Information Transmission in Political Factions*

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Abstract

Political networks are often factional: like-minded politicians form groups around a small core of leaders, and few links cross ideological collections. I develop a model in which information transmission is a main reason such networks form. Politicians are partitioned into ideologically cohesive collections, transmission is sufficiently reliable but imperfect, and link costs are neither too high nor too low. Every expert–decision-maker pair is equally likely and every politician assigns the same weight to every decision. I show that every efficient network is star-factional. In each ideological collection, politicians are connected to a unique leader, and only leaders communicate across collections. Star-factional networks arise as Nash equilibria of the bilateral network formation game and are the only equilibria immune to joint deviations. When politicians differ, efficient leadership favors those with greater expertise, authority, or relevance. If these characteristics are divided, multiple leaders may be efficient.

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1 Introduction

Networks of politicians are commonly factional. Like-minded politicians form cohesive groups around a small core of leaders. Links among peripheral politicians are less common, and few links join different political groups. This architecture appears in networks of parties, interest groups, policy organizations, local governments, and courts. For example, Koger et al. (2009) identify separate Democratic and Republican networks, each with central party organizations and a fringe whose members often have only one connection.[1]

Factionalization is not limited to a two-party division. Political networks may contain multiple factions organized around different policy objectives. For instance, California policy networks contain several cohesive groups, while regional-planning networks are generally connected through central jurisdictions.[2] Courts display a similar ideological pattern. Citation, clerk-movement, and hiring ties concentrate among culturally or ideologically close courts and judges, whereas greater political distance separates the resulting groups. These networks have a core of leaders and a larger, less-connected periphery.[3]

Political networks do more than record alliances or common positions. Information transmission is one of the principal reasons they form. Local officials seek evidence about policies tried elsewhere, advocacy organizations exchange research and legal arguments, party organizations share political intelligence, and judges consult colleagues when they face unfamiliar questions. Indeed, the research subject of many empirical studies is precisely to measure information transmission in networks.[4] The empirical literature therefore

[1]Describing their evidence, Koger et al. (2009) write that "two distinct and polarized networks are revealed." The amicus networks in Box-Steffensmeier and Christenson (2014) likewise contain tightly grouped communities organized around highly connected organizations, while Oklobdzija (2024) finds central campaign organizations connected to less central organizations. The Appendix details these and the other empirical studies discussed here.

[2]The San Francisco Bay–Delta network has four groups: water exporters, shoreline-development agencies, Bay–Delta dischargers, and environmental organizations (Zafonte and Sabatier, 1998). The Marine Life Protection Act network likewise has four groups: local governments and harbormasters; commercial fishers, divers, public agencies, and environmental organizations; recreational fishers and passenger-fishing vessels; and kelp harvesters (Weible and Sabatier, 2005). Five California regional-planning networks form geographically defined groups connected, with few exceptions, through central jurisdictions (Henry, 2011; Henry et al., 2011; Gerber et al., 2013).

[3]State supreme courts disproportionately cite prestigious, culturally close courts, a pattern consistent with approximately star-shaped citation groups (Caldeira, 1985). The federal judicial network inferred from law-clerk movements contains a dense leadership core of Supreme Court justices, with circuit judges connected to it and district judges mainly at the periphery (Katz and Stafford, 2010). Ideological proximity also helps explain clerk movements and hiring ties (Ditslear and Baum, 2001; Baum, 2014).

[4]Weible and Sabatier (2005) observe that "most of the network literature focuses on information and advice networks." In the party networks of Koger et al. (2009), "other groups serve to funnel information towards the formal party." Leifeld and Schneider (2012) find that "political actors choose contacts who minimize transaction costs while maximizing outreach and information."

establishes two closely related facts: political networks are factional, and the exchange of information is a central purpose of many such networks.

This paper connects these facts by asking how factional political networks affect information transmission and welfare. I show that, when transmission is sufficiently reliable and link costs are neither too high nor too low, the welfare-maximizing networks are exactly the star-factional networks. I then prove that the same networks form endogenously: they are equilibria of the bilateral network-formation game of Myerson (1991) and the only equilibria immune to joint deviations by multiple politicians.

The model—presented in Section 3—considers political peers with diverse ideological preferences. One politician is randomly selected to make a decision and another to hold policy-relevant evidence. The informed politician observes a binary signal about a common state of the world. The evidence is verifiable: no one can invent or alter it, but anyone who receives it may withhold it. Further, an attempted transmission may fail at any network link. Hence longer paths are less reliable. All politicians value informed decisions, while each also wants policy to move toward her own preferred position. The network is formed before the informed politician and the decision maker are selected and before the state and evidence are known. It must therefore carry information across many possible sources and destinations rather than serve one permanent expert or leader.

My analysis begins in Section 4, where Proposition 1 characterizes communication along a single path. When political disagreement is small, every politician passes on both realizations of the evidence. With intermediate and asymmetric disagreement, exactly one politician—the most biased relative to the decision maker—mixes over whether to forward the realization that runs against her preference. Greater disagreement produces one-sided disclosure: politicians disclose only the realization that favors their biases. If the informed politician and all intermediaries are biased in the same direction relative to the decision maker, they suppress the realization that counters it; in equilibrium, silence moves her decision in the opposite direction. If sufficiently biased politicians lie on both sides of the decision maker, both realizations are blocked and no evidence arrives. Thus disagreement along a path produces full forwarding, mixed disclosure, one-sided disclosure, or breakdown, in decreasing order of informativeness. When non-strategic sources of information loss are negligible, mixed and one-sided disclosure are almost as informative as full forwarding.

The network analysis considers politicians who are partitioned into distinct ideological collections according to their political preferences. Ideologies are similar within each collection and sufficiently different across them.[5] In addition, every expert–decision-maker

[5]These collections are defined by ideology, not by pre-existing factions: their members need not coordi-

pair is equally likely, and every politician assigns the same weight to every decision. No politician therefore has an ex ante claim to leadership. Nevertheless, Proposition 2 shows that, when transmission is sufficiently reliable and link costs are neither too high nor too low, every efficient network is star-factional. The members of each collection form a star: all other members are linked to its center and not to one another, while the centers of adjacent ideological collections are connected. The center becomes the leader of the resulting faction, the other politicians are its peripheral members, and only leaders communicate between ideological collections. Symmetry leaves open which member becomes the center but does not produce shared leadership.

The result rests on four observations. First, when transmission is sufficiently reliable, links beyond those needed to connect all politicians add little information, whereas removing a necessary link prevents some politicians from learning from one another. With intermediate link costs, every efficient network is therefore minimally connected. Second, Proposition 1 implies that paths in every efficient network traverse collections in ideological order. Forwarding is therefore full or one-sided for every expert–decision-maker pair and never fully blocked. A minimally connected network that violates this order instead fully blocks transmission for at least one pair. Third, each collection must be internally connected. If a path between two members left and re-entered their collection, forwarding would be at best one-sided rather than full. Minimal connectivity then leaves a single link between each pair of adjacent collections. Finally, because evidence may be accidentally lost at every link, shorter paths are more valuable. Among factional trees, organizing every ideological collection as a star minimizes total path length within collections, and using each star's center for its links to adjacent collections minimizes total path length between collections.

Having established the optimality of star-factional networks in Proposition 2, I next ask whether they form endogenously when politicians choose their links. I adopt the bilateral network-formation game of Myerson (1991). Each politician simultaneously submits a list of the other politicians with whom she attempts to establish a link. She pays the link cost for every politician on her list, whether or not that politician reciprocates; the cost represents the effort involved in attempting to establish a connection. A link forms only when two politicians include one another on their lists.

To determine which networks can emerge from this game, I consider both individual and joint deviations. A Nash equilibrium rules out any profitable deviation by one politician. Coalitional stability instead rules out any joint deviation that raises the coalition's total

nate or have recognized leaders before the network is formed.

payoff when politicians outside it keep their lists fixed.[6]

Proposition 3 shows that, when every expert–decision-maker pair is equally likely and every politician assigns the same weight to every decision, every star-factional network can be supported as a Nash equilibrium. More strongly, these are the only Nash-equilibrium networks immune to joint deviations that raise the deviating coalition's total payoff. Hence the star-factional architecture is uniquely selected among the equilibria that survive both individual and joint deviations.

This endogenous selection is especially remarkable because it does not require homophily in link costs.[7] A link costs each endpoint the same amount regardless of their ideological distance. Unequal resources, institutional position, and social status can help explain the emergence of multiple leaders. Yet Proposition 3 shows that star-factional networks are the only Nash-equilibrium networks immune to joint deviations even when politicians are homogeneous. Ideological proximity and ordering matter because they make intermediaries more willing to transmit evidence, not because like-minded politicians find links cheaper or more pleasant to maintain.

Politicians' homogeneity is not needed for the general conclusion that efficient networks are factional, but it is substantive for the equivalence between efficiency and coalitional stability. With heterogeneous political interests, all politicians still benefit from more informative decisions, but they need not receive the same share of the benefit. An endpoint that values the relevant decisions very little may cut a socially valuable link.[8]

Proposition 4 shows that the optimality of a broader factional architecture persists when politicians are heterogeneous. The members of each ideological collection remain minimally connected, a single link joins adjacent collections, and there are no other links between them. The internal network within an ideological collection need not be a star, however, and the same politician need not be the endpoint of the links to adjacent collections on both sides. Thus the pattern of leadership may change. To study the selection of a single leader separately, Proposition 5 restricts attention to star networks when all politicians

[6]This concept applies the coalitional logic of the *core* introduced by Gillies (1959). Here a coalition is any set of jointly deviating politicians, not an ideological collection or faction. Members are treated as able to compensate one another, but such compensation is not a stage of the game. The Nash requirement is the special case that considers coalitions with one member.

[7]Homophily is the tendency for ties to form disproportionately between similar politicians (Lazarsfeld and Merton, 1954; McPherson et al., 2001). Here, homophily in link costs means that links between ideologically similar politicians are cheaper or more directly valuable than links between ideologically distant politicians.

[8]The Appendix gives a two-politician example. Every coalitionally stable network remains efficient, but with heterogeneous politicians an efficient network need not be a Nash equilibrium and hence need not be coalitionally stable.

belong to the same ideological collection. Together, these results show how the likelihood of being informed and of making relevant decisions shape the efficient choice of a leader among star networks. When the expert–decision-maker pairs that matter most for welfare involve different politicians, Example 1 shows that the efficient network may instead have a small core of leaders.

Empirically observed political networks often contain additional links relative to the star-factional theoretical benchmark. Shared membership, repeated interaction, friendship, and trust can make some internal links unusually cheap or directly valuable. Remark 1 shows when these forces make additional links within factions efficient. By contrast, Proposition 6 shows why transmission failures can raise the value of additional cross-faction links when the existing paths between factions are long. The location of additional links provides a way to distinguish these explanations empirically. Multiple paths are common inside factions, while links across factions are usually scarce. Thus low or negative costs for internal links are a more plausible source of most observed redundancy than transmission failures alone.

Finally, the factionalization results require politicians to be partitioned into distinct ideological collections whose members have similar political ideologies. This is the standard empirical representation of ideological polarization, which combines relative ideological homogeneity within collections with separation across them (Rehm and Reilly, 2010; Mehlhaff, 2024).[9] Examples 2 and 3 extend the analysis to settings in which ideological boundaries are less distinct. The former contains pairs too ideologically distant for one collection but too close for separate collections, and reorganizing links to shorten paths improves welfare upon star-factional networks. In the latter, one politician is ideologically close to two others who are not close to each other, and linking her to one of them improves welfare although neither is a star center.

The paper proceeds as follows. Section 2 reviews the related literature. Section 3 introduces the model, and Section 4 studies communication along a path. Section 5 presents the efficiency and stability results for homogeneous politicians. Section 6 considers heterogeneous politicians and link costs, the effect of information transmission losses, and ideology profiles not partitioned into distinct ideological collections. Section 7 concludes. Proofs and an extended empirical review appear in the Appendix.

[9]Roll-call evidence documents this pattern among state legislators (McCarty et al., 2019).

2 Literature Review

Empirical studies of policymakers, advocates, party organizations, and judges document information and collaboration networks that combine preference homophily, a small core of leaders, a larger periphery, and few cross-faction ties (Henry, 2011; Henry et al., 2011; Gerber et al., 2013; Koger et al., 2009; Box-Steffensmeier and Christenson, 2014; Caldeira, 1985; Katz and Stafford, 2010). The Appendix reviews this evidence together with the purposes and measurement of the observed links.

Rose (1964) uses the term *tendency* for a stable pattern of political ideologies and reserves *faction* for an organized group with a recognized internal structure. Theories of factional politics commonly begin with electoral competition, bargaining, discipline, or leaders who advance a faction's interests (Janda, 1980, 1993; Persico et al., 2011). Dewan and Squintani (2016) show that ideological factions can raise welfare because politicians voluntarily transfer influence over the party manifesto to moderate, better-informed faction leaders, even in the case that factions restrict cross-faction communication. This paper gives an entirely different reason for the factions' optimality: factional networks keep evidence disclosure among politicians nearly perfectly informative while minimizing link costs.

Related lobbying work studies how ideological and procedural positions determine the value of access to legislators (Awad, 2020; Awad and Minaudier, 2026). Here the peer network forms before the relevant expert and decision maker are known. The central politician is therefore not the policymaker to target in one contest, but the person who can carry evidence efficiently across many possible sources and destinations. Work on discussion partners shows that the most knowledgeable contact need not have political ideologies closest to the choosing politician's own (Ahn et al., 2013; Ahn and Ryan, 2015); Proposition 5 explains how expertise, authority, and relevance enter the efficient choice of a center among star networks.

Methodologically, the paper joins models of political networks with the literature on verifiable disclosure. The latter studies information that can be hidden but not falsified (Milgrom, 1981; Gieczewski, 2022). The former examines learning, diffusion, cooperation, and influence through political relationships.[10] For network design, Jackson and Wolinsky (1996) show that, at intermediate link costs, a star trades off short indirect connections against the number of costly links. Galeotti et al. (2006) extend that framework to politi-

[10] Examples include Chwe (2000); Volden et al. (2008); Larson (2017); Larson and Lewis (2018); Battaglini and Patacchini (2018); Canen et al. (2023); Victor et al. (2017) provide a broad survey. Ferrali et al. (2020) show how institutions that sustain truthful communication shape peer effects in the adoption of a political technology.

cians who differ in the value and cost of links and show that highly connected positions and short distances remain robust features of equilibrium networks. Their heterogeneity is especially relevant to Section 6. However, my Proposition 1 shows that the value of a path for strategic communication also depends on the ideology of every intermediary.

The closest methodological comparison is with Squintani (2026). He shows that precise disclosure of a continuous state requires the expert's and intermediaries' ideologies to lie on the same side of the decision maker, selecting an ordered line. Here evidence is binary and the expert may be uninformed. Small bias reversals can preserve full forwarding or mixed disclosure; breakdown requires opposite sides to stop different realizations (Proposition 1). Hence an interval of close ideologies permits full forwarding within ideological collections, and efficient networks result in star-factional networks rather than only points on an ordered line (Proposition 2).

3 Model

The model has three stages. First, politicians form a network. Nature then selects an expert, a decision maker, a state, and the expert's information. Finally, evidence travels through the network and the decision maker chooses policy.

There is a finite set of politicians $\mathcal{N} = \{1, \ldots, n\}$, with $n \geq 2$. Politician i has ideological bias $b_i \in \mathbb{R}$. Biases are common knowledge and distinct. Without loss of generality, they are ordered as

$$b_1 < b_2 < \cdots < b_n.$$

The politicians are connected by an undirected network g, and $\mathcal{L}(g)$ denotes its set of links. A link $ij \in \mathcal{L}(g)$ allows i and j to pass verifiable evidence to one another. Each link costs $c > 0$ to each one of the linked politicians. The network is chosen before the expert, the decision maker, the state, and the expert's information are known.

Nature selects a pair (e, d) of distinct politicians and a state θ. Politician e is the potential expert and d is the decision maker. Each pair (e, d) is selected with a strictly positive probability π_{ed}, and $\sum_{e \neq d} \pi_{ed} = 1$. Hence, every politician may have evidence useful for every other politician's decision. The state θ is uniformly distributed on $[0, 1]$. When communication begins, everyone knows the network, the biases, and the identities of e and d.

For each politician i, let $X_i = \sum_{d \neq i} \pi_{id}$ be i's *expertise*, the probability that she is selected as the potential expert, and $A_i = \sum_{e \neq i} \pi_{ei}$ be her *authority*, the probability that she is selected as the decision maker. Expertise and authority are the marginal probabilities generated by (π_{ed}). The joint probabilities additionally describe how politicians who hold evidence are matched with the decision makers whom that evidence informs.

Independently of (e, d) and θ, the expert is informed with probability $\lambda \in (0, 1)$, which I call *evidence availability.*[11] She privately observes whether she is informed. If she is, she observes a binary signal $s_e \in \{0, 1\}$ such that $\Pr(s_e = 1|\theta) = \theta$. Hence, $\mathbb{E}[\theta|s_e = 0] = 1/3$, $\mathbb{E}[\theta|s_e = 1] = 2/3$, $\mathrm{Var}(\theta|s_e) = 1/18$. I call the pair (e, s_e) the evidence; a non-silent message carries this pair. No one else observes her informed status or signal unless the evidence reaches them. Thus politician i's unconditional probability of holding evidence is λX_i.

After communication, d chooses policy $y \in \mathbb{R}$. As is standard in spatial models of strategic communication since Crawford and Sobel (1982), each politician i's payoff is expressed by a quadratic loss function:

$$u_i(y, \theta; d) = -\alpha_{id}(y - \theta - b_i)^2.$$

The weight $\alpha_{id} > 0$ measures how much politician i cares about decisions made by d. Let $R_d = \sum_{i \in \mathcal{N}} \alpha_{id} > 0$ be the *relevance* of decisions made by d: the aggregate weight politicians place on those decisions.

Politicians are partitioned into ideological collections with similar biases within each collection and sufficient separation across collections. Formally, let $(C_1, \ldots, C_M)$ be a partition of $\mathcal{N}$ in which each C_m contains consecutive politicians in the ideological ordering, and define $\overline{\beta}(\lambda) = 1/[6(2 - \lambda)]$. The partition satisfies

$$\max_{i,j \in C_m} |b_i - b_j| \leq \frac{1}{12} \text{ for every } m, \tag{1}$$

$$\min_{i \in C_m, j \in C_{m+1}} (b_j - b_i) > \overline{\beta}(\lambda) \text{ for every } m < M. \tag{2}$$

As later shown in Propositions 2 to 4, these bounds are tight for the communication patterns underlying those results. The upper bound in (1) is the largest within-collection bias difference for which both realizations of the evidence are forwarded between every pair of politicians in the same collection. The cutoff in (2) is the smallest cross-collection threshold above which communication between adjacent ideological collections is one-sided rather than full or mixed. The conclusions of the three propositions therefore also hold when bias differences are smaller within collections or larger across collections.

Following network terminology, say that two politicians are *connected* if a sequence of links joins them. A *path* is such a sequence that never visits the same politician twice, and its length is its number of links. A path is *ordered* if the indices of the ideological collections

[11] Even when transmission is perfect—that is, every non-silent message reaches its receiver—silence is ambiguous when $\lambda < 1$: the expert may be uninformed, or evidence may have been withheld.

it visits are increasing or decreasing. A network is connected if every pair of politicians is connected. A *cycle* is a closed sequence of links that does not otherwise repeat a politician. A *tree* is a connected network without a cycle; equivalently, exactly one path joins every pair of politicians. A network is *minimally connected* if it is connected but removing any link disconnects it; equivalently, it is a tree.

On a tree, evidence simply moves along the unique path from the expert to the decision maker. In a network with cycles, e and d may be linked by several paths. Let $\mathcal{P}_{ed}(g)$ denote the set of such paths, and write each path as $p = (v_0 = e, v_1, \dots, v_{\ell(p)} = d)$. Along this path, (v_t, v_{t+1}) is the *link* at position t. The protocol moves one step along every path at each date. The first link of each path is scheduled at date zero, the second at date one, and so forth. Evidence received at date t can be forwarded from date $t + 1$ onward.

Let $J_i^t(e, d, g)$ be the set of politicians to whom i is scheduled to send at date t. Politician i chooses the vector $m_i^t = (m_{ij}^t)_{j \in J_i^t(e,d,g)}$ of all her outgoing messages. A non-silent message reaches its receiver with probability $\delta \in (0, 1]$, independently across dates and links. I refer to δ as *communication reliability.* A politician may forward only evidence that she holds, and cannot change its source or realization. If e and d are connected, the decision maker chooses policy y after all scheduled transmissions, at date $\max_{p \in \mathcal{P}_{ed}(g)} \ell(p)$. If they are disconnected, communication is skipped and she acts on the prior. The Appendix presents the exact communication protocol and strategy spaces.

The solution concept is perfect Bayesian equilibrium. For brevity, I focus on equilibria in which a politician who is indifferent between forwarding evidence and remaining silent forwards the evidence, and I refer to them as equilibria for short. This tie-breaking rule only selects among pure best replies; it does not exclude equilibria that require mixing. The Appendix gives the formal details.

An equilibrium exists for every network, realized expert–decision-maker pair (e, d), and information transmission reliability δ. I select the *most informative equilibria*, i.e., those that minimize the decision maker's expected posterior variance. This selection does not favor one politician over another. As proved in the Appendix, ex ante, all politicians prefer communication outcomes that leave the decision maker with less uncertainty.

Let $V_{ed}(g, \delta)$ be the expected posterior variance in a most informative equilibrium for pair (e, d). After dropping terms that do not depend on the network, aggregate ex-ante welfare is

$$W(g; \delta, c) = -\sum_{e \neq d} \pi_{ed} R_d V_{ed}(g, \delta) - 2c|\mathcal{L}(g)|, \tag{3}$$

where $|\mathcal{L}(g)|$ is the number of links in g. A network is *efficient* if it maximizes $W(g; \delta, c)$.

The results in Section 5 use the following symmetry assumptions. Politicians are *homogeneous* when every expert–decision-maker pair is equally likely, so that $\pi_{ed} = 1/[n(n-1)]$ for every $e \neq d$, and when weights are the same across identities, so that $\alpha_{id} = \alpha > 0$ for every i, d. Consequently, $X_i = A_i = 1/n$, $R_i = n\alpha$ for every i: homogeneous politicians have equal expertise, authority, and relevance.[12] Ideological biases remain distinct.

A *factional network* for $(C_1, \ldots, C_M)$ connects all members of each C_m, has exactly one link between C_m and C_{m+1} for every $m < M$, and has no other links between ideological collections. The network among the members of an ideological collection may contain cycles. A *factional tree* is a factional network that is also minimally connected. A factional tree is *star-factional* if the members of every C_m are linked to a center z_m and neighboring centers z_m and z_{m+1} are linked.

4 Strategic disclosure along a path

I begin by considering an expert and a decision maker connected in a network through a unique path $p = (e, \ldots, d)$ of length ℓ. If everyone forwards the evidence, the chance that it is observed by e and it reaches d is $r_\ell = \lambda\delta^\ell$. The following cutoffs will determine whether a politician forwards evidence that runs against her preference:

$$\underline{\beta}(r_\ell) = \frac{1 - r_\ell}{6(2 - r_\ell)}, \qquad \overline{\beta}(r_\ell) = \frac{1}{6(2 - r_\ell)}.$$

Because $r_\ell \in (0, 1)$, it is the case that $0 < \underline{\beta}(r_\ell) < 1/12 < \overline{\beta}(r_\ell) < 1/6$.

Only the most left- and right-biased politicians on the path matter for the equilibrium boundaries. Relative to decision maker d, write

$$\overline{x}_p = \max_{j \in p \setminus \{d\}} (b_j - b_d), \qquad \underline{x}_p = \min_{j \in p \setminus \{d\}} (b_j - b_d).$$

The path has a *bias reversal* when it contains politicians on both sides of d, so that $\underline{x}_p < 0 < \overline{x}_p$, whereas it has a *strong bias reversal* when these politicians also cross the two blocking cutoffs:

$$\overline{x}_p > \overline{\beta}(r_\ell), \qquad \underline{x}_p < -\overline{\beta}(r_\ell).$$

A path lies in the *right-mixed region* if

$$\underline{\beta}(r_\ell) < \overline{x}_p < \frac{1}{12}, \qquad \overline{x}_p - \frac{1}{6} \leq \underline{x}_p < -\frac{1}{12}.$$

[12] Uniformity of the pair is stronger than equal expertise and authority: it also rules out correlation between expert and decision-maker identities. Similarly, $\alpha_{id} = \alpha$ is stronger than equal relevance: it gives every politician the same stake in every decision.

It lies in the *left-mixed region* if

$$-\frac{1}{12} < \underline{x}_p < -\underline{\beta}(r_\ell), \qquad \frac{1}{12} < \overline{x}_p \leq \underline{x}_p + \frac{1}{6}.$$

There are four possible equilibrium outcomes. Under *full forwarding*, every politician passes both realizations of s_e. Under *mixed disclosure*, exactly one politician mixes, and only over one realization. In the right-mixed region, a politician attaining $\overline{x}_p$ mixes when $s_e = 0$; in the left-mixed region, a politician attaining $\underline{x}_p$ mixes when $s_e = 1$. Every other politician passes both realizations. In the final two cases, politicians biased to the right of the decision maker pass $s_e = 1$, which moves policy to the right, and block $s_e = 0$. Politicians biased to the left pass $s_e = 0$ and block $s_e = 1$. Under *one-sided disclosure*, only one realization can reach the decision maker. Communication *breaks down* when neither realization ever reaches her. The exact boundaries of the equilibrium regions are shown in Figure 1 and stated in the Appendix.

Proposition 1 *Suppose expert e and decision maker d are joined by the unique path $p = (e, \ldots, d)$.*

(a) Full forwarding is an equilibrium if and only if $\max_{j \in p \setminus \{d\}} |b_j - b_d| \leq 1/12$, *and then it is the most informative equilibrium.*

(b) If p lies in a mixed region, a most informative equilibrium has mixed disclosure.

(c) A strong bias reversal produces communication breakdown in every equilibrium.

(d) In the remaining regions, a most informative equilibrium has one-sided disclosure.

The full-forwarding cutoff $1/12$ has a simple interpretation. When both realizations of s_e are forwarded, silence does not favor either one, and the decision maker therefore keeps the prior mean $1/2$. A right-biased politician who observes $s_e = 0$ compares the policy based on that evidence with the policy chosen after silence. She prefers to forward when her bias relative to the decision maker is below $1/12$ and is indifferent at equality. The argument for a left-biased politician who observes $s_e = 1$ is symmetric.

Why does mixing arise? In the right-mixed region, the most right-biased politician withholds $s_e = 0$ with positive probability, thereby making silence informative and keeping the most left-biased politician willing to forward $s_e = 1$. The most right-biased politician's indifference therefore determines the mixing probability. The left-mixed case works symmetrically. When $\overline{x}_p > \overline{\beta}(r_\ell)$ and $\underline{x}_p < -\overline{\beta}(r_\ell)$, the most right-biased politician blocks realization 0 and the most left-biased politician blocks realization 1. Neither realization can therefore reach the decision maker.

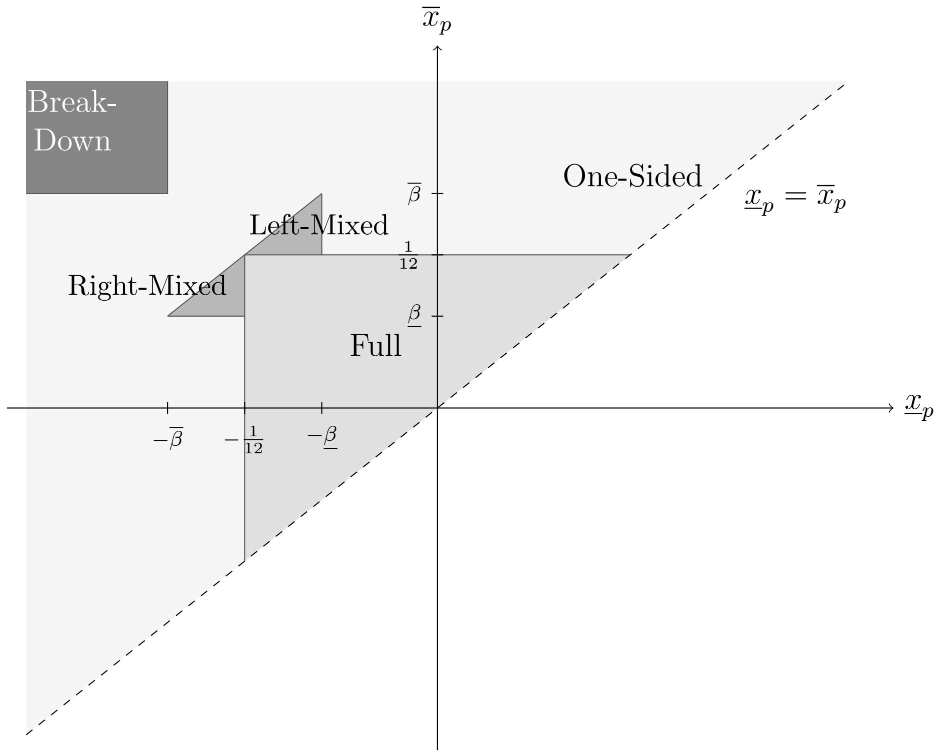


Figure 1: The disclosure outcome in a most informative equilibrium as a function of the most left- and right-biased politicians on the path. The feasible set satisfies $\underline{x}_p \leq \overline{x}_p$ and lies above the diagonal. Here $\underline{\beta} = \underline{\beta}(r_\ell)$ and $\overline{\beta} = \overline{\beta}(r_\ell)$.

Mixed disclosure does not arise in the network analysis of Sections 5 and 6 because, as explained later, the bounds defining ideological collections rule it out. Under full forwarding, evidence s_e arrives with probability r_ℓ; under one-sided disclosure, the only forwarded realization arrives with probability $r_\ell/2$. As proved in the Appendix, the expected posterior variances under full forwarding, one-sided disclosure, and breakdown are

$$V_F(r_\ell) = \frac{3 - r_\ell}{36}, \qquad V_O(r_\ell) = \frac{3 - 2r_\ell}{18(2 - r_\ell)}, \qquad V_B = \frac{1}{12}. \tag{4}$$

Full forwarding is best, one-sided disclosure is less informative, and breakdown is worst; mixed disclosure lies between full forwarding and one-sided disclosure. These rankings are strict because silence may reflect an uninformed expert, withheld evidence, or evidence that failed to arrive; they underlie the network comparisons in the following sections.

5 Network analysis: The homogeneous case

Efficiency. To isolate the effect of ideology on network structure, assume that politicians are homogeneous. The politicians remain ideologically distinct but have equal expertise,

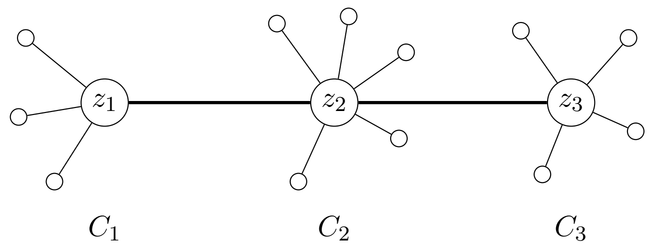


Figure 2: A star-factional network. The members of each ideological collection are organized around a center, and only the centers communicate between adjacent collections.

authority, and relevance.

Conditions (1)–(2) distinguish the communication possible within and across ideological collections. Both realizations of s_e can be forwarded between members of the same collection. When the expert and decision maker belong to different collections, no path can induce full forwarding between them. Along an ordered path across collections, one realization is forwarded at every link, while the other is stopped somewhere along the path.

Figure 2 illustrates a star-factional network. Each ideological collection forms a faction organized as a star. Its center is the faction leader: she receives information from the other members and is the only member linked to the leaders of neighboring factions.

Proposition 2 *Suppose politicians are homogeneous and a partition $(C_1, \ldots, C_M)$ satisfies (1)–(2). There exist $\bar{\delta}_S \in (0,1)$ and thresholds $\underline{c}(\delta) < \bar{c}(\delta)$ such that, for every $\delta \in (\bar{\delta}_S, 1)$ and $c \in \big(\underline{c}(\delta), \bar{c}(\delta)\big)$, the efficient networks are exactly the star-factional networks.*

To gather intuition for the result, first let $\mathcal{I}$ contain the pairs (e, d) of distinct politicians in the same ideological collection, let $\mathcal{X}$ contain those in different collections, and let $d_g(e, d)$ denote the length of the path from e to d. When communication is perfectly reliable, every factional tree achieves

$$\bar{V}_{ed} = \begin{cases} V_F(\lambda), & e, d \in C_m \text{ for some } m, \\ V_O(\lambda), & e \in C_m,\ d \in C_r,\ m \neq r. \end{cases} \tag{5}$$

No network can do better. Full forwarding is the most informative feasible outcome within a collection. Across collections, one realization is stopped somewhere along the path, while the other cannot arrive more often than with one-sided forwarding.

When transmission is sufficiently reliable and link costs are neither too high nor too low, every efficient network is a factional tree, as it achieves near perfect disclosure. The gain from extra links relative to an efficient tree vanishes as communication reliability approaches one, whereas fewer links disconnect the network and prevent some pairs from communicating. Among trees, a path between members of a collection that leaves and re-enters it permits at best one-sided disclosure, while joining collections out of ideological order blocks both realizations for some pair.

For a factional tree, define

$$D_S(g) = \frac{\lambda}{36} \sum_{(e,d)\in\mathcal{I}} d_g(e,d) + \frac{\lambda}{18(2-\lambda)^2} \sum_{(e,d)\in\mathcal{X}} d_g(e,d). \tag{6}$$

As communication reliability approaches one, the ratio between the loss of informational welfare relative to perfect reliability and $(1-\delta)D_S(g)$ converges to a positive constant common to all factional trees. The Appendix proves that $D_S(g)$ is minimized when every ideological collection forms a star and the center of each star is used for all its links to adjacent collections. Every member has the same expertise, authority, and relevance. Symmetry therefore leaves the leader's identity open but selects exactly one leader in each collection.[13]

The papers discussed in the introduction document a recurring structure: politicians with similar ideologies form factions around a small core of leaders, most peripheral politicians have few links, and only a small number of links cross factional boundaries.

Proposition 2 makes the separate normative claim that star-factional networks maximize welfare, as they deliver near perfect disclosure at minimal cost. Its welfare advantage does not depend on leaders having greater expertise, authority, or relevance, or on their links being cheaper. The result applies whether the factions are formal political organizations or recurring patterns of interaction among policymakers, advocates, and judges. Concentrating internal links on one member allows every peripheral politician to communicate with the rest of the faction while maintaining only one link. Using that same member for communication with adjacent factions also keeps cross-faction paths short. Observed leadership cores may be denser than literal stars, but the star-factional network isolates the informational value of their common sparse architecture.

Coalitional stability. Having established in Proposition 2 which networks maximize welfare, I ask whether star-factional networks form endogenously when politicians choose their own links. I adopt the bilateral proposal game of Myerson (1991). Network formation takes place before the expert, the decision maker, and the state are known. Simultaneously, each politician i submits a list $L_i \subseteq \mathcal{N} \setminus \{i\}$ of politicians with whom she attempts to

[13] The only exception occurs when there are two politicians in a single ideological collection. The network then consists of one link, which is a star centered at either politician, so it does not distinguish a unique leader.

establish a link. Politician i pays c for every name on her list, whether or not the other politician reciprocates. The cost represents the effort involved in attempting to establish a connection.

Link ij forms if and only if $j \in L_i$ and $i \in L_j$. Let $L = (L_i)_{i \in \mathcal{N}}$ denote the proposal profile and $g(L)$ the network it induces. Omitting terms that do not depend on the proposal profile, politician i's ex-ante payoff is

$$U_i(L) = -\sum_{e \neq d} \pi_{ed} \alpha_{id} V_{ed}(g(L), \delta) - c|L_i|. \tag{7}$$

Communication in the chosen network follows a most informative equilibrium. For any network f, its *associated proposal profile* L^f is such that L_i^f contains all and only the politicians linked to i in f. At this profile, both endpoints of every link incur the proposal cost, so the aggregate payoff of all politicians coincides with the welfare criterion in (3).

In terms of solution concept, note that Nash equilibrium allows coordination failure: two politicians may both reject a mutually valuable link because each expects the other to reject it. I use a stronger coalitional test. Take any coalition K of politicians and let its members revise their proposals while politicians outside the coalition keep their proposals unchanged. The term coalition refers only to a set of politicians considering a joint change; it does not refer to a collection in the ideological partition or a faction. Write L_{-K} for the list profile of the politicians outside K.

A proposal profile L is *coalitionally stable* if no such coalition K can raise its total payoff. Formally, there is no nonempty $K \subseteq \mathcal{N}$ and no alternative list profile L'_K such that

$$\sum_{i \in K} U_i\big(L'_K, L_{-K}\big) > \sum_{i \in K} U_i(L). \tag{8}$$

Members may compensate one another, so a deviation is profitable when it raises their total payoff even if one member would otherwise lose.[14] Because a coalition may contain only one politician, coalitional stability implies Nash equilibrium. A network is coalitionally stable if some proposal profile inducing it is stable.

Proposition 3 *Suppose politicians are homogeneous and a partition $(C_1, \dots, C_M)$ satisfies (1)–(2). There exist $\bar{\delta}_C \in (0,1)$ and thresholds $\underline{c}^C(\delta) < \overline{c}^C(\delta)$ such that, for every $\delta \in (\bar{\delta}_C, 1)$ and $c \in \big(\underline{c}^C(\delta), \overline{c}^C(\delta)\big)$, the coalitionally stable networks are exactly the star-factional networks.*

When politicians are homogeneous, every politician receives the same share of total informational welfare. Fix an efficient star-factional tree f, its associated proposal profile L^f,

[14] These transfers are only a way to evaluate deviations; they are not another stage of the model.

a deviating coalition K, an alternative profile L'_K, and the induced network $g = g(L'_K, L^f_{-K})$. Any unreciprocated proposal by a coalition member is wasteful and can be removed. Moreover, every link added to f must have both endpoints in K, so the coalition pays both endpoint costs while receiving only its members' share of the informational gain.

If g remains connected, it has at least $n - 1$ links, the same number as f. Efficiency rules out a gain for the grand coalition, and the smaller informational share of any proper coalition rules out such a gain for that coalition. If g is disconnected, at least one possible expert–decision-maker pair cannot communicate. The upper cost threshold makes the coalition's share of this loss greater than its largest possible cost saving. Conversely, the grand coalition can replace any inefficient network with an efficient one.

Propositions 2 and 3 make distinct claims. Proposition 2 is normative: star-factional networks maximize welfare. Proposition 3 is positive: star-factional networks arise when politicians choose their links, and they are the only Nash-equilibrium networks immune to joint deviations by multiple politicians that raise the deviating politicians' aggregate payoff.

The positive result also helps explain why leadership roles may emerge without being assigned by a formal organization. Moving a faction's cross-faction connection to another member, duplicating that connection, or disconnecting from the leader weakly lowers the joint payoff of the politicians who undertake the change. This mechanism applies most directly to policy and judicial networks in which coordination is decentralized and leadership is not imposed by a party constitution.

The equivalence between efficiency and coalitional stability does not extend to heterogeneous politicians. A politician who places little weight on political decisions may prefer to sever a link whose informational benefit to others makes it socially valuable. Hence an efficient network need not even be a Nash equilibrium.[15] The reverse implication survives: if a network is inefficient, the grand coalition can replace it with an efficient one.

6 Extensions and generalizations

When politicians are homogeneous, every efficient network is star-factional: each faction has one leader, and only leaders communicate across factions. This section asks when departures from this benchmark can raise welfare.

The first part studies who should lead. Proposition 4 shows that efficient networks remain factional trees when politicians differ in expertise, authority, or relevance. Propo-

[15]The Appendix gives a two-politician counterexample.

sition 5 characterizes efficient centers among stars, while Example 1 shows how a network with two leaders can be uniquely efficient.

The political factions discussed in the introduction often contain links beyond those in the theoretical star-factional benchmark. The second part of the section asks whether these additional links can raise welfare, again allowing politicians to differ. Multiple paths within factions improve welfare when link costs are lower within ideological collections (Remark 1), whereas substantial information-transmission losses can make additional cross-faction links welfare-improving (Proposition 6).

The final part extends the analysis to the theoretical possibility that ideological proximity does not partition politicians into distinct collections. Although this is less central to the empirical applications considered here, Examples 2 and 3 suggest that preserving full forwarding between ideologically close politicians may require longer paths elsewhere, whereas shortening many paths may require links between politicians whose ideologies are further apart.

Optimal factional trees without homogeneity. Proposition 4 below shows that efficient networks are factional also when politicians are heterogeneous. What changes is the precise shape of the tree connecting the members of each ideological collection.

Proposition 4 *Suppose $\pi_{ed} > 0$ for every $e \neq d$, $R_d > 0$ for every d, and a partition $(C_1, \dots, C_M)$ satisfies* (1)–(2)*. There exist $\bar{\delta}_F \in (0,1)$ and thresholds $\underline{c}(\delta) < \overline{c}(\delta)$ such that, for every $\delta \in (\bar{\delta}_F, 1)$ and $c \in \big(\underline{c}(\delta), \overline{c}(\delta)\big)$, every efficient network is a factional tree.*

The result in Proposition 4 preserves many features of star-factional networks. The members of every ideological collection remain connected, exactly one link joins each pair of adjacent collections, and there are no other links between them. However, the internal tree need not be a star, and the same politician need not be the endpoint of the cross-collection links on both sides. Write $w_{ed} = \pi_{ed} R_d$ for the welfare weight on communication from expert e to decision maker d: it measures how important communication from expert e to decision maker d is for welfare.[16] When politicians are homogeneous, w_{ed} is the same for every pair, so shortening any two paths with the same forwarding outcome by one link yields approximately the same welfare gain. When politicians differ, w_{ed} is larger for pairs selected more often and for decision makers with greater relevance. Other things equal,

[16] Expertise and authority are the marginals of π_{ed}, but the joint probability also records which experts are matched with which decision makers. Hence expertise, authority, and relevance do not alone determine w_{ed}.

shortening the path from e to d has a larger welfare benefit when w_{ed} is larger. The efficient internal tree balances these benefits across all pairs.

Observed factions commonly have a small core of leaders rather than a single center. Proposition 4 makes a separate normative claim. When politicians differ in expertise, authority, relevance, or expert–decision-maker matching, a sparse multi-faction network remains efficient. The weights w_{ed} determine which paths are most valuable to shorten and may therefore make it efficient for a faction to have several leaders.

Who should be the faction leader? The welfare case for leadership may rest on expertise, authority, or relevance. Suppose all politicians belong to the same ideological collection, and compare stars using the welfare weights $w_{ed} = \pi_{ed} R_d$ defined above. In a star centered at z, the path from expert e to decision maker d has length one when $z \in \{e, d\}$ and length two otherwise. Define

$$\Gamma_z = \sum_{d \neq z} w_{zd} + \sum_{e \neq z} w_{ez}. \tag{9}$$

Thus Γ_z is the sum of the welfare weights for the pairs whose path has length one in the star centered at z; every other pair has path length two.

Lemma 1 *Suppose all politicians belong to the same ideological collection and $\delta \in (0, 1)$. Among star networks, the efficient centers are* $\arg\max_{z \in \mathcal{N}} \Gamma_z$.

Lemma 1 is intuitive. Because shorter paths transmit evidence more reliably, the efficient center of a star network is the politician for whom these direct paths have the greatest combined welfare weight. Other things equal, greater expertise, authority, or relevance strengthens the case for selecting a politician as the center. In general, however, none of these characteristics is sufficient by itself: Γ_z combines all three with expert–decision-maker matching.

To isolate how expertise, authority, and relevance affect the efficient choice of a leader, rewrite Γ_z as

$$\Gamma_z = X_z \left(\sum_{d \neq z} \frac{\pi_{zd}}{X_z} R_d \right) + A_z R_z.$$

The first term is z's expertise multiplied by the average relevance of the decisions for which she is the expert. The second is her authority multiplied by her own relevance.[17] Further,

[17] When relevance is common across politicians, efficient centers among stars maximize $X_z + A_z$, expertise plus authority. When every expert–decision-maker pair is equally likely and $n \geq 3$, efficient centers among stars maximize R_z, relevance.

suppose the pair probabilities π_{ed} admit the multiplicative factorization

$$\pi_{ed} = \frac{p_E(e)p_D(d)}{\sum_{r \neq s} p_E(r)p_D(s)}, \qquad e \neq d, \tag{10}$$

where $p_E(i) > 0$ and $p_D(i) > 0$ are politician i's source-selection and decision-maker-selection weights, respectively.

Proposition 5 *Suppose all politicians belong to the same ideological collection, $\delta \in (0,1)$, and the pair probabilities satisfy* (10)*. The efficient centers among star networks are*

$$\arg\max_{z \in \mathcal{N}} \left\{ p_E(z) \sum_{d \neq z} p_D(d) R_d + p_D(z) R_z \sum_{e \neq z} p_E(e) \right\}. \tag{11}$$

If $n \geq 3$, the source-selection weights, decision-maker-selection weights, and relevance terms are common across all politicians other than i, while i has weakly higher values with at least one strictly higher, then i is the unique efficient center among stars.

The above result distinguishes the source-selection weight p_E governing expertise, the decision-maker-selection weight p_D governing authority, and relevance R_d as three reasons why placing a politician at the center may maximize welfare. The welfare comparisons have clear implications. Other things equal, expertise makes paths from a politician more likely, authority makes paths to her more likely, and relevance makes evidence reaching her more valuable.

Proposition 5 identifies the efficient center within the class of star networks. The following example shows why a network with several leaders can outperform every star. When high-weight expert–decision-maker pairs involve different politicians, requiring the network to be a star may lengthen paths important for welfare because each path must include the single center. Multiple leaders can thus be efficient within one ideological collection because of the distribution of welfare weights, not weak ideological cohesion.

Example 1 *Let g^L be the four-politician line with $\mathcal{L}(g^L) = \{12, 23, 34\}$. Suppose all four politicians belong to the same ideological collection and their welfare weights $w_{ed} = \pi_{ed} R_d$ are such that*

$$w_{ed} = \begin{cases} 1, & ed \in \mathcal{L}(g^L), \\ \varepsilon, & ed \notin \mathcal{L}(g^L), \end{cases} \qquad e \neq d, \qquad 0 < \varepsilon < \frac{1}{7}.$$

There exist $\bar{\delta}_L \in (0,1)$ and thresholds $\underline{c}_L(\delta) < \bar{c}_L(\delta)$ such that, for every $\delta \in (\bar{\delta}_L, 1)$ and $c \in \big(\underline{c}_L(\delta), \bar{c}_L(\delta)\big)$, the line g^L is the unique efficient network. Its two middle politicians, 2 *and* 3*, form a two-politician leadership core.*

To compare welfare across trees, consider the welfare-weighted total path length

$$D(g) = \sum_{e \neq d} w_{ed} d_g(e,d).$$

In g^L, the six unit-weight pairs are adjacent, while the distances of the other six pairs sum to 14. Hence $D(g^L) = 6 + 14\varepsilon < 8$. Any other tree g omits some link ij of g^L. The unit-weight pairs (i,j) and (j,i) then contribute at least 4, and the other four unit-weight pairs contribute at least another 4. The positive weights on the remaining pairs imply $D(g) > 8$. Thus g^L uniquely minimizes the first-order welfare loss among trees. For $\delta < 1$ sufficiently close to one, it therefore has the highest welfare among trees. The stated bounds on link costs ensure that disconnected networks and networks with more than three links yield lower welfare, so g^L is the unique efficient network.

When additional links are efficient. Two mechanisms can make it efficient to add links to a star-factional tree. The first is homophily in relationship costs: links between ideologically similar politicians may be cheaper or directly valuable. Remark 1 formalizes the case of directly valuable relationships. Common committee service, shared staff, repeated interaction, organizational membership, friendship, and trust can generate such homophily. Politicians balance the cost of a tie against the information it provides, and personal relationships shape how officials respond to political messages (Leifeld and Schneider, 2012; Grose et al., 2022).

Let $c_{ij} = c_{ji}$ be the total cost borne by the two endpoints of link ij. It may be negative when the relationship provides a direct benefit. Welfare becomes

$$W(g; \delta, \{c_{ij}\}) = -\sum_{e \neq d} \pi_{ed} R_d V_{ed}(g, \delta) - \sum_{ij \in \mathcal{L}(g)} c_{ij}. \tag{12}$$

Remark 1 *Suppose all politicians belong to the same ideological collection and communication is perfectly reliable. Every link with $c_{ij} < 0$ belongs to every efficient network. If the negative-cost links contain a cycle, no efficient network is a tree. If they contain a clique, that clique belongs to every efficient network.*[18]

Under Remark 1, an added internal link preserves full forwarding, cannot reduce information in the most informative equilibrium, and strictly raises welfare through its direct benefit. Thus negative or very low relationship costs can make dense factional networks efficient even when strategic communication alone selects a star.

[18] A *clique* is a set of politicians in which every pair is linked.

When link costs are positive, a link raises welfare only if its informational benefit exceeds its cost. Cost heterogeneity can therefore make additional links within ideological collections efficient, including links between peripheral politicians or between a peripheral politician and the core of leaders. An unusually cheap cross-collection relationship can likewise make an additional cross-faction link efficient. If relationship costs exhibit homophily, however, cheap or negative cost links are more common within ideological collections.

A second reason for networks to contain more links than a star-factional tree is that communication is not perfectly reliable. Even when all links cost the same, an additional link may improve transmission by shortening paths or giving evidence another chance to reach the decision maker.

For a factional tree, let $\mathcal{I}$ contain the pairs (e, d) with $e \neq d$ whose members belong to the same ideological collection, and let $\mathcal{X}$ contain those whose members belong to different ideological collections. Define

$$D(g) = \kappa_F \sum_{(e,d)\in\mathcal{I}} w_{ed} d_g(e, d) + \kappa_O \sum_{(e,d)\in\mathcal{X}} w_{ed} d_g(e, d), \tag{13}$$

where $\kappa_F = \lambda/36$ and $\kappa_O = \lambda/[18(2-\lambda)^2]$. $D(g)$ is the network's welfare-weighted total path length. Each distance is weighted by the pair's welfare importance w_{ed} and by κ_F or κ_O, according to whether the pair communicates within an ideological collection under full forwarding or across collections under one-sided disclosure. When communication is highly reliable, $D(g)$ measures the first-order welfare loss caused by longer paths; hence a lower $D(g)$ implies higher welfare among factional trees.

For the full and one-sided transmission equilibrium $\mathcal{E}$, further define

$$\Delta_{\mathcal{E}}(\ell, \delta) = V_{\mathcal{E}}(\lambda\delta^{\ell+1}) - V_{\mathcal{E}}(\lambda\delta^{\ell}) \tag{14}$$

as the increase in expected posterior variance when the path is lengthened from ℓ to $\ell + 1$ links.

For a network g without link ij, write $g + ij$ for the network obtained by adding it, and define the link's gross informational benefit by

$$B_{ij}(g, \delta) = \sum_{e\neq d} w_{ed} \left[V_{ed}(g, \delta) - V_{ed}(g + ij, \delta)\right]. \tag{15}$$

Proposition 6 *The welfare effect of adding links to networks is characterized as follows.*[19]

[19] The notation $O((1-\delta)^2)$ denotes terms that vanish at least as fast as $(1-\delta)^2$ as reliability approaches one; the coefficient on $1-\delta$ is the first-order effect.

(a) *Suppose* $ij \notin \mathcal{L}(g)$. *Adding* ij *to* g *raises welfare if and only if* $B_{ij}(g,\delta) > c_{ij}$.

(b) *Suppose* $(C_1,\ldots,C_M)$ *satisfies* (1)–(2), *and let* g *and* h *be factional trees. If* $D(g) < D(h)$, *there exists* $\bar{\delta}_{gh} \in (0,1)$ *such that for common link cost* c $W(g;\delta,c) > W(h;\delta,c)$ *for every* $\delta \in (\bar{\delta}_{gh},1)$.

(c) *For both the full and one-sided transmission equilibrium* $\mathcal{E}$. $\Delta_{\mathcal{E}}(\ell,\delta) = \kappa_{\mathcal{E}}(1-\delta) + O((1-\delta)^2)$. *For equally weighted paths, shortening a one-sided path from* $\ell+1$ *to* ℓ *links has a larger first-order informational benefit than shortening a full-forwarding path if and only if* $\lambda > 2-\sqrt{2}$.

Part (a) is intuitive. $B_{ij}(g,\delta)$ is the sum of the reductions in expected posterior variance across (e,d) pairs, weighted by w_{ed}, so a link is more valuable when it improves transmission for many pairs or those with large welfare weights.

Parts (b) and (c) show that, when reliability is close to one, shorter paths matter more for pairs with larger welfare weights and, if $\lambda > 2-\sqrt{2}$, yield a larger first-order gain under one-sided disclosure than under full forwarding for equally weighted paths. Part (a) also captures gains when an added link gives evidence another chance to reach the decision maker without shortening an existing path.

Proposition 6 is also useful to compare the effect of information transmission failure and heterogeneous relationship costs on the efficiency of adding links to networks. Transmission failures affect the informational benefit $B_{ij}(g,\delta)$ of adding the link, whereas heterogeneous relationship costs affect its cost c_{ij}. Lower reliability can increase $B_{ij}(g,\delta)$ by raising the value of shorter paths and additional opportunities for evidence to arrive. Homophily in relationship costs instead lowers c_{ij} mainly within ideological collections, making internal links more likely to satisfy the condition even when they duplicate short paths.

When ideologies do not form distinct collections. An ideology profile admits distinct ideological collections if there is a partition $(C_1,\ldots,C_M)$ satisfying conditions (1)–(2). I now consider profiles for which no such partition exists. The examples illustrate two ways this can occur. Two politicians may be too ideologically distant to belong to the same collection but insufficiently separated to belong to different collections: their bias difference lies between 1/12 and $\overline{\beta}(\lambda)$, as in Example 2. Alternatively, ideological proximity may be nontransitive. Formally, letting $H_i = \{j \in \mathcal{N} \setminus \{i\} : |b_j - b_i| \le 1/12\}$, there may be three distinct politicians i,j,k such that $j \in H_i$, $k \in H_j$, but $k \notin H_i$, as in Example 3.[20]

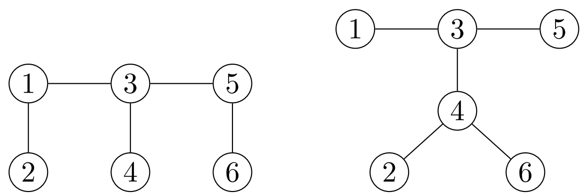
[20]Detailed analysis of these examples is in the Appendix.

Example 2 *Let six homogeneous politicians have biases satisfying* $1/12 < b_2 - b_1 < \overline{\beta}(\lambda)$, $b_3 - b_2 > \overline{\beta}(\lambda)$, $1/12 < b_4 - b_3 < \overline{\beta}(\lambda)$, $b_5 - b_4 > \overline{\beta}(\lambda)$, *and* $1/12 < b_6 - b_5 < \overline{\beta}(\lambda)$. *For the candidate partition* $\{1,2\},\{3,4\},\{5,6\}$, *I compare the star-factional tree* g *with tree* h *defined by* $\mathcal{L}(g) = \{13, 35, 12, 34, 56\}$, *and* $\mathcal{L}(h) = \{13, 35, 34, 24, 46\}$.

Because politicians are homogeneous, all ordered expert–decision-maker pairs have the same welfare weight. For any $\delta < 1$ *close to one, both trees have one-sided disclosure for every pair* (e,d) *and the same total link costs. So, near perfect reliability, the sum of the paths' lengths determines the welfare ranking to first order.*

$$\sum_{e \neq d} d_g(e,d) = 62, \qquad \sum_{e \neq d} d_h(e,d) = 58.$$

Hence h *yields greater welfare than* g *for every such* δ.

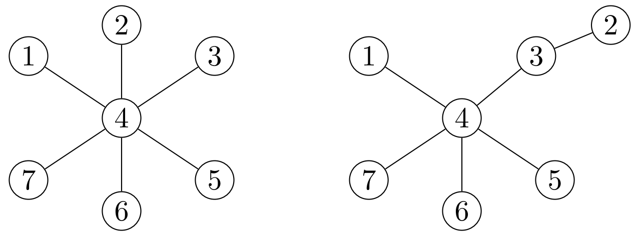


(a) Tree g (b) Tree h

Figure 3: The two trees in Example 2: h has a smaller total path length and yields greater welfare.

The second tree can be drawn as two linked stars, but the politicians in its stars are not members of distinct ideological collections. The links within these stars join politically distant politicians, while the link between these stars joins politicians who are politically close. Calling these stars factions would therefore misdescribe the underlying ideological relationships. For imperfect reliability sufficiently close to one, tree h beats g because its total path length is smaller, not because its apparent stars are ideologically coherent factions.

Example 3 *Let seven homogeneous politicians have biases satisfying* $b_2 - b_1 > \overline{\beta}(\lambda)$, $b_3 - b_2 < 1/12$, $b_4 - b_3 < 1/12$, $1/12 < b_4 - b_2 < \overline{\beta}(\lambda)$, $b_6 - b_4 < 1/12$, $1/12 < b_5 - b_3 < \overline{\beta}(\lambda)$, *and* $b_7 - b_6 > \overline{\beta}(\lambda)$. *Let* g *be the star centered at politician* 4, *and let* h *have links* $4i$ *for* $i \in \{1,3,5,6,7\}$ *and link* 23. *The direct link* 23 *restores full forwarding from politician* 3 *to politician* 2*; the path* $3-4-2$ *in* g *provides only one-sided disclosure. Evidence from politician* 2 *to politician* 3 *is already fully forwarded along* $2-4-3$ *in* g *because the biases of politicians* 2 *and* 4 *lie within* $1/12$ *of* b_3. *For every* $\delta < 1$ *sufficiently close to one, the* *informational gain for the pair* $(3, 2)$ *exceeds the loss from the longer paths in* h*, so* h *yields greater welfare than* g.



(a) Tree g (b) Tree h

Figure 4: The two trees in Example 3. Tree g is a star. In tree h, politicians 2 and 3 are directly linked, preserving full forwarding for this ideologically close pair.

Tree h is not star-factional: politician 4 is linked directly to every politician except 2, while link 23 preserves full forwarding between the ideologically close politicians 2 and 3. Thus one politician may be linked to almost everyone even when ideologies do not form distinct collections.

Together, the examples suggest that, when politicians are not partitioned into distinct ideological collections, preserving full forwarding between ideologically close politicians and keeping paths short need not favor the same tree. In Example 2, h has a shorter total path length than the star-factional network g, while a most informative equilibrium in each tree has the same disclosure outcome for every pair; in Example 3, tree h preserves full forwarding from politician 3 to 2 relative to the star g, although it lengthens other expert–decision-maker paths.

7 Conclusion

Political networks often divide into factions: politicians with similar ideologies communicate through a few leaders, and relatively few relationships cross factional boundaries. This paper asks which arrangement best transmits policy-relevant evidence net of relationship costs and, separately, which arrangements can persist when politicians choose their relationships.

Evidence may have to pass through several politicians before reaching the decision maker. Each intermediary may suppress evidence, and transmission may also fail for non-strategic reasons. In equilibrium, all evidence is passed on when disagreement is small. Greater disagreement leads politicians to withhold evidence against their bias relative to the decision maker. A well-designed network must therefore avoid paths on which politicians biased in opposite directions relative to the decision maker block evidence, keep paths to more relevant decision makers short, and avoid relationships whose benefits do not justify their costs.

Suppose the politicians' ideologies are partitioned into distinct collections, messages are sufficiently likely to arrive, and relationship costs are neither very high nor very low. I first compare networks by the quality of the decisions they support, net of relationship costs. When every expert–decision-maker pair is equally likely and every politician places the same weight on every decision, the best network is star-factional. There is a unique leader in each ideological collection, each other member of the collection is connected to that leader, and the only links across factions join the leaders of ideologically neighboring factions.

A separate result concerns how networks form. Suppose a relationship exists only when both politicians propose it, and each politician bears the cost of every proposal she makes. Star-factional networks form endogenously although politicians begin with equal expertise, authority, political relevance, and relationship costs. Among the Nash equilibrium networks, only star-factional networks also withstand profitable joint deviations by multiple politicians. Leadership can therefore emerge without being assigned by a formal organization.

I then identify the efficient leader among star networks when all politicians belong to one ideological collection but differ in expertise, authority, or relevance. The case for selecting a politician as the center strengthens with her likelihood of possessing relevant evidence, her likelihood of making decisions, and the relevance of those decisions. If these roles belong to different people, a network with several leaders may outperform any star network centered on one person. This can occur even when faction members hold similar ideologies, because consequential communication involves different members.

Relative to the star-factional theoretical benchmark, additional relationships can be socially valuable for two reasons. Relationships between like-minded politicians may cost less or be valuable in themselves, justifying more connections within a faction. Alternatively, when messages are highly likely but not certain to arrive, additional relationships across factions may shorten paths or give evidence another chance to reach the decision maker.

The paper also explores less empirically central cases in which politicians' ideologies do not form clearly separated ideological collections. Some politicians may be too far apart for all evidence to pass but not far enough apart to belong to different ideological collections. Alternatively, one politician may be close to a second and the second to a third, even though the first and third are far apart. Keeping communication open between close pairs and shortening paths for others may then favor networks that are not star-factional.

The model deliberately considers decentralized political decisions: one politician chooses policy, while another may hold useful evidence. This fits networks of policymakers across jurisdictions, interest groups and advocates, and judges, as well as many activities undertaken individually by legislators. Members of Congress gather and share policy research, draft and sponsor proposals, question witnesses, intervene in committees, contact agencies, communicate with constituents, and allocate staff attention. Information transmission is a central function of their networks (Fiellin, 1962; Ringe and Victor, 2013; Bratton and Rouse, 2011); seating proximity affects cue-taking (Masket, 2008), and staff ties are related to legislative effectiveness and voting (Montgomery and Nyhan, 2017). Collective legislative decisions such as votes raise additional questions not covered by the model.

The analysis also takes ideological collections as given and leaves political identities and organizational competition outside the model. In practice, politicians may often choose platforms, join factions, or seek leadership, while network position may affect their future recognition, authority, or expertise. The present results provide a benchmark for studying those choices: ideological proximity determines which paths can carry evidence, while the distribution of expertise, political relevance, and relationship costs helps determine who occupies the center of the resulting factions.

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Supplementary Appendix to
Information Transmission in Political Factions

August 2026

A Proofs

The main paper states the results and explains their logic. This appendix gives the formal proofs and uses the same notation. For a unique path of length ℓ between expert e and decision maker d, recall that

$$r_\ell = \lambda\delta^\ell.$$

Let $\mathcal{B}(g, e, d, \delta)$ denote the set of all perfect Bayesian equilibria of the communication game. Here a perfect Bayesian equilibrium is an assessment in which strategies are sequentially rational at every information set and beliefs satisfy Bayes' rule at every information set reached with positive probability. At any information set at which a politician observes verifiable evidence (e, s), her conditional distribution of θ is the Bayesian posterior induced by s, even if that information set has probability zero. Beliefs are otherwise unrestricted at information sets reached with probability zero. A most informative equilibrium minimizes the decision maker's expected posterior variance over this full set. The forwarding convention stated in the main paper applies only to the pure-strategy implementations presented below; it does not restrict $\mathcal{B}(g, e, d, \delta)$ or the minimization. Feasible message vectors are ordered componentwise, with forwarding above silence; whenever a construction requires a pure best reply, a componentwise-maximal one is selected. Thus an indifferent politician forwards in the stated pure path implementations, while equilibria requiring nondegenerate mixing remain admissible.

Formal communication protocol. Throughout the appendix, a path never visits the same politician twice. Fix a network g and a realized pair (e, d). Let $\mathcal{P}_{ed}(g)$ be the set of paths from e to d in g, written

$$p = (v_0 = e, v_1, \ldots, v_{\ell(p)} = d),$$

and define the communication horizon by

$$H_{ed}(g) = \begin{cases} 0, & \mathcal{P}_{ed}(g) = \varnothing, \\ \max_{p \in \mathcal{P}_{ed}(g)} \ell(p), & \mathcal{P}_{ed}(g) \neq \varnothing. \end{cases}$$

If $H_{ed}(g) = 0$, communication is skipped and d chooses from the prior. Otherwise, at each date $t = 0, \ldots, H_{ed}(g) - 1$, the ordered links used at position t on at least one path are

$$\mathcal{A}_t(e, d, g) = \{(i, j) : i = v_t,\ j = v_{t+1} \text{ on some } p \in \mathcal{P}_{ed}(g) \text{ with } t < \ell(p)\}.$$

A pair in $\mathcal{A}_t(e, d, g)$ is used once at date t, even if it occupies that position on several paths. The same pair may be used again at another date if it occupies a different position on another path.

For politician i, let

$$J_i^t(e, d, g) = \{j : (i, j) \in \mathcal{A}_t(e, d, g)\}.$$

At date t, politician i chooses the vector

$$m_i^t = (m_{ij}^t)_{j \in J_i^t(e,d,g)}$$

of all messages that she sends at that date. All politicians move simultaneously, and all transmission outcomes are realized before date $t + 1$. Each component is either silence or the evidence

(e, s_e), and politician i may send the evidence only if she is the informed expert or received it at an earlier date. She cannot change its source or realization. A non-silent transmission succeeds with probability δ, independently across dates and ordered links. If it fails, the receiver observes silence, whereas the sender does not observe the failure. Evidence received at date t can be used only from date $t+1$ onward.

A politician's private history records whether she is the informed expert and, if so, her realized signal; each outcome she has observed on a scheduled incoming transmission; each scheduled opportunity at which she was called to send or receive; and each message vector that she previously chose. A strategy maps each such history into a distribution over feasible message vectors, so the game has perfect recall. At date $H_{ed}(g)$, the decision maker chooses policy based on her private history. Since repeated copies of the same evidence add no information, one copy is sufficient once it has arrived.

The protocol permits repeated, time-respecting attempts. Evidence is not permanently assigned to the path along which it first travels: a copy received through one path may use a later transmission opportunity scheduled through another.

Existence and attainment.

Lemma 1 *For every network g, realized pair (e, d), and $\delta \in (0, 1]$, the set $\mathcal{B}(g, e, d, \delta)$ is nonempty and compact. The decision maker's expected posterior variance therefore attains a minimum on $\mathcal{B}(g, e, d, \delta)$.*

Proof. Fix a network g, a realized pair (e, d), and $\delta \in (0, 1]$. If e and d are disconnected, no communication takes place and the decision maker chooses from the prior, which gives the unique equilibrium assessment. Suppose, therefore, that at least one path connects them.

Claim 1: Reduce the state space. For strategic purposes, integrate out θ conditional on the three payoff-relevant expert types u, 0, and 1. Let $\omega \in \{u, 0, 1\}$, where u means that the expert is uninformed and 0 or 1 records her signal. Ex ante,

$$\Pr(\omega = u) = 1 - \lambda, \qquad \Pr(\omega = 0) = \Pr(\omega = 1) = \frac{\lambda}{2}.$$

Let ν_ω be the conditional distribution of θ: the uniform prior for $\omega = u$, density $2(1-\theta)$ for $\omega = 0$, and density 2θ for $\omega = 1$. Messages and transmission shocks depend on θ only through ω. If the terminal policy is y, politician i's conditional payoff is

$$-\alpha_{id} \int_0^1 (y - \theta - b_i)^2 \, \mathrm{d}\nu_\omega(\theta).$$

After integrating out θ, the communication game has a finite horizon, finitely many information sets, finite communication actions, and perfect recall. The only continuous action is therefore the decision maker's terminal policy, and her best reply always lies in

$$Y_d = [b_d, b_d + 1].$$

Consequently, the one-shot deviation principle allows communication behavior to be checked one message vector at a time, holding continuation behavior fixed.

Claim 2: The PBE set is nonempty. For each positive integer m, replace Y_d by a finite grid Y_d^m whose mesh tends to zero. The resulting game is finite and has perfect recall, so it has a sequential equilibrium. Write one such assessment as $(\psi^m, \tau^m, \zeta^m)$, where ψ^m is communication behavior, $\tau_I^m \in \Delta(Y_d^m)$ is terminal behavior at decision-maker information set I, and ζ_I^m is the belief over the nodes in I.

View each τ_I^m as a distribution on Y_d. Since the product of the relevant strategy, belief, and terminal-action spaces is compact, there is a subsequence along which

$$(\psi^m, \tau^m, \zeta^m) \longrightarrow (\psi, \tau, \zeta).$$

At any information set, the continuation payoff from a feasible message vector is a finite sum of integrals of the conditional payoff above. Its weights depend continuously on communication behavior, chance probabilities, and beliefs. Weak convergence of terminal behavior therefore implies convergence of the continuation payoff from every feasible vector.

The finite-action best-response correspondence has a closed graph. Any message vector used with positive probability in the limit remains a best reply, and the one-shot deviation principle makes the limiting communication behavior sequentially rational. Bayes consistency passes to the limit through the identities used in Claim 3 below.

At a terminal information set I, the unique best policy is

$$y_I = b_d + \sum_{h \in I} \zeta_I(h) \int_0^1 \theta \, \mathrm{d}\nu_{\omega(h)}(\theta).$$

In the grid games, every policy in the support of τ_I^m is a closest grid point to the corresponding best policy. Since the posterior means converge and the grid becomes arbitrarily fine, τ_I places probability one on the policy displayed above. Thus the limiting assessment belongs to $\mathcal{B}(g, e, d, \delta)$.

Claim 3: The PBE set is compact. Represent an assessment by its communication probabilities, beliefs at every information set, and deterministic terminal policies $y_I \in Y_d$. The ambient product space is compact.

Bayes consistency is a closed condition. Let $Q_\psi^I(h)$ be the unconditional probability of node $h \in I$ and let

$$P_\psi(I) = \sum_{h \in I} Q_\psi^I(h).$$

Bayes' rule can be written without division as

$$P_\psi(I)\zeta_I(h) = Q_\psi^I(h) \qquad \text{for every } I \text{ and } h \in I.$$

If $P_\psi(I) > 0$, these equalities are Bayes' rule. If $P_\psi(I) = 0$, both sides are zero and beliefs over the feasible nodes in I are unrestricted; the evidence-consistency requirement is already incorporated in the reduced node space. Since node probabilities vary continuously with communication behavior, the equalities define a closed set.

Sequential rationality of communication is also closed because, at each information set, the finitely many continuation payoffs depend continuously on behavior, beliefs, and terminal policies. Requiring every action in the support of a strategy to be a best reply is therefore a closed condition, while the one-shot deviation principle converts these local conditions into sequential rationality.

Terminal sequential rationality is the closed condition

$$y_I = b_d + \sum_{h \in I} \zeta_I(h) \int_0^1 \theta \; \mathrm{d}\nu_{\omega(h)}(\theta).$$

The PBE set $\mathcal{B}(g, e, d, \delta)$ is thus closed in a compact space and is itself compact, while Claim 2 shows that it is nonempty.

Claim 4: The minimum over PBEs is attained. Let z range over the finite terminal nodes and let $I(z)$ be the decision maker's terminal information set. In equilibrium, $y_{I(z)} - b_d$ is her posterior mean. Expected posterior variance is

$$\sum_z P_\psi(z) \int_0^1 \left[\theta - (y_{I(z)} - b_d)\right]^2 \mathrm{d}\nu_{\omega(z)}(\theta).$$

The terminal-node probabilities are continuous in ψ, and each integral is continuous in the terminal policy. Expected posterior variance is therefore continuous on the compact set $\mathcal{B}(g, e, d, \delta)$ and attains its minimum. ■

Common ranking of communication equilibria.

Lemma 2 *In every communication equilibrium, the decision maker chooses*

$$y_d = b_d + \mathbb{E}[\theta | \mathcal{I}_d].$$

Politician i's expected policy loss is

$$\alpha_{id} \left[(b_i - b_d)^2 + \mathbb{E} \operatorname{Var}(\theta | \mathcal{I}_d)\right].$$

All politicians therefore prefer equilibria with a lower expected posterior variance.

Proof. At any terminal information set $\mathcal{I}_d$, the decision maker d chooses y to minimize

$$\mathbb{E}[(y - \theta - b_d)^2 | \mathcal{I}_d].$$

The unique minimizer is $y_d = b_d + \mathbb{E}[\theta | \mathcal{I}_d]$. For politician i,

$$\begin{aligned} \mathbb{E}[(y_d - \theta - b_i)^2] &= \mathbb{E}\left[(\mathbb{E}[\theta|\mathcal{I}_d] - \theta + b_d - b_i)^2\right] \\ &= \mathbb{E}\left[(\mathbb{E}[\theta|\mathcal{I}_d] - \theta)^2\right] + (b_d - b_i)^2 \\ &\quad + 2(b_d - b_i)\mathbb{E}\left[\mathbb{E}[\theta|\mathcal{I}_d] - \theta\right]. \end{aligned}$$

The last term is zero by the law of iterated expectations, and

$$\mathbb{E}\left[(\mathbb{E}[\theta|\mathcal{I}_d] - \theta)^2\right] = \mathbb{E} \operatorname{Var}(\theta | \mathcal{I}_d).$$

Multiplying by α_{id} gives the expression in the lemma. The bias term is independent of the communication equilibrium, so every politician ranks equilibria by expected posterior variance. ■

Disclosure along a path. Suppose $p = (e, \dots, d)$ is the unique path from e to d, with length ℓ. Let $q_s \in [0, \delta^\ell]$ be the equilibrium probability that signal realization $s \in \{0,1\}$ reaches d, conditional on the expert being informed and observing s. Define the delivery coefficient $r_s = \lambda q_s \in [0, r_\ell]$. The ex ante probability that the specified realization s arrives is $r_s/2$. Conditional on state θ, no evidence arrives with probability

$$1 - r_1\theta - r_0(1-\theta) = 1 - r_0 + (r_0 - r_1)\theta.$$

Bayes' rule therefore gives the posterior mean after no arrival as

$$\mu(r_0, r_1) = \frac{\int_0^1 \theta[1 - r_0 + (r_0 - r_1)\theta]\, \mathrm{d}\theta}{\int_0^1 [1 - r_0 + (r_0 - r_1)\theta]\, \mathrm{d}\theta} = \frac{\frac{1}{2} - \frac{1}{6}r_0 - \frac{1}{3}r_1}{1 - \frac{1}{2}(r_0 + r_1)}. \tag{1}$$

The posterior mean is maximized at $(r_0, r_1) = (r_\ell, 0)$ and minimized at $(0, r_\ell)$. Hence

$$\mu_L(r_\ell) := \frac{3 - 2r_\ell}{3(2 - r_\ell)} \le \mu(r_0, r_1) \le \frac{3 - r_\ell}{3(2 - r_\ell)} =: \mu_H(r_\ell). \tag{2}$$

Suppose politician j has observed $s = 0$ and is *pivotal* for that realization, meaning that changing her forwarding choice can change the probability that the signal reaches d. Conditional on $s = 0$, the decision maker chooses $b_d + 1/3$ if the signal arrives and $b_d + \mu(r_0, r_1)$ if it does not. Since the positive change in arrival probability factors out of the payoff comparison, politician j prefers forwarding if and only if

$$\mathbb{E}\left[\left(b_d + \frac{1}{3} - \theta - b_j\right)^2 |s = 0\right] \le \mathbb{E}[(b_d + \mu(r_0, r_1) - \theta - b_j)^2 | s = 0],$$

which simplifies to

$$b_j - b_d \le \frac{\mu(r_0, r_1) - \frac{1}{3}}{2}. \tag{3}$$

Similarly, after observing $s = 1$, politician j forwards if and only if

$$b_j - b_d \ge \frac{\mu(r_0, r_1) - \frac{2}{3}}{2}. \tag{4}$$

Lemma 3 *Let r_0 and r_1 be the delivery coefficients of $s=0$ and $s=1$, and let $\mu(r_0,r_1)$ be the posterior mean after no arrival. In every equilibrium,*

$$r_0 = r_\ell \Longrightarrow \overline{x}_p \leq \frac{\mu(r_0,r_1)-\frac{1}{3}}{2}, \qquad 0<r_0<r_\ell \Longrightarrow \overline{x}_p = \frac{\mu(r_0,r_1)-\frac{1}{3}}{2},$$
$$r_0 = 0 \Longrightarrow \overline{x}_p \geq \frac{\mu(r_0,r_1)-\frac{1}{3}}{2}, \tag{5}$$
$$r_1 = r_\ell \Longrightarrow \underline{x}_p \geq \frac{\mu(r_0,r_1)-\frac{2}{3}}{2}, \qquad 0<r_1<r_\ell \Longrightarrow \underline{x}_p = \frac{\mu(r_0,r_1)-\frac{2}{3}}{2},$$
$$r_1 = 0 \Longrightarrow \underline{x}_p \leq \frac{\mu(r_0,r_1)-\frac{2}{3}}{2}. \tag{6}$$

For a pure implementation, the rule that an indifferent politician forwards makes the two zero-delivery inequalities strict; the maximal-delivery inequalities remain weak.

In particular, if both delivery coefficients are interior, then

$$\underline{x}_p = \overline{x}_p - \frac{1}{6}.$$

Proof. Equation (3) identifies the forwarding cutoff after $s=0$ whenever a politician's choice can affect arrival. If $r_0 = r_\ell$, every politician forwards with probability one and is pivotal, giving the first implication in (5). If $0<r_0<r_\ell$, every politician forwards with positive probability and at least one mixes. All are therefore pivotal. Any mixer must be indifferent, and monotonicity of (3) in $b_j - b_d$ implies that at least one politician attaining $\overline{x}_p$ mixes and satisfies the equality in (5).

If $r_0 = 0$, choose, among the politicians who forward $s=0$ with probability zero, the one closest to d. Every politician downstream forwards with positive probability, so, conditional on the selected politician having received the evidence, changing her action from withholding to forwarding gives $s=0$ a positive probability of reaching d. She is therefore pivotal and weakly prefers withholding, which gives the third implication in (5). In a pure implementation satisfying the forwarding convention, the inequality is strict because an indifferent politician forwards. The same argument using (4) proves (6). Finally, if both delivery coefficients are interior, the most left- and right-biased politicians are indifferent, and subtracting their indifference conditions gives $\underline{x}_p = \overline{x}_p - 1/6$. ∎

Lemma 4 *In every equilibrium, a politician with $b_j \geq b_d$ forwards $s_e = 1$ whenever forwarding can affect the decision maker's information, and a politician with $b_j \leq b_d$ forwards $s_e = 0$ whenever forwarding can affect it. If $b_j - b_d > \overline{\beta}(r_\ell)$, politician j blocks $s_e = 0$ whenever she is pivotal, and $s_e = 0$ cannot reach d. If $b_j - b_d < -\overline{\beta}(r_\ell)$, the symmetric conclusion holds for $s_e = 1$.*

Proof. Equation (2) implies

$$\frac{1}{3} < \mu(r_0,r_1) < \frac{2}{3}.$$

Hence, if $b_j \geq b_d$, the right-hand side of (4) is negative and j forwards $s=1$ whenever her action can affect arrival. If $b_j \leq b_d$, the right-hand side of (3) is positive and j forwards $s=0$ whenever her action can affect arrival.

For a right-biased politician observing $s = 0$, the largest possible forwarding threshold is obtained at $\mu_H(r_\ell)$:

$$\frac{\mu_H(r_\ell) - \frac{1}{3}}{2} = \frac{1}{6(2 - r_\ell)} = \overline{\beta}(r_\ell).$$

Thus a politician with $b_j - b_d > \overline{\beta}(r_\ell)$ strictly withholds $s = 0$ whenever she is pivotal. If she is not pivotal, the signal has already been stopped downstream, so in either case, $s = 0$ cannot reach d along the unique path. The symmetric calculation uses

$$\frac{\mu_L(r_\ell) - \frac{2}{3}}{2} = -\frac{1}{6(2 - r_\ell)} = -\overline{\beta}(r_\ell)$$

and proves the conclusion for a sufficiently left-biased politician observing $s = 1$. ■

Proposition 1 *Suppose $p = (e, \dots, d)$ is the unique path in the network from expert e to decision maker d, and let its length be ℓ.*

(a) Full forwarding is an equilibrium if and only if

$$\max_{j \in p \setminus \{d\}} |b_j - b_d| \leq \frac{1}{12}.$$

Whenever it exists, every full-forwarding equilibrium is a most informative equilibrium.

(b) If

$$\underline{\beta}(r_\ell) < \overline{x}_p < \frac{1}{12}, \qquad \overline{x}_p - \frac{1}{6} \leq \underline{x}_p < -\frac{1}{12},$$

a most informative equilibrium has mixed disclosure. Every politician forwards $s_e = 1$, and exactly one politician attaining $\overline{x}_p$ forwards $s_e = 0$ with probability

$$\sigma(\overline{x}_p, r_\ell) = \frac{6(2 - r_\ell)\overline{x}_p - (1 - r_\ell)}{6 r_\ell \overline{x}_p}.$$

If instead

$$-\frac{1}{12} < \underline{x}_p < -\underline{\beta}(r_\ell), \qquad \frac{1}{12} < \overline{x}_p \leq \underline{x}_p + \frac{1}{6},$$

the symmetric conclusion holds: a most informative equilibrium has mixed disclosure, and exactly one politician attaining $\underline{x}_p$ mixes over $s_e = 1$.

(c) If $\overline{x}_p > \overline{\beta}(r_\ell)$ and $\underline{x}_p < -\overline{\beta}(r_\ell)$, neither realization reaches d in any equilibrium.

(d) In every other region, an equilibrium with one-sided disclosure is most informative.

Proof of Proposition 1 in the main paper. *Part (a): full forwarding.* In a full-forwarding profile, $r_0 = r_1 = r_\ell$. Non-arrival is independent of the signal realization, so (1) gives $\mu(r_0, r_1) = 1/2$. Conditions (3)–(4) reduce to

$$b_j - b_d \leq \frac{1}{12}, \qquad b_j - b_d \geq -\frac{1}{12}.$$

Full forwarding is therefore sequentially rational if and only if

$$\max_{j\in p\setminus\{d\}} |b_j - b_d| \le \frac{1}{12}.$$

At equality, every most-biased politician whose constraint binds is indifferent and therefore forwards under the stated tie-breaking rule. Because $\lambda < 1$, the no-arrival history occurs with positive probability on the equilibrium path, and Bayes' rule determines its posterior.

Full forwarding also gives whichever realization the expert observes its maximal feasible arrival probability δ^ℓ, conditional on the expert being informed. Any alternative distribution of the decision maker's observations can be obtained from full forwarding by randomly suppressing some delivered copies of $s = 0$, $s = 1$, or both. It is therefore a Blackwell garbling of full forwarding. Expected posterior variance weakly rises under such a garbling, so full forwarding is most informative whenever it exists.

Part (b): mixed disclosure.

Right-mixed disclosure.

Construction and sequential rationality. Since $\underline{\beta}(r_\ell) < \overline{x}_p < 1/12$, the probability

$$\sigma(\overline{x}_p, r_\ell) = \frac{6(2-r_\ell)\overline{x}_p - (1-r_\ell)}{6r_\ell\overline{x}_p}$$

lies strictly between zero and one. Accordingly, prescribe forwarding of $s = 1$ by every politician. Select one politician $\overline{j}$ attaining $\overline{x}_p$. After $s = 0$, prescribe forwarding by every other politician and let $\overline{j}$ forward with probability $\sigma(\overline{x}_p, r_\ell)$. A politician who has not observed the evidence instead sends silence. The resulting delivery coefficients are

$$(r_0, r_1) = \big(\sigma(\overline{x}_p, r_\ell)r_\ell, r_\ell\big),$$

and Bayes' rule gives

$$\mu\big(\sigma(\overline{x}_p, r_\ell)r_\ell, r_\ell\big) = \frac{3 - r_\ell[2+\sigma(\overline{x}_p, r_\ell)]}{3\{2 - r_\ell[1+\sigma(\overline{x}_p, r_\ell)]\}} = \frac{1}{3} + 2\overline{x}_p. \tag{7}$$

Hence the forwarding threshold for $s = 0$ is $\overline{x}_p$: politician $\overline{j}$ is indifferent and uses the nondegenerate mixture required by the equilibrium, while every other politician forwards whenever pivotal. The threshold for $s = 1$ is instead $\overline{x}_p - 1/6$. The condition $\underline{x}_p \ge \overline{x}_p - 1/6$ therefore makes the expert and every intermediary willing to forward whenever pivotal; at equality, forwarding is a best reply. Thus the assessment is a perfect Bayesian equilibrium, whereas full forwarding is unavailable because $\underline{x}_p < -1/12$.

Equilibria with one maximal delivery coefficient. In the right-mixed region, suppose an equilibrium has $r_1 = r_\ell$ and $0 < r_0 < r_\ell$. The delivery-coefficient lemma implies

$$\overline{x}_p = \frac{\mu(r_0, r_\ell) - \frac{1}{3}}{2},$$

and (1) then uniquely determines $r_0 = \sigma(\overline{x}_p, r_\ell)r_\ell$. An equilibrium with $r_0 = r_\ell$ and $0 < r_1 < r_\ell$ would have $\mu(r_\ell, r_1) > 1/2$. Its $s = 1$ indifference condition would imply $\underline{x}_p > -1/12$, contrary to the parameter restrictions.

The knife edge with two interior delivery coefficients. The next two arguments first identify the indifference locus and then select its most informative point. Both delivery coefficients can be interior only when $\underline{x}_p = \overline{x}_p - 1/6$.

Indifference locus. On this knife edge, the pivotal most left- and right-biased politicians require the same no-arrival posterior. Their common indifference condition defines a one-dimensional set of equilibrium delivery coefficients. Solving (1) for r_0 gives

$$r_0 = \frac{(1 - 6\overline{x}_p)r_1 + 12\overline{x}_p - 1}{6\overline{x}_p}.$$

Because $\overline{x}_p < 1/12$, r_0 increases with r_1. Moving along the locus toward a larger r_1 therefore also increases delivery of $s = 0$.

Selection on the locus. Every point on the locus induces the same no-arrival posterior mean. The variance of posterior means is

$$\frac{r_0 + r_1}{72} + \left[1 - \frac{r_0 + r_1}{2}\right]\left(2\overline{x}_p - \frac{1}{6}\right)^2.$$

Its derivative with respect to $r_0 + r_1$ is

$$\frac{1}{72} - \frac{1}{2}\left(2\overline{x}_p - \frac{1}{6}\right)^2 > 0.$$

Expected posterior variance equals prior variance minus the variance of posterior means. The most informative point on the locus therefore maximizes $r_0 + r_1$. In the right-mixed region this is the endpoint $r_1 = r_\ell$, which yields $r_0 = \sigma(\overline{x}_p, r_\ell)r_\ell$.

Comparison with less informative outcomes. The posterior-variance calculation below gives

$$V_O(r_\ell) - V_M(r_\ell, \overline{x}_p) = \frac{1 - r_\ell}{3}[\overline{x}_p - \underline{\beta}(r_\ell)] > 0.$$

Thus the right-mixed equilibrium is strictly more informative than either equilibrium with one-sided disclosure, and communication breakdown is less informative still.

Left-mixed disclosure. This case is symmetric. Select one politician $\underline{j}$ attaining $\underline{x}_p$. Every politician forwards $s = 0$, politician $\underline{j}$ forwards $s = 1$ with probability $\sigma(-\underline{x}_p, r_\ell)$, and every other politician forwards $s = 1$. The delivery coefficients are

$$(r_0, r_1) = \big(r_\ell, \sigma(-\underline{x}_p, r_\ell)r_\ell\big).$$

The symmetric versions of the construction, knife-edge, and informativeness arguments prove the claim.

Part (c): breakdown. If $\overline{x}_p > \overline{\beta}(r_\ell)$ and $\underline{x}_p < -\overline{\beta}(r_\ell)$, the forwarding-bounds lemma implies that $s = 0$ cannot pass the sufficiently right-biased politician and $s = 1$ cannot pass the sufficiently left-biased politician. Since evidence must traverse the unique path, neither realization reaches d in any equilibrium.

Part (d): one-sided disclosure in pure strategies and exhaustion. In a right one-sided profile, $(r_0, r_1) = (0, r_\ell)$ and the no-arrival mean is $\mu_L(r_\ell)$. Every politician forwards $s = 1$ if and only if

$$b_j - b_d \geq \frac{\mu_L(r_\ell) - \frac{2}{3}}{2} = -\overline{\beta}(r_\ell).$$

The weak boundary is included because the stated tie-breaking rule makes an indifferent politician forward. At least one politician blocks $s = 0$ if and only if

$$\overline{x}_p > \frac{\mu_L(r_\ell) - \frac{1}{3}}{2} = \underline{\beta}(r_\ell).$$

The inequality is strict because an indifferent politician who uses a pure best reply forwards. Thus a pure right one-sided equilibrium in which indifferent politicians forward exists exactly under the conditions stated in the corollary. The symmetric calculation for $(r_0, r_1) = (r_\ell, 0)$ gives the pure left one-sided conditions.

Outside the full-forwarding and breakdown regions, at least one pure one-sided equilibrium in which indifferent politicians forward exists. To see this, suppose neither exists. Failure of the right one-sided conditions means that either $\underline{x}_p < -\overline{\beta}(r_\ell)$ or $\overline{x}_p \leq \underline{\beta}(r_\ell)$, whereas failure of the left one-sided conditions means that either $\overline{x}_p > \overline{\beta}(r_\ell)$ or $\underline{x}_p \geq -\underline{\beta}(r_\ell)$. The first condition from each pair gives breakdown, while the crossed alternatives are infeasible. Hence the only remaining combination,

$$\overline{x}_p \leq \underline{\beta}(r_\ell) \quad \text{and} \quad \underline{x}_p \geq -\underline{\beta}(r_\ell),$$

lies inside the full-forwarding region because $\underline{\beta}(r_\ell) < 1/12$, a contradiction.

Exhaustion. It remains to compare the other possible delivery coefficients. Suppose first that $r_1 = r_\ell$ and $0 < r_0 < r_\ell$. The delivery-coefficient lemma and (1) imply

$$\underline{\beta}(r_\ell) < \overline{x}_p < \frac{1}{12}, \qquad \underline{x}_p \geq \overline{x}_p - \frac{1}{6}.$$

If $\underline{x}_p \geq -1/12$, full forwarding exists and is a Blackwell improvement. If $\underline{x}_p < -1/12$, the path lies in the right-mixed region, so the calculation in part (b) applies. The case $r_0 = r_\ell$ and $0 < r_1 < r_\ell$ is symmetric: either full forwarding exists or the path lies in the left-mixed region.

Suppose next that $0 < r_0 < r_\ell$ and $r_1 = 0$. The delivery-coefficient lemma implies

$$\frac{1}{12} < \overline{x}_p < \overline{\beta}(r_\ell), \qquad \underline{x}_p < -\underline{\beta}(r_\ell).$$

A left one-sided equilibrium then raises the delivery coefficient of the same realization from r_0 to r_ℓ and is a strict Blackwell improvement. The case $r_0 = 0$ and $0 < r_1 < r_\ell$ is symmetric.

If both delivery coefficients are interior, the delivery-coefficient lemma gives $\underline{x}_p = \overline{x}_p - 1/6$. Because both coefficients are interior, the inequalities in (2) are strict. The indifference condition therefore gives

$$\overline{x}_p = \frac{\mu(r_0, r_1) - \frac{1}{3}}{2}, \qquad \underline{\beta}(r_\ell) < \overline{x}_p < \overline{\beta}(r_\ell).$$

The locus calculation above shows that its most informative feasible point is the right-mixed endpoint $r_1 = r_\ell$ when $\overline{x}_p < 1/12$, the left-mixed endpoint $r_0 = r_\ell$ when $\overline{x}_p > 1/12$, and full forwarding when $\overline{x}_p = 1/12$.

Finally, an equilibrium with $r_0 = r_1 = 0$ is not most informative outside part (c): full forwarding, one of the mixed equilibria constructed in part (b), or one of the one-sided equilibria constructed above strictly improves on V_B. Cases in which both coefficients are maximal are covered by part (a), and cases in which exactly one is maximal and the other is zero are one-sided. Thus, in every region not covered by parts (a)–(c), an equilibrium with one-sided disclosure is most informative. ■

Corollary 1 *A pure right one-sided equilibrium in which indifferent politicians forward exists if and only if*

$$\underline{x}_p \geq -\overline{\beta}(r_\ell) \quad \textit{and} \quad \overline{x}_p > \underline{\beta}(r_\ell).$$

A pure left one-sided equilibrium in which indifferent politicians forward exists if and only if

$$\overline{x}_p \leq \overline{\beta}(r_\ell) \quad \textit{and} \quad \underline{x}_p < -\underline{\beta}(r_\ell).$$

Proof. The necessary and sufficient conditions for the pure right and left one-sided equilibria in which indifferent politicians forward were derived in part (d) of the proof of Proposition 1. They concern equilibrium coexistence rather than which equilibrium is most informative; the tie-breaking rule does not exclude the two nondegenerate mixtures used in part (b). ■

Posterior variances. For $r \in [0,1]$, under full forwarding the total probability that evidence arrives is r, while each realization arrives with ex ante probability $r/2$. Non-arrival is uninformative. Therefore

$$V_F(r) = \frac{r}{18} + \frac{1-r}{12} = \frac{3-r}{36}.$$

Consider a right-mixed equilibrium with delivery coefficient r and most right-biased position x, so the mixing probability is $\sigma(x,r)$. Signal 0 arrives with probability $\sigma(x,r)r/2$, signal 1 arrives with probability $r/2$, and no signal arrives with probability $1 - r[1+\sigma(x,r)]/2$. The posterior means after these events are $1/3$, $2/3$, and $1/3 + 2x$, respectively. By the law of total variance,

$$\begin{aligned} V_M(r,x) &= \frac{1}{12} - \frac{\sigma(x,r)r}{2}\left(\frac{1}{3}-\frac{1}{2}\right)^2 - \frac{r}{2}\left(\frac{2}{3}-\frac{1}{2}\right)^2 \\ &\quad - \left[1 - \frac{r[1+\sigma(x,r)]}{2}\right]\left(\frac{1}{3}+2x-\frac{1}{2}\right)^2 \\ &= \frac{2-r-6(1-r)x}{18}. \end{aligned}$$

The left-mixed equilibrium gives the same expression with $x = -\underline{x}_p$.

When only $s=0$ is forwarded, its delivery coefficient is r, so the ex ante probability that $s=0$ arrives is $r/2$. If it arrives, posterior variance is $1/18$, whereas if it does not, the posterior density is proportional to $1 - r(1-\theta)$. The posterior means are therefore $1/3$ after arrival and $\mu_H(r)$ after non-arrival, with probabilities $r/2$ and $(2-r)/2$. Hence

$$\begin{aligned} \mathrm{Var}\big(\mathbb{E}[\theta|\mathcal{I}_d]\big) &= \frac{r}{2}\left(\frac{1}{3}-\frac{1}{2}\right)^2 + \frac{2-r}{2}\left(\mu_H(r)-\frac{1}{2}\right)^2 \\ &= \frac{r}{36(2-r)}. \end{aligned}$$

Since the prior variance is $1/12$,

$$V_O(r) = \frac{1}{12} - \frac{r}{36(2-r)} = \frac{3-2r}{18(2-r)}.$$

Breakdown leaves the prior unchanged, so $V_B = 1/12$. Direct subtraction yields

$$\begin{aligned} V_M(r,x) - V_F(r) &= \frac{1-r}{3}\left(\frac{1}{12} - x\right), \\ V_O(r) - V_M(r,x) &= \frac{1-r}{3}\left[x - \underline{\beta}(r)\right], \\ V_O(r) - V_F(r) &= \frac{r(1-r)}{36(2-r)}, \\ V_B - V_O(r) &= \frac{r}{36(2-r)}. \end{aligned}$$

All four differences have the signs stated in the main paper for $r \in (0,1)$ and $x \in (\underline{\beta}(r), 1/12)$.

A graph lemma. The *induced subnetwork* on a set of politicians contains those politicians and all links between them. A *connected component* is a maximal connected part of a network. In graph-theoretic language, vertices are politicians and edges are links; in a quotient, a vertex may instead represent a collapsed collection. Let $C_1, \ldots, C_M$ be the ideological collections. If a tree g induces a connected subnetwork on every C_m, replace each such subnetwork by a single vertex m. The resulting *quotient* $Q(g)$ is a tree on the ordered vertices $1, \ldots, M$.

Lemma 5 *If Q is a tree on $\{1, \ldots, M\}$ other than the ordered line $1 - 2 - \cdots - M$, then there exist a destination m and a source r such that the path from r to m contains a vertex below m and a vertex above m.*

Proof. If Q were to contain the edge $m(m+1)$ for every $m < M$, it would have exactly the $M-1$ edges of the ordered line. Therefore, because Q is not the ordered line, there is an m such that m and $m+1$ are not adjacent, and their unique path contains some vertex $k \notin \{m, m+1\}$. If $k < m$, consider the path from source $m+1$ to destination m, which contains $m+1 > m$ and $k < m$. Alternatively, if $k > m+1$, consider the path from source m to destination $m+1$, which contains $m < m+1$ and $k > m+1$. Thus, in either case, the path contains vertices on both sides of the destination. ∎

Optimality of factional trees. At zero link cost, welfare is

$$W(g; \delta, 0) = -\sum_{e \neq d} \pi_{ed} R_d V_{ed}(g, \delta),$$

and total welfare at cost c is $W(g; \delta, c) = W(g; \delta, 0) - 2c|\mathcal{L}(g)|$. Let $\mathcal{F}$ be the finite set of factional trees. Define the full/one-sided zero-cost welfare upper bound

$$\overline{W} = -\sum_{e \neq d} \pi_{ed} R_d \bar{V}_{ed}, \qquad \bar{V}_{ed} = \begin{cases} V_F(\lambda), & e, d \text{ belong to the same ideological collection,} \\ V_O(\lambda), & e, d \text{ belong to different ideological collections.} \end{cases} \tag{8}$$

Proof of Proposition 4 in the main paper. *Part 1: the upper bound.* In a factional tree, the unique path between two politicians in the same ideological collection remains inside that collection, so part (a) of Proposition 1 gives full forwarding. If $e \in C_r$ and $d \in C_m$ with $r < m$, every politician before the path enters C_m is strongly to the left of d, while politicians inside C_m are within $1/12$ of d. Part (d) of Proposition 1 gives one-sided disclosure in which $s_e = 0$ is forwarded. The case $r > m$ is symmetric. At $\delta = 1$, every factional tree therefore attains $\overline{W}$.

I next show that $\overline{W}$ is an upper bound for every network and every δ. Fix a network and a pair (e, d). For $s \in \{0, 1\}$, let $q_s \in [0, 1]$ be the probability that at least one copy of the evidence reaches d, conditional on the expert being informed and observing s, and set $r_s = \lambda q_s \in [0, \lambda]$. Since duplicate copies are informationally redundant, the posterior mean after no arrival is still $\mu(r_0, r_1)$ in (1). Applying (2) with the upper bound λ gives

$$\mu_L(\lambda) \leq \mu(r_0, r_1) \leq \mu_H(\lambda).$$

Suppose e belongs to an ideological collection to the left of d. Then $b_e - b_d < -\overline{\beta}(\lambda)$. If $q_1 > 0$, the expert can deviate jointly in all her date-zero messages and send silence everywhere after observing $s = 1$. Conditional on $s = 1$, this changes the probability of the decision maker's action based on $2/3$ from q_1 to zero. The no-arrival history occurs with positive probability on the equilibrium path because the expert is uninformed with positive probability, so the decision after no arrival remains $b_d + \mu(r_0, r_1)$. The expert would be willing to generate positive arrival only if

$$b_e - b_d \geq \frac{\mu(r_0, r_1) - \frac{2}{3}}{2} \geq \frac{\mu_L(\lambda) - \frac{2}{3}}{2} = -\overline{\beta}(\lambda),$$

which is false. The joint deviation is therefore strictly profitable, a contradiction. Hence $q_1 = 0$. When the expert lies to the right of the decision maker, the symmetric argument uses $\mu(r_0, r_1) \leq \mu_H(\lambda)$ and shows that $q_0 = 0$.

Across ideological collections, at most one realization can arrive and its delivery coefficient cannot exceed λ. The ex ante probability that this specified realization arrives cannot exceed $\lambda/2$. Since $V_O(r)$ decreases in r, the posterior variance is at least $V_O(\lambda)$. Within an ideological collection, the Blackwell-garbling argument in part (a) of the proof of Proposition 1 shows that no feasible experiment can improve on full arrival of both realizations, whose variance is $V_F(\lambda)$. It follows that $W(g; \delta, 0) \leq \overline{W}$ for every network g and every δ.

Part 2: non-factional trees. Let g be a tree that is not factional. If the subnetwork induced by some C_m is disconnected, choose $e, d \in C_m$ in different induced components. Their unique path leaves C_m and contains a politician whose bias relative to d has absolute value greater than $\overline{\beta}(\lambda)$, and therefore greater than the relevant $\overline{\beta}(\lambda\delta^\ell)$. One realization cannot reach d, so the path yields at best one-sided disclosure. At $\delta = 1$, the posterior-variance loss relative to a factional tree is at least

$$V_O(\lambda) - V_F(\lambda) = \frac{\lambda(1-\lambda)}{36(2-\lambda)} > 0.$$

Suppose instead that every ideological collection is internally connected, so contracting collections yields a quotient tree. If g is not factional, the quotient is not the ordered line. Lemma 5 therefore identifies an expert–decision-maker path containing a collection below and a collection above the decision maker's collection. The actual path then contains politicians with biases below $-\overline{\beta}(\lambda)$ and above $\overline{\beta}(\lambda)$ relative to d, so communication breaks down. The loss relative to a factional tree is at least

$$V_B - V_O(\lambda) = \frac{\lambda}{36(2-\lambda)} > 0.$$

Because every pair has positive weight and the set of trees is finite, every factional tree strictly dominates every non-factional tree at $\delta = 1$ by a difference bounded away from zero. On each tree, the strict cross-collection inequalities and the within-collection forwarding condition keep the forwarding outcome fixed for δ sufficiently close to one, and the explicit variance formulas are continuous in δ. The strict ranking therefore persists near one.

Part 3: connected networks with additional links. Write $[x]_+ = \max\{x, 0\}$. The largest gross informational advantage over the best factional tree per excess link is

$$\max_{\substack{g \text{ connected} \\ |\mathcal{L}(g)|>n-1}} \frac{[W(g;\delta,0) - \max_{f\in\mathcal{F}} W(f;\delta,0)]_+}{2[|\mathcal{L}(g)| - (n-1)]},$$

with the maximum of an empty set defined as zero. The denominator converts the largest gross informational advantage over the best factional tree per excess link into an endpoint cost because both endpoints pay c. Part 1 gives $W(g;\delta,0) \le \overline{W}$ for every g, while the explicit factional-tree payoffs imply

$$\max_{f\in\mathcal{F}} W(f;\delta,0) \longrightarrow \overline{W} \quad \text{as } \delta \to 1.$$

Hence

$$0 \le [W(g;\delta,0) - \max_{f\in\mathcal{F}} W(f;\delta,0)]_+ \le \overline{W} - \max_{f\in\mathcal{F}} W(f;\delta,0) \longrightarrow 0$$

uniformly across the finite set of networks. Hence the displayed quantity converges to zero. When c is above it, the cost of every excess link exceeds the largest gross advantage per excess link, so every connected network with additional links is dominated by a best factional tree.

Part 4: disconnected networks. Every disconnected network leaves at least one positive-weight expert–decision-maker pair unable to communicate. That pair has posterior variance V_B, while no other pair can beat its component of the upper bound. Therefore, for every disconnected g and every δ,

$$\overline{W} - W(g;\delta,0) \ge \min_{e\neq d} \pi_{ed} R_d \min\{V_B - V_F(\lambda),\, V_B - V_O(\lambda)\} > 0.$$

Since $\max_{f\in\mathcal{F}} W(f;\delta,0) \to \overline{W}$, for δ sufficiently close to one,

$$\max_{f\in\mathcal{F}} W(f;\delta,0) - W(g;\delta,0) \ge \frac{1}{2} \min_{e\neq d} \pi_{ed} R_d \min\{V_B - V_F(\lambda),\, V_B - V_O(\lambda)\}$$

for every disconnected network. A disconnected network with at least $n-1$ links is then dominated without a cost restriction. For those with fewer links, let $b_D(\delta)$ denote the smallest gross advantage of a best factional tree per saved link:

$$b_D(\delta) = \min_{\substack{g \text{ disconnected} \\ |\mathcal{L}(g)|<n-1}} \frac{\max_{f\in\mathcal{F}} W(f;\delta,0) - W(g;\delta,0)}{2[(n-1) - |\mathcal{L}(g)|]}.$$

The denominator converts the smallest gross advantage of a best factional tree per saved link into an endpoint cost. The preceding bound implies

$$\liminf_{\delta\to 1} b_D(\delta) \ge \frac{\min_{e\neq d} \pi_{ed} R_d \min\{V_B - V_F(\lambda),\, V_B - V_O(\lambda)\}}{4(n-1)} > 0.$$

If $c < b_D(\delta)$, every disconnected network is dominated by a best factional tree.

For δ sufficiently close to one, Parts 3 and 4 identify a nonempty interval of costs that excludes networks with additional links and disconnected networks. Part 2 then implies that every efficient tree is factional. This proves the proposition. ■

Star-factional networks. When politicians are homogeneous, expert–decision-maker pairs have equal welfare weights. Let $d_g(u,v)$ be the length of the path between u and v in g. The first-order expansion in the main paper implies that, among factional trees, the relevant distance objective is, up to a positive common factor,

$$D_S(g) = 2\kappa_F \sum_m \sum_{\{u,v\}\subseteq C_m} d_g(u,v) + 2\kappa_O \sum_{m<r} \sum_{u\in C_m} \sum_{v\in C_r} d_g(u,v), \tag{9}$$

where $\kappa_F, \kappa_O > 0$. The coefficients are

$$\kappa_F = \frac{\lambda}{36}, \qquad \kappa_O = \frac{\lambda}{18(2-\lambda)^2},$$

and the factor 2 converts each unordered politician pair in this display into the two expert–decision-maker pairs used in the definition in the main paper. A common probability-and-relevance factor multiplies $D_S(g)$ in the welfare expansion but does not affect the ranking.

Lemma 6 *Fix an ideological collection C_m and write $n_m = |C_m|$. Define*

$$n_m^- = \sum_{r<m} |C_r|, \qquad n_m^+ = \sum_{r>m} |C_r|.$$

Let g_m be the internal tree and d_m its distance. When $n_m^- > 0$, let $z_m^- \in C_m$ be the endpoint of the cross-collection link to the left; when $n_m^+ > 0$, let $z_m^+ \in C_m$ be the endpoint of the link to the right.

If $n_m^- > 0$ and $n_m^+ > 0$, after dividing by the common factor 2, the terms in $D_S(g)$ that depend on C_m are, up to an additive constant,

$$\kappa_F \,\mathrm{WI}(g_m) + \kappa_O \left[n_m^- \sum_{u\in C_m} d_m(u, z_m^-) + n_m^+ \sum_{u\in C_m} d_m(u, z_m^+) \right] + \kappa_O n_m^- n_m^+ d_m(z_m^-, z_m^+). \tag{10}$$

For an endpoint collection, or when there is only one ideological collection, the corresponding expressions are

$$\begin{aligned}
&\kappa_F \,\mathrm{WI}(g_m) + \kappa_O n_m^+ \sum_{u\in C_m} d_m(u, z_m^+) && \text{if } n_m^- = 0 < n_m^+,\\
&\kappa_F \,\mathrm{WI}(g_m) + \kappa_O n_m^- \sum_{u\in C_m} d_m(u, z_m^-) && \text{if } n_m^+ = 0 < n_m^-,\\
&\kappa_F \,\mathrm{WI}(g_m) && \text{if } n_m^- = n_m^+ = 0.
\end{aligned} \tag{11}$$

Here $\mathrm{WI}(g_m) = \sum_{\{u,v\}\subseteq C_m} d_m(u,v)$ is the Wiener index, the sum of the path lengths between all unordered pairs in the internal tree. Each expression is minimized exactly by a star in which every endpoint of a cross-collection link coincides with the center.

Proof. For each link $uv \in \mathcal{L}(g_m)$, let s_{uv} be the number of politicians in one of the two components created by deleting that link. Since the path between a pair contains uv exactly when its members lie in different components,

$$\mathrm{WI}(g_m) = \sum_{uv \in \mathcal{L}(g_m)} s_{uv}(n_m - s_{uv}) \geq (n_m - 1)^2,$$

because each of the $n_m - 1$ terms is at least $n_m - 1$. Equality holds exactly when every link leaves a single politician on one side, which characterizes a star. For every $z \in C_m$,

$$\sum_{u \in C_m} d_m(u, z) \geq n_m - 1,$$

with equality exactly when z is adjacent to every other vertex, that is, when g_m is a star centered at z. When $n_m^- > 0$ and $n_m^+ > 0$, $d_m(z_m^-, z_m^+) \geq 0$, with equality exactly when $z_m^- = z_m^+$. Since every coefficient displayed in the relevant case is positive, all lower bounds are attained simultaneously exactly when g_m is a star and every endpoint of a cross-collection link coincides with its center. ■

Proof of Proposition 2 in the main paper. Let $\mathcal{F}$ be the set of factional trees. Define[1]

$$\underline{c}(\delta) = \max_{\substack{g \text{ connected} \\ |\mathcal{L}(g)| > n-1}} \frac{[W(g; \delta, 0) - \max_{f \in \mathcal{F}} W(f; \delta, 0)]_+}{2[|\mathcal{L}(g)| - (n-1)]},$$

$$\overline{c}(\delta) = \min_{\substack{g \text{ disconnected} \\ |\mathcal{L}(g)| < n-1}} \frac{\max_{f \in \mathcal{F}} W(f; \delta, 0) - W(g; \delta, 0)}{2[(n-1) - |\mathcal{L}(g)|]}.$$

The proof of Proposition 4 in the main paper shows that $\underline{c}(\delta) \to 0$ and $\liminf_{\delta \to 1} \overline{c}(\delta) > 0$. Hence these thresholds are ordered when transmission is sufficiently reliable.

The Taylor expansions of $V_F(\lambda\delta^\ell)$ and $V_O(\lambda\delta^\ell)$ are uniform because path lengths are bounded by $n - 1$. Their positive common probability-and-relevance factor does not affect the ranking, so the first-order ranking of factional trees is exactly the ranking generated by $D_S(g)$ in (9). By Lemma 6, precisely the star-factional trees minimize $D_S(g)$. When politicians are homogeneous, changing the identity of an ideological collection's center does not change this minimum. Except when $n = 2$ and $M = 1$, the model therefore leaves open who leads, but every minimizing network still contains exactly one center in each collection. In the exceptional case, the network's single link does not distinguish its endpoints. If there are factional trees that are not star-factional, finiteness gives a positive gap between their values of D_S and this common minimum, and the uniform second-order remainder cannot reverse the ranking when $\delta < 1$ is sufficiently close to one. If there are none, the same conclusion is immediate. Combining this result with Proposition 4 in the main paper proves that every efficient network is star-factional on such an interval.

To prove the converse, observe that all star-factional trees have the same exact welfare. Each has $n-1$ links. Within C_m, exactly $2(n_m-1)$ ordered pairs have distance one and $(n_m-1)(n_m-2)$ have distance two. For every $m < r$, among the ordered pairs with one politician in C_m and the other in C_r, two have distance $r-m$, $2(n_m+n_r-2)$ have distance $r-m+1$, and $2(n_m-1)(n_r-1)$

[1] The maximum over an empty set is defined as zero.

have distance $r - m + 2$. Thus these distance distributions depend only on collection sizes, not on the identities of the centers. Every within-collection pair has full forwarding and every cross-collection pair has one-sided disclosure, while homogeneity gives all ordered pairs the same welfare weight. Hence all star-factional trees have the same gross informational welfare and the same link cost. Since an efficient network exists and every efficient network is star-factional, every star-factional network is efficient. Therefore, the efficient networks are exactly the star-factional networks. At $\delta = 1$, path length is irrelevant, and any factional tree that is not star-factional ties with the star-factional trees; this is why Proposition 2 in the main paper requires imperfect transmission. ■

Coalitionally stable networks. When politicians are homogeneous, every politician's gross informational payoff is

$$-\frac{\alpha}{n(n-1)} \sum_{e \neq d} V_{ed}(g, \delta) = \frac{W(g; \delta, 0)}{n}.$$

Proof of Proposition 3 in the main paper. Fix an efficient network f and a reliability level δ in the interval identified by Proposition 2 in the main paper. Thus, after increasing the reliability cutoff if necessary, f is a star-factional tree. Implement it through its exact proposal profile L^f. Consider a nonempty coalition K, alternative lists L'_K, the profile $L' = (L'_K, L^f_{-K})$, and the induced network $g = g(L')$.

Three observations drive the proof. First, homogeneity gives each coalition member the same $1/n$ share of aggregate gross informational welfare. Second, a proposal that is not reciprocated costs its sender without changing the network. Any profitable deviation can therefore be replaced by one in which coalition members make no unreciprocated proposals. Third, outsiders retain their lists, so an absent link involving an outsider cannot form because that outsider does not add the proposed partner to her list. Every link added by the deviation must therefore have both endpoints in K, and the coalition bears both proposal costs. The proof treats connected and disconnected deviations separately.

Cost accounting. After removing unreciprocated proposals by coalition members, the change in the coalition's aggregate proposal expenditure is

$$2c\big(|\mathcal{L}(g)| - |\mathcal{L}(f)|\big) + c\,|\{ij \in \mathcal{L}(f) \setminus \mathcal{L}(g) : |\{i,j\} \cap K| = 1\}|\,. \tag{12}$$

The first term is the full proposal cost borne by both endpoints for the coalition's net increase in links. The second reflects that deleting a boundary link saves the coalition only one proposal cost; the outsider keeps the unreciprocated proposal and continues to bear its cost.

Connected deviations. Suppose g is connected. Since f is a tree,

$$|\mathcal{L}(g)| \geq n - 1 = |\mathcal{L}(f)|.$$

Efficiency of f implies

$$W(g; \delta, 0) - W(f; \delta, 0) \leq 2c\big(|\mathcal{L}(g)| - |\mathcal{L}(f)|\big).$$

Therefore

$$\begin{aligned}\sum_{i\in K}[U_i(L')-U_i(L^f)] &= \frac{|K|}{n}[W(g;\delta,0)-W(f;\delta,0)]-2c\big(|\mathcal{L}(g)|-|\mathcal{L}(f)|\big)\\ &\quad -c\,|\{ij\in\mathcal{L}(f)\setminus\mathcal{L}(g):|\{i,j\}\cap K|=1\}|\\ &\le -2c\left(1-\frac{|K|}{n}\right)\big(|\mathcal{L}(g)|-|\mathcal{L}(f)|\big)\\ &\quad -c\,|\{ij\in\mathcal{L}(f)\setminus\mathcal{L}(g):|\{i,j\}\cap K|=1\}|\le 0.\end{aligned}$$

No coalition has a profitable connected deviation. The inequality covers link additions and connected rewiring at the same time.

Disconnected deviations. Every disconnected network leaves at least one positive-weight expert–decision-maker pair unable to communicate, so, for every δ,

$$\overline{W}-W(g;\delta,0)\ge\min_{e\neq d}\pi_{ed}R_d\min\{V_B-V_F(\lambda),V_B-V_O(\lambda)\}>0.$$

The gross welfare of every star-factional tree converges uniformly to $\overline{W}$. After increasing $\bar{\delta}_C$ if necessary,

$$W(f;\delta,0)-W(g;\delta,0)\ge\frac{1}{2}\min_{e\neq d}\pi_{ed}R_d\min\{V_B-V_F(\lambda),V_B-V_O(\lambda)\}$$

for every star-factional tree f and every disconnected g. The coalition bears at least a $1/n$ share of this gross informational loss, while its proposal-cost saving cannot exceed the total cost $2c(n-1)$ of the exact profile L^f. Define

$$c^D=\frac{\min_{e\neq d}\pi_{ed}R_d\min\{V_B-V_F(\lambda),V_B-V_O(\lambda)\}}{4n(n-1)}.$$

No disconnected deviation is profitable when $c<c^D$. Thus one may take

$$\underline{c}^C(\delta)=\underline{c}(\delta),\qquad\overline{c}^C(\delta)=\min\{\overline{c}(\delta),c^D\}.$$

After increasing $\bar{\delta}_C$ if necessary, these thresholds are ordered and have the stated limits. The exact proposal profile associated with every efficient network is coalitionally stable.

Inefficient networks are blocked. Suppose a proposal profile L induces a network g that is not efficient. Every formed link requires two proposals, while additional unreciprocated proposals only raise expenditure. Therefore

$$\sum_{i\in\mathcal{N}}U_i(L)\le W(g;\delta,0)-2c|\mathcal{L}(g)|=W(g;\delta,c).$$

The grand coalition $K=\mathcal{N}$ can replace all proposal lists with the exact proposal profile of an efficient network f. At that profile, aggregate payoff equals welfare. Hence efficiency gives

$$\sum_{i\in\mathcal{N}}U_i(L^f)=W(f;\delta,c)>W(g;\delta,c)\ge\sum_{i\in\mathcal{N}}U_i(L).$$

The grand coalition blocks L, contradicting stability. Hence the coalitionally stable networks are exactly the efficient networks, which are exactly the star-factional networks by Proposition 2 in the main paper. ■

Why homogeneity matters for stability. The implication from coalitional stability to efficiency does not require homogeneity: the grand coalition can always replace an inefficient network with an efficient one. The reverse implication can fail because a politician need not receive a fixed share of aggregate informational welfare.

Efficiency need not imply Nash equilibrium. With heterogeneous welfare weights, efficiency may fail to imply even Nash equilibrium. Consider two politicians with

$$b_1 = -\frac{1}{100}, \qquad b_2 = \frac{1}{100}, \qquad \lambda = \frac{1}{2}, \qquad \delta = 1,$$

and set

$$\pi_{12} = \pi_{21} = \frac{1}{2}, \qquad c = \frac{1}{1000}.$$

Politician 1 assigns weight 1/100 to either politician's decision, while politician 2 assigns weight 1 to either decision:

$$\alpha_{11} = \alpha_{12} = \frac{1}{100}, \qquad \alpha_{21} = \alpha_{22} = 1.$$

The two biases are close enough that the linked network supports full forwarding for either expert–decision-maker pair. Since

$$V_B - V_F\left(\frac{1}{2}\right) = \frac{1}{72},$$

the aggregate informational gain from the link is

$$\left[\frac{1}{2}\left(\frac{1}{100}+1\right) + \frac{1}{2}\left(\frac{1}{100}+1\right)\right]\frac{1}{72} = \frac{101}{7200}.$$

The aggregate link cost is

$$2c = \frac{1}{500}.$$

Thus its net social gain is

$$\frac{101}{7200} - \frac{1}{500} = \frac{433}{36000} > 0.$$

Because the empty and linked networks are the only possibilities, the linked network is uniquely efficient.

Any proposal profile that induces the link must have both politicians list one another. Politician 1's private informational gain from the link is only

$$\left[\frac{1}{2}\frac{1}{100} + \frac{1}{2}\frac{1}{100}\right]\frac{1}{72} = \frac{1}{7200} < c.$$

By removing politician 2 from her list, politician 1 severs the link and raises her payoff by

$$c - \frac{1}{7200} = \frac{31}{36000} > 0.$$

Thus no proposal profile inducing the unique efficient network is a Nash equilibrium. The example isolates the distributive problem: the link's informational benefit to politician 2 makes it socially valuable, but politician 1 bears an endpoint cost that exceeds her own small informational gain.

Proof of Lemma 1 in the main paper. Suppose all politicians belong to one ideological collection and $\delta < 1$. In a star centered at z, every pair supports full forwarding. Its path length is one when $e = z$ or $d = z$, and two otherwise. All stars have $n - 1$ links and therefore the same total link cost. Gross welfare is

$$W(z; \delta, 0) = -V_F(\lambda\delta) \left(\sum_{d \neq z} w_{zd} + \sum_{e \neq z} w_{ez} \right) \\ - V_F(\lambda\delta^2) \sum_{\substack{e \neq d \\ e,d \neq z}} w_{ed}.$$

Since $\sum_{e \neq d} w_{ed}$ is independent of z,

$$W(z; \delta, 0) = \text{constant} + [V_F(\lambda\delta^2) - V_F(\lambda\delta)]\Gamma_z.$$

The bracketed coefficient is strictly positive when $\delta < 1$, so efficient centers maximize Γ_z, whereas at $\delta = 1$ it is zero and all centers tie. When $n = 2$, the single link does not distinguish a center. When $n \geq 3$ and politicians are homogeneous, all candidate centers likewise tie, but each efficient star has exactly one center. Thus symmetry creates multiplicity in the leader's identity across networks; it does not create shared leadership within an efficient network. ■

Proof of Proposition 5 in the main paper. If

$$\pi_{ed} = \frac{p_E(e)p_D(d)}{Z}, \qquad Z = \sum_{r \neq s} p_E(r)p_D(s),$$

then direct substitution gives

$$\Gamma_z = \frac{1}{Z} \left[p_E(z) \sum_{d \neq z} p_D(d)R_d + p_D(z)R_z \sum_{e \neq z} p_E(e) \right].$$

Because diagonal pairs are excluded before normalization, p_E and p_D are selection weights and need not equal the resulting marginals.

For the uniqueness claim, common rescalings of the source-selection weights, decision-maker-selection weights, and relevance values do not affect the maximizing center. Set $z = i$. Normalize their common values for all politicians other than z to one and write

$$p_E(z) = \eta_E \geq 1, \qquad p_D(z) = \eta_D \geq 1, \qquad R_z = \eta_R \geq 1,$$

with at least one inequality strict. For any other candidate j,

$$Z\Gamma_z = (n-1)(\eta_E + \eta_D\eta_R), \qquad Z\Gamma_j = \eta_E + \eta_D\eta_R + 2(n-2).$$

Hence

$$Z(\Gamma_z - \Gamma_j) = (n-2)(\eta_E + \eta_D\eta_R - 2) > 0$$

when $n \geq 3$. Thus z is the unique maximizer and, by Lemma 1 in the main paper, the unique efficient center among stars. ■

Proof of Example 1 in the main paper. For a tree t, every expert–decision-maker path lies within the single ideological collection and therefore has full forwarding. Write

$$D_L(t) = \sum_{e \neq d} w_{ed} d_t(e, d).$$

Uniformly over the finite set of four-politician trees,

$$W(t; \delta, 0) = -V_F(\lambda) \sum_{e \neq d} w_{ed} - \kappa_F (1-\delta) D_L(t) + O((1-\delta)^2), \qquad \kappa_F = \frac{\lambda}{36}.$$

In g^L, the six ordered pairs corresponding to its links have unit weight and distance one, while the distances of the other six ordered pairs sum to 14. Hence

$$D_L(g^L) = 6 + 14\varepsilon < 8.$$

Any other tree t omits at least one link ij of g^L. The two unit-weight pairs (i, j) and (j, i) then contribute at least 4 to $D_L(t)$, and the other four unit-weight pairs contribute at least another 4. Since every remaining pair has positive weight, $D_L(t) > 8$. The expansion therefore implies that g^L strictly maximizes gross informational welfare among trees whenever $\delta < 1$ is sufficiently close to one.

Let

$$\overline{W}_L = -V_F(\lambda) \sum_{e \neq d} w_{ed}.$$

Full arrival of both realizations is the most informative feasible experiment, so $W(t; \delta, 0) \leq \overline{W}_L$ for every network t and every δ, while $W(g^L; \delta, 0) \to \overline{W}_L$ as $\delta \to 1$. Every disconnected network leaves at least one positive-weight ordered pair unable to communicate. Since $\min_{e \neq d} w_{ed} = \varepsilon$,

$$\overline{W}_L - W(t; \delta, 0) \geq \varepsilon [V_B - V_F(\lambda)] > 0$$

for every disconnected t. Thus, after increasing the reliability cutoff if necessary, g^L strictly dominates every disconnected network with three links.

For the remaining networks, define

$$\underline{c}_L(\delta) = \max_{|\mathcal{L}(t)| > 3} \frac{[W(t; \delta, 0) - W(g^L; \delta, 0)]_+}{2[|\mathcal{L}(t)| - 3]},$$

$$\bar{c}_L(\delta) = \min_{|\mathcal{L}(t)| < 3} \frac{W(g^L; \delta, 0) - W(t; \delta, 0)}{2[3 - |\mathcal{L}(t)|]},$$

where $[x]_+ = \max\{x, 0\}$. Every network with more than three links is connected, and the upper-bound argument gives $\underline{c}_L(\delta) \to 0$. Every network with fewer than three links is disconnected, so the preceding uniform gap gives $\liminf_{\delta \to 1} \bar{c}_L(\delta) > 0$. The thresholds are therefore ordered when reliability is sufficiently close to one. If $\underline{c}_L(\delta) < c < \bar{c}_L(\delta)$, the first threshold makes g^L dominate every network with more than three links, and the second makes it dominate every network with fewer than three links. It also dominates every other three-link network: connected three-link networks are trees, and disconnected three-link networks were covered above. Hence g^L is uniquely efficient. ∎

Welfare from an additional link. For a factional tree g, write $w_{ed} = \pi_{ed} R_d$. As in the main paper, let $\mathcal{I}$ contain the pairs (e, d) with $e \neq d$ whose members belong to the same ideological collection and let $\mathcal{X}$ contain those whose members belong to different ideological collections. Define

$$D(g) = \kappa_F \sum_{(e,d)\in\mathcal{I}} w_{ed} d_g(e, d) + \kappa_O \sum_{(e,d)\in\mathcal{X}} w_{ed} d_g(e, d), \tag{13}$$

where

$$\kappa_F = \frac{\lambda}{36}, \qquad \kappa_O = \frac{\lambda}{18(2-\lambda)^2}.$$

Proof of Proposition 6 in the main paper. *Part (a).* The difference in informational welfare between $g + ij$ and g is $B_{ij}(g, \delta)$. With heterogeneous relationship costs, adding the link increases total link costs by c_{ij}, so the welfare difference is

$$B_{ij}(g, \delta) - c_{ij}.$$

Thus adding the link raises welfare if and only if $B_{ij}(g, \delta) > c_{ij}$, proving part (a).

Part (b). Every path within an ideological collection has full forwarding, and every path across ideological collections has one-sided disclosure in the direction determined by their ideological ordering. Uniformly over the finite set of path lengths,

$$V_F(\lambda\delta^\ell) = V_F(\lambda) + \kappa_F \ell(1-\delta) + O((1-\delta)^2).$$

The analogous expansion holds with V_O and κ_O under one-sided disclosure. All factional trees have $n - 1$ links, so their link costs are equal. After multiplying each path loss by w_{ed} and summing, the welfare difference between g and h is

$$(1-\delta)[D(h) - D(g)] + O((1-\delta)^2).$$

The first-order term is strictly positive when $D(g) < D(h)$ and dominates the remainder for every $\delta < 1$ sufficiently close to one. At $\delta = 1$, the two trees tie because transmission does not decay with path length.

Part (c). The expansion in part (b) gives

$$V_F(\lambda\delta^{\ell+1}) - V_F(\lambda\delta^\ell) = \kappa_F(1-\delta) + O((1-\delta)^2).$$

The analogous expression holds with V_O and κ_O under one-sided disclosure. Thus the first-order coefficient on one additional link in the path is κ_F under full forwarding and κ_O under one-sided disclosure. Their ratio is

$$\frac{\kappa_O}{\kappa_F} = \frac{2}{(2-\lambda)^2}.$$

Therefore $\kappa_O > \kappa_F$ if and only if $(2-\lambda)^2 < 2$, or equivalently $\lambda > 2 - \sqrt{2}$. ∎

B Ideology profiles without distinct collections

This section verifies the two examples in the main paper. They retain ordered ideologies and homogeneous politicians but do not admit a partition into distinct ideological collections. The first profile contains adjacent pairs whose bias differences lie between the within-collection and across-collection bounds. In the second, ideological proximity is nontransitive. The examples show that keeping paths short between politicians with similar biases and minimizing path lengths across all politicians need not favor the same tree.

Example 2 *Let six homogeneous politicians have biases satisfying the strict inequalities*

$$\begin{aligned}
&\frac{1}{12} < b_2 - b_1 < \overline{\beta}(\lambda), && b_3 - b_2 > \overline{\beta}(\lambda),\\
&\frac{1}{12} < b_4 - b_3 < \overline{\beta}(\lambda), && b_5 - b_4 > \overline{\beta}(\lambda),\\
&\frac{1}{12} < b_6 - b_5 < \overline{\beta}(\lambda).
\end{aligned}$$

For the candidate partition $\{1,2\},\{3,4\},\{5,6\}$*, let* g *be the star-factional tree and* h *the alternative with*

$$\mathcal{L}(g) = \{13, 35, 12, 34, 56\},$$
$$\mathcal{L}(h) = \{13, 35, 34, 24, 46\}.$$

Because politicians are homogeneous, all ordered expert–decision-maker pairs have the same welfare weight. At $\delta = 1$*, both trees have one-sided disclosure for every pair, all forwarded evidence arrives, and their five links have the same total cost. They therefore yield equal welfare. For every* $\delta < 1$ *sufficiently close to one, disclosure remains one-sided, and welfare can be compared by adding the lengths of all expert–decision-maker paths. These sums are* 62 *for* g *and* 58 *for* h*, so* h *yields greater welfare.*

Proof. Every adjacent gap exceeds $1/12$. Hence $|b_e - b_d| > 1/12$ for every $e \neq d$, ruling out full forwarding. It also rules out mixed disclosure because each mixed region requires a politician on one side of the decision maker to lie strictly within $1/12$ of her. At $\delta = 1$, the paths with a bias reversal correspond to the pairs

$$(3,2), (4,2), (5,2), (6,2), (5,4), (6,4)$$

in g, and

$$(1,2), (2,3), (5,4), (6,5)$$

in h. In every such path, the politicians on one side of the decision maker are separated from her by one of the three moderate gaps $b_2 - b_1$, $b_4 - b_3$, or $b_6 - b_5$, each of which is below $\overline{\beta}(\lambda)$. No path therefore has a strong bias reversal. Proposition 1 gives one-sided disclosure for every pair in both trees. Because the relevant inequalities are strict and there are finitely many paths, the same conclusion holds for every $\delta < 1$ sufficiently close to one.

Because all ordered pairs have the same welfare weight, the relevant first-order comparison is their total path length. Direct calculation gives

$$\sum_{e \neq d} d_g(e,d) = 62, \qquad \sum_{e \neq d} d_h(e,d) = 58.$$

The first-order expansion of $V_O(\lambda\delta^\ell)$ therefore implies that h has greater gross informational welfare for every $\delta < 1$ sufficiently close to one. Both trees contain five links, so their cost terms are equal. ∎

Tree h can be drawn as the two three-politician stars $1-3-5$ and $2-4-6$, joined by 34. Yet politicians joined within these apparent stars are separated by one of the large ideological gaps, while link 34 joins a moderately close pair. Hence the two stars cannot be interpreted as ideological factions.

Example 3 *Let seven homogeneous politicians have biases satisfying*

$$\begin{gathered} b_2 - b_1 > \overline{\beta}(\lambda), \qquad b_3 - b_2 < \frac{1}{12}, \qquad b_4 - b_3 < \frac{1}{12}, \\ \frac{1}{12} < b_4 - b_2 < \overline{\beta}(\lambda), \qquad b_6 - b_4 < \frac{1}{12}, \qquad \frac{1}{12} < b_5 - b_3 < \overline{\beta}(\lambda), \\ b_7 - b_6 > \overline{\beta}(\lambda). \end{gathered}$$

Let g be the star centered at politician 4. Let h have links $4i$ for every $i \in \{1,3,5,6,7\}$ and the additional link 23. For reliability sufficiently close to one, h yields strictly greater informational welfare than g.

Proof. In the star g, the path from expert 3 to decision maker 2 passes through politician 4. Although politicians 2 and 3 are within $1/12$ of one another, politician 4 is more than $1/12$ to the right of politician 2. Proposition 1 therefore gives one-sided disclosure on this path. From politician 2 to politician 3, both senders on the path $2-4-3$ lie within $1/12$ of the decision maker, so g already has full forwarding. In h, politicians 2 and 3 are linked, giving full forwarding in both directions.

The trees differ only by replacing link 24 with link 23, and paths not involving politician 2 are unchanged. For $(2,4)$, the path in g has only sender 2, while the path in h adds politician 3 as an intermediary. Because $b_4 - b_2 > 1/12$ and $b_4 - b_3 < 1/12$, Proposition 1 gives one-sided disclosure in both trees. For every remaining pair involving politician 2, politician 3 lies between politicians 2 and 4 when 2 is the expert, or is closer to decision maker 2 than politician 4 when 2 is the decision maker. She therefore cannot cross a cutoff not already crossed in g. Proposition 1 gives the same outcome in both trees for every other pair when reliability is sufficiently close to one.

Replacing 24 with 23 lengthens some paths, but the resulting informational loss converges to zero as δ approaches one. By contrast, the improvement from one-sided disclosure to full forwarding for $(3,2)$ remains strictly positive at $\delta = 1$ because $\lambda < 1$. Hence the gain eventually dominates the path-length loss. The two trees contain the same number of links, so h also yields greater welfare. ∎

The alternative tree is not star-factional. Politicians 2 and 3 are linked, preserving full forwarding for this close pair, even though politician 4 remains the main center for the other politicians. This is the simplest way in which nontransitive ideological proximity produces a leader-centered tree that is not factional.

C Counterexamples outside the main cost region

The cost interval in Proposition 4 in the main paper is substantive. The next two examples retain a common positive cost and distinct ideological collections but show that, at the two ends of the cost range, an efficient network need not be factional. The final argument in this section instead departs from the assumptions that link costs are common and positive. It shows that direct benefits from relationships forming a cycle make every efficient network contain that cycle.

Example 4 *Let $C_1 = \{1, 2\}$ and $C_2 = \{3, 4\}$ be distinct ideological collections, and fix $\delta \in (0, 1)$. There are full-support probabilities π and a positive common link cost c such that no factional network is efficient. In particular, the non-factional network $g^\circ = \{12, 23, 34, 14\}$ strictly dominates every factional network.*

Proof. Fix $\delta \in (0, 1)$ and let g° have links 12, 23, 34, and 14. For the pair $(e, d) = (1, 4)$, the protocol schedules the path $1 \to 4$ and the three-link path $1 \to 2 \to 3 \to 4$.

Message vectors on the equilibrium path. After observing $s = 0$, expert 1 chooses the date-zero vector that sends the evidence to both 4 and 2. Politicians 2 and 3 forward $s = 0$ whenever they receive it. After observing $s = 1$, the expert sends silence directly to 4 but sends the evidence to 2. If the strategy of politician 3 prescribes forwarding $s = 1$ when pivotal, politician 2 is pivotal and withholds it. If the strategy instead prescribes blocking $s = 1$, politician 2's choice cannot affect whether the realization reaches 4, so the stated tie-breaking rule makes politician 2 forward it, after which politician 3 blocks it. Thus $s = 1$ never reaches 4. A politician who has not received evidence sends silence.

The asymmetric vector after $s = 1$ satisfies the stated tie-breaking rule. Whereas sending $s = 1$ directly to 4 is strictly harmful to the expert, sending it to 2 is payoff-irrelevant because at least one of politicians 2 and 3 blocks it before it can reach 4. Among the expert's pure best-response vectors, the prescribed vector is therefore componentwise maximal in forwarding.

Beliefs. Verifiability identifies the expert and realization. Any politician who receives $s = 0$ therefore has posterior mean 1/3, and any politician who receives $s = 1$ has posterior mean 2/3. Decision maker 4 uses posterior mean 1/3 after receiving $s = 0$, posterior mean 2/3 after an off-path receipt of $s = 1$, and posterior mean $\mu_H(\lambda q_{14})$ after receiving no evidence, where

$$q_{14} = 1 - (1 - \delta)(1 - \delta^3).$$

Bayes' rule determines every positive-probability belief. At zero-probability information sets, these realization-specific beliefs and arbitrary beliefs over payoff-irrelevant duplicate-copy histories complete the perfect Bayesian assessment.

Intermediaries' incentives. After $s = 0$, every politician on either path is weakly to the left of 4 and forwards whenever her action can affect arrival. After receiving $s = 1$, an intermediary whose action can affect arrival forwards if and only if

$$b_r - b_4 \geq \frac{\mu_H(\lambda q_{14}) - \frac{2}{3}}{2}, \qquad r \in \{2, 3\}.$$

At equality, the stated tie-breaking rule makes an indifferent politician forward. If a component cannot affect arrival, the same rule selects forwarding because doing so makes the best-response vector maximal in forwarding. Separation gives $b_2 - b_4 < -\overline{\beta}(\lambda)$, while

$$\frac{\mu_H(\lambda q_{14}) - \frac{2}{3}}{2} > -\overline{\beta}(\lambda),$$

so politician 2 strictly withholds $s = 1$ whenever pivotal. If politician 3 satisfies the displayed forwarding condition, politician 2 is pivotal and blocks the realization. If politician 3 does not satisfy it, politician 3 strictly blocks, so politician 2's choice cannot affect arrival and the stated tie-breaking rule makes politician 2 forward. In both cases, $s = 1$ is blocked before reaching 4.

The expert's incentives. After $s = 0$, each path gives an additional positive chance of moving the decision from the no-arrival posterior toward 1/3, so the expert strictly prefers to use both components of her date-zero vector. After $s = 1$, the expert evaluates any direct transmission to 4 that creates a positive probability of arrival by comparing the policy based on 2/3 with the no-arrival policy based on $\mu_H(\lambda q_{14})$. Since

$$b_1 - b_4 < -\overline{\beta}(\lambda) < \frac{\mu_H(\lambda q_{14}) - \frac{2}{3}}{2},$$

such a transmission is strictly unprofitable. The message to 2 cannot affect terminal information because either politician 2 or politician 3 blocks it. The expert's prescribed vector is therefore a pure best response and is componentwise maximal in forwarding.

Delivery and comparison. The two paths for $s = 0$ use disjoint transmission shocks. Conditional on the expert being informed and observing $s = 0$, at least one copy arrives with probability

$$q_{14} = 1 - (1 - \delta)(1 - \delta^3) > \delta.$$

The constructed assessment has posterior variance $V_O(\lambda q_{14})$. Since $V_{14}(g^\circ, \delta)$ minimizes posterior variance over equilibria for this pair and network,

$$V_{14}(g^\circ, \delta) \leq V_O(\lambda q_{14}) < V_O(\lambda\delta).$$

With two politicians in each ideological collection, a factional network must contain links 12 and 34 and exactly one of the four cross-collection links. It is therefore a tree. For every such network f, the unique path from 1 to 4 has length $d_f(1, 4) \geq 1$ and has one-sided disclosure in which $s_e = 0$ is forwarded. Hence

$$V_{14}(f, \delta) = V_O(\lambda\delta^{d_f(1,4)}) \geq V_O(\lambda\delta).$$

Choose $\pi_{14} = 1 - \varepsilon$ and distribute $\varepsilon > 0$ across all remaining pairs with full support. Since every posterior variance lies in $[0, 1/12]$, for every factional network f,

$$W(g^\circ; \delta, 0) - W(f; \delta, 0) \geq (1 - \varepsilon)R_4[V_O(\lambda\delta) - V_O(\lambda q_{14})] - \frac{\varepsilon \max_d R_d}{12}.$$

The first bracket is strictly positive. Choose ε so that the displayed lower bound, denoted B_ε, is positive, and then choose

$$0 < c < \frac{B_\varepsilon}{2}.$$

Because g° has one more link than every factional network,

$$W(g^\circ; \delta, c) > W(f; \delta, c)$$

for all factional networks f. Thus no factional network is efficient, and at least one efficient network is non-factional. ■

Example 5 *Fix an ideology profile with at least two politicians that admits distinct ideological collections* $(C_1, \ldots, C_M)$*, and fix any* $\delta \in (0, 1]$*. There is a positive common link cost c for which the empty network is the unique efficient network. This efficient network is not factional for the specified collection partition.*

Proof. Let $\varnothing$ denote the empty network. The finite set of networks and bounded posterior variance imply that

$$B = \max_g \{W(g; \delta, 0) - W(\varnothing; \delta, 0)\} < \infty.$$

For every nonempty network g,

$$W(g; \delta, c) - W(\varnothing; \delta, c) = W(g; \delta, 0) - W(\varnothing; \delta, 0) - 2c|\mathcal{L}(g)| \le B - 2c.$$

Thus, for every $c > B/2$, the empty network strictly dominates every nonempty network and is uniquely efficient. Because every factional network on at least two politicians contains a link, the empty network is not factional. ■

Directly valuable relationships. A clique is a set of politicians in which every pair is linked.

Proof of Remark 1 in the main paper. All politicians belong to one ideological collection, so $|b_r - b_d| \le 1/12$ for every politician r and every possible decision maker d. The full-forwarding profile is an equilibrium after any internal link is added. At $\delta = 1$, adding such a link cannot reduce informational welfare. If an efficient network omitted a link ij with $c_{ij} < 0$, adding it would raise total welfare by at least $-c_{ij} > 0$, contradicting efficiency. Every negative-cost link must therefore be present. The cycle and clique conclusions follow immediately. ■

D Relation to an earlier disclosure model

The model in Squintani (2026) shares the basic idea that politically biased actors may hide evidence, but it asks a different network question. That model studies a continuous state that an informed actor observes exactly and allows partial disclosure. Its path condition is simple: exact disclosure to decision maker d is possible if and only if the expert and every intermediary lie weakly on the same side of d. If the path contains actors on both sides, they suppress different parts of the state and exact disclosure fails. Under that model's network-design and cost conditions, this condition selects the ordered ideological line as the unique efficient and coalitionally stable network.

The present paper studies binary evidence and allows the expert to be uninformed. These assumptions create three cutoffs, $\underline{\beta}(r)$, 1/12, and $\overline{\beta}(r)$, rather than a single sign condition. When every actor on a path lies within 1/12 of the decision maker, both realizations can be forwarded. In two asymmetric regions, exactly one politician—the most biased relative to the decision maker—mixes over the realization that runs against her bias. Larger disagreement on one side gives one-sided disclosure in pure strategies. A bias reversal by itself need not cause breakdown; the actors on both sides must also cross the blocking cutoffs. The within-collection bound permits actors on both sides of a decision maker while preserving full forwarding for every within-collection expert–decision-maker pair.

The network-design results therefore differ. In the model of Squintani (2026), the no-bias-reversal condition selects a line. In the present model, the efficiency result selects factional trees under the stated cost, reliability, and ideological-separation conditions; when politicians are homogeneous, the internal network of each ideological collection is a star, and adjacent stars are joined through their centers. The present model also studies how expertise, authority, relevance, relationship costs, and transmission failures produce departures from this structure.

Both models use the same definition of coalitional stability: any set of actors may coordinate a deviation, which is evaluated by its total payoff. Squintani (2026) applies this test to the ordered line under that model's cost conditions. The present model instead applies it to the bilateral proposal game, in which the networks selected by the efficiency result are also coalitionally stable. Within the present model, decentralized link choice can therefore sustain internal centers and sparse links between neighboring collections.

The communication protocol follows the path schedule in Squintani (2026). Different paths may create repeated, time-respecting transmission attempts, and evidence may reach the decision maker through scheduled links drawn from more than one original path. The formal protocol is stated at the beginning of this Supplementary Appendix and summarized in the main paper. The optimality proof does not need a closed-form delivery probability for every network with cycles. It uses the welfare bound under full and one-sided disclosure and the fact that factional-tree payoffs approach that bound when transmission becomes reliable.

E Extended discussion of political peer networks

The main paper uses evidence from several kinds of political networks, and this section gives the longer discussion. It does not claim that every observed tie exists only to transmit verifiable evidence. Political networks also support coalitions, monitoring, resource exchange, and social relations. The evidence is nevertheless closely related to the model because the measured ties often represent information or advice, and actors often cite information exchange as a principal reason for maintaining them. Moreover, the same architecture recurs across settings: several groups are differentiated by preferences, each has a small leadership core and a larger periphery, and relatively few links connect different groups.

Policy networks. The San Francisco Bay–Delta network studied by Zafonte and Sabatier (1998) provides a clear example of several policy factions rather than a two-party split. The authors identify four cohesive groups. The first contains state and local agencies concerned with exporting water from the Delta. The second contains local agencies concerned with developing the Bay shoreline. The third contains local governments and organizations associated with discharges into the Bay–Delta. The fourth contains organizations primarily concerned with environmental management. A less cohesive residual group combines some environmental-protection organizations with organizations linked to developers who also value environmental quality. Coordination reflects shared beliefs as well as the need to manage interdependence, while the separation among the groups follows their different policy objectives.

The advice and information network surrounding California's Marine Life Protection Act also contains four relatively separate subnetworks (Weible and Sabatier, 2005). One brings together local governments and harbormasters. A second contains commercial fishers and divers together with state and federal agencies and environmental groups. A third contains recreational fishers and commercial passenger fishing vessels. A fourth consists of kelp harvesters. The alliance and coordination network is somewhat more integrated than the information and advice network, but fundamental policy beliefs and preference similarity predict both kinds of ties. This distinction is especially useful for the present paper: the multi-faction structure is visible in a network whose stated function is information and advice, not only in a network of declared political alliances.

California regional-planning studies supply evidence about leadership within these groups. Henry et al. (2011) find that policy actors with similar beliefs are more likely to coordinate and that ideological coalitions organize around policy leaders and brokers. Henry (2011) and Gerber et al. (2013) study local institutions in five California regions. The regional networks form geographically defined groups that range from stars to networks containing every possible internal link, with intermediate cases as well. With limited exceptions in San Diego and Riverside, these groups are linked through their central jurisdictions. Political distance is associated with a lower probability of collaboration even after demographic, socioeconomic, and geographic proximity are taken into account. Thus these studies contain all three features of the structure studied here: differentiated groups, central actors inside them, and concentrated links between them.

The star captures this architecture; it does not imply that every peripheral tie is absent. Feiock et al. (2012), for example, find hierarchical local-government networks in metropolitan Orlando, where an actor's contacts also tend to be linked to one another and some peripheral actors are linked directly. Schneider et al. (2003) find denser networks in estuaries participating in the National Estuary Program, while preference similarity and perceived influence remain important for collaboration. In Swedish wildlife management, policy beliefs rather than perceived influence drive coordination (Matti and Sandström, 2011). Across these cases, political camps contain leaders and peripheral members, but repeated contact and institutional arrangements can add links beyond the pure star.

Other work helps separate the mechanisms that place an actor at short network distances from many others. Leifeld and Schneider (2012) describe policy actors as balancing the cost of maintaining a tie against its informational value. Fischer and Sciarini (2016) find that preference similarity and perceived power both shape collaboration in major Swiss policy processes, while Metz and Brandenberger (2023) show that institutions affect power concentration and collaboration across opposing camps. Boehmke et al. (2006) connect information sources to the choice of policy venue. Hence an actor may occupy such a position because she has greater expertise, authority, or relevance, or because relationships with her are inexpensive. These mechanisms correspond to distinct parts of the heterogeneous analysis.

Policy-diffusion research provides a related perspective. Volden et al. (2008) model how governments learn from policy experiments in other jurisdictions. Desmarais et al. (2015) infer networks of policy influence among U.S. states and find that ideology, population, and geography help explain which states follow one another. These are not direct measures of interpersonal communication, but the findings are consistent with policy information moving selectively through recognizable channels rather than being broadcast uniformly.

Interest groups and party organizations. Information exchange is central to networks of lobbyists, interest groups, party organizations, and policy advocates. Carpenter et al. (2004) document actors who connect otherwise separated contacts and a tendency for an actor's contacts to be linked to one another in Washington information networks. Their earlier work explains why weak ties matter: they carry information across lobbying communities that would otherwise remain separated (Carpenter et al., 1998). König and Bräuninger (1998) find that political proximity strongly predicts ties among interest groups, trade unions, government agencies, and legislators in German labor policy. Their networks contain ideologically homogeneous, star-like groups in which powerful actors occupy central positions. Personal relationships also shape how officials respond to political messages (Grose et al., 2022).

The party networks in Koger et al. (2009) contain two largely separate and polarized groups, one Democratic and one Republican. Each group has a dense inner set of organizations and a fringe whose members often have only one connection. Formal party organizations are central. Media outlets and interest groups both communicate with those organizations and share information among themselves, creating additional links within the leadership core and periphery. The network is therefore not a literal pair of stars, but it closely resembles two factional leadership cores surrounded by less connected actors, with very little communication across the party boundary.

The amicus network studied by Box-Steffensmeier and Christenson (2014) provides a multi-faction comparison. Co-signing relations reveal a collection of tightly grouped ideological communities organized around highly connected organizations. Some communities are highly centralized, others contain almost every possible internal link, and still others lie between these forms. The most connected organizations tend to be larger in membership, budget, or activity. Over time the network becomes more connected and more hierarchical. Thus the data again place many peripheral actors around a few leaders, while the number of links within factions varies. Oklobdzija (2024) finds a related structure with a leadership core and a periphery among nonparty campaign organizations, with central dark-money groups connected to more peripheral organizations.

These networks clarify the distinction between ideological collections and factions. Rose (1964) uses *tendency* for a pattern of political views and *faction* for an organized group. In the paper, actors with sufficiently similar political views form an ideological collection. Janda (1980, 1993) document organized party factions around identifiable leaders. The model begins only with ideological collections. It then produces a communication structure in which one actor becomes the center and the endpoint of the links to adjacent factions. Electoral organization or discipline may reinforce that position, but it is not needed to create it.

Formal lobbying models show why network position cannot be reduced to preference similarity. Awad (2020) shows that an allied legislator can carry an interest group's argument to less aligned colleagues, although excessive closeness may reduce credibility. Awad and Minaudier (2026) show that the value of access to one policymaker depends on ideology and procedural power elsewhere in the institution. These papers take legislative access as given. The present model instead characterizes which peer relationships maximize its aggregate-payoff objective when the source and destination of evidence are not known ex ante.

Judicial networks. Courts are especially relevant because judges make individual decisions, consult peers, and exchange material whose source and content are usually verifiable. Weiser (2015) reports that judges in the Southern District of New York frequently seek informal advice from other jurists. Citation networks provide a systematic record of the authorities judges invoke. A judge may select which precedents, statutes, records, or procedural histories to cite, but ordinarily cannot fabricate their content without detection. Hence the strategic choice is what authority to transmit or emphasize.

Caldeira (1985) shows that state supreme courts are more likely to cite courts that are culturally close and prestigious. Geography and political similarity also matter, but cultural proximity and the prestige of the cited court are especially important. This pattern is consistent with approximately star-shaped citation groups in which courts direct citations toward prestigious, culturally close peers. Similarity therefore helps determine the relevant group, whereas prestige favors a position with many incoming citations.

Clerk mobility reveals a related professional hierarchy. Katz and Stafford (2010) identify a densely connected leadership core of socially prominent judges. Supreme Court justices occupy the leadership core, circuit judges are connected to it, and district judges are mainly in the periphery. Ideological proximity also predicts clerk hiring and professional ties (Ditslear and Baum, 2001; Baum, 2014). These patterns correspond closely to the heterogeneous model. Jurisdictional authority makes a judge an important destination; experience and influential precedents make a judge a valuable source; and ideology affects which actors trust the information that travels through the relationship.

The judicial evidence also motivates allowing a small leadership core rather than a single leader. Different dimensions of prestige, subject-matter expertise, jurisdiction, and ideology may make different communication paths especially important.

In the model's welfare analysis, Lemma 1 in the main paper identifies the efficient center among stars, while the four-politician example shows that heterogeneous welfare weights can instead make a network with a leadership core efficient. Whether observed judicial leadership cores arise from corresponding communication demands is a separate empirical question.

Legislative networks and the model's boundary. Networks in Congress help legislators filter the information they receive (Fiellin, 1962). Legislative member organizations, friendship networks, and cosponsorship networks display ideological homophily and large differences in how many links actors have and how close they lie to others (Ringe and Victor, 2013; Bratton and Rouse, 2011; Canen et al., 2023). Cosponsorship networks combine locally dense relationships with short paths across the wider network, a pattern related to legislative production (Cho and Fowler, 2010). Seating proximity affects cue-taking (Masket, 2008), staff ties are associated with legislative effectiveness and voting (Montgomery and Nyhan, 2017), and social relations change the allocation of influence (Battaglini and Patacchini, 2018). Thus information clearly travels through congressional relationships.

The decision problem in a legislature is nevertheless different from the one modeled here. A policymaker, judge, advocate, or interest-group representative can often take an individual action after consulting peers. A legislator casts one vote, while the collective outcome depends on other votes, the agenda, and institutional rules. Communication may therefore change beliefs and coalition incentives simultaneously. The current framework does not capture this interaction; extending it to legislatures would require combining strategic disclosure with collective choice.

Assessment. Across the reviewed studies, political networks often contain several factions, each with a small leadership core and a larger periphery, while relatively few links cross factional boundaries. The number of factions varies from two in U.S. party networks to four California environmental-policy groups, five regional communities, and many amicus communities.

The efficiency result gives this pattern a normative interpretation. Under the model's cost, reliability, and ideological-separation conditions, every efficient network is star-factional when politicians are homogeneous. Except when two politicians form a single ideological collection, each faction has one leader, and only the leaders have cross-faction links. Internal centralization keeps paths short without requiring every pair to maintain a relationship, while sparse links between leaders preserve feasible cross-faction communication.

The formation result is positive and separate: in the bilateral proposal game, the networks selected by the efficiency result are exactly the coalitionally stable networks. Thus decentralized link choice can support the same networks.

The reviewed studies also find that additional links are concentrated within factions. This pattern is consistent with trust, repeated contact, common membership, or direct organizational benefits making internal relationships less costly or directly valuable.

The model separately shows that, when reliability is close to one, shortening a path by one link reduces expected posterior variance by $\kappa_F(1-\delta)+O((1-\delta)^2)$ under full forwarding and by $\kappa_O(1-\delta)+O((1-\delta)^2)$ under one-sided disclosure. This local comparison does not generally rank additional internal and cross-faction links because a new link can alter many paths and delivery opportunities. Link locations and reliability alone therefore do not identify whether relationship costs or transmission losses explain an observed link.

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