Available online at www.sciencedirect.com
SciVerse ScienceDirect
Journal of Economic Theory 148 (2013) 1751–1769
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Strategic information transmission networks ☆

Andrea Galeotti [a], Christian Ghiglino [a,*], Francesco Squintani [b]

[a] *Department of Economics, University of Essex, Wivenhoe Park, Colchester CO4 3SQ, Essex, UK*
[b] *Department of Economics, University of Warwick, Coventry CV4 7AL, UK*

Received 15 September 2011; final version received 30 March 2013; accepted 1 April 2013

Available online 25 April 2013

Abstract

We study a model of multi-player communication. Privately informed decision makers have different preferences about the actions they take, and communicate to influence each others' actions in their favor. We prove that the equilibrium capability of any player to send a truthful message to a set of players depends not only on the preference composition of those players, but also on the number of players truthfully communicating with each one of them. We establish that the equilibrium welfare depends not only on the number of truthful messages sent in equilibrium, but also on how evenly truthful messages are distributed across decision makers.

JEL classification: D82; D85

Keywords: Cheap talk; Multi-agent communication; Networks; Organizations; Strategic communication

1. Introduction

This paper studies multi-player strategic information transmission. We consider a setting in which multiple decision makers have private incomplete information about a state of the world, which influences all players' utilities. But, given the state, the decision makers have different preferences over the actions they take. Before making a decision, players communicate with

☆ The first draft of this paper has circulated under the title "Strategic information transmission in networks" since January 2009.

* Corresponding author. Fax: +44 1206 872724.
E-mail addresses: agaleo@essex.ac.uk (A. Galeotti), cghig@essex.ac.uk (C. Ghiglino), f.squintani@warwick.ac.uk (F. Squintani).

0022-0531/$ – see front matter
http://dx.doi.org/10.1016/j.jet.2013.04.016

each other, but the information transmitted is not verifiable. Our analysis can be applied to several economic and political scenarios. In many organizations, decision making is decentralized at the division level, but these divisions do not necessarily share the same preferences over the optimal course of action.[1] Before making decisions, the division leaders may communicate their information to each other. In international organizations, national leaders retain control of their own policy choice (such as national environmental, military, or economic policies), but the implementation of such policies may have spillovers on other States. Different States may have different preferences on policies. Before making decisions, the leaders communicate to each other within the context of international organizations.

We develop a natural extension of the uniform-quadratic version of the model of cheap talk by Crawford and Sobel [9]. There are n players, and an unknown state of the world θ, uniformly distributed on the interval $[0, 1]$. Each player i chooses an action y_i, that influences the utility of all players. Each player i would like that each player j's action y_j were as close as possible to $\theta + b_i$, where b_i represents player i's bias relative to the common bliss point θ; specifically, player i's payoff is $-\sum_j (y_j - \theta - b_i)^2$. Each player i is privately informed of a signal s_i, which takes the value of one with probability θ and the value of zero with complementary probability. Before players choose their actions, they simultaneously send messages to each other. A player can differentiate her message only across *audiences*, where the set of a player's audiences is a partition of the set of all the other players. Our model covers both the case of *private communication*, where every player can send a message privately to every other player, and the case of *public communication*, where every player's message is common to all other players.

A communication strategy profile is described by a (directed) network in which each link represents a truthful message, termed a *truthful network*. Our first result derives the equilibrium condition for truthful communication of player i with audience J. The characterization identifies the following equilibrium effects. First, each player's incentive to misreport a low signal in order to raise the action of lower bias opponents is tempered by the loss incurred from the increase in actions of all higher bias players who belong to the same audience J. Second, the composition of these gains and losses depends on the number of players truthfully communicating in equilibrium with each player in audience J. The reason for this is that the influence that player i's message has on player j's decision depends on player j's equilibrium information, i.e., on the number of truthful messages received by j in equilibrium. Third, an increase in the number of truthful messages received by a player j in an audience J has an ambiguous effect on i's capability to truthfully communicate with the audience J in equilibrium. If communication from player i to player j is private there is a stark congestion effect: the willingness of player i to communicate truthfully with player j declines with the number of players communicating with j. However, if j is part of a larger audience and her preferences are distant from i's preferences, an increase in the number of truthful messages received by j decreases the influence that i has on j's decision. Hence, player i capability to be truthful depends now more on the effect of her message on the other players in J, who have a bias closer to i's bias, than j's bias. As a result, player i may be more willing to communicate truthfully with audience J, than when j receives fewer truthful messages.

In our framework, an equilibrium maximizes the *ex-ante* utility of a player if and only if it maximizes the *ex-ante* utility of each one of the players. We find that each player i's ex-ante

[1] For example, the priorities of a marketing division may be different from the ones of the R&D division, when developing a new product.

payoff induced by a player j's choice is an increasing and concave function of the number of players who truthfully communicate with j (the in-degree of j in the truthful network). Hence, equilibria can be ranked in the *ex-ante* Pareto sense on the basis of the in-degree distribution that they generate in their corresponding equilibrium networks. If the in-degree distribution of an equilibrium first order stochastic dominates the in-degree distribution of another equilibrium, the former is more efficient than the latter. Moreover, if the in-degree distribution of an equilibrium network is a mean preserving spread of the in-degree distribution of another equilibrium network, then the latter is more efficient than the former.

We then apply our results to some specialized environments for which we can obtain sharp properties of the associated equilibrium networks. The first environment comprises two groups of players. Preferences are the same within groups, but different across groups. Focusing on private communication, we find that truthful communication across groups requires that the two groups are sufficiently small. Furthermore, since larger groups endogenously access more information within their own group than smaller groups, communication across groups influence the choice of larger groups less than the choice of smaller groups. This results in higher incentives for small group members to misreport information to large group members, than *vice-versa*. In this environment, we also compare the maximal welfare achieved under private and public communication.

In the second environment, players' biases are distributed evenly on the real line. Each player i has bias $b_i = i\beta$, where the constant β describes the preference distance across players. Focusing on private communication, we find that equilibrium networks are highly decentralized, links connecting players with a small bias difference are reciprocal, whereas links between players with a very different bias may be not reciprocal. Specifically, the equilibrium behavior is different for moderate bias players (i.e., players with a bias close to the average player's bias), and for extreme bias players (i.e. players with a bias significantly different from the average bias). Moderate bias players influence the decision of extreme bias players through truthful communication, whereas extreme bias players do not influence the decision of moderate bias players.[2]

Our specific results are derived in a simple quadratic-loss Beta-binomial model, but they deliver insights based on general properties of Bayesian models and of utility functions in the Crawford and Sobel [9] framework. Specifically, these general properties are (i) the assumptions that utility functions are single-peaked, strictly concave and ordered through a single-crossing condition, and (ii) the fact that the effect of a signal on the posterior update decreases with the precision of prior, i.e., in a multi-player communication model, that the effect of a player's truthfully reported signal decreases with the number of truthful messages received from other players.[3,4]

---

[2] These results can be related to the large literature that studies endogenous network formation, see, e.g., Bala and Goyal [3], Bloch and Dutta [6], Calvo-Armengol et al. [8], Dessein et al. [12], Jackson and Wolinsky [20], Galeotti et al. [14] and Hojman and Szeidl [19]. Our main departure from that literature is that we study the formation of endogenous communication networks in a context where information transmission is costless and non-verifiable.

[3] The decreasing marginal effect of signals on the posteriors is a general property of Bayesian updating, in environments where signals are identically distributed and independent conditionally on the state of nature. When signals are not identically distributed, this property need not hold. For example, see McGee and Yang [25], who consider a multi-dimensional state and different signals that convey complementary information across different dimensions.

[4] However, our specific welfare results also depend on the *anonymity* assumption that each player i's payoff depends on each player j's action through the same quadratic-loss function. See also discussion in Section 3.2.

Our paper relates to the literature on cheap talk, which builds on Crawford and Sobel [9]. This literature is too vast to be fully surveyed here, and we discuss only the papers closer to ours.[5] The first one is the study of many-to-one communication by Morgan and Stocken [26].[6] One of their findings is that each sender's incentives to reveal information declines with the number of senders communicating with the receiver. The other one, by Farrell and Gibbons [13], considers a sender communicating privately or publicly with two receivers. In the former case, they highlight that the gains induced by lying to bias one receiver may be tempered by the loss induced by biasing also the other receiver.[7]

Our analysis first describes the implications of these two findings for many-to-many communication (Theorem 1). Then, we uncover further novel insights. Unlike in Morgan and Stocken [26], we find that an increase in the number of truthful messages received by a player has an ambiguous effect on the other players' capabilities to truthfully communicate with her (Example 1). Further, we advance the findings by Farrell and Gibbons [13], by showing that the composition of the gains and losses induced by misreporting a signal to an audience depends on the number of players truthfully communicating in equilibrium with each player in that audience (Theorem 1). Finally, our welfare results (Theorem 2, Corollary 2 and Example 2) are entirely novel in the cheap talk literature.

Our paper is also related to a recent paper by Hagenbach and Koessler [18], who also investigate strategic communication with multiple decision makers. In that paper, players have incentives to coordinate their actions, the common state of the world equals the sum of each player i's individual binary signal, and signals are ex-ante independent across players. Under these assumptions the marginal effect of one truthful message on the action chosen by a player is constant in the number of truthful messages she receives. Hence, the incentives for a player's truthful equilibrium communication do not depend on the other players' equilibrium strategies. We abstract from coordination motives, but we consider a (standard) statistical model in which signals are independent only conditionally on the state of the world, and hence are ex-ante correlated. As a result, the players' incentives to communicate in equilibrium depend not only on the players' biases but also on the other players' communication strategies.[8]

The rest of the paper is organized as follows. Section 2 develops the basic framework and Section 3 presents the general results on equilibrium and welfare. Section 4 studies the two special environments discussed above. Section 5 concludes. All the proofs of the results presented in Section 3 are in Appendix A; the proofs of the results presented in Section 4 are in an on-line Appendix.

2. Model

There are n players, $N = \{1, 2, \dots, n\}$, and player $i \in N$ has bias b_i, with $b_1 \leqslant b_2 \leqslant \cdots \leqslant b_n$; the vector $\mathbf{b} = \{b_1, \dots, b_n\}$ is common knowledge. The state of the world θ is uniformly dis-

[5] Other influential works include Ambrus and Takahashi [1], Austen-Smith [2], Battaglini [4,5], Gilligan and Krehbiel [15,16], Kartik, Ottaviani and Squintani [21], Krishna and Morgan [23,24], Wolinsky [27].

[6] Indeed, our equilibrium characterization for the special case of private communication (Corollary 1) can be described as a generalization of their Proposition 2.

[7] More recent work on cheap talk game with multiple receivers and a single sender includes Goltsman and Pavlov [17], Koessler and Martimort [22], and Caillaud and Tirole [7].

[8] Despite this difference, it is interesting to note that some of our results in specialized environments are qualitatively similar to the ones by Hagenbach and Koessler [18]. We will discuss these similarities in details in Section 4.

tributed on $[0, 1]$. Every player i receives a private signal $s_i \in \{0, 1\}$ on the realization of θ, where $s_i = 1$, with probability θ. Signals are independent across players, conditionally on θ.

Communication among players is restricted by a communication mode, which describes to what extent messages can be targeted to a subset of other players. The communication mode available to i is $\mathcal{N}_i$, a partition of $N_{-i} = N \setminus \{i\}$, with the interpretation that player i must send the same message $m_{iJ} \in \{0, 1\}$ to all players $j \in J$, for any group of players $J \in \mathcal{N}_i$; we refer to each set J as an audience. A communication strategy for player i specifies, for every $s_i \in \{0, 1\}$, a vector $\mathbf{m}_i(s_i) = \{m_{iJ}(s_i)\}_{J \in \mathcal{N}_i}$; $\mathbf{m} = (\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_n)$ denotes a communication strategy profile. Let $\hat{\mathbf{m}}_i$ be the messages that agent i sends, and $\hat{\mathbf{m}} = (\hat{\mathbf{m}}_1, \hat{\mathbf{m}}_2, \ldots, \hat{\mathbf{m}}_n)$.

After communication occurs, each player i chooses an action $\hat{y}_i \in \Re$. Agent i's action strategy depends on agent i's signal and the other agents' messages, i.e., $y_i : \{0, 1\}^n \to \Re$. Let $\mathbf{y} = \{y_1, \ldots, y_n\}$ denote an action strategy profile. Given the state of the world θ, the payoff of player i facing a profile of actions $\hat{\mathbf{y}}$ is

$$u_i(\hat{\mathbf{y}}|\theta) = -\sum_{j \in N} (\hat{y}_j - \theta - b_i)^2.$$

Agent i's payoff depends on how close her own action y_i, and the actions of the other players, are to her ideal action $b_i + \theta$. The underlying assumption is that players' interaction is of a global nature. Depending on the particular context, a model where each player is affected only by the actions taken by a subset of the population may be more plausible. A way of incorporating this is to assume that $u_i(\hat{\mathbf{y}}|\theta) = -\sum_{j \in N_i} (\hat{y}_j - \theta - b_i)^2$, for any player i, where N_i is a subset of N. It can be easily appreciated that our method of analysis and our equilibrium results can be extended to these settings, with minor modifications.

The equilibrium concept is Perfect Bayesian Equilibrium. We restrict attention to pure strategy equilibria.[9] Hence (up to relabelling of messages), each agent i's communication strategy $\mathbf{m}$ with an audience $J \in \mathcal{N}_i$, may take one of only two forms: the truthful one, $m_{iJ}(s_i) = s_i$ for all s_i, and the babbling one, $m_{iJ}(0) = m_{iJ}(1)$.

Given her own signal and the received messages $\hat{\mathbf{m}}_{N_{-i},i} = \{\hat{\mathbf{m}}_{j,i}\}_{j \in N_{-i}}$, by sequential rationality, agent i chooses y_i to maximize her expected payoff. Agent i's optimization then reads

$$\max_{y_i} \left\{ -E\left[\sum_{j \in N} (y_j - \theta - b_i)^2 \middle| s_i, \hat{\mathbf{m}}_{N_{-i},i} \right] \right\} = \max_{y_i} \left\{ -E\left[(y_i - \theta - b_i)^2 \middle| s_i, \hat{\mathbf{m}}_{N_{-i},i} \right] \right\}.$$

Hence, agent i chooses

$$y_i(s_i, \hat{\mathbf{m}}_{N_{-i},i}) = b_i + E[\theta | s_i, \hat{\mathbf{m}}_{N_{-i},i}], \tag{1}$$

where the expectation is based on equilibrium beliefs: all the messages $\hat{m}_{ji}$ received by an agent j who adopts a babbling strategy are disregarded as uninformative, and all the messages $\hat{m}_{ji}$ received by an agent j who adopts a truthful strategy are taken as equal to s_j. Hereafter,

[9] The existence of mixed strategies equilibria cannot be ruled out, as is customary in communication models with discrete signal spaces. A partial characterization result can be proved, stating that each player i would randomize for at most one signal s_i, in any mixed strategy equilibrium. The full characterization is cumbersome because the capability of a player to truthfully communicate with an audience J depends on the equilibrium information held by each player in J. In a mixed strategy equilibrium, this will depend on the exact randomized communication strategy of each other player with any player in J. In the conclusion we provide further insights on mixed strategy equilibrium; the discussion is based on a formal analysis developed in an on-line Appendix.

whenever we refer to a strategy profile $(\mathbf{m}, \mathbf{y})$, each element of $\mathbf{y}$ is assumed to satisfy condition (1).

Players' updating is based on the standard Beta-binomial model. Suppose that an agent i holds k signals, i.e., she holds the signal s_i and $k-1$ players truthfully reveal their signal to her. Let l denote the number of such signals that equals 1. If l out of such k signals equal 1, then the conditional pdf is

$$f(l|\theta, k) = \frac{k!}{l!(k-l)!}\theta^l(1-\theta)^{(k-l)},$$

and her posterior is

$$h(\theta|l, k) = \frac{(k+1)!}{l!(k-l)!}\theta^l(1-\theta)^{(k-l)}.$$

Consequently, $f(l|\theta, k) = h(\theta|l, k)/(k+1)$ and $E[\theta|l, k] = (l+1)/(k+2)$.

In the first stage of the game, in equilibrium, each agent i adopts either truthful communication or babbling communication with each audience $J \in \mathcal{N}_i$, correctly formulating the expectation on the action chosen by each player $j \in J$ as a function of her message $\hat{m}_{iJ}$ and with the knowledge of the equilibrium strategies $\mathbf{m}_{-i}$ of the opponents.

We remark that our framework encompasses two widely studied modes of communication: *private communication* and *public communication*. The model of private communication obtains when for each player i, the partition $\mathcal{N}_i$ is composed of singleton sets. The model of public communication obtains when, for each player i, $\mathcal{N}_i$ consists of the trivial partition $\{N_{-i}\}$.[10]

3. General results

We first characterize equilibria for arbitrary modes of communication. We then show that the characterization takes a simple form under private communication. Lastly, we investigate the relationship between equilibrium and Pareto efficiency.

3.1. Equilibrium networks

A strategy profile $(\mathbf{m}, \mathbf{y})$ induces a network, in which a link from a player i to another player j is associated with truthful communication from i to j. We refer to this network as the *truthful network* and denote it by $\mathbf{c}(\mathbf{m}, \mathbf{y})$. Formally, $\mathbf{c}(\mathbf{m}, \mathbf{y})$ is a binary directed graph where $c_{ij}(\mathbf{m}, \mathbf{y}) = 1$ if and only if j belongs to some element $J \in \mathcal{N}_i$ and $m_{iJ}(s) = s$, for every $s = \{0, 1\}$, with the convention that $c_{ii}(\mathbf{m}, \mathbf{y}) = 0$. The in-degree of player j is the number of players who send a truthful message to j, and it is denoted by $k_j(\mathbf{c}(\mathbf{m}, \mathbf{y}))$. When $(\mathbf{m}, \mathbf{y})$ is an equilibrium, we refer to $\mathbf{c}(\mathbf{m}, \mathbf{y})$ as to the *equilibrium network*. Our first result provides the equilibrium condition for truthful communication of an agent i with an audience J.

Theorem 1. *Consider a collection of communication modes $\{\mathcal{N}_i\}_{i\in N}$. The strategy profile $(\mathbf{m}, \mathbf{y})$ is an equilibrium if and only if for every truthful message from a player i to an audience $J \in \mathcal{N}_i$,*

$$2\left|b_i - \sum_{j\in J} b_j \gamma_j\big(\mathbf{c}(\mathbf{m}, \mathbf{y})\big)\right| \leqslant \sum_{j\in J} \frac{1}{(k_j(\mathbf{c}(\mathbf{m}, \mathbf{y})) + 3)}\gamma_j\big(\mathbf{c}(\mathbf{m}, \mathbf{y})\big), \tag{2}$$

[10] However, unlike Goltsman and Pavlov [17], our model of communication does not allow players to send both a public and a private message to any given audience.

where for every $j \in J$, with $J \in \mathcal{N}_i$,

$$\gamma_j\big(\mathbf{c}(\mathbf{m},\mathbf{y})\big) = \frac{1/(k_j(\mathbf{c}(\mathbf{m},\mathbf{y})) + 3)}{\sum_{j' \in J} 1/(k_{j'}(\mathbf{c}(\mathbf{m},\mathbf{y})) + 3)}.$$

The left-hand side of condition (2) tells us that player i's ability to truthfully communicate with audience J depends on the difference between her own bias b_i and the weighted average $\sum_{j \in J} b_j \gamma_j$ of the audience's biases (we discuss the weights γ_j below). This reflects the fact that, when contemplating whether to deviate from truthful reporting, player i can only influence the action of all players in audience J in the same direction. When, for example, $s_i = 0$ and she reports $\hat{m}_{iJ} = 1$, player i will gain by biasing upwards the action of every player $j \in J$ with bias $b_j < b_i$, but, at the same time, she will lose by increasing the action of every $j \in J$ with bias $b_j > b_i$.

The right-hand side of condition (2) is a weighted average of the expected action change of each player j in the audience J of player i, that is induced by an equilibrium deviation by player i at the communication stage. Specifically, the absolute expected change of each player j's action equals the quantity $1/(k_j + 3)$. To see this, note that sequential rationality, condition (1), implies that player j matches her action y_j to her expected value of $\theta + b_j$, and in the Beta-binomial model $E[\theta|l, k] = (l + 1)/(k + 2)$, where l is the number of signals that take value 1 out of k truthful signals. So, if agent j is informed of her own signal $s_j = 1$, and l out of the $k_j - 1$ truthful messages received from players other than i takes the value 1, an equilibrium deviation by player i at the communication stage leads to an absolute expected change in j's action of $1/(k_j + 3)$.

Each specific weight γ_j in both the left-hand side and right-hand side of condition (2) represents the influence that communication of player i has on player j's action, $1/(k_j + 3)$, relative to the aggregate influence that communication of player i has on the whole audience, $\sum_{j' \in J} 1/(k_{j'} + 3)$. In particular, the numerator $1/(k_j + 3)$ decreases in the number of players truthfully communicating with j in equilibrium. The reason for this is that the more player j is informed in equilibrium, the less the message $\hat{m}_{iJ}$ will change her final action. Therefore, when contemplating a deviation, player i can gain or lose less in absolute terms by influencing j relative to the other players in J. As a result, player i will give less weight γ_j to player j in the weighted average, relative to the other players in audience J.

In sum, there is an equilibrium where player i truthfully communicates with audience J if and only if twice the absolute difference between her bias and the weighted average of their biases is smaller than the weighted average change of the actions in audience J induced by an equilibrium deviation by player i at the message stage.

The characterization in Theorem 1 dramatically simplifies when communication is private.

Corollary 1. *Under private communication a strategy profile* $(\mathbf{m}, \mathbf{y})$ *is an equilibrium if and only if for every* (i, j) *with* $c_{ij}(\mathbf{m}, \mathbf{y}) = 1$,

$$|b_i - b_j| \leqslant \frac{1}{2[k_j(\mathbf{c}(\mathbf{m},\mathbf{y})) + 3]}. \tag{3}$$

Under private communication, the willingness of a player i to credibly communicate with a player j displays a simple dependence on their bias difference $|b_i - b_j|$ and on the number of players truthfully communicating with j. In particular, a high in-degree k_j prevents communication from i to j to be truthful. To see this, suppose that i is biased upwards relative to j, i.e.,

$b_i > b_j$. When many opponents truthfully communicate with j, this player is well informed. In this case, if player i deviates from the truthful communication strategy and reports $\hat{m}_{ij} = 1$ when $s_i = 0$, she will induce a small increase of j's action. Such a small increase in j's action is always beneficial in expectation to i, as it brings j's action closer to i's (expected) bliss point. Hence, player i will not be able to truthfully communicate the signal $s_i = 0$. In contrast, when j has a low in-degree, then i's report $\hat{m}_{ij} = 1$ moves j's action upwards significantly, possibly over i's bliss point. In this case, biasing upwards j's action may result in a loss for player i. Hence, she does not deviate from the truthful communication strategy.

The equilibrium effects derived in Theorem 1 and Corollary 1 are driven by the fact that the marginal return to receiving an additional signal is strictly decreasing on the number of signals that a player has already received. This is different from Hagenbach and Koessler [18], where, because of the statistical model they adopt, the equilibrium communication strategy of a player is independent of the communication strategy played by all other players.

We conclude this part of the section by returning to general communication modes. We show that, unlike in the case of private communication, a player i's willingness to truthfully communicate with another player j need not to decrease in the in-degree of j. We show this in the example below.

Example 1. Let $N = \{1, 2, 3, 4\}$ and $\mathbf{b} = \{-1, 0, \beta, \beta + c\}$, where $\beta > 1$ and c is a small positive constant. Suppose also that player 2 must send the same message to player 1 and 3, otherwise communication is private. We construct two equilibria and compare them.

In the first equilibrium player 1 and player 4 babble with player 3, and player 4 and player 3 babble with player 1. In this case, player 2 assigns the same weight to player 1 and player 3, i.e., $\gamma_1 = \gamma_3 = 1/2$. The communication strategy in which player 2 sends a truthful public message to 1 and 3 is then part of an equilibrium whenever $\beta \leqslant 5/4$.

In the second equilibrium we still consider that player 1 babbles with player 3 and player 3 babbles with player 1. However, now player 4 communicates truthfully with 3, which is always consistent with equilibrium for sufficiently small c. In this case, player 2 gives a higher weight to player 1 who is less informed than player 3, i.e., $\gamma_1 = 5/9 > 4/9 = \gamma_3$. The communication strategy in which player 2 sends a truthful public message to 1 and 3 is then part of an equilibrium whenever $\beta \leqslant 241/160$.

We conclude that if the difference between the bias of player 2 and the bias of player 3 is $\beta \in (5/4, 241/160]$, then player 2 is capable of reporting a truthful public message to player 1 and player 3, only if player 4 also communicates truthfully with player 3.

3.2. Welfare

We now consider equilibrium welfare. Because of the quadratic utility formulation, if we let $\sigma_j^2(\mathbf{m}, \mathbf{y})$ be the residual variance of θ that player j expects to have once communication has taken place, we can write player i's expected utility in equilibrium $(\mathbf{m}, \mathbf{y})$ as follows

$$EU_i(\mathbf{m}, \mathbf{y}) = -\left[\sum_{j \in N} (b_j - b_i)^2 + \sum_{j \in N} \sigma_j^2(\mathbf{m}, \mathbf{y})\right].$$

This is an extension of the welfare characterization by Crawford and Sobel [9] to multiple senders and multiple receivers for the uniform-quadratic version of their model. A nice feature of our model is that we can express the sum of residual variances of θ as a function of a simple property of the equilibrium network, namely its in-degree distribution. Indeed, using the properties of the Beta-binomial updating, we have that[11]

$$\sigma_j^2(\mathbf{m},\mathbf{y}) = \int_0^1 \sum_{l=0}^{k_j(\mathbf{c}(\mathbf{m},\mathbf{y}))+1} \big[E\big[\theta\big|l, k_j\big(\mathbf{c}(\mathbf{m},\mathbf{y})\big)+1\big]-\theta\big]^2 f\big(l\big|k_j\big(\mathbf{c}(\mathbf{m},\mathbf{y})\big)+1,\theta\big)\,d\theta$$
$$= \frac{1}{k_j(\mathbf{c}(\mathbf{m},\mathbf{y}))+2} \sum_{l=0}^{k_j(\mathbf{c}(\mathbf{m},\mathbf{y}))+1} Var\big(\theta\big|l, k_j\big(\mathbf{c}(\mathbf{m},\mathbf{y})\big)+1\big)$$
$$= \frac{1}{6[k_j(\mathbf{c}(\mathbf{m},\mathbf{y}))+3]},$$

and, by rearranging, we obtain

$$\sum_{j\in N} \sigma_j^2(\mathbf{m},\mathbf{y}) = \frac{n}{6}\sum_{k=0}^{n-1} \frac{1}{k+3} P\big(k\big|\mathbf{c}(\mathbf{m},\mathbf{y})\big), \tag{4}$$

where $P(k|\mathbf{c}(\mathbf{m},\mathbf{y}))$ is the fraction of players with in-degree k in the equilibrium network $\mathbf{c}(\mathbf{m},\mathbf{y})$, and $P(\cdot|\mathbf{c}(\mathbf{m},\mathbf{y}|)) : \{0,\dots,n-1\} \to [0,1]$ is its in-degree distribution.

Inspection of the above equation shows that *an equilibrium* $(\mathbf{m},\mathbf{y})$ *yields a higher ex-ante expected utility to a player* i *than another equilibrium* $(\mathbf{m}',\mathbf{y}')$ *if and only if* $(\mathbf{m},\mathbf{y})$ *yields higher ex-ante expected utility than* $(\mathbf{m}',\mathbf{y}')$ *to all players* $j \in N$. Hence, ranking equilibria in the Pareto sense is equivalent to ranking them in the sense of utility maximization for all players. We can now state the following result.

Theorem 2. *Equilibrium* $(\mathbf{m},\mathbf{y})$ *Pareto dominates equilibrium* $(\mathbf{m}',\mathbf{y}')$ *if and only if*

$$\sum_{k=0}^{n-1} \frac{1}{k+3} P\big(k\big|\mathbf{c}(\mathbf{m},\mathbf{y})\big) < \sum_{k=0}^{n-1} \frac{1}{k+3} P\big(k\big|\mathbf{c}\big(\mathbf{m}',\mathbf{y}'\big)\big). \tag{5}$$

Condition (5) allows us to rank equilibria in the Pareto sense based on stochastic dominance relations between the in-degree distributions of their corresponding equilibrium networks.[12]

Corollary 2. *Suppose that* $(\mathbf{m},\mathbf{y})$ *and* $(\mathbf{m}',\mathbf{y}')$ *are equilibria.*

1. *If* $P(k|\mathbf{c}(\mathbf{m},\mathbf{y}))$ *first order stochastic dominates* $P(k|\mathbf{c}(\mathbf{m}',\mathbf{y}'))$ *then equilibrium* $(\mathbf{m},\mathbf{y})$ *Pareto dominates equilibrium* $(\mathbf{m}',\mathbf{y}')$.
2. *If* $P(k|\mathbf{c}(\mathbf{m}',\mathbf{y}'))$ *is a mean preserving spread of* $P(k|\mathbf{c}(\mathbf{m},\mathbf{y}))$ *then equilibrium* $(\mathbf{m},\mathbf{y})$ *Pareto dominates equilibrium* $(\mathbf{m}',\mathbf{y}')$.

---

[11] Recall from Section 2 that in the Beta-binomial model if l out of k signals equal 1 then $f(l|k,\theta) = h(\theta|l,k)/(k+1)$. Furthermore, $Var(\theta|l,k) = [(l+1)(k-l+1)]/[(k+2)^2(k+3)]$.

[12] The fact that only the distribution of in-degrees and not the identity of players matters for equilibrium welfare relies on the anonymity assumption embedded in the payoff specifications $u_i(\hat{\mathbf{y}}|\theta) = -\sum_{j\in N}(\hat{y}_j - \theta - b_i)^2$, for all i. For example, if each player i cared more about her own action y_i, than about others' actions $\mathbf{y}_{-i}$, then equilibria could not be Pareto ranked on the basis of their in-degree distributions. However, a result analogous to Theorem 2 would still hold if we focused on the utilitarian welfare criterion, and if the effect of the action y_j of each player $j \neq i$ on any player i did not depend on i and j.

To illustrate the first part of Corollary 2, consider an equilibrium in which i babbles with j and another equilibrium in which the only difference is that player i communicates truthfully with j. The presence of this additional truthful message only alters the equilibrium action of player j. In particular, player j's action becomes more precise, increasing the utility of each player. A direct consequence of this result is that if $(\mathbf{m}, \mathbf{y})$ and $(\mathbf{m}', \mathbf{y}')$ are two distinct equilibria and $\mathbf{c}(\mathbf{m}', \mathbf{y}')$ is a subgraph of $\mathbf{c}(\mathbf{m}, \mathbf{y})$, then equilibrium $(\mathbf{m}, \mathbf{y})$ Pareto dominates equilibrium $(\mathbf{m}', \mathbf{y}')$.

The second part of Corollary 2 compares equilibria that have the same number of truthful communication links. It shows that equilibria in which truthful messages are distributed evenly across players Pareto dominate equilibria where few players receive many truthful messages, while others receive only a few. The reason is that the residual variance of θ associated with every player j is decreasing and *convex* in j's in-degree.

Theorem 2 and Corollary 2 suggest the possibility that an equilibrium that sustains a low number of truthful messages may Pareto dominate an equilibrium with a high number of truthful messages, as long as its messages are distributed more evenly across players. We now develop an example in which this is the case.

Example 2 *(Evenly distributed truthful messages vs. total number of truthful messages).* Suppose that $n = 5$, $b_{i+1} - b_i = \beta$, for $i = 1, 2, 3, 4$, and assume private communication. When $\beta \leqslant 1/28$ the following two networks are equilibrium networks. The first one has four truthful links: each agent sends a truthful message to agent 3, and there are no other truthful messages. The in-degree distribution of the equilibrium network is then: $P(0) = 4/5$, $P(4) = 1/5$, and $P(k) = 0$, $k = 1, 2, 3$. The other equilibrium network has three truthful links: agent 3 sends a truthful message to players 1, 2 and 4, and there are no other truthful messages. The in-degree distribution associated to this equilibrium is: $\tilde{P}(0) = 2/5$, $\tilde{P}(1) = 3/5$ and $\tilde{P}(k) = 0$, $k = 2, 3, 4$.

Note that P and $\tilde{P}$ cannot be ranked in terms of first order or second order stochastic dominance relations. However, applying condition (5), it is easy to check that

$$\sum_{k=0}^{n-1} \tilde{P}(k) \frac{1}{k+3} = \frac{17}{60} < \frac{31}{105} = \sum_{k=0}^{n-1} P(k) \frac{1}{k+3}.$$

Hence, the second equilibrium Pareto dominates the first one, despite its lower number of truthful messages.

4. Two simple environments

In this section we apply our theorems to two interesting environments for which we derive detailed equilibrium properties. We shall focus on the set of utility-maximizing equilibria, i.e., equilibria that maximize the ex-ante expected utility of all players. As noted in Section 3.2, in our setting a utility-maximizing equilibrium weakly Pareto dominates every other equilibrium.

4.1. Communication across groups

We specialize our analysis to the case in which players are divided in two groups with homogeneous biases. The set of players is partitioned into a group 1 of size n_1 and a group 2 of size n_2; without loss of generality, we assume that $n_1 \geqslant n_2 \geqslant 1$. Players in group 1 have bias 0 and players in group 2 have a bias $b > 0$.

We first consider the case of private communication. In an on-line Appendix we provide a full characterization of utility-maximizing equilibria. Here, we focus on a natural subclass where there is complete intra-group communication: each player in group G communicates truthfully with each other player in group G.[13]

Proposition 1. *Consider private communication. In every utility-maximizing equilibrium network with complete intra-group communication,*

1. *if* $b < \frac{1}{2(n+2)}$ *there is complete communication across groups*;
2. *if* $b \in [\frac{1}{2(n+2)}, \frac{1}{2(n_2+3)}]$ *the number of truthful messages that each player in group G sends to group* G' *declines with the size of group* G'*; furthermore, the number of truthful messages that each player in the larger group* 1 *sends to the smaller group* 2 *is larger than the number of truthful messages that each player in the smaller group* 1 *sends to the larger group* 2;
3. *if* $b > \frac{1}{2(n_2+3)}$*, there is no communication across groups.*

Proposition 1 highlights how the size of the two groups affects communication across groups. Members of larger groups endogenously access more information within their own group than members of smaller groups. In view of Corollary 1 this implies that communication across groups influences the choice of larger group members less than the choice of smaller group members. As a consequence, members of small groups have higher incentives to misreport information to members of larger groups, than *vice-versa*.[14]

We now investigate the effect that different modes of communication have on equilibrium welfare. We say that a communication mode dominates another mode if the utility-maximizing equilibria under the former Pareto dominate utility-maximizing equilibria under the latter. As a first step we compare private communication and public communication. For expositional simplicity, we assume that the two groups are of equal size, $n_1 = n_2 = n/2$.

Proposition 2. *Assume that* $n_1 = n_2 = n/2$*. There exists* $\frac{1}{2(n+2)} < \hat{b}(n) < \infty$*, such that*:

1. *If* $b \leqslant \frac{1}{2(n+2)}$*, then utility-maximizing equilibria under private and public communication yield the same utility to all players*;
2. *If* $b \in (\frac{1}{2(n+2)}, \hat{b}(n)]$*, then public communication dominates private communication*;
3. *If* $b > \hat{b}(n)$*, then private communication dominates public communication.*

If the conflict between the two groups is sufficiently small, regardless of the communication mode, all players truthfully communicate in the utility-maximizing equilibrium. As the level of

[13] These equilibria are robust to the introduction of infinitesimal group-sensitive preferences. For example, we can slightly modify the model so that the utility of every player i in group G is: $-(1+\epsilon)\sum_{j\in N_G}(\hat{y}_j - \theta - b_i)^2 - \sum_{j\in N_{G'}}(\hat{y}_j - \theta - b_i)^2$, where ϵ is a small positive constant. Furthermore, this class of utility-maximizing equilibria coincides with the set of utility-maximizing equilibria as long as the conflict of interest between the two groups is not too low.

[14] Despite the significant modelling differences that we discussed earlier, Hagenbach and Koessler [18] find a similar result. In their model, players wish to coordinate their actions. So, the capability of a player to communicate truthfully increases with the number of players with whom she is truthful. Consequently, an increase in the size of, say, group 1, increases truthful communication from group 1 to group 2.

conflict across groups increases, we know from Proposition 1 that communication across groups eventually breaks down under private communication. However, as long as the level of conflict is not too high, player i has no incentive to misreport information, under public communication. This is because such a deviation will also change the action of players in her own group. In this case public communication dominates private communication. As the conflict of interest becomes large, public communication prevents truthful reporting, while under private communication players communicate within their own group, and hence private communication dominates public communication.

In light of Theorem 1, when public communication fails, an intuitive way to restore some amount of equilibrium communication across groups is to lower the average bias difference between the player and her audience, by reducing the number of players from the other group in the audience. This observation leads us to consider modes of communication where each player's audience is composed of all the fellow players in her group, and of a subset of the players in the other group. Such mixed modes of communication may dominate private and public communication, as we show next.

Proposition 3. *Assume that $n_1 = n_2 = n/2$. Consider the mixed communication modes family, parametrized in k, such that each player i in group $G = 1, 2$ has an audience composed by all members of group G and players $i, i + 1 \bmod (n/2), \ldots, i + k \bmod (n/2)$ in opposite group $G' \neq G$, and sends private messages to the remaining players.*[15] *For any bias value b, there exists a k such that the associated mixed mode weakly dominates the private and the public communication mode. The dominance is strict for $b > \hat{b}(n)$, as long as b is not too large.*

This result is intuitive. Because these mixed communication modes generalize public communication, evidently they cannot be dominated by the latter. Further, for $b \geqslant \hat{b}(n)$, public communication fails, and communication is truthful to a set of players larger than under private communication, because it mixes members of the different groups in the same audience.[16]

4.2. Equidistant bias

The equilibrium characterization in Theorem 1 and Corollary 1 does not provide specific insights on the properties of the equilibrium network. To make progress on this issue we focus on private communication and assume that any players i and $i + 1$ have equidistant biases, i.e., $b_{i+1} - b_i = \beta$, for all $i = 1, \ldots, n - 1$. In an on-line Appendix we characterize a subclass of utility-maximizing equilibria, which coincides with the whole set for a wide range of values of the parameter β. In what follows we summarize the properties of the equilibrium networks associated to the equilibria in this subclass.

[15] The notation $a + b \bmod (c)$ stands for $a + b$ if $a + b \leqslant c$ and $a + b - c$ otherwise.

[16] Proposition 3 identifies a simple communication mode in which each player in the same group truthfully communicates to an audience composed of the same number of players in the two groups. It would be interesting to also characterize optimal audience structures. But, even in the simplified example that we are considering, this may require studying communication modes in which the audience structure differs across players even within the same group. We postpone this analysis to further research.

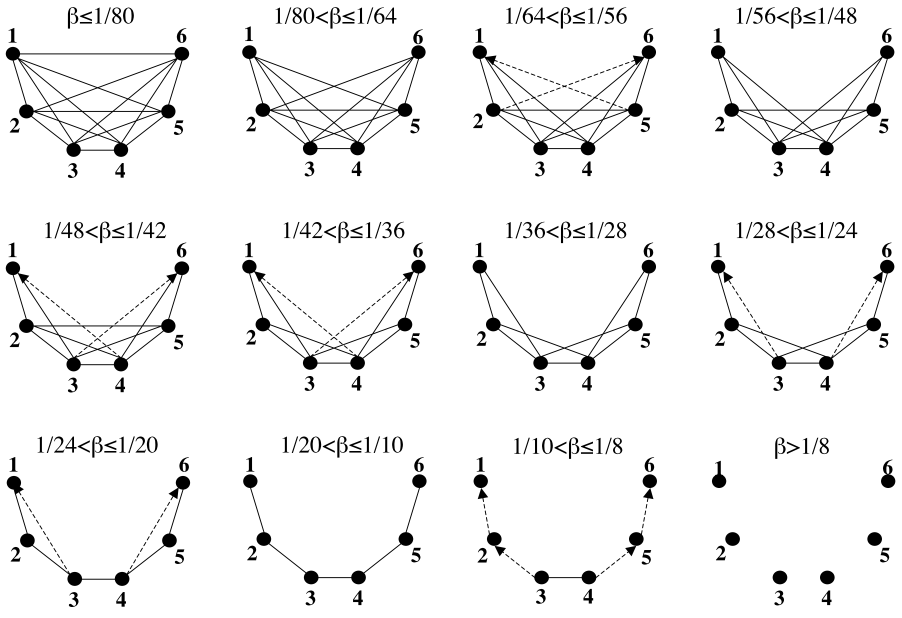

Fig. 1. Equilibrium networks in Proposition 4, $n = 6$.

Proposition 4. *For every β, there exists a utility-maximizing equilibrium with the following properties*:

1. Localization. *If a player i communicates with j, then so do all players l such that $|l - j| < |i - j|$.*
2. Asymmetric Communication. *For every $i < j \leqslant \lfloor \frac{n+1}{2} \rfloor$ or $i > j \geqslant \lfloor \frac{n+1}{2} \rfloor$, it can be the case that j truthfully communicates with i and yet i does not truthfully communicate with j, but not vice-versa.*
3. Decentralization. *There exists an integer k_β such that the fraction of players with in-degree k_β converges to one as $n \to +\infty$.*

Fig. 1 illustrates equilibrium networks associated to utility-maximizing equilibria for the case of $n = 6$ and for different values of β. In the figure a solid line linking i and j means that i and j communicate truthfully with each other; a dashed line starting from i with an arrow pointing at j means that only player i truthfully communicates with j. Communication is localized in the sense that each player i communicates truthfully to players j whose indexes are consecutive numbers. Due to the congestion effect, communication may be asymmetric, from moderate bias to extreme bias players, but not *vice-versa*. Finally, the players' equilibrium network in-degrees are fairly similar across players, as they do not differ by more than two. Indeed Proposition 4 shows that the proportion of players with the same in-degree converges to one as n tends to infinity.[17]

[17] Interestingly, moderate bias players communicate more than extreme bias players also in Hagenbach and Koessler [18]. However, this occurs because of the assumption of coordination motives. In our case, instead, moderate bias players communicate more than extreme bias players because of the congestion effect. Furthermore, the equilibrium network localization property we uncover here is entirely novel.

5. Conclusion

In this paper we have studied a model of multi-player communication. Our analysis has clarified the forces that determine the possibility of information aggregation in equilibrium and its consequences for welfare.

Our analysis has focused on pure strategy equilibrium. An on-line Appendix explores the characteristics of mixed strategy equilibrium. We omit formal derivations, here, and only summarize the main insights. By fully characterizing mixed strategy equilibrium in the case of two players, we show that mixed strategy equilibria are Pareto dominated. In fact, we prove that, whenever there exists an equilibrium in which player i randomizes her message, then there is also an equilibrium in which player i is truthful. Truthful communication yields both players' higher expected utility than player i's randomization. Because our analysis focuses on Pareto dominant equilibria, we conclude that the restriction to pure strategy is without loss of generality, in the two-player case. But then, we show that this result does not extend to settings with more than two players, by providing an example with three players and private communication. For some range of the bias parameters, there is an equilibrium in which agent 1 communicates truthfully to agent 2 and agent 3 randomizes when communicating to agent 2. However, for the same range of parameters, there is no equilibrium in which both agent 1 and agent 3 communicate truthfully to agent 2.[18]

Beyond the analysis of mixed strategies, our work can be extended in other directions. For instance, in our model, each player takes an action, but in many settings (such as in governments or in organizations), the allocation of decision making authority is often the result of the solution of a design problem. This question is investigated in Dewan et al. [10], who apply our findings to the institutional design of governments. Further, Dewan and Squintani [11] consider the possibility that decision making authority is voluntarily delegated to players that are more informed in equilibrium, in the context of faction formation in parties. Another possibility is to relax the assumption, made here, that there is only one round of communication. This is a good approximation when decisions must be taken urgently, and there is no time for lengthy meetings. It would interesting to extend the model to allow for repeated communication. Finally, in our setting the communication mode is given exogenously, but in many applications, a player or a designer can choose whether to make communications private (e.g., by organizing bilateral meetings), public (by organizing public meetings), or more generally, could select appropriate audiences for each player. While we have provided some results on this question in Section 4.1 for the case of two groups, the general model analysis is left for further research.

Acknowledgments

We thank an Editor and two anonymous referees for valuable comments. We thank Daron Acemoglu, Attila Ambrus, David Austen-Smith, Drew Fudenberg, Johannes Horner, Frederic Koessler, Alessandro Pavan, Wolfgang Pesendorfer, Andrea Prat, Andrew Postlewaite, Larry Samuelson, Joel Sobel, Peter Sorensen, Adam Szeidl, Joel Watson, Asher Wolinsky, Leaat Yariv, and participants at theory seminars of University of Basel, University of California at Berkeley,

[18] The intuition for this result is based on the congestion effect that we describe in Corollary 1. When player 3 adopts a mixed strategy, the information that player 2 receives from player 3 is noisier than when player 3 communicates truthfully to player 2. Hence, player 1 has a stronger incentive to be truthful to player 2 when player 3 randomizes, than when 3 is truthful to 2.

Caltech, Cambridge University, Columbia University, Copenhagen University, CORE, Edinburgh University, Essex University, London School of Economics, University of California at Los Angeles, MIT/Harvard University, University College London, SAET Conference 2009, and Decentralization Conference 2010 for valuable suggestions. Andrea Galeotti is grateful to the European Research Council for support through ERC-starting grant (award No. 283454) and to The Leverhulme Trust for support through the Philip Leverhulme Prize.

Appendix A

All the proofs of the results presented in Section 3 are in this appendix while the proofs of the results presented in Section 4 are in an on-line Appendix.

Proof of Theorem 1. Suppose that all players in J believe that player i reports her signal s_i truthfully. We henceforth simplify notation and denote s_i simply as s. Let s_R be a vector containing the (truthful) signals that each j has received and her own signal. With some abuse of notation, we denote the in-degree of j in truthful network $\mathbf{c}$ by k_j. Let also $y_{s_R,s}$ be the action that j would take if she has information s_R and player i has sent signal s; analogously, $y_{s_R,1-s}$ is the action that j would take if she has information s_R and player i has sent signal $1-s$. Player i reports signal s truthfully to a collection of players J if and only if

$$-\int_0^1 \sum_{j\in J} \sum_{s_R\in\{0,1\}^{k_j}} \left[(y_{s_R,s}-\theta-b_i)^2-(y_{s_R,1-s}-\theta-b_i)^2\right] f(\theta,s_R|s)\,d\theta \geqslant 0,$$

and using the identity $a^2-b^2=(a-b)(a+b)$ we get

$$-\int_0^1 \sum_{j\in J} \sum_{s_R\in\{0,1\}^{k_j}} \left[(y_{s_R,s}-y_{s_R,1-s})\left(\frac{y_{s_R,s}+y_{s_R,1-s}}{2}-(\theta+b_i)\right)\right] f(\theta,s_R|s)\,d\theta \geqslant 0.$$

Next, observing that $y_{s_R,s}=E[\theta+b_j|s_R,s]$ we obtain

$$\begin{aligned}-\int_0^1 \sum_{j\in J} \sum_{s_R\in\{0,1\}^{k_j}} &\Bigg[\big(E[\theta+b_j|s_R,s]-E[\theta+b_j|s_R,1-s]\big)\\ &\cdot\left(\frac{E[\theta+b_j|s_R,s]+E[\theta+b_j|s_R,1-s]}{2}-(\theta+b_i)\right)\Bigg] f(\theta,s_R|s)\,d\theta \geqslant 0.\end{aligned}$$

Denote $\Delta(s_R,s)=(E[\theta|s_R,s]-E[\theta|s_R,1-s])$. Observing that $f(\theta,s_R|s)=f(\theta|s_R,s)\times P(s_R|s)$, and simplifying, we get

$$\begin{aligned}-\sum_{j\in J} \sum_{s_R\in\{0,1\}^{k_j}} \int_0^1 &\left[\Delta(s_R,s)\left(\frac{E[\theta|s_R,s]+E[\theta|s_R,1-s]}{2}+b_j-b_i-\theta\right)\right]\\ &\cdot f(\theta|s_R,s)P(s_R|s)\,d\theta \geqslant 0.\end{aligned}$$

Furthermore,

$$\int_0^1 \theta f(\theta|s_R, s)\, d\theta = E[\theta|s_R, s],$$

and

$$\int_0^1 P(\theta|s_R, s)E[\theta|s_R, s]\, d\theta = E[\theta|s_R, s],$$

because $E[\theta|s_R, s]$ does not depend on θ. Therefore, we obtain

$$\begin{aligned}
&-\sum_{j\in J}\sum_{s_R\in\{0,1\}^{k_j}}\left[\Delta(s_R, s)\left(\frac{E[\theta|s_R, s] + E[\theta|s_R, 1-s]}{2} + b_j - b_i - E[\theta|s_R, s]\right)\right]P(s_R|s) \\
&\quad = -\sum_{j\in J}\sum_{s_R\in\{0,1\}^{k_j}}\left[\Delta(s_R, s)\left(-\frac{E[\theta|s_R, s] - E[\theta|s_R, 1-s]}{2} + b_j - b_i\right)\right]P(s_R|s) \geqslant 0.
\end{aligned}$$

Now, note that

$$\begin{aligned}
\Delta(s_R, s) &= E[\theta|s_R, s] - E[\theta|s_R, 1-s] \\
&= E[\theta|l+s, k_j+1] - E[\theta|l+1-s, k_j+1] \\
&= (l+1+s)/(k_j+3) - (l+2-s)/(k_j+3) \\
&= \begin{cases} -1/(k_j+3) & \text{if } s = 0, \\ 1/(k_j+3) & \text{if } s = 1, \end{cases}
\end{aligned}$$

where l is the number of digits equal to one in s_R. Hence, we obtain that player i is willing to communicate the signal $s = 0$ to player j if and only if

$$-\sum_{j\in J}\left(\frac{-1}{k_j+3}\right)\left(-\frac{-1}{2(k_j+3)} + b_j - b_i\right) \geqslant 0,$$

or

$$\sum_{j\in J}\frac{b_j - b_i}{k_j+3} \geqslant -\sum_{j\in J}\frac{1}{2(k_j+3)^2}.$$

Note that this condition is redundant if $\sum_{j\in J} b_j - b_i > 0$. On the other hand, she is willing to communicate the signal $s = 1$ to player j if and only if

$$-\sum_{j\in J}\left(\frac{1}{k_j+3}\right)\left(-\frac{1}{2(k_j+3)} + b_j - b_i\right) \geqslant 0,$$

or

$$\sum_{j\in J}\frac{b_j - b_i}{k_j+3} \leqslant \sum_{j\in J}\frac{1}{2(k_j+3)^2}.$$

Note that this condition is redundant if $\sum_{j\in J} b_j - b_i < 0$. Collecting the two conditions

$$\left|\sum_{j\in J}\frac{b_j - b_i}{k_j+3}\right| \leqslant \sum_{j\in J}\frac{1}{2(k_j+3)^2}.$$

This completes the proof of Theorem 1. □

Proof of Corollary 1. Corollary 1 is a special case of Theorem 1, in which for every $i \in N$ the partition $\mathcal{N}_i$ is composed of singleton sets. □

Proof of Theorem 2. Assume $(\mathbf{m},\mathbf{y})$ is an equilibrium. Select an arbitrary player i. The ex-ante expected utility of i is

$$EU_i(\mathbf{m},\mathbf{y}) = -E\left[\sum_{j=1}^{n}(y_j-\theta-b_i)^2 \middle| \{0,1\}^{k_j(\mathbf{c})+1}\right] \tag{6}$$

$$= -\sum_{j=1}^{n} E\left[(y_j-\theta-b_i)^2 \middle| \{0,1\}^{k_j(\mathbf{c})+1}\right], \tag{7}$$

where (with some abuse of notation) $k_j(\mathbf{c})$ indicates j's in-degree in truthful network $\mathbf{c}(\mathbf{m},\mathbf{y})$.

Consider an arbitrary j with in-degree $k_j(\mathbf{c})$ and let l be the number of digits equal to one in a realized information vector $\{0,1\}^{k_j(\mathbf{c})+1}$. Then, we obtain

$$\begin{aligned}
&E\left[(y_j-\theta-b_i)^2 \middle| \{0,1\}^{k_j(\mathbf{c})+1}\right]\\
&= \int_0^1 \sum_{l=0}^{k_j(\mathbf{c})+1}\left(E\left[\theta \middle| l,k_j(\mathbf{c})+1\right]+b_j-\theta-b_i\right)^2 f\left(l \middle| k_j(\mathbf{c})+1,\theta\right) d\theta\\
&= \int_0^1 \sum_{l=0}^{k_j(\mathbf{c})+1}\left(E\left[\theta \middle| l,k_j(\mathbf{c})+1\right]+b_j-\theta-b_i\right)^2 \frac{h(\theta|l,k_j(\mathbf{c})+1)}{k_j(\mathbf{c})+1+1}\, d\theta,
\end{aligned}$$

where the second equality follows from $f(l|k_j(\mathbf{c})+1,\theta) = h(\theta|l,k_j(\mathbf{c})+1)/(k_j(\mathbf{c})+2)$. Let $\Pi = (E[\theta|l,k_j(\mathbf{c})+1]-\theta)^2$. Then we have

$$\begin{aligned}
&E\left[(y_j-\theta-b_i)^2 \middle| \{0,1\}^{k_j(\mathbf{c})+1}\right]\\
&= \frac{1}{k_j(\mathbf{c})+2}\int_0^1 \sum_{l=0}^{k_j(\mathbf{c})+1}\left(\Pi+(b_j-b_i)^2+2(b_j-b_i)\left(E\left[\theta \middle| l,k_j(\mathbf{c})+1\right]-\theta\right)\right)\\
&\quad\cdot h\left(\theta \middle| l,k_j(\mathbf{c})+1\right) d\theta\\
&= (b_j-b_i)^2+\frac{1}{k_j(\mathbf{c})+2}\left[\int_0^1 \sum_{l=0}^{k_j(\mathbf{c})+1}\left(\Pi+2(b_j-b_i)\left(E\left[\theta \middle| l,k_j(\mathbf{c})+1\right]-\theta\right)\right)\right.\\
&\quad\left.\cdot h\left(\theta \middle| l,k_j(\mathbf{c})+1\right) d\theta\right]\\
&= (b_j-b_i)^2+\frac{1}{k_j(\mathbf{c})+2}\left[\sum_{l=0}^{k_j(\mathbf{c})+1}\left(\int_0^1\left(E\left[\theta \middle| l,k_j(\mathbf{c})+1\right]-\theta\right)^2 f\left(\theta \middle| l,k_j(\mathbf{c})+1\right) d\theta\right)\right].
\end{aligned}$$

Next, let $V(\theta|l,k)$ be the variance of a beta distribution with parameters l and k, i.e.,

$$V(\theta|l,k)=\int_0^1 \big(E[\theta|l,k]-\theta\big)^2 h(\theta|l,k)\,d\theta.$$

It is well known that

$$V(\theta|l,k)=\frac{(l+1)(k-l+1)}{(k+2)^2(k+3)}.$$

Hence,

$$\begin{aligned}
E\big[(y_j-\theta-b_i)^2\big|\{0,1\}^{k_j(\mathbf{c})+1}\big] &= (b_j-b_i)^2+\frac{1}{k_j(\mathbf{c})+2}\left[\sum_{l=0}^{k_j(\mathbf{c})+1} V\big(\theta\big|l,k_j(\mathbf{c})+1\big)\right] \\
&= (b_j-b_i)^2+\sum_{l=0}^{k_j(\mathbf{c})+1}\frac{(l+1)(k_j(\mathbf{c})-l+2)}{(k_j(\mathbf{c})+2)(k_j(\mathbf{c})+3)^2(k_j(\mathbf{c})+4)} \\
&= (b_j-b_i)^2+\frac{1}{6(k_j(\mathbf{c})+3)}.
\end{aligned}$$

We can then write the ex-ante expected utility of player i in equilibrium $(\mathbf{m},\mathbf{y})$ as follows

$$\begin{aligned}
EU_i(\mathbf{m},\mathbf{y}) &= -\sum_{j=1}^{n}\left[(b_j-b_i)^2+\frac{1}{6(k_j(\mathbf{c})+3)}\right] \\
&= -\sum_{j=1}^{n}(b_j-b_i)^2-\frac{1}{6}\sum_{j=1}^{n}\frac{1}{k_j(\mathbf{c})+3} \\
&= -\sum_{j=1}^{n}(b_j-b_i)^2-\frac{1}{6}\sum_{k=0}^{n-1}\frac{|I(k|\mathbf{c}(\mathbf{m},\mathbf{y}))|}{k+3},
\end{aligned}$$

where $|I(k|\mathbf{c}(\mathbf{m},\mathbf{y}))|$ is the set of players with in-degree k, i.e., $I(k|\mathbf{c}(\mathbf{m},\mathbf{y}))=\{i\in N: k_i(\mathbf{c}(\mathbf{m},\mathbf{y}))=k\}$. Therefore,

$$EU_i(\mathbf{m},\mathbf{y})\geqslant EU_i\big(\mathbf{m}',\mathbf{y}'\big)$$

if and only if

$$\sum_{k=0}^{n-1}\frac{|I(k|\mathbf{c}(\mathbf{m},\mathbf{y}))|}{k+3}\leqslant\sum_{k=0}^{n-1}\frac{|I(k|\mathbf{c}'(\mathbf{m}',\mathbf{y}'))|}{k+3},$$

which is equivalent to

$$\sum_{k=0}^{n-1}P\big(k\big|\mathbf{c}(\mathbf{m},\mathbf{y})\big)\frac{1}{k+3}\leqslant\sum_{k=0}^{n-1}P\big(k\big|\mathbf{c}'\big(\mathbf{m}',\mathbf{y}'\big)\big)\frac{1}{k+3}.$$

This concludes the proof of Theorem 2. □

Proof of Corollary 2. The proof of Corollary 2 follows from standard arguments of stochastic dominance, the details are omitted. □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jet.2013.04.016.

References

[1] A. Ambrus, S. Takahashi, Multi-sender cheap talk with restricted state spaces, Theor. Econ. 3 (1) (2008) 1–27.
[2] D. Austen-Smith, Interested experts and policy advice: Multiple referrals under open rule, Games Econ. Behav. 5 (1) (1993) 3–43.
[3] V. Bala, S. Goyal, A non-cooperative model of network formation, Econometrica 68 (2000) 1181–1230.
[4] M. Battaglini, Multiple referrals and multidimensional cheap talk, Econometrica 70 (4) (2002) 1379–1401.
[5] M. Battaglini, Policy advice with imperfectly informed experts, Adv. Theor. Econ. 4 (1) (2004) 1–32.
[6] F. Bloch, B. Dutta, Communication networks with endogenous link strength, Games Econ. Behav. 66 (1) (2009) 39–56.
[7] B. Caillaud, J. Tirole, Building consensus: How to persuade a group, Amer. Econ. Rev. 97 (5) (2007) 1877–1900.
[8] A. Calvo-Armengol, J. de Martí, A. Prat, Endogenous communication in complex organizations, Mimeo, 2009.
[9] V. Crawford, J. Sobel, Strategic information transmission, Econometrica 50 (1982) 1431–1451.
[10] T. Dewan, A. Galeotti, C. Ghiglino, F. Squintani, Information aggregation and optimal structure of the executive, Mimeo, 2011.
[11] T. Dewan, F. Squintani, Party factions and platform design: An information aggregation approach, Mimeo, 2012.
[12] W. Dessein, A. Galeotti, T. Santos, Rational inattention and organizational focus, Mimeo, 2012.
[13] J. Farrel, R. Gibbons, Cheap talk with two audiences, Amer. Econ. Rev. 79 (1989) 1214–1223.
[14] A. Galeotti, S. Goyal, J. Kamphorst, Network formation with heterogeneous players, Games Econ. Behav. 54 (2) (2006) 353–372.
[15] T. Gilligan, K. Krehbiel, Collective decision making and standing committees: An informational rationale for restrictive amendment procedures, J. Law, Econ., Organ. 3 (2) (1987) 287–335.
[16] T.W. Gilligan, K. Krehbiel, Asymmetric information and legislative rules with a heterogeneous committee, Am. J. Polit. Sci. 33 (2) (1989) 459–490.
[17] M. Goltsman, G. Pavlov, How to talk to multiple audiences, Games Econ. Behav. 72 (1) (2011) 100–122.
[18] J. Hagenbach, F. Koessler, Strategic communication networks, Rev. Econ. Stud. 77 (2010) 1072–1099.
[19] D. Hojman, A. Szeidl, Core and periphery in networks, J. Econ. Theory 139 (1) (2008) 295–309.
[20] M.O. Jackson, A. Wolinsky, A strategic model of social and economic networks, J. Econ. Theory 71 (1) (1996) 44–74.
[21] N. Kartik, M. Ottaviani, F. Squintani, Credulity, lies and costly talk, J. Econ. Theory 134 (1) (2006) 93–116.
[22] F. Koessler, D. Martimort, Multidimensional communication mechanisms: Cooperative and conflicting designs, Mimeo, Toulouse School of Economics, 2008.
[23] V. Krishna, J. Morgan, A model of expertise, Quart. J. Econ. 116 (2) (2001) 747–775.
[24] V. Krishna, J. Morgan, Asymmetric information and legislative rules: Some amendments, Amer. Polit. Sci. Rev. 95 (2) (2001) 435–452.
[25] A. McGee, H. Yang, Cheap talk with two senders and complementary information, Mimeo, 2009.
[26] J. Morgan, P. Stocken, Information aggregation in polls, Amer. Econ. Rev. 98 (3) (2008) 864–896.
[27] A. Wolinsky, Eliciting information from multiple experts, Games Econ. Behav. 41 (1) (2002) 141–160.