (2007), and Gerardi and Yariv (2007)). Such models focus on the cases where the individual choices are aggregated in a single final action through voting. Inspired by the study of intergovernmental organizations, instead, we assume that individuals retain control of their own policy choices, but consider externalities among the politicians' choices.

We first characterize the truthful communication network for private communication. We find that endogenous communication networks emerging from strategic communication are highly decentralized: all moderate bias players have the same in-degree, while the in-degree declines slowly as the biases become more extreme. Further, links between similar bias players are reciprocal, whereas links between players with a very different bias may be not reciprocal. In that case, moderate bias players influence the decision of extreme bias players through truthful communication, while extreme bias players do not influence the decision of moderate bias players.[5]

Second, we characterize public communication truthful networks and then numerically determine whether private or public communication is more effective. We find that, in most cases, public broadcasting of information, through public announcements or through the organization of public meetings outperforms the private exchange of information through bilateral

[5]There is a large literature which studies endogenous communication networks, see, e.g., Bala and Goyal (2000), Bloch and Dutta (2007), Calvo et al. (2009), Galeotti et al. (2006) and Hojman and Szeidl (2008) and Jackson and Wolinsky (1996). Our main departure from that literature is that we study the formation of endogenous communication network in a context where information transamission is costless and non-verifiable.

closed-door meetings. This result can be seen as a theoretical justification for the common practice of large intergovernmental meetings, such as the European Council, the G20, or the meetings of the general assembly of the United Nations.[6]

Related Literature. Our paper also relates to the literature on strategic information transmission, which builds on the classical model of cheap talk by Crawford and Sobel (1982). This literature is too vast for us to fully survey here, and we discuss only the papers that are more closely related to our work.[7]

The main contribution of our paper lies in bringing two basic insights derived by the cheap talk literature into network economics. The first insight is about strategic communication in a many-to-one communication model: the incentives of a sender to reveal information to the receiver declines with the number of senders communicating with the receiver. This effect has been emphasized by Morgan and Stocken (2008).[8] The second insight is about strategic communication in a one-to-many communication model: when a sender communicates publicly with two receivers, the gains induced by misreporting a signal to bias one receiver may be tempered by the loss induced by biasing the other receiver, see Farrell and Gibbons (1989).[9] Our equilibrium characterization shows that these two basic insights are key to understand strategic communication in a communication network. In fact, we show that the incentives of a player to communicate truthfully with an audience depends on the gains and losses induced by misreporting a signal and that the weights of such gains and losses depend on the number of players truthfully communicating with the audience.

Our paper is close in spirit to a recent paper by Hagenbach and Koessler (2010) but, as explained below, there are major differences. Furthermore, our model turns out to allow a wider

[6]Indeed, while bilateral international summits may often occur, the discussions held in such meeting are seldom held with closed doors.

[7]Other influential works include Ambrus and Takahashi (2008), Austen Smith (1993), Battaglini (2002, 2004), Gilligan and Krehbiel (1987, 1989), Krishna and Morgan (2001a, 2001b), Wolinsky (2002).

[8]Indeed, our equilibrium characterization for the special case of private communication can be described as a generalization of their Proposition 2.

[9]More recent work on cheap talk game with multiple receivers and a single sender includes Goltsmann and Pavlov (2009), Koessler and Martimort (2008), and Caillaud and Tirole (2007).

analysis, particularly concerning the welfare implications of the different modes of communication and the role of diffusion of information. Both articles investigate strategic communication in networks, but they explore different modeling assumptions, which, in turn, generate distinct incentives and predictions. Hagenbach and Koessler (2010) studies an environment where the state of the world is additive in players' signals and players have incentives to coordinate their actions. We abstract away from any coordination motives, but we focus on the effect of correlation between players' signals. This is a key difference as we now elaborate. Under the assumption that the state of the world θ equals the sum of each player i's individual binary signal (and signals are independent across players), the marginal effect of one truthful message on the action chosen by a receiver is constant in the number of truthful messages received. So, unless players have some motive for coordinating their actions, strategic communication between player i and player j would only depend on their relative distance in preferences. In contrast, as long as players' signals are correlated (but conditionally independent across players), even if players' actions are strategic independent, players' incentives to communicate will depend on their differences in preferences as well as on the communication strategy of other players; understanding this dependence is the focus of our analysis. It is interesting to note that, even if the economic forces at work are different, some of the conclusions in the model of two communities are qualitatively similar; we discuss the similarity and the differences in the respective sections.

The rest of the paper is organized as follows. Section 2 develops the basic framework and section 3 presents the basic results on equilibrium and welfare. Section 4 studies the situation in which an external agent targets a player in a network, Section 5 explores communication within and across two communities, and section 6 considers information aggregation in intergovernmental organizations. Section 7 concludes, and all proofs are in an Appendix.

2 Basic Model

The set of players is $N = \{1, 2, ..., n\}$, player i's individual bias is b_i and $b_1 \leq b_2 \leq ... \leq b_n$. The vector of biases $\mathbf{b} = \{b_1, ..., b_n\}$ is common knowledge. The state of the world θ is uniformly distributed on $[0, 1]$. Every player i receives a private signal $s_i \in \{0, 1\}$ on the realization of the state of the world, where $s_i = 1$, with probability θ.

Communication among players is exogenously restricted by a communication network and a communication mode. The former describes who can exchange information with whom, while the latter describes to what extend the technology of communication allows to target messages. Specifically, a communication network $\mathbf{g} \in \{0, 1\}^{n \times n}$ is a (possibly directed) graph: i can send his own signal to j whenever $g_{ij} = 1$. We assume that $g_{ii} = 0$ for all $i \in N$. The communication neighborhood of i is the set of players to whom i can send his signal and it is denoted by $N_i(\mathbf{g}) = \{j \in N : g_{ij} = 1\}$. The communication mode available to i is $\mathcal{N}_i(\mathbf{g})$, a partition of the communication neighborhood of i with the interpretation that player i must send the same message m_{iJ} to all agents $j \in J$, for any group of agents $J \in \mathcal{N}_i(\mathbf{g})$, where each J is called an audience.

A communication strategy of a player i specifies, for every $s_i \in \{0, 1\}$, a vector $\mathbf{m}_i(s_i) = \{m_{iJ}(s_i)\}_{J \in \mathcal{N}_i(\mathbf{g})}$; $\mathbf{m} = \{\mathbf{m}_1, \mathbf{m}_2, ..., \mathbf{m}_n\}$ denotes a communication strategy profile. We let $\hat{\mathbf{m}}_i$ be the messages sent by agent i to his communication neighborhood, and $\hat{\mathbf{m}} = (\hat{\mathbf{m}}_1, \hat{\mathbf{m}}_2, ..., \hat{\mathbf{m}}_n)$.

After communication occurs, each player i chooses an action $\hat{y}_i \in \Re$. Let $N_i^{-1}(\mathbf{g}) = \{j \in N : g_{ji} = 1\}$ be the set of players communicating with agent i. Then, agent i's action strategy is $y_i : \{0, 1\}^{|N_i^{-1}(\mathbf{g})|} \times \{0, 1\} \rightarrow \Re$; $\mathbf{y} = \{y_1, ..., y_n\}$ denotes an action strategy profile. Given the state of the world θ, the payoffs of i facing a profile of actions $\hat{\mathbf{y}}$ is:

$$u_i(\hat{\mathbf{y}}|\theta) = -\sum_{j \in N} (\hat{y}_j - \theta - b_i)^2 .$$

Agent i's payoffs depend on how his own action y_i and the actions taken by other players are close to his ideal action $b_i + \theta$. The underlying assumption here is that players' interaction is of a global nature. Depending on the particular context, a model where each player is affected only by the actions taken by a subset of the population may be more plausible. A way of incorporating this is to assume that $u_i(\hat{\mathbf{y}}|\theta) = -\sum_{j\in N} \alpha_{ij} \left(\hat{y}_j - \theta - b_i\right)^2$. Our method of analysis and our equilibrium results can easily be extended to these settings (see also footnote 11 for further discussion).

The equilibrium concept is Perfect Bayesian Equilibrium. We restrict attention to pure-strategy equilibria. Hence, up to relabeling of messages, each agent i's communication strategy $\mathbf{m}$ with an audience $J \in \mathcal{N}_i(\mathbf{g})$, may take only two forms: the truthful one, $m_{iJ}(s_i) = s_i$ for all s_i, and the babbling one, $m_{iJ}(0) = m_{iJ}(1)$.[10]

Given the received messages $\hat{\mathbf{m}}_{N_i^{-1}(\mathbf{g}),i}$, by sequential rationality, agent i chooses y_i to maximize his expected payoff. Therefore, agent i's optimization reads

$$\begin{aligned} & \max_{y_i} \left\{ -E\left[\sum_{j\in N} \left(y_j - \theta - b_i\right)^2 \middle| s_i, \hat{\mathbf{m}}_{N_i^{-1}(\mathbf{g}),i} \right] \right\} \\ = \; & \max_{y_i} \left\{ -E\left[\left(y_i - \theta - b_i\right)^2 \middle| s_i, \hat{\mathbf{m}}_{N_i^{-1}(\mathbf{g}),i} \right] \right\}. \end{aligned}$$

Hence, agent i chooses

$$y_i\left(s_i, \hat{\mathbf{m}}_{N_i^{-1}(\mathbf{g}),i}\right) = b_i + E\left[\theta \middle| s_i, \hat{\mathbf{m}}_{N_i^{-1}(\mathbf{g}),i}\right], \tag{1}$$

where the expectation is based on equilibrium beliefs: All the messages $\hat{m}_{ji}$ received by an agent j who adopts a babbling strategy are disregarded as uninformative, and all $\hat{m}_{ji}$ received by an agent j who adopts a truthful strategy are taken as equal to s_j. Hereafter, whenever we refer to a strategy profile $(\mathbf{m}, \mathbf{y})$, each element of $\mathbf{y}$ is assumed to satisfy condition 1.

We further note that the agents' updating is based on the standard Beta-binomial model. So, suppose that an agent i holds k signals, i.e., he holds the signal s_i and $k-1$ neighbors

[10] We focus the analysis on pure strategy equilibria. However, it can be shown that in every equilibrium, a player may either play a babbling strategy, or a truthful strategy, or a semi-truthful strategy which mixes among the two messages for one of the signals, and sends one message with probability one for the other signal.

truthfully reveal their signal to him. If l out of such k signals equal 1, then the conditional pdf is:

$$f\left(l|\theta,k\right)=\frac{k!}{l!\left(k-l\right)!}\theta^{l}\left(1-\theta\right)^{\left(k-l\right)},$$

and his posterior is:

$$f\left(\theta|l,k\right)=\frac{\left(k+1\right)!}{l!\left(k-l\right)!}\theta^{l}\left(1-\theta\right)^{\left(k-l\right)}.$$

Consequently, $f(l|\theta,k)=f(\theta|l,k)/(k+1)$ and $E\left[\theta|l,k\right]=\left(l+1\right)/\left(k+2\right).$

In the first stage of the game, in equilibrium, each agent i adopts either truthful communication or babbling communication with each group of agents $J\in\mathcal{N}_i(\mathbf{g})$, correctly formulating the expectation on the action chosen by agent $j\in J$ as a function of his message $\hat{m}_{iJ}$ and with the knowledge of the equilibrium strategies $\mathbf{m}_{-i}$ of the opponents.

We finally note that our framework encompasses two widely studied modes of communication: *private communication* and *public communication*. The model of private communication obtains when for each agent i the partition $\mathcal{N}_i\left(\mathbf{g}\right)$ of $N_i(\mathbf{g})$ is composed of singleton sets. In this case each agent has the possibility to communicate privately with each of his neighbors. The opposite polar case is when, for each player i, $\mathcal{N}_i\left(\mathbf{g}\right)$ consists of the trivial partition $\{N_i(\mathbf{g})\}$, which corresponds to a model of public communication.

3 Basic Results

We first characterize equilibria for arbitrary modes of communication. We then show that the characterization takes a simple form under private communication. We finally investigate the relationship between equilibrium communication and Pareto efficiency.

3.1 Equilibrium Networks

A communication network $\mathbf{g}$ together with a strategy profile $(\mathbf{m},\mathbf{y})$ induces a subgraph of $\mathbf{g}$, in which each link involves truthful communication. We refer to this network as the *truthful network* and denote it by $\mathbf{c}(\mathbf{m},\mathbf{y}|\mathbf{g})$. When $(\mathbf{m},\mathbf{y})$ is equilibrium, we refer to the induced truthful network $\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})$ as to the *equilibrium network*. Formally, $\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})$ is a binary directed graph where $c_{ij}(\mathbf{m}, \mathbf{y}|\mathbf{g}) = 1$ if and only if j belongs to some element $J \in \mathcal{N}_i(\mathbf{g})$ and $m_{iJ}(s) = s$, for every $s = \{0, 1\}$. Given a truthful network $\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})$, let $k_j(\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g}))$ be the number of agents who send a truthful message to j. We term $k_j(\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g}))$ the in-degree of player j.

Out first result provides the equilibrium condition for truthful communication of an agent i with an audience J.

Theorem 1 *Consider a communication network* $\mathbf{g}$ *and a collection of communication modes* $\{\mathcal{N}_i(\mathbf{g})\}_{i \in N}$. *The strategy profile* $(\mathbf{m}, \mathbf{y})$ *is an equilibrium if and only if for every truthful message from a player* i *to an audience* $J \in \mathcal{N}_i(\mathbf{g})$,

$$2\left|b_i - \sum_{j \in J} b_j \gamma_j(\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g}))\right| \leq \sum_{j \in J} \frac{1}{(k_j(\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})) + 3)} \gamma_j(\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})); \qquad (2)$$

where for every $j \in J$, *with* $J \in \mathcal{N}_i(\mathbf{g})$,

$$\gamma_j(\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})) = \frac{1/(k_j(\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})) + 3)}{\sum_{j' \in J} 1/(k_{j'}(\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})) + 3)}$$

Condition (2) may be interpreted as follows. The left-hand side of condition (2) tells us that player i is willing to truthfully communicate with all players in J if and only if the weighted average $\sum_{j \in J} b_j \gamma_j$ of their biases b_j is not too different from his own bias b_i. This reflects the fact that, when contemplating whether to deviate from truthful reporting, player i can only influence the action of all players in audience J in the same direction. When, for instance, $s_i = 0$ and he reports $\hat{m}_{iJ} = 1$, he will gain by biasing upwards the action of every player $j \in J$ with bias $b_j < b_i$, but, at the same time, he will lose by increasing the action of every $j \in J$ with bias $b_j > b_i$. Overall, player i's deviation from truthful reporting is deterred if and only if, on average, losses outweigh gains.

The right-hand side of condition (2) is a weighted average, with the same weights as the average in the left-hand side, of the quantities $1/(k_j + 3)$ for $j \in J$. The quantity $1/(k_j + 3)$

is the absolute difference of the expected value of θ induced by the change of one signal realization, when knowing k_j further signal realizations. Because player j is informed of signal s_j, receives $k_j - 1$ truthful messages from players other than i, and matches his action y_j to his expected value of θ, the quantity $1/(k_j + 3)$ corresponds to the action change of player j that follows from a message deviation from equilibrium by player i.

Turning to the specific weights γ_j in both the left-hand side and right-hand side of condition 2, we observe the following. The numerator $1/(k_j + 3)$ decreases in the number of players truthfully communicating with j in equilibrium. The reason for this is that the more player j is informed in equilibrium, the less the message $\hat{m}_{iJ}$ will change his final action. Therefore, when contemplating a deviation, player i can gain or lose less in absolute terms by influencing j relative to the other players in J. As a result, player i will give less weight γ_j to player j in the weighted average, relative to the other players in audience J.

In sum, there is an equilibrium where player i truthfully communicates with audience J if and only if twice the absolute difference between his bias and the weighted average of their biases is smaller than the weighted average change of the action in audience J induced by a message deviation by i. In fact, when this condition holds, player i suffers an overall loss by deviating from the truthful equilibrium strategy.

The characterization in Theorem 1 simplifies substantially in the specific case of private communication.

Corollary 1 *Consider a communication network* $\mathbf{g}$. *Under private communication a strategy profile* $(\mathbf{m}, \mathbf{y})$ *is equilibrium if and only if for every* (i, j) *with* $c_{ij}(\mathbf{m}, \mathbf{y}|\mathbf{g}) = 1$,

$$|b_i - b_j| \leq \frac{1}{2\left[k_j(\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})) + 3\right]}. \tag{3}$$

Under private communication, the willingness of a player i to credibly communicate with a player j displays a simple dependence on their bias difference $|b_i - b_j|$ and on the number of players truthfully communicating with j. In particular, a high in-degree k_j prevents communication from i to j to be truthful. To see this, suppose that $b_i > b_j$, so that j is biased upwards relative to i. When many opponents truthfully communicate with j, this player is well informed. In this case, if player i deviates from the truthful communication strategy and reports $\hat{m}_{ij} = 1$ when $s_i = 0$, he will induce a small increase of j's action. Such a small increase in j's action is always beneficial in expectation to i, as it brings j's action closer to i's (expected) bliss point. Hence, player i will not be able to truthfully communicate the signal $s_i = 0$. In contrast, when j has a low in-degree, then i's report $\hat{m}_{ij} = 1$ moves j's action upwards significantly, possibly over i's bliss point. In this case, biasing upwards j's action may result in a loss for player i. Hence, he does not deviate from the truthful communication strategy.

We observe that the equilibrium effects derived in Theorem 1 and Corollary 1 are driven by the fact that the marginal returns to receive an additional signal is strictly decreasing on the amount of signals that a player has already received. This is different from Hagenbach and Koessler (2010), where, because of the assumption of additive state, the equilibrium communication strategy of a player is independent of the communication strategy played by all other agents.

Returning to general communication modes, we conclude this section by showing that, unlike in the case of private communication, a player i's willingness to truthfully communicate with another player j needs not to decrease in the in-degree of j.

Example 1. Let $N = \{1, 2, 3, 4\}$ and $\mathbf{b} = \{-1, 0, \beta, \beta + c\}$, where $\beta > 1$ and c is a small positive constant. Consider the following communication network $\mathbf{g}$: $g_{21} = g_{23} = g_{43} = 1$ and no other communication links. Suppose also that player 2 must send the same message to his neighbors 1 and 3.

First, suppose that player 4 babbles to player 3. In this case, player 2 assigns the same weight to player 3 and player 4, i.e., $\gamma_1 = \gamma_3 = 1/2$. The communication strategy in which player

2 sends a truthful public message to 1 and 3 is part of an equilibrium whenever $\beta \leq 5/4$. Second, suppose that player 4 communicates truthfully with 3 (which is always possible in equilibrium for sufficiently small c). In this case, player 2 gives a higher weight to player 1 who is less informed than player 3, i.e., $\gamma_1 = 5/9 > 4/9 = \gamma_3$. The communication strategy in which player 2 sends a truthful public message to 1 and 3 is part of an equilibrium whenever $\beta \leq 241/169$. Hence, when $\beta \in (5/4, 241/160]$ player 2 is able to report a truthful public message to his neighbors 1 and 3, only if player 4 also communicates truthfully with 3. ■

3.2 Welfare

We now consider equilibrium welfare. Because of the quadratic utility formulation, if we let $\sigma_j^2(\mathbf{m}, \mathbf{y})$ be the residual variance of θ that player j expects to have once communication has occurred, we can write player i's expected utility in equilibrium $(\mathbf{m}, \mathbf{y})$ as follows:

$$EU_i(\mathbf{m}, \mathbf{y}) = -\left[\sum_{j \in N}(b_j - b_i)^2 + \sum_{j \in N} \sigma_j^2(\mathbf{m}, \mathbf{y})\right].$$

This is an extension of the welfare characterization by Crawford and Sobel [1982] to multiple senders and multiple receivers. A nice feature of our model is that we can express the sum of residual variances of θ as a function of a simple property of the equilibrium network, namely its distribution of in-degrees. That is

$$\sum_{j \in N} \sigma_j^2(\mathbf{m}, \mathbf{y}) = \frac{1}{6} \sum_{k=0}^{n-1} \frac{1}{k+3} P(k|\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})), \tag{4}$$

where $P(k|\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g}))$ is the proportion of players with in-degree k in the equilibrium network $\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})$, and $P(\cdot|\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})) : \{0, ..., n-1\} \to [0, 1]$ is its in-degree distribution.

Inspection of the above equation shows that an equilibrium $(\mathbf{m}, \mathbf{y})$ yields a higher ex-ante utility to a player i than another equilibrium $(\mathbf{m}', \mathbf{y}')$ if and only if $(\mathbf{m}, \mathbf{y})$ yields higher ex-ante utility than $(\mathbf{m}', \mathbf{y}')$ to all other players j. Hence, ranking equilibria in the Pareto sense is equivalent to ranking them in the sense of utility maximization of all players. We can now state the following result.

Theorem 2 *Consider communication networks* $\mathbf{g}$ *and* $\mathbf{g}'$*. Suppose that* $(\mathbf{m}, \mathbf{y})$ *and* $(\mathbf{m}', \mathbf{y}')$ *are equilibria in* $\mathbf{g}$ *and* $\mathbf{g}'$*, respectively. Equilibrium* $(\mathbf{m}, \mathbf{y})$ *Pareto dominates equilibrium* $(\mathbf{m}', \mathbf{y}')$ *if and only if*

$$\sum_{k=0}^{n-1} \frac{1}{k+3} P(k|\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})) < \sum_{k=0}^{n-1} \frac{1}{k+3} P(k|\mathbf{c}'(\mathbf{m}', \mathbf{y}'|\mathbf{g}')). \tag{5}$$

A simple inspection of condition 5 allows us to rank equilibria in the Pareto sense based on stochastic dominance relations between the in-degree distributions of their corresponding equilibrium networks.[11]

Corollary 2 *Consider communication networks* $\mathbf{g}$ *and* $\mathbf{g}'$*. Suppose that* $(\mathbf{m}, \mathbf{y})$ *and* $(\mathbf{m}', \mathbf{y}')$ *are equilibria in* $\mathbf{g}$ *and* $\mathbf{g}'$*, respectively.*

1. *If* $P(k|\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g}))$ *first order stochastic dominates* $P(k|\mathbf{c}'(\mathbf{m}', \mathbf{y}'|\mathbf{g}'))$ *then equilibrium* $(\mathbf{m}, \mathbf{y})$ *Pareto dominates equilibrium* $(\mathbf{m}', \mathbf{y}')$*.*

2. *If* $P(k|\mathbf{c}'(\mathbf{m}', \mathbf{y}'|\mathbf{g}'))$ *is a mean preserving spread of* $P(k|\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g}))$ *then equilibrium* $(\mathbf{m}, \mathbf{y})$ *Pareto dominates equilibrium* $(\mathbf{m}', \mathbf{y}')$

To illustrate the first part of Corollary 2, consider an equilibrium in which i babbles with j and another equilibrium in which the only difference is that player i communicates truthfully with j. The presence of this additional truthful message only alters the equilibrium action of player j. In particular, player j's action becomes more precise and therefore the utility of each player increases. A direct consequence of this result is that if $(\mathbf{m}, \mathbf{y})$ and $(\mathbf{m}', \mathbf{y}')$ are two distinct equilibria in a communication network $\mathbf{g}$ and $\mathbf{c}'(\mathbf{m}', \mathbf{y}'|\mathbf{g})$ is a subgraph of $\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})$, then equilibrium $(\mathbf{m}, \mathbf{y})$ Pareto dominates equilibrium $(\mathbf{m}', \mathbf{y}')$.

[11] The fact that only the distribution of in-degrees and not the identity of players matters for equilibrium welfare relies on the anonymity assumption embedded in the payoff specifications $u_i(\hat{\mathbf{y}}|\theta) = -\sum_{j \in N} (\hat{y}_j - \theta - b_i)^2$, for all i. For example, if each player i cared more about his own action y_i, than about others' actions y_{-i}, then equilibria could not be Pareto ranked on the basis of their in-degree distributions. However, a result analogous to Theorem 2 would still hold if we focused on the utilitarian welfare criterion, and if the effect of the action y_j of each player $j \neq i$ on any player i did not depend on i and j.

The second part of Corollary 2 allows to compare equilibria that have the same number of truthful communication links. It shows that equilibria in which truthful messages are distributed evenly across players Pareto dominate equilibria where few players receive many truthful messages, while others receive only a few. The reason is that the residual variance of θ associated to every player j is (decreasing) and *convex* in his in-degree.

Theorem 2 and Corollary 2 suggest the possibility that an equilibrium that sustains a low number of truthful messages may Pareto dominate an equilibrium with a high number of truthful messages, as long as its messages are distributed more evenly across players. We now develop an example in which this is the case.

Example 2: Evenly distributed truthful messages vs. total number of truthful messages. Suppose that $n = 5$ and that $b_{i+1} - b_i = \beta$, for $i = 1, 2, 3, 4$. Let $\mathbf{g}$ be a star network, player 3 be the center and consider the case of private communication. When $\beta \leq 1/28$ the following two equilibrium networks are part of equilibrium. One equilibrium network sustains four truthful links: each peripheral player sends a truthful message to the center, and there are no other truthful messages. The in-degree distribution of the equilibrium truthful network is then: $P(0) = 4/5$, $P(4) = 1/5$, and $P(k) = 0$, $k = 1, 2, 3$. The other equilibrium network sustains three truthful links: the center sends a truthful message to players 1, 2 and 4, and there are no other truthful messages. The in-degree distribution associated to this equilibrium is: $\tilde{P}(0) = 2/5$, $\tilde{P}(1) = 3/5$ and $\tilde{P}(k) = 0$, $k = 2, 3, 4$.

Note that P and $\tilde{P}$ cannot be ranked in terms of first order or second order stochastic dominance relations. However, applying condition 5, it is easy to check that

$$\sum_{k=0}^{n-1} \tilde{P}(k) \frac{1}{k+3} = \frac{17}{60} < \frac{31}{105} = \sum_{k=0}^{n-1} P(k) \frac{1}{k+3}.$$

Hence, the second equilibrium Pareto dominates the former equilibrium, despite it sustains a lower number of truthful messages. ■

This result concludes our basic equilibrium and welfare analysis of strategic information transmission in networks. In the next sections, we apply our findings to the study of three distinct economic questions. We shall focus on utility-maximizing equilibria. A utility-maximizing equilibrium is an equilibrium that maximizes the utility of all players, across all equilibria.[12]

4 Targeting the influencer

The growth of the internet, along with the other advances in information technology, has generated a wide interest on how governments and private institutions can harness the power of social networks to promote social welfare and private goals. There is by now a large popular literature on this and to related questions, and also a sizable academic literature across different disciplines such as computer science, marketing and public health.[13] Organizations often incorporate peer effects in designing policies to promote behavior change, such as reducing smoking and reducing risk behavior that can lead to sexually transmitted diseases.[14] Political campaign organisers are quickly recognising the possible advantages of using social network in influencing voters' behaviour. A recent example is given by the campaign strategies of Democrats and Republicans in the last presidential USA elections, where both Barack Obama's "Neighbor-to-Neighbor" and John McCain's "Voter-to-Voter" strategies rely on their supporter to reach out to their neighbours and social networks.

We study the problem of an external agent $\mathcal{E}$ who sends information to a member in the network with a view to influencing members of the network to choose certain actions. The external agent has a bias $b_{\mathcal{E}} = 0$, while all agents in the network share the same bias $b > 0$. At the beginning of the game, all players (including the external agent) get a signal about the

[12]As we noted earlier, in our setting a utility-maximizing equilibrium Pareto dominates every other equilibrium.

[13]The advent of new communication tools such as email, chat rooms and web sites has led marketers to re-evaluate the impact of network-based marketing strategy as an effective way to market products. For a discussion on the empirical importance of viral marketing see Leskovec, Adamic and Huberman (2007) and Godes and Mayzlin (2004). See also Rosen (2000) for case studies in which commercial organizations target key individuals in social contexts to generate desirable outcomes.

[14]See Rogers (2003) for an illustration of different case studies. See Valente et al. (2003) and Kelly et al. (1991) for a discussion about the empirical importance of peer-leading interventions.

state of the world, which is private information. Then, the external agent $\mathcal{E}$ chooses a player i in the network and sends to him a message. Finally, players in the network engage in T periods of conversations where in every period each player sends a private message to each of his neighbours. At the end of period T, each player in the network takes an action.

Our model is in line with a few recent articles in economics who address somewhat related questions, see Ballester, Calvo-Armengol, and Zenou (2006), Campbell (2010), Chatterjee and Dutta (2010), Galeotti and Goyal (2010). The main departure of our work with the existing literature is our focus on the implication of strategic communication in order to determine the optimal targeting strategy. This leads to two new insights. The first insight is that an increase in connectivity of the network and/or an increase in the quality of information diffusion within the network may break the ability of governmental and private agents to use social networks in order to influence individuals' behaviour. The second insight is that key players in the network are those who diffuse information to many other agents, who, in turn, are poorly connected. Both insights are novel in the economic literature.

The general model developed in section 2 and 3 of this paper studies single round communication. We now show that our framework can be used to analyze a situation with T periods of communication in the following sense. Since all agents in the network have the same bias, there is always an equilibrium where messages are truthful in each of the T periods of conversation and this equilibrium maximizes social welfare. Hereafter, we assume that agents play this equilibrium. As a consequence, before taking an action, each agent i in the network will receive a message from each player who is at most at distance T in the network. So, T is a measure of how information diffuses in the network and we call T the diffusion radius.

Formally, let $d(i, j; \mathbf{g})$ be the geodesic distance between i and j in network $\mathbf{g}$; by convention $d(i, i; \mathbf{g}) = 0$ for all i and $d(i, j; \mathbf{g}) = \infty$ whenever there is not a path between i and j.[15] Given $\mathbf{g}$, let $\mathbf{g}(T)$ be a network where a link between i and j is present if and only if the

[15]The geodesic distance $d(i, j; \mathbf{g})$ is the number of links in the shortest path between i and j.

geodesic distance between i and j is at most T in network $\mathbf{g}$, i.e., $g_{ij}(T) = 1$ if and only if $d(i, j : \mathbf{g}) \leq T$. The set of players who send a message to agent i at the end of period T is then $N_i(\mathbf{g}(T)) = \{j \in N : g_{ij}(T) = 1\}$. Similarly, when the external agent $\mathcal{E}$ targets his message to agent $i \in N$, he anticipates that the message will reach agent i and also all agents at most at distance T from agent i. That is, the audience of external agent $\mathcal{E}$ when he targets agent i is $N_{\mathcal{E}}(i; \mathbf{g}, T) = N_i(\mathbf{g}(T)) \cup \{i\}$. We finally denote the number of messages that agent j receives in network $\mathbf{g}$ when the external agent target agent i and the diffusion radius is T as $k_j(i; \mathbf{g}, T) = |N_j(\mathbf{g}(T))| + \mathbf{1}_{j \in N_{\mathcal{E}}(i; \mathbf{g}, T)}$, where $\mathbf{1}_{j \in N_{\mathcal{E}}(i; \mathbf{g}, T)}$ is an indicator function that takes value 1 if $j \in N_{\mathcal{E}}(i; \mathbf{g}, T)$ and 0 otherwise.

We can now use Theorem 1 to obtain the following result.

Proposition 1 *Consider a network* $\mathbf{g}$ *and diffusion radius* T. *There exists an equilibrium where the external agent* $\mathcal{E}$ *communicates truthfully to agent* $i \in N$ *if and only if*

$$\sum_{j \in N_{\mathcal{E}}(i; \mathbf{g}, T)} \frac{1}{k_j(i; \mathbf{g}, T) + 3} \left(\frac{1}{2(k_j(i; \mathbf{g}, T) + 3)} - b \right) \geq 0. \tag{6}$$

Proof: The proof follows directly by Theorem 1. QED.

We illustrate the equilibrium condition 6 within a class of symmetric graphs: regular trees. A tree is a network $\mathbf{g}$ where there are no cycles. A regular tree with degree d is a tree where players in the network can be partitioned in two sets: agents with $d > 1$ connections and agents with one connection. The latter agents are called end-agents or agents at the boundary of the network. Suppose the external agent $\mathcal{E}$ may target any agent i far away from the boundary of the network (formally $d(i, j; \mathbf{g}) < 2T$, where j is an end agent). It is easy to check that $|N_{\mathcal{E}}(i; \mathbf{g}, T)| = \sum_{x=0}^{T} d^x = (d^T - 1)/(d - 1)$ and that each agent $j \in N_{\mathcal{E}}(i; \mathbf{g}, T)$ receives $k_j(i; \mathbf{g}, T) = (d^T - 1)/(d - 1)$ messages. Using Proposition 1, we conclude that the external agent is able to send a truthful message to player i if and only if

$$b \leq \frac{d - 1}{2(d^{T+1} + 3d - 4)}.$$

We first observe that the LHS is decreasing in the degree of the regular tree, d, and in the radius of diffusion T. So, *an increase in the degree of the network or an increase in the diffusion radius may break down the ability of the external agent to influence agents in the network.* This is related to the congestion effect that has been illustrated in our general Theorem 1.

The second observation is that for every two agents i and i' (distant enough from the boundary of the network) we have that $|N_{\mathcal{E}}(i; \mathbf{g}, T)| = |N_{\mathcal{E}}(i'; \mathbf{g}, T)|$ and that each agent targeted directly or indirectly via i or i', has the same degree in the network $\mathbf{g}(T)$. This implies that the external agent is indifferent between targeting the message to i or i'. This observation depends on the assumed symmetry of the network. In general, the choice of the external agent to whom to target is not trivial, as the next proposition illustrates. Note that from Theorem 2 we know that maximizing the welfare of all players is equivalent to maximize the expected utility of an arbitrary player.

Proposition 2 *Consider a network* $\mathbf{g}$ *and a diffusion radius* T*. Welfare is maximized when the external agent targets the message to player* i^**, where player* i^* *solves:*

$$\min_{i \in \mathcal{I}(b,T,\mathbf{g})} \sum_{j \in N_{\mathcal{E}}(i;\mathbf{g},T)} \frac{1}{|N_j(\mathbf{g}, T)| + 4} + \sum_{j \in N \setminus N_{\mathcal{E}}(i;\mathbf{g},T)} \frac{1}{|N_j(\mathbf{g}, T)| + 3}, \tag{7}$$

where $\mathcal{I}(b, T, \mathbf{g}) = \{i \in N : \text{condition 6 holds}\}$

The optimal agent to target diffuses the message in a way to minimize the residual variance in the society (expression 4), subject to the condition that the external agent is able to communicate truthfully to that agent (i.e., the agent belongs to $\mathcal{I}$). Consequently, to determine who is the key player to target, the external agent considers the distribution of degrees of all the agents that receive the message via the targeted agent. In particular, expression 7 reveals two things. First, the welfare increases if we add players in the set $N_{\mathcal{E}}(i; \mathbf{g}, T)$. Hence, by targeting the message to a well connected agent welfare improves. This observation is akin to the first order stochastic property of welfare illustrated in the first part of Corollary 2.

Second, welfare improves if we exchange one player with high degree from the set of players who receive the message of the external agent (the set $N_{\mathcal{E}}(i; \mathbf{g}, T)$) with one player who has low degree and does not receive the message from the external player. In other words, welfare can be improved if the external agent targets the message to a player who diffuses information to poorly connected agents. This observation is akin to the second order stochastic dominance property of the welfare highlighted in the second part of Corollary 2. It then follows that an influential agent is one who spreads the message directly or indirectly to many other agents in the network, who, in turn, are poorly connected.

5 Communication Across Communities

This section offers a new perspective to the study of homophily and segregation in communities, by studying equilibrium information transmission within and across groups with different preferences. A form of homophily is already present in the seminal work of Crawford and Sobel (1982): information transmission is more truthful if the principal and the agent have similar preferences. The added value of our analysis is that, by studying strategic information transmission in a multi-player environment, we provide insights on how the size of a community affects the incentives to exchange information truthfully across communities. We first characterize equilibria for the case of private communication, and show that (i) there is less truthful exchange of information across large communities than across small communities; and (ii) communication may be asymmetric: from large communities to small ones. We then compare the welfare of equilibria under public communication and private communication, and explore intermediate modes of communication.

5.1 Private Communication: homophily and the size of communities

We study strategic communication between two communities.[16] The set of players is partitioned in two groups, N_1 and N_2, with size n_1 and n_2, respectively. Without loss of generality, we assume that $n_1 > n_2 \geq 1$. Each member of group 1 has a bias which is normalized to 0; players in group 2 have a bias $b > 0$. Players can send a message to every other player.

Let k_i be the in-degree of an arbitrary player in group i in a utility-maximizing equilibrium network. Then $k_i = k_{ii} + k_{ij}$, where k_{ii} is the number of truthful messages that a player of group i receives from members of the same group, whereas k_{ij} is the number of truthful messages that a player in group i receives from members of the opposite community. So k_{ii} measures the level of intra-group communication and k_{ij} measures the level of inter-group communication.

In the appendix we provide a full characterization of utility-maximizing equilibrium networks. Here, we focus on the natural subclass where there is complete intra-group communication.[17] Within this class, the only parameters that must be determined are the levels of inter-group communication. This allows us to parsimoniously describe communication across communities as follows. Since $k_{ii} = n_i - 1$ and since the in-degree within groups is the same across players, condition (3) in Corollary 2 implies that k_{ij} must satisfy

$$k_{ij} \leq \left\lfloor \frac{1}{2b} - n_i - 2 \right\rfloor,$$

where $\lfloor x \rfloor$ denotes the largest integer smaller than x. Furthermore, in view of Theorem 2, in a utility-maximizing equilibrium network k_{ij} will be the highest possible subject to the above equilibrium condition and that $k_{ij} \leq n_j$ and $k_{ij} \geq 0$. We can now state the following result.

[16]The main results we present in this section are robust to a setting with an arbitrary number of communities.

[17]First, these equilibria are robust to the introduction of infinitesimal group-sensitive preferences. For example, we can slightly modify the model so that the utility of every player l in group i is: $-(1+\epsilon)\sum_{l'\in N_i}(\hat{y}_{l'} - \theta - b_l)^2 - \sum_{l'\in N_j}(\hat{y}_{l'} - \theta - b_l)^2$, where ϵ is a small positive constant. Second, this class of utility-maximizing equilibria coincides with the set of utility-maximizing equilibria as long as the conflict of interest between the two groups is not too low. A formal result is in Appendix, see Proposition A.

Proposition 3 *In every utility-maximizing equilibrium network with complete intra-group communication, the levels of communication across communities are:*

$$k_{ij} = \max\left\{\min\left\{\left\lfloor \frac{1}{2b} - n_i - 2 \right\rfloor, n_j\right\}, 0\right\}, i, j = 1, 2, i \neq j.$$

1. *If* $b < \frac{1}{2(n+2)}$ *there is complete inter-group communication;*
2. *If* $b \in \left[\frac{1}{2(n+2)}, \frac{1}{2(n_2+3)}\right]$ *the level of truthful communication from group* j *to group* i*,* k_{ij}*, declines with the size of group* i*, and the level of communication from the large group* 1 *to the small group* 2 *is higher than the level of communication from group* 2 *to group* 1*;*
3. *If* $b > \frac{1}{2(n_2+3)}$*, there is not communication across communities.*

The proposition shows that as communities grow larger, inter-group communication declines. In particular, complete inter-group communication cannot be sustained if the population is large enough. Furthermore, communication across communities is more pervasive from large to small groups, than *vice-versa.* The intuition for this result relies on the congestion effect that is illustrated in Corollary 2. Members of larger groups access more information within their own group than member of smaller groups and this implies that communication across communities influence the choice of larger group members less than the choice of smaller group members. As a consequence, members of small groups have higher incentives to misreport information to members of larger groups, than vice-versa.

In a different model, Hagenbach and Koessler (2010) find a similar result: intergroup information transmission is higher for players in the larger group than for players in the smaller group. In our model this is driven by the congestion effect illustrated in Corollary 1. In their model, the result follows because, due to the need of players to coordinate their actions, the ability of a player to communicate truthfully increases with the number of other agents to whom he is already transmitting information. Therefore, as truthful communication within group is always equilibrium, an increase in the size of, say, community 1, relaxes the informational incentive constraint of group 1's members, thereby increasing communication from group 1 to group 2.

5.2 Comparison of Modes of Communication.

We now investigate the implication that different modes of communication have on equilibrium welfare. As a first step we compare the welfare generated by utility-maximizing equilibria under private communication and public communication, for different levels of conflict between communities. We then show that there are natural mixed modes of communication which outperform both private and public communication.

We start by comparing the players' ex-ante utility in utility-maximizing equilibria under public communication and private communication. We show that public communication outperforms private communication when the level of conflict between groups is moderate, whereas private communication outperforms public communication when the level of conflict between the two groups is high. For expositional simplicity, we assume that the two groups are of equal size, i.e., $n_1 = n_2 = n/2$. Let

$$\hat{b}(n) = \frac{2(n^3 + 9n^2 + 14n - 36)}{n(n^3 + 15n^2 + 74n + 120)}.$$

Proposition 4 *Suppose $n_1 = n_2 = n/2$.*

1. *If $b \leq \frac{1}{2(n+2)}$, then private and public communication utility-maximizing equilibria yield the same utility to all players;*

2. *If $b \in \left(\frac{1}{2(n+2)}, \hat{b}(n)\right]$, then public communication utility-maximizing equilibria outperform private communication utility-maximizing equilibria;*

3. *If $b > \hat{b}(n)$, then private communication utility-maximizing equilibria fare better than public communication utility-maximizing equilibria;*

If the conflict between the two communities is sufficiently small, regardless of the communication mode, all players truthfully communicate in the utility-maximizing equilibrium. As the level of conflict across groups increases, we know from Proposition 3 that eventually any player i has an incentive to misreport his information to a player j in the opposite community, under private communication. But if player i must send a public message, then, as detailed in Theorem 1, player i's incentives to misreport his signal depends on the difference between his own bias and the average bias of the population. This difference is clearly smaller than the difference between player i's bias and the bias of any player in the opposite community. That is, for intermediate levels of b, public communication allows to aggregate more information than private communication. Finally, as the conflict of interest becomes sufficiently large, public communication prevents truthful reporting, while under private communication players are always able to communicate truthfully within their own group.

In light of Theorem 1, when public communication fails, an intuitive way to restore some amount of equilibrium communication is to lower the average bias difference between the player and his audience, by reducing the number of players from the opposite community in the audience. This observation leads us to consider modes of communication where each player i's audience is composed of all the fellow players in his community, and of a subset of the players in the opposite community. Clearly, these modes of communication weakly improve welfare upon public communication. They also weakly improve upon private communication. In fact, even including in the audience all players from the opposite community that receive information privately from i, the average bias in the community is still closer to player i's bias than the bias of any player in the opposite community. The following is an example of such a mixed mode of communication.

Example. Suppose that $n_1 = n_2 = n/2$ and consider a communication mode where each player l in community i is required to send the same message to all the $n/2 - 1$ members of community i and to the players $l, l+1 \mod (n/2), ..., l+k \mod (n/2)$ in population j.[18] This defines a family of modes parameterized by k. Note that with this mode of communication the in-degree of all players in either community is the same, and equals $n/2 - 1 + k$. This is also the out-degree of any player, regardless of the community to which he belongs to. From

[18]Here, the notation $a + b \mod (c)$ stands for $a + b$ if $a + b \leq c$ and $a + b - c$ otherwise.

Theorem 1 we know that the condition for player i to truthfully communicate is:

$$2kb \leq \frac{n/2 - 1 + k}{n/2 + 2 + k}, \tag{8}$$

because $k_j = n/2 - 1 + k$ and there are k players with bias difference equal to b. Evidently, the maximal welfare is achieved by maximizing k. Formally this is obtained by taking the integer part of the positive root of the quadratic equation defined by inequality (8), that is

$$k^* = \max\left\{0, \min\left\{\left\lfloor \frac{1 - 4b - bn - \sqrt{1 - 16b + 2bn + 16b^2 + 8b^2 n + b^2 n^2}}{4b} \right\rfloor, n/2\right\}\right\}.$$

6 Information Aggregation: Private Meetings *vs.* Public Speeches

This section considers the information aggregation problem in a model with players with heterogeneous equidistant biases. We first characterize the set of private communication equilibria for a large class of economies. Then, by comparing the welfare of equilibria under private and public communication, we establish that public communication usually outperforms private communication in such an environment.

We believe our model naturally describes an environment in which there is a set of heterogenous ideological politicians whose choices affect each others. Each politician would like that all policy choices match his bliss point, which consists of the common state of the world plus his own idiosyncratic ideological bias. Our analysis proposes a distinct contribution to the political economy literature on information aggregation in committees that we quoted in the introduction. Unlike such models, we do not presume that individual choices are aggregated through voting. Instead, we let politicians retain control of their own policy choices, but study the effect of ideological externalities among the politicians' choices.

Formally, we assume that each player can communicate to all other individuals and players have equidistant bias, i.e., $b_{i+1} - b_i = \beta$, for all $i = 1, ..., n-1$. We first consider private communication. The next proposition precisely characterizes a class of utility-maximizing equilibrium networks, which for a wide range of the parameter β can be shown to coincide with the utility-maximizing equilibria.

Proposition 5 *Let* $V(\beta) = \left\lfloor \frac{1}{2} \min\left\{ \left(\frac{\sqrt{\beta+\beta^2}}{\beta} - 1 \right), n-1 \right\} \right\rfloor$. *For every* β*, under private communication there exists an utility-maximizing equilibrium such that:*

1. *Every player* $j \in \{V(\beta)+1, ..., n-V(\beta)\}$ *receives truthful information from* i *if* $|i-j| < V(\beta)$ *and from no players* i *such that* $|i-j| > V(\beta)$*;*

   – *if* $\beta > \frac{1}{2(2V(\beta)+3)V(\beta)}$*, then* j *receives truthful information from exactly one player* i *such that* $|i-j| = V(\beta)$*;*

   – *if* $\beta \leq \frac{1}{2(2V(\beta)+3)V(\beta)}$*, then* j *receives truthful information from both players* i *such that* $|i-j| = V(\beta)$*;*

2. *For all players* $j \in \{1, ..., V(\beta)\} \cup \{n-V(\beta)+1, ..., n\}$*,* j *receives truthful information from* i *if and only if*
   $|i-j| \leq M(j,\beta) \equiv \min\left\{ \left\lfloor \frac{\sqrt{2\beta+(3\beta+\beta\min\{j-1,n-j\})^2}-(3\beta+\beta\min\{j-1,n-j\})}{2\beta} \right\rfloor, n-1 \right\}.$

Figure 3 illustrates the equilibrium characterization obtained in Proposition 5 for the case of $n = 6$ and for different values of β. In the figure a solid line linking i and j signifies that i and j communicate truthfully with each other; a dashed line starting from i with an arrow pointing at j means that only player i truthfully communicates with j. The equilibria described in Proposition 5 are characterized by three properties; localization in bias differences, decentralization and asymmetric communication. Indeed, in Corollary 3 in the appendix it is formally shown that for every β, there exists an utility-maximizing equilibrium such that (i) each individual communicates only with his bias neighbors, and hence communication is localized; (ii) there are no major variations in the number of messages received by different players – and indeed the proportion of players with the same in-degree converges to one as n diverges to infinity, so that communication is decentralized; and (iii) communication may be asymmetric, from moderate bias to extreme bias players, but not vice-versa.[19]

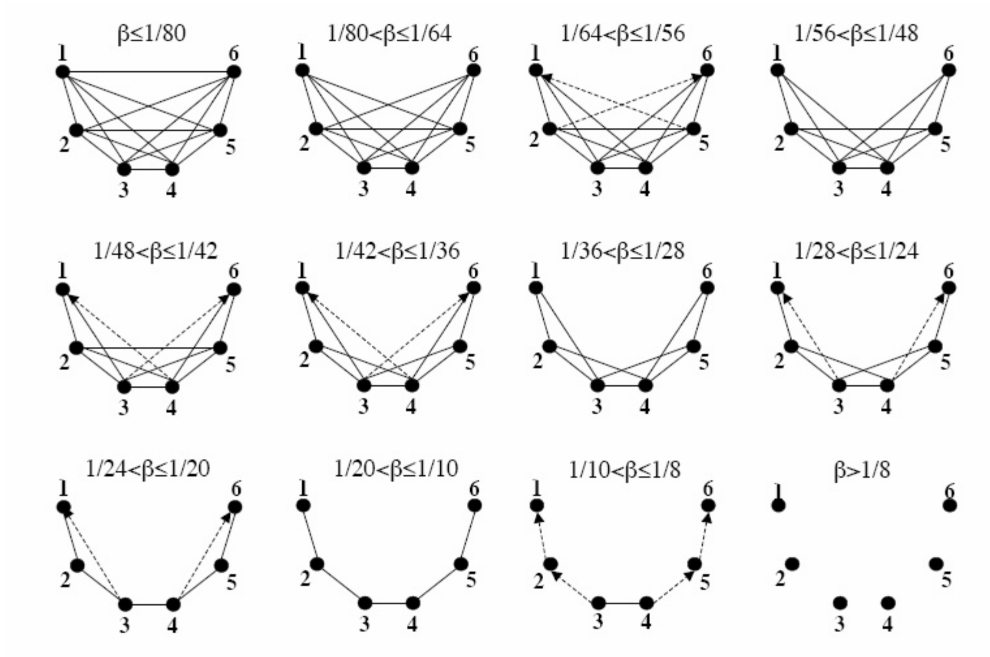

Figure 1: Equilibrium Networks in Proposition 5, $n = 6$.

We now describe a class of public communication utility-maximizing equilibria. In the definition, we make use of the following two functions. For any index $l = 1, ..., \lfloor n/2 \rfloor$, let

$$f(l,n) = \frac{\frac{n-2l+1}{2(n-2l+4)^2} + \frac{l-1}{(n-2l+5)^2}}{(n-2l+1)\left[\frac{n+2-2l}{2(n-2l+4)} + \frac{l-1}{n-2l+5}\right]}$$

and

$$g(l,n) = \frac{\frac{n-2l}{2(n-2l+3)^2} + \frac{2l-1}{2(n-2l+4)^2}}{(n-2l+1)\left[\frac{n-2l}{2(n-2l+3)} + \frac{l}{n-2l+4}\right]}.$$

For $l = 0$, the function $g(\cdot,\cdot)$ is defined by $g(0,n) = 0$ for all n.

Proposition 6 *Suppose that $b_{i+1} - b_i \equiv \beta$ for all players $i = 1, ..., n-1$, and that communication is public. There exists a class of utility-maximizing equilibria such that:*

[19] We note that also in Hagenbach and Koessler (2010) moderate bias players communicate more than extreme bias players. These results are of independent interest as they obtain from two different reasons: congestion effect in our case and the coordination motive in their case. Furthermore, the localization property of equilibrium network described above is novel.

1. *For any* $l = 1, ..., \lfloor n/2 \rfloor$,

   - *if* $g(l-1, n) < \beta \leq f(l, n)$ *the players who communicate truthfully are* $\{l, ..., n-l+1\}$;
   - *if* $f(l, n) < \beta \leq g(l, n)$ *the players who communicate truthfully are either* $\{l, ..., n-l\}$ *or* $\{l+1, ..., n-l+1\}$.

2. *If* $\beta > g(\lfloor n/2 \rfloor, n)$, *then no player truthfully communicates when* n *is even, otherwise player* $(n+1)/2$ *truthfully communicates.*

Proposition 6 fully characterizes this specific class of equilibria. The main interest of the result is that it allows to compare the two modes of communication. Indeed, because all utility-maximizing equilibria yield the same ex-ante utility to all players, the welfare comparison of private and public communication only requires calculating the welfare of the equilibria described in Proposition 5 and 6, through equation (4). Inspection of these results reveals the following trade-off. On the one hand, public communication may outperform private communication as a player's incentive to misreport a low signal in order to raise the action of lower bias players is tempered by the loss derived from the action increase by all higher bias players. But on the other hand, when a player fails to truthfully communicate publicly, he may still communicate with some players under private communication, as he can discriminate among his receivers and send a truthful message only to those with a closer bias.

This basic trade-off was early identified by Farrell and Gibbons (1989) in a cheap talk game with one sender and two receivers, where each receiver chooses between two actions. In our model, we can refine the insights by Farrell and Gibbons (1989), by identifying the sender's bias relative to the average bias as the key element to settle the welfare comparison between public and private communication. Specifically, we observe that, if there is only one sender i, he will truthfully communicate under public communication if and only if

$$\left| b_i - \sum_{j \in N \setminus \{i\}} \frac{b_j}{n} \right| = \beta \left| i - \frac{n+1}{2} \right| \leq \frac{1}{8}, \tag{9}$$

i.e., if and only if the sender's bias is sufficiently moderate relative to the other players' biases. Evidently, public communication weakly outperforms private communication if and only if the condition (9) holds.

This result suggests that, when all players are simultaneously senders and receivers of communication, it would be effective to make moderate bias players communicate publicly, and extreme bias players communicate privately. But the precise characterization of this intermediate mode of communication is in general complex because the equilibrium conditions for truthful communication to any audience depend on the information received in equilibrium by each player in the audience. Furthermore, this intermediate mode of communication may often not be viable, as it conditions each player's communication mode on his ideological bias, which is unknown behind a veil of ignorance. Hence, we propose a simpler and more robust normative analysis, for the comparison between public and private communication.

The analysis is numerical, and conducted for all $n \leq 500$. Figure 2 represents the case in which the number of players is even. The first graph represents the aggregate action precision $-\sum_{j\in N} \sigma_j^2 (\mathbf{m}, \mathbf{y})$, for any utility-maximizing equilibrium $(\mathbf{m}, \mathbf{y})$ under private and public communication, as a function of the bias β, for $n = 16$. Because the ex-ante utility of each player i induced by an equilibrium $(\mathbf{m}, \mathbf{y})$ is $-\sum_{j\in N} \left[(b_i - b_j) + \sigma_j^2 (\mathbf{m}, \mathbf{y})\right]$, evidently, all players are better off under equilibria with higher aggregate action precision $-\sigma_j^2 (\mathbf{m}, \mathbf{y})$. The second graph represents in white the regions of the bias β and number of agents n, for which utility-maximizing public communication equilibria Pareto dominate utility-maximizing private communication equilibria. The regions for which the two communication modes give the same welfare are represented in grey. Hence, we conclude that when the number of player is even, public communication always weakly Pareto dominates private communication, and that the pattern is consistent as the number of agents increases.

Figure 3 considers the case of an odd number of players. The first graph shows $-\sum_{j\in N} \sigma_j^2 (\mathbf{m}, \mathbf{y})$, for any utility-maximizing equilibrium $(\mathbf{m}, \mathbf{y})$ under private and public communication, as a function of the bias β, for $n = 15$. The second graph shows the comparison between public and private communication welfare as a function of β and n, for n odd. Here, for low and high values of β, public communication always Pareto dominates private communication. For intermediate values of β, private communication may occasionally outperform public communication, in the area represented in black. Hence, we conclude that, also in the case of odd number of players, public communication usually Pareto dominates private communication, and that, when this is not the case, the welfare difference between the two modes is small.

Figure 4 shows the comparison between public and private communication from an ex-ante perspective behind a veil of ignorance concerning the value of the bias difference and the individual bias allocation. From a normative viewpoint this notion is interesting as it allows to evaluate the relative performance of public vs. private communication independently of the specificity of the information aggregation problem. Inspired by the principle of insufficient reasoning, we conduct the ex-ante analysis with β uniformly distributed on $[0, 1/8]$.[20] The first graph represents, as a function of the number of agents n, assumed to be odd, the average of the aggregate action precision $-\frac{1}{n}\sum_{j\in N}\sigma_j^2(\mathbf{m}, \mathbf{y})$ of the utility-maximizing equilibrium $(\mathbf{m}, \mathbf{y})$ under private and public communication. The second graph depicts the analogous relationship when n is even. We find that, expect for $n = 3$ and $n = 5$, public communication dominates ex-ante, for $n = 5$ it is equivalent to private communication, and for $n = 3$ private communication strictly dominates.

In sum, having compared numerically the welfare of utility-maximizing equilibria under private and public communication for all values of β and for n ranging from 3 to 500, we have established the following result.

Result 1 *Suppose that* $b_{i+1} - b_i = \beta$ *for all players* $i = 1, ..., n-1$, *and* $n \leq 500$.

[20] For $\beta > 1/8$, public communication always Pareto dominates private communication. Indeed, in equilibrium, there is no truthful private communication, whereas player $(n+1)/2$ always truthfully publicly communicates. Therefore, any upper bound larger than 1/8 would be more favorable to public communication.

1. *When n is even, public communication always yields weakly higher welfare than private communication;*

2. *When n is odd, private communication may occasionally outperform public communication only for intermediate values of β; otherwise, it is dominated by public communication;*

3. *Public communication strictly dominates private communication for all $n > 5$ in ex-ante sense, when β is uniformly distributed on $[0, 1/8]$. For $n = 3$ private communication dominates, for $n = 5$ the two modes give the same ex-ante welfare, and for $n = 4$ public communication strictly dominates.*

7 Conclusion

This paper provides a tractable model to study multi-person environments where players can strategically transmit their private information to individuals who are connected to them in a communication network. The players' message may either be different for each linked player (private communication), or be common among them (public communication). The first basic insight that emerges from the analysis is that whether truthful communication can be sustained in equilibrium or not does not only depend on the conflict of interest between players. It also depends on the architecture of the communication network, and on the allocation of players within the network. In particular, under private communication, the willingness of a player to communicate with another individual decreases with the number of players communicating with the individual. Under public communication, the composition of biases of linked players determines whether a player is willing to communicate to them. The second basic insight is that, in a multi-person environment, equilibrium welfare does not only depend on the amount of information aggregated in the network, but also on how evenly truthful information transmission is distributed across players. We demonstrate the relevance of our basic results by addressing a number of theoretical economic questions: optimal targeting strategy in networks,

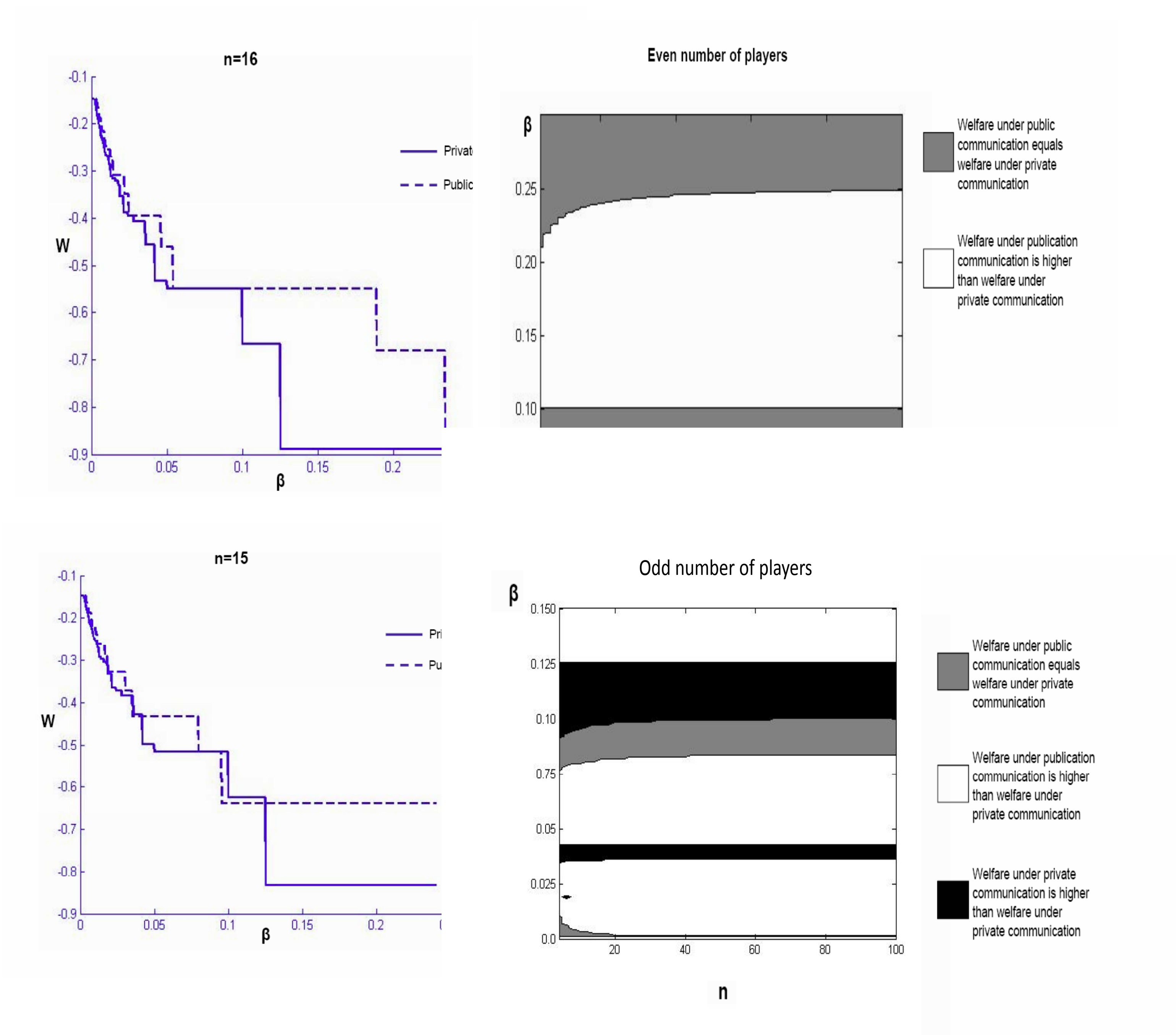

Figure 2: Welfare under Private Communication *vs.* Public Communication

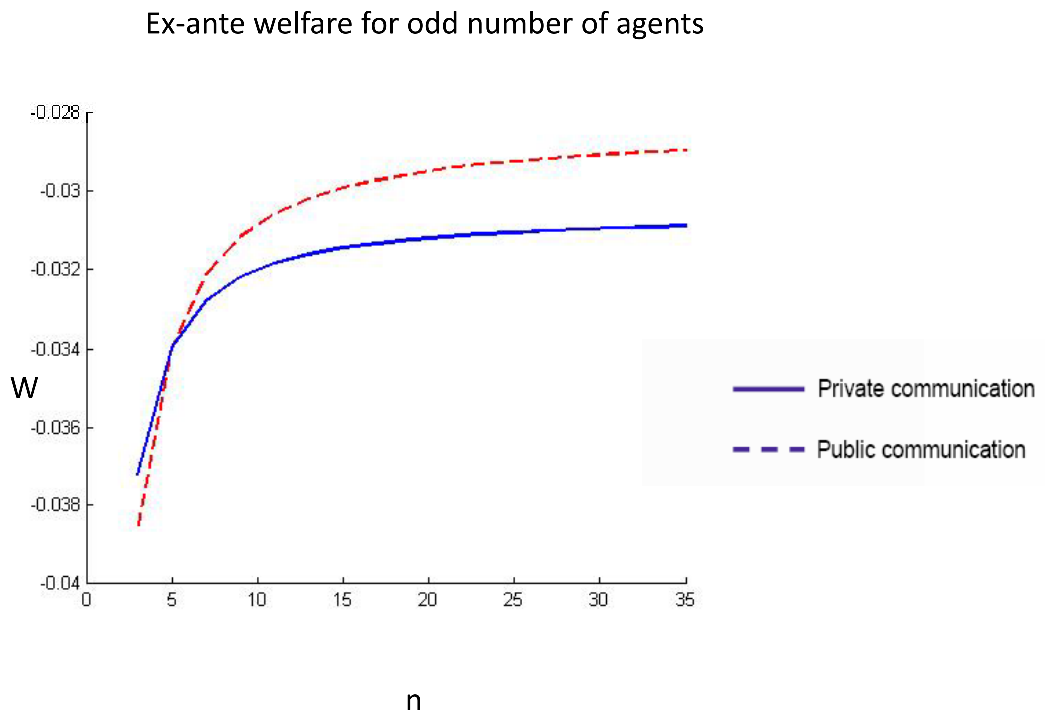

Figure 3: Ex-ante welfare

strategic communication between communities and the problem of information aggregation in intergovernmental organizations.

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Appendix

Proof of Theorem 1 Suppose that all agents in J believe that agent i reports his signal s truthfully. Let s_R be a vector containing the (truthful) signals that each j has received from his communication neighbors, i.e, from every $j' \in C_j(\mathbf{c}) \setminus \{i\}$, and his own signal. With some abuse of notation, we denote the in-degree of j in truthful network $\mathbf{c}$ by $k_j = |C_j(\mathbf{c})|$. Let also $y_{s_R,s}$ be the action that j would take if he has information s_R and player i has sent signal s; analogously, $y_{s_R,1-s}$ is the action that j would take if he has information s_R and player i has sent signal $1-s$. Agent i reports truthfully signal s to a collection of agents J if and only if

$$-\int_0^1 \sum_{j\in J} \sum_{s_R\in\{0,1\}^{k_j}} \left[(y_{s_R,s} - \theta - b_i)^2 - (y_{s_R,1-s} - \theta - b_i)^2\right] f(\theta, s_R|s)d\theta \geq 0,$$

and using the identity $a^2 - b^2 = (a-b)(a+b)$ we get:

$$-\int_0^1 \sum_{j\in J} \sum_{s_R\in\{0,1\}^{k_j}} \left[(y_{s_R,s} - y_{s_R,1-s})\,(\frac{y_{s_R,s} + y_{s_R,1-s}}{2} - (\theta + b_i))\right] f(\theta, s_R|s)d\theta \geq 0.$$

Next, observing that

$$y_{s_R,s} = E\left[\theta + b_j|s_R, s\right],$$

we obtain

$$\begin{aligned} &-\int_0^1 \sum_{j\in J} \sum_{s_R\in\{0,1\}^{k_j}} [(E[\theta + b_j|s_R, s] - E[\theta + b_j|s_R, 1-s]) \\ &\cdot \left(\frac{E[\theta + b_j|s_R, s] + E[\theta + b_j|s_R, 1-s]}{2} - (\theta + b_i)\right)\Bigg] f(\theta, s_R|s)d\theta \geq 0. \end{aligned}$$

Denote

$$\Delta\left(s_R, s\right) = \left(E\left[\theta|s_R, s\right] - E\left[\theta|s_R, 1-s\right]\right).$$

Observing that:

$$f(\theta, s_R|s) = f(\theta|s_R, s)P(s_R|s),$$

and simplifying, we get:

$$-\sum_{j\in J} \sum_{s_R\in\{0,1\}^{k_j}} \int_0^1 \left[\Delta\left(s_R, s\right)\left(\frac{E\left[\theta|s_R, s\right] + E\left[\theta|s_R, 1-s\right]}{2} + b_j - b_i - \theta\right)\right] f(\theta|s_R, s)P(s_R|s)d\theta \geq 0.$$

Furthermore,

$$\int_0^1 \theta f(\theta|s_R, s)d\theta = E[\theta|s_R, s],$$

and

$$\int_0^1 P(\theta|s_R, s)E[\theta|s_R, s]d\theta = E[\theta|s_R, s],$$

because $E[\theta|s_R, s]$ does not depend on θ. Therefore, we obtain:

$$\begin{aligned} & -\sum_{j\in J}\sum_{s_R\in\{0,1\}^{k_j}}\left[\Delta(s_R,s)\left(\frac{E[\theta|s_R,s]+E[\theta|s_R,1-s]}{2}+b_j-b_i-E[\theta|s_R,s]\right)\right]P(s_R|s) \\ = & -\sum_{j\in J}\sum_{s_R\in\{0,1\}^{k_j}}\left[\Delta(s_R,s)\left(-\frac{E[\theta|s_R,s]-E[\theta|s_R,1-s]}{2}+b_j-b_i\right)\right]P(s_R|s)\geq 0. \end{aligned}$$

Now, note that:

$$\begin{aligned} \Delta(s_R,s) &= E[\theta|s_R,s]-E[\theta|s_R,1-s] \\ &= E[\theta|l+s,k_j+1]-E[\theta|l+1-s,k_j+1] \\ &= (l+1+s)/(k_j+3)-(l+2-s)/(k_j+3) \\ &= \begin{cases} -1/(k_j+3) & \text{if } s=0 \\ 1/(k_j+3) & \text{if } s=1. \end{cases} \end{aligned}$$

where l is the number of digits equal to one in s_R. Hence, we obtain that agent i is willing to communicate to agent j the signal $s = 0$ if and only if:

$$-\sum_{j\in J}\left(\frac{-1}{k_j+3}\right)\left(-\frac{-1}{2(k_j+3)}+b_j-b_i\right)\geq 0,$$

or

$$\sum_{j\in J}\frac{b_j-b_i}{k_j+3}\geq -\sum_{j\in J}\frac{1}{2(k_j+3)^2}$$

Note that this condition is redundant if $\sum_{j\in J} b_j - b_i > 0$. On the other hand, she is willing to communicate to agent j the signal $s = 1$ if and only if:

$$-\sum_{j\in J}\left(\frac{1}{k_j+3}\right)\left(-\frac{1}{2(k_j+3)}+b_j-b_i\right)\geq 0,$$

or

$$\sum_{j\in J}\frac{b_j-b_i}{k_j+3}\leq\sum_{j\in J}\frac{1}{2(k_j+3)^2}.$$

Note that this condition is redundant if $\sum_{j\in J} b_j - b_i < 0$. Collecting the two conditions:

$$\left|\sum_{j\in J} \frac{b_j - b_i}{k_j + 3}\right| \leq \sum_{j\in J} \frac{1}{2\left(k_j+3\right)^2}.$$

This completes the proof of Theorem 1. ■

Proof of Corollary 1. Corollary 1 is a special case of theorem 1, in which for every $i \in N$ the partition $\mathcal{N}_i(\mathbf{g})$ of i's communication neighborhood, $N_i(\mathbf{g})$, is composed of singleton sets. ■

Proof of Theorem 2. Assume $(\mathbf{m}, \mathbf{y})$ is equilibrium in communication network $\mathbf{g}$. Select an arbitrary player i. The ex-ante expected utility of i is:

$$\begin{aligned} EU_i(\mathbf{m}, \mathbf{y}) &= -E\left[\sum_{j=1}^{n} (y_j - \theta - b_i)^2 |\{0,1\}^{k_j(\mathbf{c}))+1}\right] \quad (10)\\ &= -\sum_{j=1}^{n} E\left[(y_j - \theta - b_i)^2 |\{0,1\}^{k_j(\mathbf{c})+1}\right], \quad (11)\end{aligned}$$

where, with some abuse of notation, $k_j(\mathbf{c})$ indicates j's in-degree in truthful network $\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})$.

Consider an arbitrary j with in-degree $k_j(\mathbf{c})$ and let l be the number of digits equal to one in a realized information vector $\{0,1\}^{k_j(\mathbf{c})+1}$. Then, we obtain:

$$\begin{aligned} E\left[(y_j - \theta - b_i)^2 |\{0,1\}^{k_j(\mathbf{c})+1}\right] &= \int_0^1 \sum_{l=0}^{k_j(\mathbf{c})+1} \left(E\left[\theta|l, k_j(\mathbf{c})+1\right] + b_j - \theta - b_i\right)^2 f(l|k_j(\mathbf{c})+1, \theta) d\theta \\ &= \int_0^1 \sum_{l=0}^{k_j(\mathbf{c})+1} \left(E\left[\theta|l, k_j(\mathbf{c})+1\right] + b_j - \theta - b_i\right)^2 \frac{f\left(\theta|l, k_j(\mathbf{c})+1\right)}{k_j(\mathbf{c})+1+1} d\theta, \end{aligned}$$

where the second equality follows from $f(l|k_j(\mathbf{c})+1, \theta) = f(\theta|l, k_j(\mathbf{c})+1)/(k_j(\mathbf{c})+2)$. Let

$\Pi = \left(E\left[\theta|l, k_j(\mathbf{c})+1\right] - \theta\right)^2$. Then we have:

$$
\begin{aligned}
& E\left[(y_j - \theta - b_i)^2 \left| \{0,1\}^{k_j(\mathbf{c})+1}\right.\right] \\
= \quad & \frac{1}{k_j(\mathbf{c})+2} \int_0^1 \sum_{l=0}^{k_j(\mathbf{c})+1} \left(\Pi + (b_j - b_i)^2 + 2(b_j - b_i)\left(E\left[\theta|l, k_j(\mathbf{c})+1\right] - \theta\right)\right) f\left(\theta|l, k_j(\mathbf{c})+1\right) d\theta \\
= \quad & (b_j - b_i)^2 + \frac{1}{k_j(\mathbf{c})+2}\left[\int_0^1 \sum_{l=0}^{k_j(\mathbf{c})+1} \left(\Pi + 2(b_j - b_i)\left(E\left[\theta|l, k_j(\mathbf{c})+1\right] - \theta\right)\right) f\left(\theta|l, k_j(\mathbf{c})+1\right) d\theta\right] \\
= \quad & (b_j - b_i)^2 + \frac{1}{k_j(\mathbf{c})+2}\left[\sum_{l=0}^{k_j(\mathbf{c})+1} \left(\int_0^1 \left(E\left[\theta|l, k_j(\mathbf{c})+1\right] - \theta\right)^2 f\left(\theta|l, k_j(\mathbf{c})+1\right) d\theta\right)\right].
\end{aligned}
$$

Next, let $V(\theta|l,k)$ be the variance of a beta distribution of parameters l and k, i.e.,

$$V(\theta|l,k) = \int_0^1 (E[\theta|l,k] - \theta)^2 f(\theta|l,k) d\theta.$$

It is well known that:

$$V(\theta|l,k) = \frac{(l+1)(k-l+1)}{(k+2)^2(k+3)}.$$

Hence,

$$
\begin{aligned}
E\left[(y_j - \theta - b_i)^2 \left| \{0,1\}^{k_j(\mathbf{c})+1}\right.\right] &= (b_j - b_i)^2 + \frac{1}{k_j(\mathbf{c})+2}\left[\sum_{l=0}^{k_j(\mathbf{c})+1} V\left(\theta|l, k_j(\mathbf{c})+1\right)\right] \\
&= (b_j - b_i)^2 + \sum_{l=0}^{k_j(\mathbf{c})+1} \frac{(l+1)(k_j(\mathbf{c}) - l + 2)}{(k_j(\mathbf{c})+2)(k_j(\mathbf{c})+3)^2(k_j(\mathbf{c})+4)} \\
&= (b_j - b_i)^2 + \frac{1}{6(k_j(\mathbf{c})+3)}.
\end{aligned}
$$

We can then write the ex-ante expected utility of player i in equilibrium $(\mathbf{m}, \mathbf{y})$ as follows:

$$
\begin{aligned}
EU_i(\mathbf{m}, \mathbf{y}) &= -\sum_{j=1}^{n}\left[(b_j - b_i)^2 + \frac{1}{6(k_j(\mathbf{c})+3)}\right] \\
&= -\sum_{j=1}^{n}(b_j - b_i)^2 - \frac{1}{6}\sum_{j=1}^{n}\frac{1}{k_j(\mathbf{c})+3} \\
&= -\sum_{j=1}^{n}(b_j - b_i)^2 - \frac{1}{6}\sum_{k=0}^{n-1}\frac{|I(k|\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g}))|}{k+3},
\end{aligned}
$$

where $|I(k|\mathbf{c}(\mathbf{m},\mathbf{y}|\mathbf{g}))|$ is the set of players with in-degree k, i.e., $I(k|\mathbf{c}(\mathbf{m},\mathbf{y}|\mathbf{g})) = \{i \in N : k_i(\mathbf{c}(\mathbf{m},\mathbf{y}|\mathbf{g})) = k\}$. Therefore,

$$EU_i(\mathbf{m},\mathbf{y}) \geq EU_i(\mathbf{m}',\mathbf{y}')$$

if and only if:

$$\sum_{k=0}^{n-1} \frac{|I(k|\mathbf{c}(\mathbf{m},\mathbf{y}|\mathbf{g}))|}{k+3} \leq \sum_{k=0}^{n-1} \frac{|I(k|\mathbf{c}'(\mathbf{m}',\ \mathbf{y}'|\mathbf{g}'))|}{k+3},$$

which is equivalent to

$$\sum_{k=0}^{n-1} P(k|\mathbf{c}(\mathbf{m},\mathbf{y}|\mathbf{g}))\frac{1}{k+3} \leq \sum_{k=0}^{n-1} P(k|\mathbf{c}'(\mathbf{m}',\mathbf{y}'|\mathbf{g}'))\frac{1}{k+3}.$$

This concludes the proof of Theorem 2. ■

Proof of Corollary 2. The proof of Corollary 2 follows from standard arguments of stochastic dominance and therefore the details are omitted. ■

Proof of Proposition 3. We fist need to characterize all utility-maximizing equilibria in the two communities case developed in Section 4.1. This is done in Proposition A below.

However, before stating the result we need to introduce some definitions. A $k^1 \times k^2$-network is a network where k^x is the in-degree of players in group x, $x = 1, 2$. A segregated network is a $(n_1 - 1) \times (n_2 - 1)$-network with no links across communities. A partially segregated network is a $(n_1 - 1) \times k^2$-network where there are no links going from players in community 2 to players in community 1 and there are some links going from community 1 to community 2, i.e., $k^2 \in \{n_2, ..., n_1 - 1\}$. A complete network is a $(n - 1) \times (n - 1)$-network.

Proposition A. Consider the two-communities model.

I. The complete network is a utility-maximizing equilibrium network if and only if $b \leq \frac{1}{2(n+2)}$;

II. A $k \times k$–network with $k \in \{n_1, ..., n - 2\}$ is a utility-maximizing equilibrium if and only if $b \in \left(\frac{1}{2(k+4)}, \frac{1}{2(k+3)}\right]$;

III. A partially segregated network with $k^2 \in \{n_2, ..., n_1 - 1\}$ is a utility-maximizing equilibrium network if and only if $b \in \left(\frac{1}{2(k+4)}, \frac{1}{2(k+3)}\right]$;

IV. A segregated network is a utility-maximizing equilibrium network if and only if $b > \frac{1}{2(n_2+3)}$.

Proof of Proposition A. We first need to show the following Lemma.

Lemma 1 *Suppose* $\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})$ *is a utility-maximizing equilibrium network (UMEN). Then,* $\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})$ *is a* $k^1 \times k^2$*-communication network, i.e., all players in a group have same degree and this is larger or equal than the size of the group minus one.*

Proof of Lemma 1. With some abuse of notation we indicate a UMEN $\mathbf{c}(\mathbf{m}, \mathbf{y}|\mathbf{g})$ as to $\mathbf{c}$, and the in-degree of a player i in $\mathbf{c}$ as to k_i. Let $M(\mathbf{c})$ be the number of (directed) links in $\mathbf{c}$, i.e., the total number of truthful communications. First, note that the segregate communication network, $\mathbf{c}^s$, is always equilibrium and that $M(\mathbf{c}^s) = n_1(n_1 - 1) + n_2(n_2 - 1)$. Since $\mathbf{c}$ is a UMEN equilibrium, and each player in $\mathbf{c}^\mathbf{s}$ has the same in-degree, Theorem 2 implies that $M(\mathbf{c}) \geq M(\mathbf{c}^s)$. We now divide the analysis in two parts.

Part A. If $b > \frac{1}{2(n_2+3)}$ then $\mathbf{c} = \mathbf{c}^s$. To see this, suppose, for a contradiction, that $\mathbf{c} \neq \mathbf{c}^s$. Let $I_{12} = \{i \in N_1 : c_{ji} = 1 \text{ for some } j \in N_2\}$ and $I_{21} = \{j \in N_2 : c_{ij} = 1 \text{ for some } i \in N_1\}$. If $|I_{12}| = |I_{21}| = 0$, then, since $\mathbf{c} \neq \mathbf{c}^s$, $M(\mathbf{c}) < M(\mathbf{c}^s)$, a contradiction.

Next, assume that $|I_{12}| \neq 0$ and that $|I_{21}| \neq 0$. Since $\mathbf{c}$ is a UMEN equilibrium, it cannot be the case that $k_i < n_1 - 1$, for all $i \in I_{12}$ and $k_j < n_2 - 1$ for all $j \in I_{21}$; for otherwise $M(\mathbf{c}) < M(\mathbf{c}^s)$. Note also that for all $i \in I_{12}$ it must be the case that $k_i < n_1 - 1$. Indeed, if it exists some $i \in I_{12}$ with $k_i \geq n_1 - 1$, then, since $\mathbf{c}$ is equilibrium, it must hold that $b(k_i + 3) \leq 1/2$, which contradicts our initial hypothesis that $b(n_2 + 3) > 1/2$, because $k_i + 3 \geq n_1 - 1 + 3 \geq n_2 + 3$. These two observations imply that there must exist $j \in I_{21}$ such that $k_j \geq n_2 - 1$. Furthermore, if all these players like j have $k_j = n_2 - 1$, then $M(\mathbf{c}) < M(\mathbf{c}^s)$. So, there exists $j \in I_{21}$ such that $k_j > n_2 - 1$. In such a case, equilibrium implies that $b[k_j + 3] \leq 1/2$. But, since $k_j + 3 \geq n_2 + 3$, this contradicts our initial hypothesis that $b[n_2 + 3] > 1/2$.

Hence, it must be the case that either $|I_{12}| \neq 0$ and $|I_{21}| = 0$ or $|I_{12}| = 0$ and $|I_{21}| \neq 0$. Each of these two cases can be ruled out using the same arguments adopted for the case in which $|I_{12}| \neq 0$ and $|I_{21}| \neq 0$; details are omitted. This completes the proof of part A.

Part B. Suppose that $b(n_2 + 3) < 1/2$. We first prove that each player in group 2 must have the same in-degree, i.e., $k_i = k^2$ for all $i \in N_2$. Given $\mathbf{c}$, without loss of generality, all players in group 2 are ordered according to their in-degrees, i.e., $k_1 \leq k_2 \leq .... \leq k_{n_2}$. Assume, for a contradiction, that $k_1 < k_{n_2}$. We consider three sub-cases.

Part B, Case 1. Suppose $k_{n_2} > n_2 - 1$. This implies that $c_{jn_2} = 1$ for some $j \in N_1$, and since $\mathbf{c}$ is equilibrium, it must hold that $b[k_{n_2} + 3] \leq 1/2$. Next, since $k_1 < k_{n_2}$, it must exist a $j \in N$ such that $c_{j1} = 0$. But then the network $\mathbf{c}' = \mathbf{c} + c_{j1}$ is also equilibrium. In fact, every agent communicating in $\mathbf{c}$ with a player different from player 1 can still communicate in $\mathbf{c}'$, because the in-degrees of these players have not changed, and every agent l that was communicating with 1 in $\mathbf{c}$ still communicates in $\mathbf{c}'$ because $k_1(\mathbf{c}') = k_1(\mathbf{c} + 1) \leq k_{n_2}$ and $b[k_{n_2} + 3] \leq 1/2$. But then $\mathbf{c}$ is a subgraph of $\mathbf{c}'$, which, in view of Theorem 2, contradicts our initial hypothesis that $\mathbf{c}$ is a UMEN.

Part B, Case 2. Suppose $k_{n_2} = n_2 - 1$. We first note that $c_{jn_2} = 0$ for all $j \in N_1$; otherwise, we can replicate the argument developed in Part A, Case 1 to show a contradiction. Next, let player $l \in N_2$ such that $k_l < k_{n_2}$ and $k_{l+1} = k_{n_2}$. Note that for all $l' \in N_2$ with $l' \leq l$, there must exist some $j \in N_1$ such that $c_{jl'} = 1$. Indeed, if there exists a $l' \in N_2$ with $l' \leq l$, such that $c_{jl'} = 0$ for all $j \in N_1$, then, since $k_{l'} < n_2 - 1$, there exists a $i \in N_2$ such that $c_{il'} = 0$. But then, the network $\mathbf{c}' = \mathbf{c} + c_{il'}$ is also equilibrium, and in view of Theorem 2, this contradicts that $\mathbf{c}$ is a UMEN.

Now, for an arbitrary $l' \in N_2$ with $l' \leq l$, define $A(l')$ as the number of links that l' receives from players in group N_1. Define also $W(l')$ as the number of links that l' receives from players in group N_2. Then, the number of players in group N_2 who do not communicate with l' is

$$\bar{W}(l') = n_2 - 1 - k_{l'} + A(l') > A(l').$$

From network $\mathbf{c}$, construct $\mathbf{c}'$ in the following way: one, delete all links from group 1 to players $l' \in N_2$, with $l' \leq l$, and, two, for each $j \in N_2$ such that $c_{jl'} = 0$, $l' \in N_2$, $l' \leq l$, set $c'_{jl'} = 1$. Note that since $\mathbf{c}$ is equilibrium, then $\mathbf{c}'$ is also equilibrium, because each of the new links in $\mathbf{c}'$ are between members of the same community. Note also that

$$M(\mathbf{c}') - M(\mathbf{c}) = \sum_{l'<l, l' \in N_2} \left(\bar{W}(l') - A(l')\right) > 0,$$

and, by construction, the in-degree distribution in $\mathbf{c}'$ first order stochastic dominates the in-degree distribution of $\mathbf{c}$. Corollary 2 then implies that $\mathbf{c}'$ Pareto dominates $\mathbf{c}$, which contradicts that $\mathbf{c}$ is a UMEN equilibrium.

The final case in which $k_{n_2} < n_2 - 1$ is easy to rule out and details are omitted. We have shown that players in group 2 must have the same in-degree. The arguments developed here, can then be used to show that all players in group 1 must have the same in-degree. This concludes the proof of Lemma 1. ■

Finally, note that part IV of Proposition A follows from the proof of Part A of Lemma 1. Parts I-III of Proposition A simply follows by comparing the total number of links that can be sustained in $k^1 \times k^2$-communication network equilibrium. This concludes the proof of Proposition A. ■

The proof of Proposition 1 simply follows from Proposition A and the details are omitted. ■

Proof of Proposition 4. To proof Proposition 4 we first need a preliminary result which is summarized in Proposition B. Let

$$f(\alpha, b) = \frac{n(\alpha+2)^2 + \alpha^2 - 9 - \alpha}{2b(\alpha+2)(\alpha+3)}$$

and $\Delta(\alpha) = 2f(\alpha, b) - \alpha - n(\alpha + 2)$.

Proposition B. Consider the model with two groups, suppose that $n_1 = n_2 = n/2$ and that communication is public. If $b < \frac{4n-9}{12n}$ then in every utility-maximizing equilibrium the total number of players who communicate truthfully is $\alpha^* = \min\{\lfloor\hat{\alpha}\rfloor, n\}$, where $\hat{\alpha}$ solves $\Delta(\hat{\alpha}) = 0$. If $b \geq \frac{4n-9}{12n}$ then in every utility-maximizing equilibria there is no communication.

Proof Proposition B. Let α_i be the number of players communicating truthfully in group i. Theorem 1 implies that for (α_1, α_2) to be equilibrium we must have $\alpha_i \leq f(\alpha, b) - (\alpha_i + 1)n/2$, where $\alpha = \alpha_1 + \alpha_2$. Now suppose that (α_1, α_2) and (α_1', α_2') are both equilibria. We first note that if $\alpha_1 + \alpha_2 = \alpha_1' + \alpha_2'$ then the welfare in the two equilibria is the same. This follows by Theorem 2 and by noticing that the in-degree distribution of an equilibrium (α_1, α_2) only depends on $\alpha_1 + \alpha_2$. Furthermore, it is easy to check that if $\alpha_1 + \alpha_2 > \alpha_1' + \alpha_2'$ then the in-degree distribution of equilibrium (α_1, α_2) first order stochastic dominates the in-degree distribution of equilibrium (α_1', α_2'). In view of Theorem 2 this implies that the welfare of equilibrium (α_1, α_2) is higher than the welfare of equilibrium (α_1', α_2').

These three observations together imply that an equilibrium (α_1^*, α_2^*) is utility-maximizing if and only if (α_1^*, α_2^*) is the solution of the following problem PR1: $\max \alpha_1 + \alpha_2$ s.t. $\alpha_i \leq f(\alpha, b) - (\alpha_i + 1)n/2$, $i = 1, 2$. Consider now the problem PR2: $\max_{\alpha \in \{0 \ldots n\}} \alpha$ s.t. $\Delta(\alpha) \geq 0$ and let α^* be the solution to PR2. Define (α_1^*, α_2^*) as $\alpha_1^* = \lfloor \frac{\alpha^*}{2} \rfloor$ and $\alpha_2^* = \alpha - \alpha_1^*$. Note that by construction α_i^* satisfies the constraint of PR1, i.e., the constraint $\alpha_i \leq f(\alpha, b) - (\alpha_i + 1)n/2$. Furthermore, α^* has been obtained by maximizing the same objective of PR1, but with one less constraint. Hence, the constructed (α_1^*, α_2^*) are the solution to PR1.

We can therefore focus PR2. In particular we observe that: 1. $\lim_{\alpha \to \infty} \Delta(\alpha) = -\infty$, 2. $\Delta(\alpha)$ is concave, because it is a sum of concave functions, 3. $\Delta(0) > 0$ if and only if $b < (4n - 9)/24n$. Properties 1,2,3 together imply the first part of Proposition B. Finally, note that if $b > (4n - 9)/24n$ then $\Delta(\alpha) < 0$ for all $\alpha \geq 0$, which implies the second part of Proposition B. This concludes the proof of Proposition B. ∎

We now complete the proof of Proposition 4. First consider all $b \leq \frac{1}{2(n+2)}$; it is easy to verify that both under private and public communication in the utility-maximizing equilibrium all players truthfully communicate with all other players and therefore the welfare is the same (see Proposition 1 and Proposition B). Suppose now that $b \in (\frac{1}{2(n+2)}, \frac{1}{n+6})$, then under private communication all players have $\bar{k} \in (n/2, ..., n-2)$ links; under public communication all players have in-degree $n-1$. So, public communication dominates private communication. The same argument applies if $b \in (\frac{1}{n+6}, \frac{n-1}{n(n+2)})$. So consider $b > \frac{n-1}{n(n+2)}$. The in-degree distribution under private communication is $P(n/2) = 1$, i.e., all players have in-degree $n/2$; under public communication we have $P(\alpha^*) = \frac{n-\alpha^*}{n}$ and $P(\alpha^* - 1) = \frac{\alpha^*}{n}$. So, public communication dominates private communication if and only if $\alpha^* - 1 \geq n/2$, which requires that $\Delta(\frac{n}{2}+1) \geq 0$, which is equivalent to $b \leq \hat{b}$. This concludes the proof of Proposition 4. ■

Proof of Proposition 5. The proof proceeds in two steps. The first step of the proof shows that a strategy profile $(\mathbf{m}, \mathbf{y})$ that satisfies the conditions in Proposition 5 is equilibrium. The second step shows the set of these equilibrium strategies is a subset of the set of utility-maximizing equilibria.

Step I. Let $(\mathbf{m}, \mathbf{y})$ be a strategy profile that satisfies the conditions in Proposition 5. Note that, for all β, $V(\beta) = \max\{V \in N : \beta \leq \frac{1}{2V(2V-1+3)}\}$ and $M(\beta, j) = \max\{M \in N : \beta \leq \frac{1}{2(\min[j-1, n-j]+M+3)M}\}$. We show that $(\mathbf{m}, \mathbf{y})$ is an equilibrium. Select $l = 1, ..., 2j-n-1$. When $\beta \leq [2(j-l)(n-l+3)]^{-1}$, the strategy profile such that the $n-l$ players $\{l, ..., j-1, j+1, ..., n\}$ truthfully communicate with j is part of an equilibrium. Indeed, as $j - l > n - j$, i.e., $l < 2j - n$, it follows that

$$\{l\} = \arg \max_{i \in \{l,...,j-1,j+1,...,n\}} |b_i - b_j| \text{ and that } j - l = \max_{i \in \{l,...,j-1,j+1,...,n\}} |b_i - b_j|,$$

and Theorem 1 implies that the requirement for the strategy profile to be an equilibrium is exactly $\beta \leq [2(j-l)(n-l+3)]^{-1}$.

Next, select $l = 2j - n, ...j - 1$. For $\beta \leq [2(j-l)(2(j-l)+3)]^{-1}$ the profile such that the $2(j-l)$ players who truthfully communicate with j are $\{l, ..., j-1, j+1, 2j-l\}$ or $\{l+1, ..., j-1, j+1, 2j-l+1\}$ is part of an equilibrium. Indeed, suppose the players who truthfully communicate with j are $\{l, ..., j-1, j+1, 2j-l\}$ (the other case being symmetric). As $j - l = (2j - l) - j$ it follows that:

$$\{l, 2j-l\} = \arg \max_{i \in \{l,...,j-1,j+1,...,2j-l\}} |b_i - b_j| \text{ and that } j - l = \max_{i \in \{l,...,j-1,j+1,...,2j-l\}} |b_i - b_j|,$$

and Theorem 1 implies that the requirement for the profile to be an equilibrium is exactly $\beta \leq [2(j-l)(2(j-l)+3)]^{-1}$.

To conclude the first step, note that for $\beta \leq [2(j-l)(2(j-l)-1+3)]^{-1}$, the profile such that the $2(j-l)-1$ players who truthfully communicate with player j are $\{l+1, ..., j-1, j+1, 2j-l\}$ is part of an equilibrium. Indeed, as $j-l = (2j-l)-j$ it follows that:

$$\{2j-l\} = \arg \max_{i \in \{l+1,...,j-1,j+1,...,2j-l\}} |b_i - b_j| \text{ and that } j - l = \max_{i \in \{l+1,...,j-1,j+1,...,2j-l\}} |b_i - b_j|,$$

and theorem 1implies that the requirement for the strategy profile to be an equilibrium is exactly $\beta \leq [2(j-l)(2(j-l)-1+3)]^{-1}$.

Step II. Suppose that $(\mathbf{m}, \mathbf{y})$ belongs to the set of equilibrium strategy profiles considered in Step I above. We now show that this strategy profile is such that $\mathbf{m}$ is a utility-maximizing equilibrium. We start by noting that for every $l = 1, ..., 2j-n-1$ if a set of players C_j communicates with j and $|C_j| = n-l$, then $\beta \leq [2(j-l)(n-l+3)]^{-1}$. Indeed, since $n-l$ players communicate with j, there must be a player $i \in C_j$ such that $i \leq l$, and the equilibrium condition for player i to communicate with j is:

$$\beta \leq [2(j-i)(n-l+3)]^{-1} \leq [2(j-l)(n-l+3)]^{-1},$$

where the inequality follows because $i \leq l$. Because $[2(j-l)(n-l+3)]^{-1}$ increases in l, it follows that if a set of players C_j truthfully communicates with j and $|C_j| = n-v \geq n-l$, then $\beta \leq [2(j-v)(n-v+3)]^{-1} \leq [2(j-l)(n-l+3)]^{-1}$. Hence, for every $l = 1, ..., 2j-n-1$, if $\beta > [2(j-l)(n-l+3)]^{-1}$, then there is no equilibrium where $n-l$ players truthfully communicate to j. So, the proposed profile where $n-l-1$ players truthfully communicate to j, achieves the maximal number of communication links to player j and it is part of a utility-maximizing equilibrium.

We now turn to the case of $l = 2j - n, ..., j - 1$, and show equilibrium communication by $2(j - l)$ players to j requires that $\beta \leq [2(j-l)(2(j-l)+3)]^{-1}$. To see this, suppose that a set C_j of $2(j - l)$ players communicate with j. Then, there must be a player $i \in C_j$ such that $|j - i| \geq j - l$. Consequently, the equilibrium condition for player i to communicate with j is:

$$\beta \leq [2(j-i)(2(j-l)+3)]^{-1} \leq [2(j-l)(2(j-l)+3)]^{-1}.$$

Because $[2(j-l)(2(j-l)+3)]^{-1} < [2(j-l)(2(j-l)-1+3)]^{-1}$ holds for all l and the fact that $[2(j-l)(2(j-l)+3)]^{-1}$ increases in l, we can conclude that communication by at least $2(j-l)$ players to j requires $\beta \leq [2(j-l)(2(j-l)+3)]^{-1}$. Hence, for $\beta > [2(j-l)(2(j-l)+3)]^{-1}$, the specified strategy profile where $2(j-l)-1$ players communicate with j, is part of a utility maximizing equilibrium.

To conclude this second step we need to show that equilibrium communication by $2(j-l)-1$ players to j requires that $\beta \leq [2(j-l)(2(j-l)-1+3)]^{-1}$. Indeed, if a set C_j of $2(j-l)-1$ players communicates with j, then, there must be a player $i \in C_j$ such that $|j - i| \geq j - l$. Then, the equilibrium condition for player i to communicate with j is:

$$\beta \leq [2(j-i)(2(j-l)-1+3)]^{-1} \leq [2(j-l)(2(j-l)-1+3)]^{-1}.$$

Because $[2(j-l)(2(j-l)-1+3)]^{-1} < [2(j-(l+1))(2(j-(l+1))+3)]^{-1}$ holds and the fact that $[2(j-l)(2(j-l)-1+3)]^{-1}$ increases in l, we can conclude that communication by $2(j-l)-1$ players with j requires $\beta \leq [2(j-l)(2(j-l)-1+3)]^{-1}$. Hence, for $\beta > [2(j-l)(2(j-l)-1+3)]^{-1}$, the specified strategy profile where $2(j-(l-1))$ players communicate with j, is part of a utility-maximizing equilibrium. This concludes the proof of Proposition 5. ■

Corollary 3 *For every β, there exists an utility-maximizing equilibrium with the following properties:*

1. Localization. *If a player i communicates with j, then so do all players l such that $|l - j| < |i - j|$;*

2. Decentralization. *Every player* $j \in \{V(\beta)+1, ..., n-V(\beta)\}$ *has the same in-degree, every player* $j \in \{1, ..., V(\beta)\} \cup \{n - V(\beta) + 1, ..., n\}$ *has in-degree* $\min[j-1, n-j] + M(j, \beta)$; *hence the proportion of players with the same in-degree converges to one as* $n \to +\infty$;

3. Asymmetric Communication. *For every* $i < j \leq \lfloor \frac{n+1}{2} \rfloor$ *or* $i > j \geq \lfloor \frac{n+1}{2} \rfloor$, *it can be the case that* j *truthfully communicates with* i *and yet* i *does not truthfully communicates with* j, *but not vice-versa.*

Proof of Corollary 3. The first and third part of the corollary is obvious. We prove the second part. For all β, we show that $M(j, \beta)$ is weakly decreasing in j for $j \in \{1, ..., V(\beta)\}$. In fact, solving $2\beta(j - 1 + M + 3)M - 1 = 0$, we obtain that $M(j, \beta) = \left\lfloor \frac{1}{2}\left(-(j+2) + \sqrt{2/\beta + (j+2)^2}\right)\right\rfloor$ and that

$$\frac{dM(j, \beta)}{dj} = \frac{1}{2}\left(\frac{j+2}{\sqrt{2/\beta + (j+2)^2}} - 1\right) < 0$$

Consequently, $d(j)$ decreases in j. Furthermore, j's in-degree is equal to $j - 1 + M(j, \beta)$, and it is easy to check that it increases in j. Also, we note that, by construction, $M(V(\beta), \beta) = V(\beta)$. Hence, when $\beta > \frac{1}{2(2V(\beta)+3)V(\beta)}$, j's in-degree increases in j until reaching $2V(\beta) - 1$ for $j = V(\beta)$. On the other hand, when $\beta \leq \frac{1}{2(2V(\beta)+3)V(\beta)}$, j's in-degree increases in j until reaching $2V(\beta)$ for $j = V(\beta) + 1$ and then stays constant. ■

Proof of Proposition 6 The proof proceeds in two steps. In the first step we show that the described profile of strategies is equilibrium. The second step shows that the constructed equilibria are utility maximizing equilibria. In what follows $(\mathbf{m}, \mathbf{y})$ denotes the equilibrium, $\mathbf{c}(\mathbf{m}, \mathbf{y})$ the truthful communication network and k_j is the in-degree of j in truthful communication network $\mathbf{c}(\mathbf{m}, \mathbf{y})$. Note that, with some abuse of notation, we have suppressed the qualification that the communication network $\mathbf{g}$ is complete.

Step I. We show that the described strategy profiles are equilibria. First, note that Theorem 1 implies that, when $\beta \leq f(k, n)$, the profile $(\mathbf{m}, \mathbf{y})$ such that $c_{i,j}(\mathbf{m}, \mathbf{y}) = 1$ if and only if

$i \in \{l, ..., n-l+1\}$ is equilibrium if and only if

$$\left| \sum_{j \in N \backslash \{i\}} \frac{b_j - b_i}{k_j + 3} \right| \leq \sum_{j \in N \backslash \{i\}} \frac{1}{2\left(k_j + 3\right)^2}$$

for all $i \in \{l, ..., n-l+1\}$. To see this note that in $\mathbf{c}(\mathbf{m}, \mathbf{y})$ there are $n - 2l + 2$ players communicating truthfully, $k_j = n - 2l + 1$ for all $j \in \{l, ..., n-l+1\}$, whereas $k_j = n - 2l + 2$ for all $j \notin \{l, ..., n-l+1\}$. Because $b_j - b_i = \beta\left(j - i\right)$, the above equilibrium condition simplifies to:

$$\left| \sum_{j \in \{l,...,n-l+1\} \backslash \{i\}} \frac{\beta(j-i)}{n-2l+1+3} + \sum_{j=1}^{l-1} \frac{\beta(j-i)}{n-2l+2+3} + \sum_{j=n-l+2}^{n} \frac{\beta(j-i)}{n-2l+2+3} \right|$$
$$\leq \sum_{j \in \{l,...,n-l+1\} \backslash \{i\}} \frac{1}{2\left(n-2l+1+3\right)^2} + \sum_{j=1}^{l-1} \frac{1}{2\left(n-2l+2+3\right)^2} + \sum_{j=n-l+2}^{n} \frac{1}{2\left(n-2l+2+3\right)^2},$$

for all $i \in \{l, ..., n-l+1\}$. This condition can be further simplified as follows: $\beta \leq \min_{i \in \{l,...,n-l+1\}} \phi\left(i, l, n\right)$, where

$$\begin{aligned} \phi\left(i, l, n\right) &= \frac{\frac{n-2l+1}{2(n-2l+4)^2} + \frac{2(l-1)}{2(n-2l+5)^2}}{\left| \frac{\sum_{j \in \{l,...,n-l+1\} \backslash \{i\}}(j-i) + (n-2l+4)\left[\sum_{j \neq i}(j-i)\right]}{(n-2l+4)(n-2l+5)} \right|} \\ &= \frac{\frac{n-2l+1}{2(n-2l+4)^2} + \frac{2(l-1)}{2(n-2l+5)^2}}{\frac{1}{2}\left|n+1-2i\right| \frac{(n+(n-2l+4)(n-2l+2))}{(n-2l+4)(n-2l+5)}}. \end{aligned}$$

The numerator of this expression does not depend on i, whereas the denominator is decreasing for $i < (n+1)/2$, it is increasing for $i > (n+1)/2$ and symmetric around $(n+1)/2$. Thus, the denominator is maximized for $i = l$ and $i = n-l+1$. This implies that $\min_{i \in \{l,...,n-l+1\}} \phi\left(i, l, n\right) = \phi\left(l, l, n\right)$ and, by definition, $f\left(l, n\right) = \phi\left(l, l, n\right)$. Hence we have recovered the condition that $\beta \leq f\left(l, n\right)$. For future reference, we stress that $\min_{i \in \{l,...,n-l+1\}} \phi\left(i, l, n\right) = \phi\left(n-l+1, l, n\right) = \phi\left(l, l, n\right)$.

Next, using a similar approach, we note that when $\beta \leq g\left(l, n\right)$, the strategy profile $(\mathbf{m}, \mathbf{y})$

such that $c_{i,j}(\mathbf{m},\mathbf{y})=1$ if and only if $i\in\{l,...,n-l\}$ is equilibrium if and only if

$$\begin{aligned}&\left|\sum_{j\in\{l,...,n-l\}\backslash\{i\}}\frac{\beta(j-i)}{n-2l+3}+\sum_{j=1}^{l-1}\frac{\beta(j-i)}{n-2l+1+3}+\sum_{j=n-l+1}^{n}\frac{\beta(j-i)}{n-2l+1+3}\right|\\ \leq&\sum_{j\in\{l,...,n-l\}\backslash\{i\}}\frac{1}{2\left(n-2l+3\right)^{2}}+\sum_{j=1}^{l-1}\frac{1}{2\left(n-2l+1+3\right)^{2}}+\sum_{j=n-l+1}^{n}\frac{1}{2\left(n-2l+1+3\right)^{2}},\end{aligned}$$

for all $i\in\{l,...,n-l\}$. The above condition simplifies as: $\beta\leq\min_{i\in\{l,...,n-l\}}\gamma\left(i,l,n\right)$, where

$$\begin{aligned}\gamma\left(i,l,n\right)&=\frac{\frac{n-2l}{2(n-2l+3)^{2}}+\frac{2l-1}{2(n-2l+4)^{2}}}{\left|\frac{\sum_{j\in\{l,...,n-l\}\backslash\{i\}}(j-i)+(n-2l+3)\sum_{j\neq i}(j-i)}{(n-2l+3)(n-2l+4)}\right|}\\ &=\frac{\frac{n-2l}{2(n-2l+3)^{2}}+\frac{2l-1}{2(n-2l+4)^{2}}}{\frac{1}{2}\frac{|(n-2i)(n-2l+1)+(n-2l+3)n(n+1-2i)|}{(n-2l+3)(n-2l+4)}}.\end{aligned}$$

In $\gamma\left(i,l,n\right)$, the numerator does not depend on i; the denominator is maximal for $i=l$, because $\{l,n-l\}=\arg\max_{i\in\{l,...,n-l\}}|n-2i|$ and $\{l\}=\arg\max_{i\in\{l,...,n-l\}}|n+1-2i|$. Hence, $\min_{i\in\{l,...,n-l\}}\gamma\left(i,l,n\right)=\gamma\left(l,l,n\right)$ and, by definition, $g\left(l,n\right)=\gamma\left(l,l,n\right)$. Hence we have recovered the condition that $\beta\leq g\left(l,n\right)$.

Step II. We now show that the equilibria described are utility-maximizing equilibria. This amounts to show that: 1) when $g(l,n)<\beta$, there is no equilibrium where strictly more than $n-2l$ players truthfully communicates, and 2) when $f\left(l,n\right)<\beta$, there is no equilibrium where strictly more than $n-2l+1$ players truthfully communicate. To see that this is sufficient, note that the welfare of each player i when L players communicate truthfully is: $W_{i}\left(L\right)=-\sum_{j\in N}\left(b_{i}-b_{j}\right)^{2}-\left(n-L\right)\frac{1}{6(L+2)}-L\frac{1}{6(L-1+2)}$. Indeed, each of the L players who communicate truthfully receives $L-1$ truthful messages, whereas each of the remaining players who do not communicate truthfully receives L messages. It is easy to see that $W_{i}\left(L\right)$ is increasing in L, i.e., $W_{i}'\left(L\right)=\frac{1}{6}\frac{n(1+L)^{2}+L^{2}-2}{(L+1)^{2}(L+2)^{2}}>0$ for $n>2$.

We start by noting that because $g\left(v-1,n\right)<f\left(v,n\right)<g\left(v,n\right)$ for all $v=1,...,\left\lfloor\frac{n}{2}\right\rfloor$, it follows that for $\beta>f\left(l,n\right)$ there are no equilibria where strictly more than $n-2l+2$ players communicate, and that for $\beta > g(l,n)$ there are no equilibria where strictly more than $n-2l$ players communicate.

Next, suppose that $n-2l+2$ players communicate in an equilibrium $(\mathbf{m}', \mathbf{y}')$. Let the set of players who truthfully communicate in $(\mathbf{m}', \mathbf{y}')$ be C', so that $|C'| = n-2l+2$. Then, since $(\mathbf{m}', \mathbf{y}')$ is equilibrium, Theorem 3 implies that for all $i \in C'$ it must be that

$$\beta \leq \min_{i \in C'} \phi(i, |C'|, n) \text{ where } \phi(i, |C'|, n) = \frac{\frac{n-2l+1}{2(n-2l+4)^2} + \frac{2l-2}{2(n-2l+5)^2}}{\left| \frac{\left[\sum_{j \in N\setminus\{i\}} (j-i)\right][n-2l+4] + \left[\sum_{j \in C'\setminus\{i\}} (j-i)\right]}{[n-2l+4][n-2l+5]} \right|}.$$

We now claim that the set $C^* = \{l, ..., n-l+1\}$ has the property that

$$\{C^*\} = \arg \max_{C:|C|=n-2l+2} \min_{i \in C} \phi(i, C, n).$$

Note that this claim would imply that an equilibrium where $n-2l+2$ players communicate truthfully exists if and only if $\beta \leq \min_{i \in C^*} \phi(i, C^*, n)$. But, since we have earlier proved that $l \in \arg\min_{i \in C^*} \phi(i, |C^*|, n)$ and that $f(l,n) = \min_{i \in C^*} \phi(i, |C^*|, n)$, this implies that if $\beta > f(l,n)$ then there are no equilibria where $n-2l+2$ players communicate.

To prove the claim, first note that the numerator of $\phi(i, |C|, n)$ depends neither on i nor on $|C|$. Consider the denominator of $\phi(i, |C|, n)$, and suppose that $C \neq \{l, ..., n-l+1\}$. Let v be one of the most extreme players in C, i.e., $v \in \arg\max_{i \in C} |i - (n+1)/2|$. We must consider two sub-cases.

The first sub-case is when $v < (n+1)/2$. Here, note that

$$\sum_{j \in N\setminus\{v\}} (j-v) > \sum_{j \in N\setminus\{l\}} (j-l) > 0 \textit{ and } \sum_{j \in C\setminus\{v\}} (j-v) \geq \sum_{j \in C^*\setminus\{l\}} (j-l) > 0.$$

These inequalities follow from noticing that: 1) since $C \neq \{l, ..., n-l+1\}$ and $v \in \arg\max_{i \in C} |i-(n+1)/2|$, it must be the case that $v < l$, and, 2) since because $l = \min\{i : i \in C^*\}$ and $v = \min\{i : i \in C\}$, we have then that $j - v > 0$ for all $j \in C\setminus\{v\}$ and $j - l > 0$ for all $j \in C^*\setminus\{l\}$. Hence, we can now conclude that:

$$f(l,n) = \phi(l, |C^*|, n) = \min_{i \in C^*} \phi(i, |C^*|, n) > \phi(v, |C|, n) \geq \min_{i \in C} (i, |C|, n).$$

The sub-case when $v > (n+1)/2$, can be ruled out using similar arguments, and therefore details are omitted. Hence, we can conclude that an equilibrium where $n-2l+2$ players communicate truthfully exists if and only if $\beta \leq f(l,n)$.

Suppose now that $n-2l+1$ players communicate in equilibrium $(\mathbf{m}', \mathbf{y}')$; again, C' is the set of players communicating truthfully and $|C'| = n-2l+1$. Since $(\mathbf{m}', \mathbf{y}')$ is equilibrium, Theorem 3 implies that for all $i \in C(m,y)$ it must be that:

$$\beta \leq \min_{i \in C'} \gamma\left(i, |C'|, n\right) \text{ where } \gamma\left(i, |C'|, n\right) = \frac{\frac{n-2l}{2(n-2l+3)^2} + \frac{2l-1}{2(n-2l+4)^2}}{\left| \frac{\left[\sum_{j \in N \setminus \{i\}} (j-i)\right][n-2l+3] + \left[\sum_{j \in C' \setminus \{i\}} (j-i)\right]}{[n-2l+3][n-2l+4]} \right|}.$$

Consider the sets $C^* = \{l, ..., n-l\}$ and its symmetric counterpart around $(n+1)/2$, denoted $C^{**} = \{l+1, ..., n-l+1\}$. Let $h = n-l+1$. By symmetry, it is easy to see that

$$\min_{i \in C^*} \phi\left(i, |C^*|, n\right) = \phi\left(l, |C^*|, n\right) = \phi\left(h, |C^{**}|, n\right) = \min_{i \in C^{**}} \phi\left(i, |C^{**}|, n\right).$$

We now claim that

$$\{C^*, C^{**}\} = \arg \max_{C:|C|=n-2l+1} \min_{i \in C} \phi\left(i, |C|, n\right).$$

As in the case covered above for $f(l,n)$, this result concludes that if $\beta > g(l,n)$ then there are no equilibria where $n-2l+1$ players communicate.

To prove the claim, note that the numerator of $\gamma(i, |C|, n)$ does not depend on i nor on $|C|$. Consider the denominator of $\gamma(i, |C|, n)$. Suppose that $C \notin \{\{l, ..., n-l\}, \{l+1, ..., n-l+1\}\}$. Let v be one of the most extreme players in C, i.e., $v \in \arg\max_{i \in C} |i - (n+1)/2|$. Proceeding in exactly the same way as for the case of $f(k,n)$, we show that for $v < (n+1)/2$, $g(l,n) = \gamma(l, |C^*|, n) = \min_{i \in C^*} \gamma(i, |C^*|, n) > \gamma(v, |C|, n) \geq \min_{i \in C} \gamma(i, |C|, n)$; and that for $v > (n+1)/2$, $g(l,n) = \gamma(h, |C^*|, n) = \min_{i \in C^*} \gamma(i, |C^*|, n) > \gamma(v, |C|, n) \geq \min_{i \in C} \gamma(i, |C|, n)$. Because $g(l,n) \geq \min_{i \in C}(i, |C|, n)$ for all C such that $|C| = n-2l+1$, we conclude that an equilibrium where $n-2l+1$ players communicate truthfully exists if and only if $\beta \leq g(l,n)$.

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