Overconfidence and Asymmetric Information: 
The Case of Insurance*

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Abstract

This paper contributes to the recent behavioral economics literature by showing that whether or not overconfidence matters qualitatively in asymmetric information markets may depend on the market structure itself. We first show that overconfidence may overturn fundamental relations between observable variables in perfect-competition asymmetric information insurance markets. In monopolistic insurance markets, in constrast, we find that overconfidence may be observationally equivalent to variations in the risk composition of the economy. Our analysis provides a number of novel testable implications on (i) price heterogeneity within and across risk classes, (ii) the relationship between ex-post risk and insurance coverage, (iii) the fact that a significant fraction of agents chooses to be uninsured, and (iv) the relationship between underinsurance and age. Further, some insights from previous behavioral economics studies may also be reversed: (i) in the case of perfect competition, we show, perhaps surprisingly, that an increase in overconfidence and a reduction in risk aversion may have opposite effects on insurance coverage; (ii) we find that monopolists cannot exploit biased customers, and their profits decrease in the fraction of overconfident agents in the economy.

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1 Introduction

The recent behavioral economics literature has advanced our understanding of economic problems, often by revising fundamental insights from “fully-rational” economic theory.\(^1\) This paper delivers a basic contribution to this enterprise. We study how overconfidence changes our understanding of simple asymmetric information problems. Indeed, overconfidence is one of the most robust biases uncovered in behavioral economics.\(^2\) At the same time, asymmetric information is a core problem of modern economics, since the seminal work of Akerlof (1970), Spence (1973), and Rothschild and Stiglitz (1976). We find that whether or not overconfidence matters qualitatively in asymmetric information markets may depend on the market structure itself. The introduction of overconfident agents overturns fundamental relationships between observable variables in perfect-competition asymmetric information insurance models. In models of monopolistic insurance with asymmetric information, in contrast, the introduction of overconfident agents may be observationally equivalent to changes in the composition of risk in the economy.

Our analysis focuses on insurance markets for two main reasons. First, overconfidence is a first-order issue in insurance markets. Indeed, an extensive empirical literature finds that many individuals underestimate their health, financial, and driving risks, and this often results in underinvestment in insurance.\(^3\) Second, the seminal models of Rothschild and Stiglitz (1976) and Stiglitz (1976) provide a detailed benchmark for studying the effects of biased beliefs in asymmetric information models. This paper thus studies the implications of overconfidence in insurance markets, both under perfect competition and under monopoly. In addition to uncovering the relevance of market structure for the effect of overconfidence on insurance, this paper provides a number of testable results, thereby laying out a road map for future empirical research on the implications of overconfidence for insurance. To keep matters as transparent as possible, we build

\(^1\) A nice survey of the advances in behavioral industrial organization is provided, for example, by Ellison (2006).

\(^2\) According to De Bondt and Thaler (1995, p. 389), “perhaps the most robust finding in the psychology of judgment is that people are overconfident.”

\(^3\) An earlier account of the pervasiveness of overconfidence in insurance markets is given by Adam Smith in The Wealth of Nations (1776): “That the chance of loss is frequently under-valued, [...] we may learn from [the limited demand for insurance]. Taking the whole kingdom at an average, nineteen houses in twenty, or rather, perhaps, ninety-nine in a hundred, are not insured from fire. [...] Many sail [...] at all seasons, and even in time of war, without any insurance. [...] The neglect of insurance [...] is, in most cases, the effect [...] of mere thoughtless rashness and presumptuous contempt of the risk.” We discuss the evidence for overconfidence in section 2. The interested reader may also consult Sandroni and Squintani (2004), for a survey.
our analysis under perfect competition on the basic model of Rothschild and Stiglitz (1976), and our analysis under monopoly on the basic model of Stiglitz (1976). As in those two papers, we assume that insurance companies cannot directly observe their customers’ risk. Unlike those two papers, however, we allow for overconfident agents. Some agents believe that their risk is low, when it is in fact high; the other agents know their risk.4

In the case of perfect competition, overconfidence reverses one of the main results of the Rothschild and Stiglitz (1976) model, namely the result that the equilibrium contracts do not depend on the proportion of high-risk and low risk agents in the economy. This result holds in the Rothschild and Stiglitz (1976) model because the insurance companies can fully recover the agents’ private information (i.e., their risk level) through their choice of contract. Once this is done, no residual information about the agents is of any value to the firms. However, when we introduce biased beliefs into the Rothschild and Stiglitz (1976) model, the information that firms gather by screening consumers may be biased. Hence, the firms may need to adjust their menu of contracts, or attempt to gather hard evidence to correct for this bias. As a result, we show that the equilibrium contracts depend on the composition of perceived and actual risks among agents, when agents’ beliefs may be biased. Therefore, introducing overconfidence in perfect-competition models may have implications for the equilibrium contracts that cannot be derived by changing the proportion of unbiased high-risk agents.

In contrast, we show for the case of monopoly that the introduction of overconfident agents in the Stiglitz (1976) model is qualitatively indistinguishable from an increase in the fraction of unbiased high-risk agents. This result follows from the following two insights. First, the monopolist cannot screen between overconfident and low risk agents, because their beliefs are the same at the moment they purchase the insurance contract. Second, the incentive compatibility constraint and the individual rationality constraint must bind independently of the presence of overconfident agents in the monopolist profit maximization problem. Given that these binding constraints fully describe the space of possible profit-maximizing contracts, the introduction of

4Our simple model is inspired by the general continuum type model, where each agent is identified by a true risk $p$ and perceived risk $\hat{p}$. Such a model simultaneously allows for heterogeneity in beliefs conditional on true risk, and heterogeneity in true risk given beliefs. Further, in the general model, for any level of risk perceived by an overconfident agent, there will be an unbiased agent with the same beliefs. These key features are present also in our 3-type model, and drive most of our results.
overconfident agents is equivalent to a change in the fraction of high-risk agents in the Stiglitz (1976) model without overconfidence.

The stark contrast between the implications of overconfidence in perfect-competition insurance markets and in monopolistic insurance markets implies that the testable implications of overconfidence on asymmetric information models may also depend on market structure. Overconfidence overturns essential testable relationships in perfect competition, but does not lead to major changes in testable implications in the case of monopoly. Specifically, our analysis focuses on the following empirical matters: (i) price heterogeneity within and across risk classes, (ii) the relationship between ex-post risk and insurance coverage, (iii) the stylized fact that a significant fraction of agents chooses to be uninsured, and (iv) the relationship between underinsurance and age. We identify a number of novel testable implications, and show that overconfidence may help in reconciling theoretical predictions with empirical stylized facts.

Consider perfect competition first. In our model, price schedules depend on hard information about subscribers (i.e., their risk class).\footnote{Different subscribers are said to belong to the same risk class if they are observationally equivalent at the time they purchase the insurance contract.} This result is in stark contrast with the Rothschild and Stiglitz (1976) model of perfect competition. In that model, the pricing schedule offered by firms does not depend on the proportion of low and high risk subscribers in the risk class, as the individual risk of subscribers is recovered through the equilibrium screening pricing schedule, which is independent of the agent’s risk class. Further, in our model, agents with different levels of perceived risk choose different contracts, within each risk class. In direct contrast with models of insurance with symmetric information such as the benchmark model by Mossin (1968), we show that prices differ within risk classes when some agents are overconfident. In sum, our model accounts for price heterogeneity both within and across risk classes. These two stylized facts are seldom accommodated simultaneously in alternative models of insurance with perfect competition.

We also show that biased beliefs reverse the relationship between ex-post risk and insurance coverage in asymmetric information models.\footnote{An agent’s ex-post risk consists of the actual frequency of accidents measured after she purchased her insurance contract.} A general and robust implication of asymmetric information (without overconfidence) is a positive relationship between ex-post risk and insurance coverage.
coverage, within each risk class (Chiappori et al., 2002). When overconfidence is sufficiently pervasive, agents who perceive that their risk is low may in fact be riskier on average. In this case, the relationship between insurance coverage and ex-post risk becomes negative, and the insurance contracts display quantity discounts (as has been observed, for example, by Cawley and Philipson (1999)). Our results may account for the fact that no statistically significant relationship was found by Chiappori and Salanié (2001) in French automobile insurance data sets, by Cawley and Philipson (1999) in U.S. life insurance data sets, and by Cardon and Hendel (2001) in the 1987 National Medical Expenditure Survey. When positive and negative relations cancel each other out, the overall relationship may turn out to be statistically not significant.7

Further, when overconfidence is sufficiently pervasive in the economy, both overconfident and low risk agents choose not to buy insurance in our model. In contrast, in perfect competition models of insurance with asymmetric information such as Rothschild and Stiglitz (1976), low-risk agents may be underinsured, but still purchase positive coverage. A fortiori, our findings also differ from models of insurance with symmetric information such as Mossin (1968), where agents purchase full insurance. Our results may account for the empirical finding that a significant fraction of agents choose to be uninsured, even in competitive insurance markets. Consistently with our model, the Insurance Research Council (IRC) reports that an average of 14.9 percent of U.S. motorists were uninsured between 1989 and 1997. Similarly, an estimated 15.2 percent of the U.S. population did not have health insurance in 2002, according to U.S. Census data.8

We conclude our analysis of perfect competition by showing that, under some regularity conditions, average insurance coverage is lower in risk classes where overconfidence is more pervasive. This result provides a simple account for the empirical finding that young adults (18 to 24 years old) are less likely than any other risk class to buy motorist insurance. In fact, there is strong experimental evidence that overconfidence is particularly pervasive among young adults. In con-

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7 While the relationship between insurance coverage and ex-post risk may also be not statistically significant in symmetric information models, our results do not imply that overconfidence is observationally equivalent to symmetric information. In symmetric information perfect competition models such as Mossin (1968), all agents are fully insured. In our model, in contrast, overconfident and low-risk individuals may be severely underinsured even under perfect competition.

8 In his 1992 Econometric Society Presidential Address, Peter Diamond stated: “Except for a few totally unable to purchase insurance [...] people are without insurance because it costs more than it appears to be worth to them [...] Some are without insurance because they misperceive the risks or consequences of this decision.” Diamond (1992), p. 1236.
trast, in models of insurance with asymmetric information, such as the Rothschild and Stiglitz (1976) model, only low-risk agents are predicted to be underinsured. Hence, in the absence of overconfidence, the model by Rothschild and Stiglitz (1976) is difficult to reconcile with the empirical observations that young adults are simultaneously the riskiest motorists and the least likely agents to buy motorist insurance.

The case of monopoly stands in stark contrast with the case of perfect competition, because introducing biased beliefs in the Stiglitz (1976) model does not lead to major qualitative changes in the relationship between observable variables. In particular, the model of monopolistic insurance with asymmetric information by Stiglitz (1976) predicts (i) price heterogeneity within and across risk classes, (ii) a positive relationship between ex-post risk and insurance coverage, and (iii) uninsured low-risk agents when the fraction of high-risk agents is large enough. We show that these predictions are qualitatively unchanged when overconfident agents are introduced in the model.

However, biased beliefs are not entirely irrelevant in the context of monopolistic insurance with asymmetric information. In fact, we show that the relationship between age and insurance coverage is positive in our model, under regularity conditions that apply, for example, to the motorist insurance market. In contrast, the relationship between age and insurance coverage is undetermined in the model of Stiglitz (1976). Hence, overconfidence may help refine the predictions on the relationship between age and insurance coverage that were derived in the monopolistic insurance model with asymmetric information of Stiglitz (1976).

In addition to showing the implications of biased beliefs in asymmetric information models, our study contributes to the behavioral economics literature in another dimension, by overturning some basic findings derived in previous studies. Absent asymmetric information, in simple decision-theoretical behavioral economics models, both an increase in overconfidence and a decrease in risk aversion lead to a reduction in insurance coverage. But this result is reversed when we consider asymmetric information. We show that adding a small fraction of less risk averse individuals to the Rothschild and Stiglitz (1976) model reduces average equilibrium insurance coverage. In contrast, introducing a small fraction of overconfident individuals in the same model increases equilibrium coverage.
In direct contrast with previous findings from behavioral industrial organization, we show that monopolists cannot exploit biased consumers. In our model, monopoly profits decrease as the fraction of overconfident agents in the economy increases. Indeed, monopolists have an incentive to advertise the risk of accidents so as to reduce the overconfidence bias in the population. This is in direct contrast, for example, with the results by Ellison (2005) and Gabaix and Laibson (2005) on hidden prices. They show that when naive consumers overlook add-on prices, unshrouding add-ons is not profitable to firms.

The paper is presented as follows. After the literature review, section 3 presents the model and the equilibrium with perfect competition. Section 4 presents our main positive results with perfect competition. Section 5 derives the optimal solution for monopolistic insurance with overconfident agents. Section 6 presents our main positive results for the monopoly case. Our conclusions are in section 7 concludes, and the proofs are in the appendix.

2 Related Literature

Experimental Evidence of Overconfidence Survey studies, with the most disparate subject samples, show that a large fraction of individuals believe that they are healthier, more financially secure and better drivers than the median individual.\(^9\) Widely replicated experimental studies find evidence of overoptimism by comparing self-reported risk with objective personal risk. Kreuter and Strecher (1995) and Robb et al. (2004) found evidence of health risk underestimation in relationship to medical exams. Overconfidence has been detected by Groeger and Grande (1996) who compare drivers’ self-assessments skills with those assessed by an instructor. Walton and McKeown (2001) compare self-reported and actual speed of drivers. Hoch (1985) found that MBA students overestimate the number of job offers they will receive and the magnitude of their salary.\(^10\)

\(^9\)Such results by Svenson (1981) on overconfidence of driving ability in Sweden have been replicated in Australia, the United States, Canada, Britain, Finland, France, as well as in Germany, Spain and Brazil. The results by Weinstein (1980) that subjects overestimates of their future financial success have been replicated in the US, Sweden, New Zealand, Belgium, Morocco, Poland, the UK, Hawaii, Switzerland, and in the Netherlands. Health overconfidence has been detected in samples from the US, the UK, the Netherlands, Israel, Tanzania, and Norwegia.

\(^10\)There is also strong specific evidence that overconfidence does not vanish with learning nor with experience. For example, Dalziel and Job (1997) found that professional drivers, such as metropolitan taxi drivers from Sydney, underestimate their risk of automobile accident.
The implications of overconfidence and of the illusion of control on precautionary behavior have been confirmed in several studies. Overconfidence has been recognized as a major determinant of traffic safety in many institutional studies (e.g. the European Union projects by Hatakka et al., 2002, and by Bartl, 2000). Health risk underestimation is recognized as a major barrier preventing healthy behavior (see the survey by Hoorens, 1994). There is also evidence that overconfidence induces poor financial planning and economic decisions. Benartzi (2001) finds that employees severely underestimate the risks of their own company stock, which is over-represented in their retirement saving plans. The experimental results by Camerer and Lovallo (1999) suggest that entrepreneurs’ overconfidence of their ability is one of the main factors to explain the well-established phenomenon of excess entry in competitive markets. Babcock and Loewenstein (1997) review experimental work that suggests that parties to legal disputes are reluctant to settle out of court because they hold overly optimistic beliefs about the merits of their case.

To our knowledge, empirical testing of overconfidence in insurance markets is still underdeveloped. However, there are some studies on the subject. Spurred by proposed welfare reforms in the UK, Cebulla (1999) conducted surveys on the perception of the risk of becoming unemployed and the willingness to purchase unemployment insurance. He detected underestimation of risk by comparing self-reported assessments with statistical assessments. Risk underestimation reduced the willingness to buy insurance. Bhattacharya, Goldman and Sood (2004) study secondary life-insurance markets, where consumers with a life-threatening illness may sell their life insurance policies in return for an up-front payment. They find evidence that patients who underestimate their risk of death are unwilling to hold insurance coverage.11

We conclude this brief review with an important qualification. While there is strong evidence that subjects underestimate risk on uncertain activities that they believe are under their control, such as driving or financial planning, or that pertain to their self-image, such as health, there is no comparable empirical evidence (to our knowledge) that subjects underestimate the risk of other uncertain events such as fires, floods, earthquakes, theft, malfunctioning of durable goods etc.

11 Beyond such empirical analyses, there seems to be consensus that underestimation of health risks is one of the factors contributing to the large number of individuals without insurance in the U.S. (an estimated 15.2 percent of the population in 2002, or 43.6 million people). Also, the very limited purchase of long-term care insurance the U.S. (roughly 10% of those aged 65 purchased such insurance in 2000), may partly be blamed on the public underestimation of the risk involved in being uninsured.
Hence our analysis may not apply to such insurance markets. Among the experimental papers studying some of these markets, some suggest that subjects overinsure (e.g. Eisner at Strotz, 1961, on airplane travel insurance) and some that they underinsure (e.g. Kunreuther et al., 1978, on disaster insurance).

The aim of this paper is not to build a complete behavioral theory of insurance. We focus on implications of a well-documented and specific bias: underestimation of personal risk. We assume that subjective probabilities differ from objective ones, but our analysis is entirely within the standard expected utility representation. Among further departures from standard insurance models, one may consider prospect theory (Kahneman and Tverski 1979), and regret theory (Bell 1982). Our analysis can be extended by employing some of these non-standard theory utility representation, instead of the expected utility representation. Preliminary investigations suggest that our main results remain qualitatively unchanged (details available upon request).

**Related Work in Behavioral Economics** Our paper is most closely related to two separate branches in behavioral economics. The first branch is a growing literature which studies market interaction between sophisticated firms and behaviorally biased consumers.

Among these papers, Ellison (2005) and Gabaix and Laibson (2005) study models where naive consumers overlook add-on prices, or underestimate the chances that they will be subject to hidden fees. In equilibrium, firms only advertise low base prices. Sophisticate consumers exploit the low base prices without purchasing the add-on, and carefully avoid hidden fees. Unshrouding add-ons is not profitable to firms and the practice of hidden prices survives even in competitive markets. A similar exploitation of naive consumers by sophisticated consumers in competitive markets is shown by DellaVigna and Malmendier (2004), who study a model where consumers may have naive or sophisticated beliefs on their present-biased future tastes. Unlike these models, our biased overconfident agents cannot be separated from low-risk agents, because their beliefs are the same. Overconfident agents exert negative externalities, as they increase their insurance prices. This entails an efficiency loss, not only distributive effects. Spiegler (2006) finds an efficiency loss in a market where consumers have bounded ability to make inference on quality by sampling goods. Increased competition causes firms to increase their effort to complicate the consumer’s
inference, and market efficiency may deteriorate.

Closely related to our work, Eliaz and Spiegler (2008c) study monopolistic design of a menu of non-linear tariffs when consumers have private information optimistic beliefs regarding their future preferences. But the focus of their paper is completely different from ours: They determine the monopolist’s optimal menu of non-linear tariffs to screen consumers’ degrees of optimism, whereas we focus on the relationship between biased beliefs and observables in asymmetric information models. One further difference is that we allow for perfect competition as well as monopoly.12

Another contribution related to our paper is Sandroni and Squintani (2007). The focus of our earlier contribution is orthogonal to the analysis here. This paper studies the testable implications of overconfidence on observable variables, while our earlier contribution is concerned only with normative results. Further, this paper compares results in different market structure, i.e. perfect competition and monopoly, in contrast, our earlier paper only analyzes perfect competition.13

The second closely related branch of behavioral economics studies the economic effects of overconfidence. For brevity, we discuss only a small subset of this literature. Benabou and Tirole (2002) and Koszegi (2000) show that an overconfident time-inconsistent individual may strategically choose to ignore information about her uncertain payoff, and Benabou and Tirole (2003) characterize some incentive schemes that an individual may use to manipulate her own or her opponent’s self-confidence to her own benefit. Yildiz (2003) shows that excessive optimism can cause delays in agreement in sequential bargaining, but this result is reversed if players remain sufficiently optimistic for a sufficiently long future.

Manove and Padilla (1999) study a model of debt financing of possibly overconfident start-up entrepreneurs. Competition may lead banks to insufficiently conservative lending. Collateral requirements lower the cost of capital, thus attracting optimistic entrepreneurs and reducing credit

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12 In Eliaz and Spiegler (2007,2008a,b) multiple agents hold different priors over an unverifiable state of Nature which affects the outcome of a game they are about to play. In the first period, the agents negotiate over contracts that define side payments as a function of the second-period game outcome. Thus, contracts are essentially bets over the second-period outcome. These papers define a notion of “constrained interim-efficient bets”, characterize them and discuss their implementability in terms of the underlying game’s payoff structure.

13 Jeleva and Villeneuve (2004) study a model of monopolistic insurance with asymmetric information where agents may take two types, and their perceived risk may differ from their actual risk. Unlike our model and the continuum model, their model does not allow simultaneously for heterogeneity in risk given beliefs and for heterogeneity in beliefs given risk. This modeling feature drives most of our results, which are thus entirely novel relative to their analysis.
markets efficiency. Instead, limited liability increases the cost of credit, and helps discipline optimists. Landier and Thesmar (2003) analyze and estimate a model of optimal financial contracting. When contracting is restricted to debt, they find that overconfident entrepreneurs may borrow more short-term debt than unbiased ones.

Compte and Postlewaite (2003) show that it may be optimal to be overconfident when performance is enhanced by confidence, even though this may result in taking excessive risks. Van den Steen (2004) shows that agents with different priors may become overconfident, because they only choose actions they overestimate chances of success, and attribute failure to exogenous factors. Sobel and Pinto (2005) provide a framework that explains optimistic self-assessments. Individuals have heterogeneous production functions that determine ability as a function of multiple skills. They make skill-enhancing investments, and make ability comparisons with others using their own production function.

3 Perfect Competition: Model and Theoretical Results

This section formally introduces overconfidence in the Rothschild and Stiglitz (1976) model, derives the equilibrium of this model, and reports some useful comparative statics results. Furthermore, it presents our most important theoretical result for the case of perfect competition: The introduction of overconfident agents in the economy changes qualitatively the equilibrium of the Rothschild and Stiglitz (1976) model.

The Model We introduce overconfidence in the basic framework of competitive insurance markets with asymmetric information by Rothschild and Stiglitz (1976). Each agent has wealth $W$ and may incur an accident of damage $d$ can occur with probability $p$. An insurance contract is a pair $\alpha = (\alpha_1, \alpha_2)$ so that the individual’s wealth is $(W - \alpha_1, W - d + \alpha_2)$ when buying $\alpha$. The amount $\alpha_1$ is the premium, $\alpha_1 + \alpha_2$ is the payment, or insurance coverage, and $P = \alpha_1/(\alpha_1 + \alpha_2)$ is the price of a unit of insurance. We assume that $\alpha_1 \geq 0$, $\alpha_2 \geq 0$: individuals cannot take on more risk through an insurance contract. So, an agent’s expected utility is $V(W, d; p, \alpha) = (1 - p)U(W - \alpha_1) + pU(W - d + \alpha_2)$. We assume that $U$ is twice differentiable, that $U' > 0$ and that $U'' < 0$, so that individuals are risk averse. There are three types of agents in the
economy. High risk (type $H$) and Low risk (type $L$) agents know that their risks are $p_H$ and $p_L$, respectively, with $p_H > p_L$. Overconfident (type $O$) agents believe that their risk is $p_L$ when in fact it is $p_O \geq p_H$. We follow the wide-spread supposition that overconfident individuals are the riskiest ones. Let $\lambda \in (0, 1)$ be the fraction of low risk agents in the economy. Let $\kappa \in (0, 1)$ be the fraction of overconfident agents in the economy, so that $\kappa + \lambda \leq 1$. The insurance firms cannot observe a subscriber’s risk or beliefs, but it is assumed that they know $\kappa$ and $\lambda$. In fact, firms have access to aggregate data on customers’ risk, measured ex-post, and to the surveys on agents’ perceptions of risk that we have discussed earlier. It is assumed that this information is sufficient to correctly estimate $\kappa$ and $\lambda$.

The insurance market is a competitive industry of expected profit maximizing (risk neutral) companies. A contract $\alpha$ sold to an agent with risk $p$ yields expected profit $\pi(p, \alpha) = (1 - p)\alpha_1 - p\alpha_2$. A perfectly-competitive equilibrium is a set of contracts $A$ such that: (i) no contract $\alpha \in A$ makes strictly negative expected profits, and (ii) no contract $\alpha' \notin A$ makes strictly positive profits. As in Rothschild and Stiglitz (1976), a perfectly-competitive equilibrium may fail to exist for some parameter values. Hence, we consider the weaker concept of locally-competitive equilibrium, which always exists. A locally-competitive equilibrium is a set of contracts $A$ such that when each contract $\alpha \in A$ is available in the market, (i) no contract $\alpha \in A$ makes strictly negative expected profits, and (ii) there is an $\varepsilon > 0$ such that any contract $\alpha' \notin A$ for which $||\alpha - \alpha'|| < \varepsilon$ for any $\alpha \in A$, would not make strictly positive profits. Informally, a set of contracts $A$ is locally-competitive if the insurance firms cannot make positive profits by introducing small changes in the contracts they already offer. Any perfectly-competitive equilibrium is also locally-competitive,

14 To simplify the exposition, we focus on the case that the difference between low risk and overconfident risk is not too small relative to the damage $d$. That is, we assume that

$$\frac{(1 - p_L)/p_L}{(1 - p_O)/p_O} > \frac{U''(W - d)}{U''(W)}$$

15 As already pointed out in our literature review, overconfidence has been recognized as a major determinant of traffic safety in many institutional studies (e.g. the European Union projects by Hatakka et al., 2002, and by Bartl, 2000). At the same time, health risk underestimation is recognized as a major barrier preventing healthy behavior (see the survey by Hoorens, 1994).

16 In a general-equilibrium model, Dubey and Geanakoplos (2002) establish the general existence of a separating equilibrium that approximates the locally-competitive equilibrium. Riley (1979) shows that the locally-competitive equilibrium coincides with a “reactive” equilibrium concept where each firm, before introducing new contracts, anticipates that firms already in the market will react by offering new contracts, as long as they generate positive profits. Wilson (1977) proposes an alternative reactive equilibrium where firms anticipate that any loss-making contracts will be removed as a reaction to any newly-introduced contracts.
The Equilibrium  For future reference, we briefly consider the model without overconfidence, i.e., \( \kappa = 0 \). Rothschild and Stiglitz (1976) show that the equilibrium is separating. Subscribers are screened according to the contract they choose. High-risk individuals fully insure, i.e. \( \alpha_1^H + \alpha_2^H = d \), and choose the contract \( \alpha^H \) such that the profit \( (1 - p_H) \alpha_1^H - p_H \alpha_2^H \) equals to zero. Hence, \( \alpha^H = (p_H d, (1 - p_H) d) \). Separation requires that high-risk subscribers (weakly) prefer contract \( \alpha^H \) to the low-risk individuals’ contract \( \alpha^L \). Hence, the contract \( \alpha^L \) solves the maximization problem

\[
\max_{\alpha} V(W, d; p_L, \alpha), \tag{12}
\]

subject to the binding incentive compatibility and zero-profit conditions:

\[
V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha),
\]

\[
(1 - p_L) \alpha_1 - p_L \alpha_2 = 0.
\]

Inspection of the above maximization problem reveals that the solution does not depend on the fraction \( \lambda \) of low-risk agents in the economy, thereby delivering our first benchmark result. A basic result of the Rothschild and Stiglitz (1976) model is that the equilibrium does not depend on the proportion of high and low risk agents in the economy. This result will be overturned later, when we introduce overconfident agents in the model.

Result 1 (Rothschild and Stiglitz (1976))  The equilibrium contracts \( \alpha^H \) and \( \alpha^L \) are independent of the fraction of low-risk agents \( \lambda \).

When introducing overconfidence in the Rothschild and Stiglitz (1976) model, the core of our analysis is now based on two intuitive insights. The first one is that insurance firms cannot screen between overconfident and low-risk individuals because, at the time of purchasing insurance, both types believe that their risk is low.\(^{17}\)

\(^{17}\)This insight is quite general, for example, it extends to the case when insurance companies have administrative costs and hence in determining the insurance premium there is a positive loading factor.
the basis of their beliefs. High-risk individuals purchase the contract \( \alpha^H = (p_H d, (1 - p_H)d) \), whereas low-risk and overconfident individuals choose a different contract \( \alpha^{LO} \). The average risk of overconfident and low-risk agents is

\[
p_{LO} \equiv \frac{\kappa p_O + \lambda p_L}{\kappa + \lambda}.
\]

Perfect competition requires that the equilibrium contract \( \alpha^{LO} \) satisfies the zero-profit condition \( (1 - p_{LO})\alpha^1_{LO} - p_{LO}\alpha^2_{LO} = 0 \). Hence, low-risk and overconfident individuals choose the contract \( \alpha^{LO} \) that solves the maximization problem

\[
\max_{\alpha} V(W, d; p_L, \alpha),
\]  

subject to the non-negativity constraint \( \alpha \geq 0 \), and to the incentive compatibility and zero-profit conditions:

\[
V(W, d; p_H, \alpha^H) \geq V(W, d; p_H, \alpha),
\]

\[
(1 - p_{LO})\alpha_1 - p_{LO}\alpha_2 = 0.
\]

The above analysis allows us to conclude that, unlike in the case without overconfidence, the equilibrium contract of low-risk and overconfident agents \( \alpha^{LO} \) depends on the composition of actual and perceived risk in the economy. In fact, both the fraction of overconfident agents \( \kappa \) and the fraction of low-risk agents \( \lambda \) appear in the formula of \( p_{LO} \), which appears in the zero-profit constraint (3).

**Result 2** When there are overconfident agents, \( \kappa > 0 \), the low-risk and overconfident agents’ contract \( \alpha^{LO} \) changes as the fraction of low-risk agents \( \lambda \) and as the fraction of overconfident agents \( \kappa \) change.

The above result implies that the introduction of overconfident agents in the economy changes qualitatively the equilibrium of the Rothschild and Stiglitz (1976) model. In fact, setting a strictly positive fraction of overconfident agents \( \kappa \) yields a different equilibrium from any change in the fraction of low-risk agents \( \lambda \), holding \( \kappa = 0 \). This is because the contract of low risk and overconfident agents \( \alpha^{LO} \) is invariant in the fraction of low risk agents \( \lambda \), in the absence of overconfidence.
(i.e. $\kappa = 0$), whereas it changes in $\kappa$ and $\lambda$ when introducing overconfident agents in the economy, i.e. $\kappa > 0$. In the statement of the following result, we let $\alpha^{LO}(\kappa, \lambda)$ denote the equilibrium contract associated with the parameters $\kappa$ and $\lambda$.

**Result 3** For any $\kappa > 0$ and $\lambda > 0$ such that $\kappa + \lambda \leq 1$, there does not exist $\lambda' \in (0, 1)$ such that $\alpha^{LO}(\kappa, \lambda) = \alpha^{LO}(0, \lambda')$. The introduction of overconfident agents in the economy induces a low-risk agents’ contract $\alpha^{LO}$ that is different from the low-risk agents’ contract $\alpha^L$ obtained with any change of the fraction of low-risk agents $\lambda$ in an economy without overconfident agents.

We shall see that this result is reversed in the case of monopoly, where the introduction of overconfident agents is equivalent to a change in the risk composition in the economy. Also, the next section will explore testable implication of Result 3: We will see that it implies that overconfidence overturns the relationships between observable variables in the Rotschild and Stiglitz (1976) model.

We now conclude the equilibrium analysis and present some comparative statics results that will be the base of our study of the testable implications of overconfidence, presented in the next section. We show in the Appendix, that unlike in the case without overconfidence, incentive compatibility need not be binding in equilibrium, and specifically it does not bind when the fraction of overconfident agents is large enough relative to the fraction of low-risk agents. Formally, there are two threshold function, $\kappa_1(\lambda)$ and $\kappa_2(\lambda)$, defined in the Appendix, such that (i) the incentive compatibility condition (2) binds if and only if $\kappa < \kappa_1(\lambda)$; (ii) for $\kappa_1(\lambda) < \kappa < \kappa_2(\lambda)$, the equilibrium contract $\alpha^{LO}$ satisfies the tangency condition

$$\frac{(1 - p_L) U'(W - \alpha_1^{LO})}{p_L U'(W - d + \alpha_2^{LO})} = \frac{1 - p_{LO}}{p_{LO}};$$

and (iii) for $\kappa > \kappa_2(\lambda)$, low-risk and overconfident individuals are uninsured: $\alpha^{LO} = 0$.

We conclude the section by reporting two comparative statics results that will deliver testable implications of overconfidence in the next section.

**Result 4** As long as the fraction of overconfident agents is not too large, i.e. $\kappa < \kappa_2(\lambda)$, so that $\alpha^{LO} > 0$, the insurance price $p^{LO}$ equals $p_{LO}$ and hence increases in $\kappa$ and decreases in $\lambda$. 

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**Result 5** When the fraction of overconfident agents is small, \( \kappa < \kappa_1 (\lambda) \), the insurance coverage \( \alpha_1^{LO} + \alpha_2^{LO} \) increases in \( \kappa \) and decreases in \( \lambda \); whereas it decreases in \( \kappa \) and increases in \( \lambda \) when the fraction of overconfident agents is intermediate, \( \kappa_1 (\lambda) < \kappa < \kappa_2 (\lambda) \).

The first result follows immediately from the zero profit condition (3). To understand the second result, consider first the case \( \kappa < \kappa_1 (\lambda) \). As \( \kappa \) increases and \( \lambda \) decreases, the price \( P^{LO} \) of contract \( \alpha^{LO} \) increases. This implies that the contract \( \alpha^{LO} \) becomes less attractive to high-risk agents. Hence, the insurance coverage \( \alpha_1^{LO} + \alpha_2^{LO} \) must be restricted less in order to establish that the incentive compatibility condition (2) binds. When \( \kappa_1 (\lambda) < \kappa < \kappa_2 (\lambda) \), instead, the tangency condition (4) holds, and the choice of high-risk agents is unrelated to the choice of low-risk and overconfident agents. As the price \( P^{LO} \) of contract \( \alpha^{LO} \) increases, low-risk and overconfident agents find these contracts less attractive and choose to insure less.

### 4 Perfect Competition: Testable Implications

This section explores the testable implications of overconfidence in the Rothschild and Stiglitz (1976) model. We show that biased beliefs overturn fundamental relationships between observables in asymmetric information models, and accounts for empirical findings that are not explained by benchmark models without biased beliefs. Further, we show that our study of biased beliefs in asymmetric information models reverses some insights from previous behavioral economics studies.

**Price Heterogeneity Within and Across Risk Classes** Our model accounts for the following two stylized facts, which are seldom accommodated simultaneously in previous models of insurance with perfect competition. For any given coverage amount, the prices of insurance contracts are higher for agents who belong to riskier classes. In addition, the data also show large heterogeneity in prices within risk classes (e.g. Chiappori and Salanié, 2001).

Risk classification is introduced in our model, by assuming that each agent has a public verifiable signal \( x \): her risk class. We denote by \( \kappa (x) \), the fraction of overconfident agents in class \( x \), and by \( \lambda (x) \), the fraction of low risk agents in class \( x \). We order risk classes so that riskier
classes are indexed with a larger signal \( x \); specifically, we assume that \( \lambda(x) \) decreases in \( x \) and \( \kappa(x) \) increases in \( x \). Because the risk class \( x \) is verifiable by insurance companies, individuals in different risk classes \( x \) are offered different contracts \( \alpha(x) \), that depend on \( x \). In a competitive market, each contract \( \alpha(x) \) must make zero-profit. Hence, the equilibrium contracts \( \alpha(x) \) in each risk class \( x \) are derived as if the risk class \( x \) was a single separate insurance market.\(^{18}\) The equilibrium contracts \( \alpha(x) \) are determined by the parameters \( \kappa(x) \) and \( \lambda(x) \) through to the equilibrium characterization in the previous section.\(^{19}\)

To introduce the next result, note that high risk agents in all risk classes purchase the same contract \( \alpha^H \), with price \( P^H \). Low risk and overconfident agents choose the same contract \( \alpha^{LO}(x) \). The insurance price \( P^{LO}(x) = p^{LO}(x) \) depends on the composition of overconfident and low-risk individuals which, in turn, is different in different risk classes. The insurance price \( P^H \) for high-risk agents is different from \( P^{LO}(x) \) for every \( x \), apart from the non-generic case, knife-edge case where \( p^{LO}(x) = p^H \). At the same time, the higher the risk class \( x \), the higher is the price \( P^{LO}(x) \) of the contract purchased by low-risk and overconfident agents, because \( P^{LO}(x) \) increases in \( \kappa(x) \) and decreases in \( \lambda(x) \). Formally, we have concluded that:

**Result 6** The insurance price \( P^{LO}(x) \) of low-risk and overconfident agents increases in the risk class \( x \). Generically, for any \( x \), the high risk agents’ contract insurance price \( P^H(x) \) differs from the low-risk and overconfident agents’ contract insurance price \( P^{LO}(x) \).

This result implies that our model accounts for the two above-mentioned stylized facts. First, agents who belong to riskier classes pay more for any given coverage amount. This is in direct contrast with the findings of perfect-competition insurance models with asymmetric information such as Rothschild and Stiglitz (1976). In the equilibrium of the model by Rothschild and Stiglitz (1976), the pricing schedule offered by the firms does not depend on the proportion of low and high risk subscribers in the risk class, as the individual risk of subscribers is recovered through the equilibrium screening pricing schedule, which does not depend on the risk class. Regardless of

\(^{18}\)Note that profit maximizing behavior rules out cross-subsidization across risk classes. If a company were to make negative profit in one risk class, it may increase its profits by stopping offering such contracts.

\(^{19}\)The assumption that the two levels of risk and beliefs in our model, \( p_L \) and \( p_H \), are the same across risk classes is motivated by the general continuum type model. In the continuum model, the set of realizations for risk and beliefs is the same across risk classes, whereas the distribution of risk and beliefs may change (as in our model).
the signal $x$, high-risk agents choose contract $\alpha^H$, and low-risk ones choose $\alpha^L$. Hence insurance prices $P^L(x)$ and $P^H(x)$ are independent of the signal $x$. In contrast, in our model the price $P^{LO}(x)$ of contracts offered to low-risk and overconfident agents increases in the risk class $x$, whereas $P^H(x)$ stays constant.

Second, insurance prices differ significantly within risk classes. This is direct contrast with the findings of perfect-competition insurance models with symmetric information such as Mossin (1968). If the risk class is a sufficient statistics of each agent’s actual risk, then perfect competition implies that the equilibrium price $P(x)$ of an agent’s insurance contract $\alpha(x)$ depends only on her risk class $x$. Instead, in our model the insurance price $P^H$ for high-risk agents is higher than $P^{LO}(x)$ within each risk class $x$.

Our analysis accounts for price-heterogeneity both within and across risk classes because screening gives valuable information, but cannot fully identify agents’ risks. On the one hand, insurance firms screen agents to recover valuable information about their risk, which is correlated with their beliefs. On the other hand, agents hold biased beliefs on average, and hence it is valuable for insurance firms to make contracts depend on risk statistics, even after all private information has been recovered through screening. Instead, in models with symmetric information such as Mossin (1968), there is no reason to screen consumers, as private information is of no value to the firms. Hence, such models of symmetric information fail to account for price heterogeneity within risk classes. At the same time, in models with asymmetric information and unbiased agents, such as the model of Rothschild and Stiglitz (1976), all the relevant information about an agent’s risk can recovered through screening. Hence these models fail to account for price-schedule heterogeneity across risk classes.\footnote{However, price heterogeneity within and across risk classes can be also explained by multi-dimensional or moral hazard models of asymmetric information with unbiased agents. When asymmetric information concerns both risk and risk aversion, Smart (2000) shows that low-risk high-risk aversion agents may pool with high-risk low-risk aversion agents in equilibrium. Also, bunching can take place in equilibrium if the basic asymmetric information model is extended to incorporate moral hazard (see for instance Jullien, Salanie and Salanie, 2007). Whenever pooling takes place, the equilibrium price will eventually depend on the agents’ risk classes.}

**The Relationship between Insurance Coverage and Ex-Post Risk** Overconfidence may overturn the relationship between ex-post risk and insurance coverage in asymmetric information models. The main finding of Chiappori et al. (2006) is that, in general models with
asymmetric information that abstract from overconfidence, the relationship between insurance coverage and ex-post risk is positive. It is easy to see that this result may not hold when some agents are overconfident. There are two observable contracts $\alpha^H$ and $\alpha^{LO}$ in the market, and the insurance coverage associated with the first contract is $\alpha^H_1 + \alpha^H_2 = d$, larger than the coverage associate with the second contract, $\alpha^{LO}_1 + \alpha^{LO}_2 < d$. The prices and average ex-post risks associated with the two contracts are $P^H = p_H$ and $P^{LO} = p_{LO}$. So, we conclude the following result.

**Result 7** The relationship between insurance coverage $\alpha_1 + \alpha_2$ and ex-post risk $p$ is negative (positive) if and only if individuals who believe their risk to be lower are riskier (less risky) on average; i.e. if and only if

$$p_{LO} = \frac{\kappa p_O + \lambda p_L}{\kappa + \lambda} > (<) p_H.$$  

When $p_{LO} > p_H$, the pricing schedule displays quantity discounts, i.e. the price of an insurance contract $P = \alpha_1/(\alpha_1 + \alpha_2)$ is negatively related to the coverage $\alpha_1 + \alpha_2$.

When the fraction of overconfident individuals is large relative to low-risk individuals, we accomodate the empirical finding that the pricing schedule may display quantity discounts (Cawley and Philipson (1999)). Further, we may reverse the standard prediction of asymmetric information models that unobservable risk is positively related with insurance coverage.\textsuperscript{21} Our results may account for the lack of statistically significant relationship found by Chiappori and Salanié (2001) in French automobile insurance datasets, by Cawley and Philipson (1999) in U.S. life insurance datasets, and by Cardon and Hendel (2001) in the 1987 National Medical Expenditure Survey. By canceling positive and negative relationships, the overall relationship may easily turn out to be statistically not significant.\textsuperscript{22,23}

\textsuperscript{21} Consistently with our predictions, Chiappori and Salanié (2000) find a (non statistically significant) negative relation between unobservable risk and coverage for young drivers, who are more likely to be overconfident, whereas Cohen (2003) finds a significant and positive relation for mature drivers.

\textsuperscript{22} The relevance of overconfidence for the relationship between risk and coverage has also been independently singled out by Koufopoulos (2003). Unlike our model, his model is based on moral hazard.

\textsuperscript{23} A different explanation of the possibility of a negative relation between risk and insurance coverage has been identified by DeMeza and Webb (2001). In their model, there is ex-ante heterogeneity with respect to risk aversion, and agents may invest in precautionary activities. In equilibrium low-risk aversion agents buy less insurance and invest less in precautions than high-risk aversion agents. On a related account, Finkelstein and McGarry (2007) propose a general test for asymmetric information when agents may differ in risk aversion. Also, Netzer-Scheuer (2009) argue that a nonmonotone relationship between risk and coverage may arise due to unobservable savings (wealth heterogeneity).
While the relationship between insurance coverage and ex-post risk may not be statistically significant also under symmetric information, our results do not imply that overconfidence is observationally equivalent to symmetric information. In fact, models with symmetric information and perfect competition, such as Mossin (1968), predict that all agents purchase full insurance. Instead, our model predicts that overconfident and low-risk individuals do not ever purchase full insurance: $\alpha_{1O}^{LO} + \alpha_{2O}^{LO} < d$. Further, under symmetric information, there is an indeterminate relationship between insurance coverage and self-reported risk, measured at the time of insurance purchase. In our model, instead, this relationship is positive. The insurance coverage $\alpha_{1H} + \alpha_{2H}$ of the contract $\alpha^H$ purchased by high-risk unbiased agents is larger than the insurance coverage $\alpha_{1L} + \alpha_{2L}$ of the contract $\alpha^L$ purchased by low-risk and overconfident agents. At the time of insurance purchase, all agents who purchase contract $\alpha^L$ believe their risk to be $p_L$, whereas high-risk unbiased agents believe that their risk is $p_H$.

**Underinsurance and the Relationship between Age and Overconfidence** Overconfidence may help accommodate the empirical finding that large number of agents choose to be uninsured, even in perfectly competitive insurance markets. According to the Insurance Research Council (IRC), an average of 14.9% of motorists were uninsured between 1989 and 1997.\(^{24}\) Similarly, an estimated 15.2 percent of the U.S. population did not have health insurance in 2002, according to US Census data.\(^{25}\) In our model, low-risk and overconfident agents choose to be uninsured if they belong to risk classes $x$ where the fraction of overconfident agents $\kappa(x)$ is large enough. Naturally, overconfidence is not the only possible explanation for the uninsurance problem, but it may be a significant factor.

**Result 8** In risk classes $x$ where the fraction of overconfident agents is sufficiently large, $\kappa(x) > \kappa_2(\lambda(x))$, low-risk and overconfident individuals choose to be uninsured in equilibrium, $\alpha^{LO}(x) = 0$.

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\(^{24}\) An alternative theoretical explanation is that severely budget-constrained motorists do not to insure and declare bankruptcy in case of accident where they are at fault (see Smith and Wright, 1992). This explanation and our overconfidence hypothesis complement each other. In the Public Attitude Monitor (2000 and 2003) surveys, about 41% of the interviewed individuals reported reasons for non-insurance consistent with the limited-liability hypothesis and 31% report reasons consistent with overconfidence. The remaining 28% of interviewed individuals gave reasons consistent with both hypothesis.

\(^{25}\) An explanation alternative to overconfidence is that agents may refuse to buy health insurance as they would be able to access publicly provided basic health care in case of life-threatening situations.
Our results are in stark contrast with perfect competition models of insurance with symmetric information such as Mossin (1968), that predict that all agents purchase full insurance, and with perfect competition models of insurance with asymmetric information, such as Rothschild and Stiglitz (1976), that predict that low-risk agents be underinsured, but still purchase positive coverage.

One specific feature of the underinsurance problem that our analysis can accommodate is the relationship between young age and underinsurance. There is strong experimental evidence that overconfidence is persistent in any age group, but is particularly pervasive among young adults. At the same time, young adults (18 to 24 years old) are less likely than any other population segment to buy health or automobile insurance. This stylized fact is difficult to reconcile with asymmetric information perfect-competition models with unbiased agents, such as Rothschild and Stiglitz (1976). In that model, low-risk agents purchase less insurance than high-risk agents. This conclusion is inconsistent with the empirical findings that young motorists purchase less insurance than older motorists, and that young motorists tend to be riskier than older motorists (see for example Bartl, 2000).\footnote{However, the model of DeMeza and Webb (2001) may provide an explanation that abstracts from overconfidence. They find that low-risk aversion agents purchase less insurance and invest less in precautions, thus becoming more risky. As far as we know, the relation between risk aversion and age has not been solidly established in experiments, and this prevents us from establishing a link between underinsurance and age through risk aversion.}

In order to formalize our intuition that the fraction of uninsured agents is higher among young adults because they are more likely to be overconfident, we expand our basic model and distinguish each agent by her age, \( a \). The composition of overconfident and low-risk individuals differ across age groups: For any age group \( a \), we let \( \kappa(a) \) and \( \lambda(a) \) be the fractions of overconfident and low-risk agents respectively, among the agents with age equal to \( a \). Because age is verifiable by insurance companies, individuals of different age \( a \) are offered different contracts \( \alpha(a) \) that depend on \( a \). Hence, as in contracts that depend on risk classes, the equilibrium contracts in each age group \( a \) are derived as if the age group \( a \) was a single separate insurance market. The equilibrium contracts \( \alpha^{H}(a) \) and \( \alpha^{LO}(a) \) are determined by the parameters \( \kappa(a) \) and \( \lambda(a) \) through the equilibrium characterization in the previous section.
Let \( C(a) \equiv (\alpha_1^H + \alpha_2^H) [1 - \kappa(a) - \lambda(a)] + [\alpha_1^{LO}(a) + \alpha_2^{LO}(a)] [\kappa(a) + \lambda(a)] \) be the average insurance coverage as a function of age. Result 9 identifies sufficient conditions for \( C(a) \) to increase in age. The first condition is motivated by the experimental evidence that individuals become less overconfident with age. It states that the proportion of overconfident agents \( \kappa(a) \) among the agents who believe that their risk is low, \( \kappa(a) + \lambda(a) \), decreases with age. The second condition is that the fraction of high-risk unbiased agents increases in age, either because agents become riskier as they age (as in health insurance markets), or because some overconfident agents learn their true risk as they age (as in motorist insurance markets).

**Result 9** Suppose that in all age groups \( a \), overconfidence is sufficiently pervasive, i.e., \( \kappa(a) > \kappa_1(\lambda(a)) \). If the proportion of overconfident agents \( \kappa(a) \) among the agents who believe that their risk is low, \( \kappa(a) + \lambda(a) \), decreases with age, i.e., \( \frac{d}{da} \left( \frac{\kappa(a)}{\kappa(a) + \lambda(a)} \right) < 0 \), and if the fraction of high-risk unbiased agents increases in age, i.e. \( \kappa'(a) + \lambda'(a) < 0 \), then the average insurance coverage \( C(a) \) increases with age \( a \).

The above result accommodates the positive relationship between age and insurance coverage in both health and motorist insurance markets.

**Risk aversion and Overconfidence** Our study of biased beliefs in asymmetric information models reverses some basic insights from previous behavioral economics studies. Unlike simple decision-theoretical models, we find that an increase in overconfidence and a reduction in risk aversion may have opposite effects on insurance coverage.

We have shown in Section 3 that adding a small fraction of overconfident individuals to the Rothschild and Stiglitz (1976) model increases the equilibrium insurance coverage. In contrast, Figure 1 shows that adding a small fraction of high-risk individuals with reduced risk aversion decreases the equilibrium insurance coverage. The equilibrium of the Rothschild and Stiglitz (1976) model is \( (\alpha^L, \alpha^H) \): high-risk agents are fully insured by contract \( \alpha^H \) and constrain the choice of low-risk agents \( \alpha^L \). We extend the Rothschild and Stiglitz (1976) model by introducing less-risk averse, high-risk agents, whose indifference curves are denoted by \( L_H \). Because they are less risk averse, the curve \( L_H \) is flatter than \( I_H \) and hence it imposes a stronger constraint on the
choice of low-risk agents. Regardless of the proportion of less risk averse agents, the equilibrium coverage of low-risk agents discontinuously drops from $\alpha^L_1 + \alpha^L_2$ to $\alpha^L_1 + \alpha^L_2$. We obtain the following result.

**Result 10** Adding a small fraction of high-risk moderately risk-averse individuals to the Rothschild and Stiglitz (1976) model reduces the average insurance coverage in the economy. Instead, adding a small fraction of high-risk overconfident agents to the Rothschild and Stiglitz (1976) model increases the average insurance coverage in the economy.

The reason why introducing overconfident agents in our model may lead to an opposite implication on insurance coverage than introducing moderately risk averse agents is the following. Moderately risk-averse individuals are aware of their risk preference, whereas overconfident agents incorrectly believe that their risk is low. Agents with different levels of risk aversion may want to purchase different contracts, but overconfident agents cannot be separated from low-risk agents. As a result, introducing overconfident agents in the economy increases the equilibrium price of the low-risk and overconfident agents insurance contract. This in turn makes the incentive-compatibility constraint easier to be satisfied, so that the equilibrium insurance coverage of low-risk and overconfident agents increases. Instead, introducing moderately risk-averse agents makes the incentive-compatibility constraint more stringent, because these agents are more willing to reduce their insurance coverage for a reduction in price than agents with a higher level of risk-aversion. This results in a decrease of low-risk agents’ insurance coverages in equilibrium.

### 5 Monopoly: Model and Theoretical Results

This section introduces overconfidence in model by Stiglitz (1976), derives the solution of this model, and reports some useful comparative statics and welfare results. Furthermore, it presents our most important theoretical result for the case of monopoly competition: The introduction of overconfident agents in the economy is qualitatively indistinguishable from changes in the composition of risk in the economy.
Figure 1: Adverse Selection with less risk-averse high-risk individuals

The Model To keep matters simple, and to highlight the effect of overconfidence, we build on the celebrated benchmark model by Stiglitz (1976). The model is analogous the one presented in section 3, with the modification is that, instead of considering a perfectly-competitive market, there is now a monopolistic firm that maximizes profit.

It is also convenient that we change the parametrization of the types in the economy. There is a fraction $\eta$ of high-risk unbiased agents, and a fraction $\kappa$ of overconfident agents. Clearly $\eta + \kappa \leq 1$, and $\lambda = 1 - \eta - \kappa$. Also, we make the additional assumption that, whenever indifferent between two contracts, each agent purchases as much insurance coverage as possible. This assumption rules out implausible equilibria, where overconfident and low risk agents choose different contracts among which they are indifferent, despite the fact that they have identical beliefs at the time of purchase of the contract.

The Monopoly Solution without Overconfidence For future reference, we briefly consider the model without overconfidence, i.e., $\kappa = 0$. Stiglitz (1976) shows that the monopoly solution is separating. Subscribers are screened according to the contract they choose. The monopolist fully extracts all surplus from low-risk individuals. Hence, the contract $\alpha^L$ lies on the indifference curve $I_L$ through the no-insurance contract $0$. Incentive compatibility requires that high-risk subscribers (weakly) prefer contract $\alpha^H$ to the low-risk individuals’ contract $\alpha^L$. Hence, the contract $\alpha^H$ lies on the indifference curve $I_H$ through the low-risk agents’ contract.
\[ \alpha^H, \alpha^L \]. High-risk individuals fully insure. Their contract \( \alpha^H \) equalizes wealth across states and lies on the 45-degree line. The contract \( \alpha^L \) coincides with no insurance when the fraction of high-risk individuals \( \eta \) is sufficiently large. The monopoly solution is depicted in Figure 2.

Formally, the monopoly solution is the pair of contracts \( \alpha^H \geq 0, \alpha^L \geq 0 \) that maximizes the monopoly profit

\[
\pi(\alpha^H, \alpha^L) = (1 - \eta) \left[ (1 - p_L) \alpha^L_1 - p_L \alpha^L_2 \right] + \eta \left[ (1 - p_H) \alpha^H_1 - p_H \alpha^H_2 \right]
\]

\[ s.t. \quad V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^L) \]

\[ V(W, d; p_L, \alpha^L) = V(W, d; p_L, 0). \]

The first constraint requires that high-risk agents be indifferent between their contract \( \alpha^H \) and the contract of low-risk agents \( \alpha^L \). The second constraint requires that low-risk agents be indifferent between their contract \( \alpha^L \) and no insurance.

Inspection of the above problem immediately underlines a key difference with respect to the case of perfect competition: \textit{The contracts \( \alpha^H \) and \( \alpha^L \) that maximize the monopoly profit \( \pi(\alpha^H, \alpha^L) \) change as the fraction of high-risk agents \( \eta \) changes.} Specifically, the comparative statics of contracts \( \alpha^H \) and \( \alpha^L \) as a function of \( \eta \) is described as follows.

\textbf{Result 11 (Stiglitz (1976))} \textit{As the fraction of high-risk individuals \( \eta \) increases, the contract \( \alpha^H \) moves towards the origin, and the contract \( \alpha^L \) moves towards the contract 0 along the indifferent-}
ence curve $I_L$ through the contract $0$. When $\eta$ is sufficiently large, the solution of the monopolist’s maximization problem reaches the corner solution $\alpha^L = 0$.

The results is graphically described as follows. For any pair of contracts $(\alpha^H, \alpha^L)$ that satisfies the constraints in the maximization problem, there is a tradeoff between the profit $\pi_H = (1 - p_H) \alpha_1^H - p_H \alpha_2^H$ that can be obtained from the contract $\alpha^H$ and the profit $\pi_L = (1 - p_L) \alpha_1^L - p_L \alpha_2^L$ from the contract $\alpha^L$. In order to increase the profit $\pi_H$, the monopolist needs to increase the price $P^H$ of the contract $\alpha^H$. But an increase in $P^H$ would violate the incentive compatibility constraint, unless the monopolist reduces the insurance coverage offered to low-risk individuals, so as to make the contract $\alpha^L$ less attractive to high-risk individuals. So, to increase the profit $\pi_H$ and induce a contract $\alpha^H$ closer to the origin, the monopolist must reduce the profit $\pi_L$ and induce a contract $\alpha^L$ closer to the contract $0$ along the indifference curve $I_L$ through the contract $0$.

The Monopoly Solution with Overconfidence

We now describe the monopoly solution with overconfidence (i.e., $\kappa > 0$). The core of our analysis is based on two intuitive insights. The first one is that, as in the perfect-competition case, the monopolist cannot screen between overconfident and low-risk individuals. Under the assumption that whenever indifferent between two contracts, each agent purchases as much insurance coverage as possible, the overconfident and low-risk individuals must necessarily purchase the same insurance contract.

Given this qualification, arguments analogous to the analysis of Stiglitz (1976) allow us to conclude that in the monopoly solution, individuals are again screened on the basis of their beliefs. High-risk individuals purchase a contract $\alpha^H$, whereas low-risk and overconfident individuals choose a different contract $\alpha^{LO}$. The monopolist’s profit $\pi^{LO}$ from the contract $\alpha^{LO}$ is $(1 - p_{LO}) \alpha_1^{LO} - p_{LO} \alpha_2^{LO}$, where $p_{LO} = [\kappa p_O + (1 - \kappa - \eta) p_L]/[1 - \eta]$. Hence, the monopolist solves the maximization problem

$$\max_{\alpha^H \geq 0, \alpha^{LO} \geq 0} \pi(\alpha^H, \alpha^{LO}) = (1 - \eta) [(1 - p_{LO}) \alpha_1^{LO} - p_{LO} \alpha_2^{LO}] + \eta [(1 - p_H) \alpha_1^H - p_H \alpha_2^H]$$

(6)
\[ \text{s.t.} \quad V(W, d; p_H, \alpha^H) \geq V(W, d; p_H, \alpha^{LO}) \tag{7} \]
\[ V(W, d; p_L, \alpha^{LO}) \geq V(W, d; p_L, 0). \tag{8} \]

Unlike in the perfect competition case, the incentive compatibility constraint (7) and the individual rationality constraint (8) bind in the monopolistic solution \((\alpha^H, \alpha^{LO})\) for all parameter values \(\kappa\) and \(\lambda\). In fact, if either constraint were slack, the monopolist would not maximize profits. Suppose that constraint (8) did not bind. The monopolist could increase its profit by slightly increasing the price \(P^{LO}\) of contract \(\alpha^{LO}\) so that the low-risk and overconfident agents still prefer contract \(\alpha^{LO}\) to remaining uninsured. In the same manner, if the constraint (7) did not bind, the monopolist could increase its profit by slightly increasing the price \(P^H\) of contract \(\alpha^H\) so that the low-risk and overconfident agents still prefer contract \(\alpha^H\) to contract \(\alpha^{LO}\).

Once concluded that the constraints (7) and (8) bind for all values of \(\kappa\) and \(\lambda\), inspection of the two maximization problems (5) and (6) leads us to the second insight. Substituting the contract of low-risk agents \(\alpha^L\), with the contract of low-risk and overconfident agents \(\alpha^{LO}\), the constraints on the set of contracts among which the monopolist maximizes are the same. So, the equilibrium characterization is qualitatively the same. We have previously seen that, in the case without overconfidence, the low-risk agents contract \(\alpha^L\) lies on the indifference curve \(I_L\) through the no-insurance contract \(0\), and the high-risk agents \(\alpha^H\) lies at the crossing of the 45-degree line and the indifference curve \(I_H\) through the contract \(\alpha^{LO}\). Because the contraints (7) and (8) bind, we now conclude that, also in the case with overconfidence, the low-risk and overconfident agents contract \(\alpha^{LO}\) lies on the indifference curve \(I_L\) through contract \(0\), and the high-risk agents contract \(\alpha^H\) lies at the crossing of the 45-degree line and the indifference curve \(I_H\) through contract \(\alpha^{LO}\).

These observations yield the most important theoretical result of this section. The effect of an increase of the fraction of overconfident agents \(\kappa\) is qualitatively indistinguishable from the effect of an increase of the fraction of high-risk agents \(\eta\). We have previously seen that, as the fraction of high-risk individuals \(\eta\) increases, the contract \(\alpha^H\) moves towards the origin, and the contract \(\alpha^L\) moves towards the contract \(0\) along the indifference curve \(I_L\) through the contract \(0\). Suppose now that the fraction of overconfident agents \(\kappa\) grows. Hence, the average risk of low-risk and overconfident agents \(p_{LO}\) increases. So, the profit \(\pi_{LO}\) of any contract \(\alpha^{LO}\) decreases relative to the profit \(\pi_H\) of the associated contract \(\alpha^H\). As \(\kappa\) grows, the monopolist seeks more profit from
contract $\alpha^H$. This implies that, as in the case without overconfidence, the contract $\alpha^H$ moves towards the origin along the 45 degrees line, and the contract $\alpha^{LO}$ moves towards the contract $0$ along the indifference curve $I_L$ through the contract $0$. When the fraction of overconfident agents $\kappa$ becomes sufficiently large, and crosses the decreasing threshold $\bar{\kappa}(\eta)$ that we derive in the appendix, the solution of the monopolist’s maximization problem reaches the corner solution $\alpha^L = 0$.\footnote{Intuitively, the pool of overconfident and low-risk agents becomes so unprofitable, that the monopolist chooses to ignore them and do not offer them any insurance. This allows the monopolist to extract the full surplus from high-risk individuals, by offering them the full-insurance contract that makes them indifferent between accepting the contract and choosing to be uninsured.}

We summarize and formalize this discussion in the following result, where we denote any pair of parameters $\eta$ and $\kappa$ as \textit{admissible} if $\kappa + \eta \leq 1$, and for any admissible parameter values $\eta$ and $\kappa$, we let the associated contracts solving problem (6) be $(\alpha^H, \alpha^{LO}) [\kappa, \eta]$.

\textbf{Result 12} Consider any admissible pair of parameter values $\eta > 0$ and $\kappa \geq 0$. For any small reduction $d\eta < 0$ in the fraction of high-risk agents, there is an admissible small increase $d\kappa > 0$ in the fraction of overconfident agents such that the associated equilibrium contracts do not change: $(\alpha^H, \alpha^{LO}) [\kappa, \eta] = (\alpha^H, \alpha^{LO}) [\kappa + d\kappa, \eta - d\eta]$.

This result is striking, as it states that an increase in the fraction of overconfident agents is qualitatively indistinguishable from an increase in the fraction of high-risk agents in the model. As we later show, Result 12 implies that the relationship between observables is not qualitatively affected by the introduction of overconfidence in the Stiglitz (1976) model of monopolistic insurance with asymmetric information. This finding is in stark contrast with the case of perfect competition, where the introduction of overconfidence overturns the relationship between the same observables.

We conclude this section by considering monopoly profit. In behavioral models, a monopolist can often successfully exploits biased agents to its own advantage. Here, instead, the monopolist profit decreases in the fraction of overconfident agents $\kappa$. This result follows from a simple ‘revealed preferences’ argument. Note that, because the profit $\pi_{LO}$ decreases in $\kappa$, the profit $\pi(\alpha^H, \alpha^{LO})$ for any pair of contracts $(\alpha^H, \alpha^{LO})$ that satisfies the maximization constraints decreases in $\kappa$. As a result, when $\kappa$ decreases, the optimal pair of contracts $(\alpha^H, \alpha^{LO})$ must yield a higher profit than
the optimal pair associated to the higher initial $\kappa$. We summarize and formalize this discussion in the following result, where we let the monopoly profit be $\pi(\kappa, \eta) = \pi(\alpha^H, \alpha^{LO})$ for $(\alpha^H, \alpha^{LO})$ that solve problem (6).

**Result 13** For any $\kappa < \tilde{\kappa}(\eta)$, the monopoly profit $\pi(\kappa, \eta)$ strictly decreases in the fraction of overconfident agents $\kappa$. For $\kappa \geq \tilde{\kappa}(\eta)$, the monopoly profit $\pi(\kappa, \eta)$ is constant in $\kappa$.

This result shows that, in direct contrast with the insights from the behavioral industrial organization literature, a monopolist cannot exploit biased consumers. Indeed, monopoly profits decrease as the fraction of overconfident agents in the economy increases. Hence, monopolists have an incentive to reduce fraction of biased, overconfident agents in the population. This is in direct contrast, for example, with the results by Ellison (2005) and Gabaix and Laibson (2005) on hidden prices. They show that firms do not have any incentive to reduce the fraction of biased, naive consumers who overlook add-on prices.

6 Monopoly: Testable Implications

The purpose of this section is to revisit the positive results for the perfect competition case in Section (4) and assess the implications of overconfidence in the Stiglitz (1976) model. We find that overconfidence does not qualitatively affect the predictions of the Stiglitz (1976) with respect to (i) price heterogeneity within and across risk classes, (ii) the relationship between ex-post risk and insurance coverage, and (iii) the problem of uninsurance. However, overconfidence reverses the results of Stiglitz (1976) with respect to the relationship between age and underinsurance.

**Price Heterogeneity Within and Across Risk Classes** As in the perfect competition case, overconfidence implies price heterogeneity within and across risk classes. But in stark contrast with the case of perfect competition, *overconfidence is not needed to account for heterogeneity across and within risk classes in the case of monopoly*.

Recall that risk classification is introduced in our model by assuming that each agent has a public verifiable signal $x$: her risk class. We again assume that both $\kappa(x)$ –the fraction of overconfident agents in class $x$– and $\eta(x)$ –the fraction of high-risk unbiased agents in class $x$–.

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increase in \( x \). The monopolist is not constrained in its choice of contracts offered to different agents in different risk classes, and perfectly differentiates the contract according to the risk class of the subscriber. Hence, the monopolist can treat each risk group as a single different market. So, as in the perfect-competition case, the optimal contracts \( \alpha(x) \) are determined by the parameters \( \kappa(x) \) and \( \eta(x) \) through to the monopolist solution in Section 5. It is then easy to derive the following result.

**Result 14** As \( \kappa(x) \) and \( \eta(x) \) increase, the price of both contracts \( \alpha^H(x) \) and \( \alpha^{LO}(x) \) increase. For any class \( x \), the price of contract \( \alpha^H(x) \) is larger than the price of contract \( \alpha^{LO}(x) \).

The significance of Result 14 is that, because the price of both contracts \( \alpha^H(x) \) and \( \alpha^{LO}(x) \) increase in \( x \), there is price-schedule heterogeneity across risk classes \( x \). At the same time, because for any class \( x \), the price of contract \( \alpha^H(x) \) is larger than the price of contract \( \alpha^{LO}(x) \), there is price heterogeneity within risk classes \( x \). Hence biased beliefs imply price heterogeneity within and across risk classes. But overconfidence is not needed to account for these stylized facts. In fact, the effect of a change in the fraction of overconfident agents is qualitatively the same as the effect of a change in the fraction of high-risk agents. So, the heterogeneity of prices across and within risk classes can be accounted for by differences in the fraction of high-risk agents risk across risk classes.\(^{28}\)

**The Relationship between Insurance Coverage and Ex-Post Risk** We now turn to considering the relationship between insurance coverage and ex-post risk. In striking contrast with the perfect competition case, we find that, for the parameter range where the low-risk and overconfident agents purchase positive amount of insurance, \( \alpha^{LO} > 0 \), the relationship between insurance coverage and ex-post risk is always positive. Unlike for the case of perfect competition, overconfidence cannot account for a negative or statistically insignificant relationship between insurance coverage and ex-post risk.

\(^{28}\) Furthermore, one can extend the work of Landsberger and Meilijson (1999) to formulate an explanation of price heterogeneity within and across risk classes based on monopolistic insurance with asymmetric information with respect to risk and risk aversion.
**Result 15** Suppose that the fraction of overconfidence agents is not too high, i.e. \( \kappa < \bar{\kappa}(\lambda) \), and hence \( \alpha^{LO} > 0 \). The relationship between insurance coverage \( \alpha_1 + \alpha_2 \) and ex-post risk \( p \) is positive: i.e. \( P^H > P^{LO} \) and \( \alpha_1^H + \alpha_2^H = d > \alpha_1^{LO} + \alpha_2^{LO} \).

The explanation for this result is simple. Because the incentive compatibility constraint \( V(W, d, p_H, \alpha^H) = V(W, d, p_H, \alpha^{LO}) \) binds in the monopolistic solution, and the insurance coverage \( \alpha_1^H + \alpha_2^H \) is larger than \( \alpha_1^{LO} + \alpha_2^{LO} \), it cannot be that the price of the high-risk agents’ contract \( P^H \) is smaller than the price of the low-risk and overconfident agents’ contract \( P^{LO} \), or else high-risk agents would strictly prefer contract \( \alpha^H \) to contract \( \alpha^{LO} \). In contrast, in the equilibrium for the perfect-competition case, the incentive compatibility constraint \( V(W, d, p_H, \alpha^H) \geq V(W, d, p_H, \alpha^{LO}) \) does not bind when \( p_H < p_{LO} \), and hence there is no contradiction with the finding that the price of the high-risk agents’ contract \( P^H \) is smaller than the price of the low-risk and overconfident agents’ contract \( P^{LO} \).

The result is quite striking, as it shows that the relationship between insurance coverage and ex-post risk may depend on the market structure, in the presence of biased beliefs.

**Underinsurance: The Relationship between Age and Overconfidence** We now note that, as in the case of perfect competition, whenever the fraction of overconfident agents in the economy is large enough, low-risk and overconfident agents choose to be uninsured.

**Result 16** In risk classes \( x \) where the fraction of overconfident agents is sufficiently large, \( \kappa(x) \geq \bar{\kappa}(\eta(x)) \), low-risk and overconfident individuals choose to be uninsured \( \alpha^{LO}(x) = 0 \).

As in the case of perfect competition, overconfidence provides a possible account for the empirical finding that large number of agents choose to be uninsured. But in stark contrast with the case of perfect competition, overconfidence is not needed to account for this stylized fact. Because the effect of an increase in the fraction of overconfident agents is qualitatively the same as the effect of an increase in the fraction of high-risk agents, the presence of uninsured agents can be equivalently explained by a large fraction of high-risk agents. Hence, the effect of overconfidence on the unemployment problem depends on market structure.
We conclude this section by showing that, as in the case of perfect competition, overconfidence can account for the observed negative relationship between age and insurance coverage. We expand our basic model, by indexing each agent with her age $a$. Because age is verifiable, the monopolist can treat each age group as a single different market. The optimal contracts $\alpha(a)$ are determined by the parameters $\kappa(a)$ and $\eta(a)$ through to the monopolist solution in Section 5. Hence, we obtain the following results, where we define the average insurance coverage as $C(a) \equiv [\alpha_1^H(a) + \alpha_2^H(a)] \eta(a) + [\alpha_1^{LO}(a) + \alpha_2^{LO}(a)] [1 - \eta(a)]$. 

**Result 17** As $\kappa(a)$ and $\eta(a)$ decrease in $a$, the insurance coverage $\alpha_1^{LO}(a) + \alpha_2^{LO}(a)$ of low-risk and overconfident agents increases with $a$, whereas the coverage $\alpha_1^H(a) + \alpha_2^H(a)$ remains constant. As $\kappa(a)$ decreases in $a$, and $\eta(a)$ remains constant, the average insurance coverage $C(a)$ increases in age $a$.

The significance of this result is that overconfidence may account for the positive relationship between age and insurance coverage in monopolistic insurance markets where the fraction of high-risk unbiased agents $\eta(a)$ is not too sensitive to age. Result 17 covers insurance markets where the agents’ level of risk and overconfidence decrease with age, such as the motorist insurance market.

**7 Conclusion**

In this paper, we show that the implications of biased beliefs with respect to the insights of celebrated asymmetric information models may depend on the market structure. Overconfidence overturns essential relationships between observable variables in perfect-competition asymmetric-information insurance models. In monopolistic insurance models with asymmetric information, in contrast, overconfidence may be equivalent to variation in the risk composition in the economy.

Our analysis delivers a number of testable predictions. For the perfect competition case, we have found that overconfidence may account for a number of empirical findings such as (i) price heterogeneity within and across risk classes, (ii) a negative or statistically insignificant relationship between ex-post risk and insurance coverage, (iii) the fact that a large fraction of agents chooses to be uninsured, and (iv) the positive relationship between insurance coverage and age. In contrast, for the monopoly case, overconfidence cannot account for the negative or statistically insignificant
relationship between ex-post risk and insurance coverage. Further, overconfidence does not change the implications of asymmetric information with respect to price heterogeneity within and across risk classes, and to the uninsured phenomenon. However, overconfidence may help refine the predictions about the relationship between age and insurance coverage that we derived in the monopolistic insurance model with asymmetric information by Stiglitz (1976).

Finally, our analysis has reversed some essential findings from previous behavioral economics studies. First, in the case of perfect competition, an increase in overconfidence and a reduction in risk aversion may have opposite effects on insurance coverage. This result runs contrary to previous models that abstracted from asymmetric information, where both an increase in overconfidence and a reduction in risk aversion induce agents to take more risks. Second, monopolists cannot exploit biased customers, and have a strong incentive to reduce agents’ biases, because the monopoly profits decrease as the fraction of overconfident agents in the economy increases.

A Appendix

Perfect Competition – Equilibrium Analysis. This section formalizes the graphical equilibrium analysis of section 3. The first step in the equilibrium analysis shows that overconfident and low-risk agents pool together, and together they separate from high-risk agents. For future reference, we define the marginal rate of substitution associated to contract $\alpha$ and risk $p$, as:

$$M(\alpha, p) = \frac{(1 - p)U'(W - \alpha_1)}{pU'(W - d + \alpha_2)}.$$  

Proposition A.1 In the unique locally-competitive equilibrium, high-risk individuals choose the contract $\alpha^H = (p_H d, (1 - p_H)d)$. Low-risk and overconfident individuals choose the contract $\alpha^{LO}$ that solves the maximization problem

$$\max_{\alpha} V(W, d; p_L, \alpha),$$

subject to the non-negativity constraint $\alpha \geq 0$, and to the incentive compatibility and zero-profit conditions:

$$V(W, d; p_H, \alpha^H) \geq V(W, d; p_H, \alpha),$$  \hspace{1cm} (A.2)  

$$ (1 - p_{LO}) \alpha_1 - p_{LO} \alpha_2 = 0.$$  \hspace{1cm} (A.3)

As long as $\alpha^{LO} > 0$, the insurance price $p^{LO}$ equals $p_LO$ and increases in $\kappa$. The insurance coverage is such that $\alpha_1^{LO} + \alpha_2^{LO} < \alpha_1^H + \alpha_2^H = d$.  

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**Proof.** Step 1. In equilibrium, types $L$ and $O$ pool on the same contract $\alpha^{LO}$, type $H$ chooses a different contract $\alpha^H$.

For any contract $\alpha$, bought by types $H$, $L$ and $O$ with probabilities $\sigma_H^\alpha$, $\sigma_L^\alpha$, and $\sigma_O^\alpha$, respectively, let the average risk be:

$$p_\alpha = \frac{p_O \kappa \sigma_O^\alpha + p_H (1 - \kappa - \lambda) \sigma_H^\alpha + p_L \lambda \sigma_L^\alpha}{\kappa \sigma_O^\alpha + (1 - \kappa - \lambda) \sigma_H^\alpha + \lambda \sigma_L^\alpha}.$$

Consider any equilibrium contract $\alpha$ such that $\sigma_H^\alpha + \sigma_O^\alpha > 0$. Hence, $V(W, d; p_L, \alpha) = V(W, d; p_L, \beta)$ for any equilibrium contract $\beta$ such that $\sigma_H^\beta + \sigma_O^\beta > 0$, and $V(W, d; p_H, \alpha) = V(W, d; p_H, \beta)$ for any contract $\beta$ such that $\sigma_H^\beta > 0$. Further, competition requires that $\pi(\alpha) \equiv (1 - p_\alpha) a_1 - p_\alpha a_2 = 0$, or else there is a local profitable deviation, by continuity. Suppose by contradiction that $p_\alpha > p_{LO}$. Because $(1 - p_L)/p_L > (1 - p_H)/p_H$, it follows that $M(\alpha, p_L) > M(\alpha, p_H)$. Since $U$ is twice differentiable, there is an $\varepsilon > 0$ small enough such that for any $m \in (M(\alpha, p_H), M(\alpha, p_L))$, the contract $\alpha - \varepsilon (1, m)$ is purchased by all type $L$ and $O$ agents but not by type $H$ agents. Hence, $\alpha - \varepsilon (1, m)$ yields expected profit $(1 - p_{LO})(\alpha_1 - \varepsilon) - p_{LO}(\alpha_2 - \varepsilon m)$, which is strictly bigger than $\pi(\alpha) = 0$ for $\varepsilon$ small enough because $p_\alpha > p_{LO}$. Because $\alpha - \varepsilon (1, m)$ is a local profitable deviation, $\alpha$ cannot be an equilibrium contract.

Conversely, consider any equilibrium contract $\alpha$ such that $\sigma_H^\alpha > 0$. Hence, $V(W, d; p_L, \alpha) \leq V(W, d; p_L, \beta)$ for any equilibrium contract $\beta$ such that $\sigma_H^\beta + \sigma_O^\beta > 0$, and $V(W, d; p_H, \alpha) = V(W, d; p_H, \beta)$ for any contract $\beta$ such that $\sigma_H^\beta > 0$. Again, competition requires that $\pi(\alpha) \equiv (1 - p_\alpha) a_1 - p_\alpha a_2 = 0$. Suppose by contradiction that $p_\alpha > p_H$. There is an $\varepsilon > 0$ small enough such that for any $m \in (M(\alpha, p_H), M(\alpha, p_L))$, the contract $\alpha + \varepsilon (1, m)$ is purchased by all type $H$ agents but not by type $L$ or $O$ agents. Hence, again, $\alpha + \varepsilon (1, m)$ is a profitable deviation because $p_\alpha > p_H$.

The results that $p_\alpha \leq p_{LO}$ for all $\alpha$ such that $\sigma_H^\alpha + \sigma_O^\alpha > 0$ and that $p_\alpha \leq p_H$ for all $\alpha$ such that $\sigma_H^\alpha > 0$ imply that (i) $\sigma_H^\alpha = 0$ and $p_\alpha = p_{LO}$ for all $\alpha$ such that $\sigma_H^\alpha + \sigma_O^\alpha > 0$, (ii) $\sigma_L^\alpha + \sigma_L^{LO} = 0$ and $p_\alpha = p_H$ for all $\alpha$ such that $\sigma_H^\alpha > 0$.

Because $\pi(\alpha) = 0$ for all equilibrium contracts, and $U'' < 0$, there are therefore at most two equilibrium contracts $\alpha, \beta$, with $\alpha > \beta$, such that $\sigma_H^\alpha + \sigma_O^\alpha > 0$ and $\sigma_L^\alpha + \sigma_L^{LO} > 0$. Because $M(\alpha, p_L) > M(\alpha, p_H)$, it must be that for any $\gamma$ such that $\sigma_H^\gamma > 0$, $V(W, d; p_H, \gamma) \geq V(W, d; p_H, \alpha) > V(W, d; p_H, \beta)$, or else agents of type $H$ purchase contract $\alpha$. Hence, there is an $\varepsilon > 0$ small enough such that for any $m \in (M(\alpha, p_L), (1 - p_{LO})/p_{LO})$, the contract $\beta + \varepsilon (1, m)$ is purchased by all type $L$ and $O$ agents but not by type $H$ agents. The profit $\pi(\beta + \varepsilon (1, m))$ is strictly positive because $m < (1 - p_{LO})/p_{LO}$. This concludes that types $L$ and $O$ must pool on the same contract $\alpha^{LO}$.

Analogously, there are therefore at most two equilibrium contracts $\alpha, \beta$, with $\alpha > \beta$, such that $\sigma_H^\alpha > 0$ and $\sigma_H^\beta > 0$. Because $M(\beta, p_L) > M(\beta, p_H)$, it must be that for any $\gamma$ such that $\sigma_L^\gamma + \sigma_O^\gamma > 0$, $V(W, d; p_L, \gamma) \geq V(W, d; p_L, \beta) > V(W, d; p_L, \alpha)$, or else agents of type $L$ purchase contract $\beta$. Hence, there is an $\varepsilon > 0$ small enough such that for any $m \in ((1 - p_H)/p_H, M(\alpha, p_H))$,
the contract $\alpha - \varepsilon (1, m)$ is purchased by all type $H$ agents but not by type $L$ or $O$ agents. The profit $\pi(\alpha - \varepsilon (1, m))$ is strictly positive because $m > (1 - p_H)/p_H$. This concludes that types $H$ must pool on the same contract $\alpha^H$.

**Step 2.** There exists a unique locally-competitive equilibrium, characterized in the statement of Proposition A.1.

By Step 1, if a locally-competitive equilibrium exists, it is a pair of distinct contracts $\alpha^H, \alpha^{LO}$ such that $\alpha^{LO} \in \arg\max_{\alpha} V(W, d; p_L, \alpha)$ s.t. $\alpha \geq 0$, $(1 - p_{LO}) \alpha_1 - p_{LO} \alpha_2 = 0$, $V(W, d; p_H, \alpha^H) \geq V(W, d; p_H, \alpha)$; and $\alpha^H \in \arg\max_{\alpha'} V(W, d; p_H, \alpha')$, s.t. $\alpha' \geq 0$, $(1 - p_H) \alpha'_1 - p_H \alpha'_2 = 0$, $V(W, d; p_L, \alpha^{LO}) \geq V(W, d; p_L, \alpha')$. By construction, any other pair of contracts admits local profitable deviations, whereas these contracts do not admit any local deviations.

Suppose by contradiction that the constraint $V(W, d; p_L, \alpha^{LO}) \geq V(W, d; p_L, \alpha^H)$ binds in the solution of the $\alpha^H$-maximization problem. Because $M(\alpha, p_H) < M(\alpha, p_L)$ for all $\alpha$ and $V(W, d; p_H, \alpha^H) \geq V(W, d; p_H, \alpha^{LO})$, it follows that $\alpha^H > \alpha^{LO}$. But this and $V(W, d; p_L, \alpha^{LO}) = V(W, d; p_L, \alpha^H)$ are incompatible with $(1 - p_{LO}) \alpha_1^{LO} - p_{LO} \alpha_2^{LO} = 0$ and $(1 - p_H) \alpha_1^H - p_H \alpha_2^H = 0$. Because $U$ is twice differentiable and $U'' < 0$, the solution to the $\alpha^H$-maximization problem is $\alpha^H = (p_H d, (1 - p_H)d)$. A solution to the $\alpha^{LO}$-maximization problem exists and is unique because $U'' < 0$ and $M(\alpha, p_H) < M(\alpha, p_L)$ for all $\alpha'$.

To show that the insurance coverage $\alpha_1^{LO} + \alpha_2^{LO} < d$, suppose the contrary and note that, because $(1 - p_{LO}) \alpha_1^{LO} - p_{LO} \alpha_2^{LO} = 0$, this would imply $\alpha^{LO} = (p_{LO} d, (1 - p_{LO}) d)$. If $p_{LO} \geq p_H$, then there is an $\varepsilon > 0$ small enough such that for any $m \in ((1 - p_H)/p_H, (1 - p_L)/p_L)$, the contract $\alpha^{LO} - \varepsilon (1, m)$ is purchased by all type $L$ and $O$ agents but not by type $H$ agents. The profit $\pi(\alpha^{LO} - \varepsilon (1, m))$ is strictly positive because $m > (1 - p_H)/p_H \geq (1 - p_{LO})/p_{LO}$. If $p_H > p_{LO}$, then agents of type $H$ would purchase contract $\alpha^{LO}$ contradicting the result that they separate from agents of type $L$ and $O$ in equilibrium.

Finally, we note that, because $p_O > p_L$, $d p_{LO}/d \lambda < 0$ and $d p_{LO}/d \kappa > 0$. By condition (A.3), the price $P^{LO} = \alpha_1^{LO}/(\alpha_1^{LO} + \alpha_2^{LO})$ equals $p_{LO}$, and hence it increases in $\kappa$.

The equilibrium characterization is completed in the Proposition A.2 below, which also reports our comparative statics results 4 and 5, and determines perfect-competitive equilibrium existence. For any parameter constellation $(W, d, p_H, p_L)$, the thresholds $\kappa_1$ and $\kappa_2$, functions of $\lambda$, uniquely solve respectively:

$$V(W, d; p_H, \alpha) = U(W - p_H d), \quad p_{LO} \alpha_2 - (1 - p_{LO}) \alpha_1, \quad M(\alpha, p_L) = (1 - p_{LO})/p_{LO}; \quad \text{(A.4)}$$

$$M(0, p_L) = (1 - p_{LO})/p_{LO}. \quad \text{(A.5)}$$

where the variables $\kappa$ and $\lambda$ are embedded in the expression $p_{LO} = (kp_L + \lambda p_O) / (\kappa + \lambda)$.

**Proposition A.2** The incentive compatibility condition (A.2) binds if and only if $\kappa < \kappa_1(\lambda)$. In this case, the insurance coverage $\alpha_1^{LO} + \alpha_2^{LO}$ increases in $\kappa$ and decreases in $\lambda$. For $\kappa_1(\lambda) < \kappa <
\( \kappa_2(\lambda) \), the equilibrium contract \( \alpha^{LO} \) satisfies the tangency condition

\[
M(\alpha, p_{LO}) = (1 - p_{LO})/p_{LO}.
\] (A.6)

Hence the insurance coverage \( \alpha_1^{LO} + \alpha_2^{LO} \) decreases in \( \kappa \) and increase in \( \lambda \), as long as the Relative Risk Aversion coefficient of \( U \) is bounded by \((W - d)/d\). For \( \kappa > \kappa_2(\lambda) \), low-risk and overconfident individuals are uninsured: \( \alpha^{LO} = 0 \). The locally-competitive equilibrium \( (\alpha^H, \alpha^{LO}) \) is also perfectly competitive if and only if \( \lambda > \lambda_0(\kappa) \), where the function \( \lambda_0 \) is such that \( \lambda_0^{-1} < \kappa_1 \).

**Proof.** Note that the set of contracts \( \alpha \) that satisfy (A.2) with equality is such that \((1 - p_H)\alpha_1 - p_H\alpha_2 < 0\). Hence, if \( p_H < p_{LO} \), then there is no such contract \( \alpha \geq 0 \) that satisfies conditions (A.3) and (A.2) with equality. Suppose that \( p_H > p_{LO} \). Let \( \tilde{\alpha} = (\tilde{\alpha}_1, \tilde{\alpha}_2) \) be the contract pinned down by condition (A.3) and (A.2) with equality. Differentiating these equations, we obtain:

\[
\frac{d\tilde{\alpha}_1}{dp_{LO}} = \frac{(\tilde{\alpha}_1 + \tilde{\alpha}_2)p_HU'(W - d + \tilde{\alpha}_2)}{\Delta} > 0, \quad \frac{d\tilde{\alpha}_2}{dp_{LO}} = \frac{(\tilde{\alpha}_1 + \tilde{\alpha}_2)(1 - p_H)U'(W - \tilde{\alpha}_1)}{\Delta} > 0, \quad (A.7)
\]

where the quantity \( \Delta = (1 - p_{LO})p_HU'(W - d + \tilde{\alpha}_2) - p_{LO}(1 - p_H)U'(W - \tilde{\alpha}_1) \) is positive because \( U'' < 0 \), \(-\tilde{\alpha}_1 > -d + \tilde{\alpha}_2 \) and \( p_H > p_{LO} \). Because \( dp_{LO}/d\lambda < 0 \) and \( dp_{LO}/d\kappa > 0 \), we obtain that \( d\tilde{\alpha}_1/d\kappa > 0, d\tilde{\alpha}_1/d\lambda < 0, d\tilde{\alpha}_2/d\kappa > 0, \) and \( d\tilde{\alpha}_2/d\lambda < 0 \).

Let \( \chi = (1 - p_{LO})/p_{LO} \). Because

\[
dM(\tilde{\alpha}, p_L) = \frac{1 - p_L}{p_L} \left[ -\frac{U''(W - \tilde{\alpha}_1)}{U''(W - d + \tilde{\alpha}_2)}d\tilde{\alpha}_1 - \frac{U''(W - d + \tilde{\alpha}_2)U'(W - \tilde{\alpha}_1)}{(U''(W - d + \tilde{\alpha}_2))^2}d\tilde{\alpha}_2 \right],
\]

we obtain: \( dM(\tilde{\alpha}, p_L)/d\kappa > 0 \). Because \( d\chi/dp_{LO} < 0 \) and \( dp_{LO}/d\kappa > 0 \), we have shown that for any \( \lambda \), there is a unique threshold \( \kappa_1 \) pinned down by system (A.4) and that \( M(\tilde{\alpha}, p_L) > (1 - p_{LO})/p_{LO} \) if and only if \( \kappa > (\kappa_1) \). Furthermore, for \( \kappa = \kappa_1(\lambda) \), \( p_{LO} < p_H \). Because \( d\chi/d\lambda < 0, d\chi/dp_{LO} < 0 \) and \( dp_{LO}/d\lambda < 0 \), \( \kappa_1 \) is strictly increasing in \( \lambda \) by the implicit function theorem.

Suppose that \( \kappa < \kappa_1(\lambda) \), and that, by contradiction, condition (A.2) does not bind in equilibrium: \((1 - p_H)U(W - \alpha_1^{LO}) + p_HU(W - d + \alpha_2^{LO}) < U(W - p_Hd)\). Since \( U'' < 0 \), and both \( \tilde{\alpha} \) and \( \alpha^{LO} \) satisfy condition (A.3), it must be that \( \tilde{\alpha} < \alpha^{LO} \) and hence that \( M(\alpha^{LO}, p_L) < M(\tilde{\alpha}, p_L) < (1 - p_{LO})/p_{LO} \). Because \( U \) is twice differentiable, there is an \( \varepsilon > 0 \) small enough such that for any \( m \in (M(\alpha^{LO}, p_L), (1 - p_{LO})/p_{LO}) \), the contract \( \alpha^{LO} + \varepsilon(1, m) \) is chosen by type \( L \) and \( O \) but not by type \( H \), and makes strictly positive profit. This concludes that for \( \kappa < \kappa_1(\lambda) \), \( \alpha^{LO} = \tilde{\alpha} \).

Suppose that \( \kappa > \kappa_1(\lambda) \), and hence that \( M(\tilde{\alpha}, p_L) > (1 - p_{LO})/p_{LO} \). In the case that \( p_H < p_{LO} \), there is no such contract \( \alpha \geq 0 \) that satisfies conditions (A.3) and (A.2) with equality. So, it cannot be that \( \alpha^{LO} = \tilde{\alpha} \) in equilibrium. Suppose that \( p_H > p_{LO} \), and, by contradiction, that \( \alpha^{LO} = \tilde{\alpha} \) in equilibrium. Note that \( M(\tilde{\alpha}, p_H) < (1 - p_H)/p_H < (1 - p_{LO})/p_{LO} \). Since \( U'' < 0 \)
and $U$ is smooth, for any $\varepsilon > 0$ small enough, and $m \in ((1 - p_{LO})/p_{LO}, M(\alpha, p_L))$, the contract $\alpha - \varepsilon (1, m)$ is chosen only by types $L$ and $O$, and not by type $H$, and yields strictly positive profit. This proves that condition (A.2) does not bind in equilibrium.

Since $dp_{LO}/d\kappa > 0$, for any $\lambda$ there is a unique threshold $\kappa_2(\lambda)$ such that $M(0, p_L) > (\kappa_1(\lambda))$(1 - p_{LO})/p_{LO}$ if and only if $\kappa > (\kappa_1(\lambda))$. When $\kappa > \kappa_2(\lambda)$, the constraint $\alpha \geq 0$ binds in equilibrium, whereas when $\kappa_1(\lambda) < \kappa < \kappa_2(\lambda)$, the equilibrium contract $\alpha^{LO}$ is pinned down by condition (A.3) and by the tangency condition (A.6). Since $dp_{LO}/d\lambda < 0$, the function $\kappa_2$ is increasing in $\lambda$.

We differentiate conditions (A.3) and (A.6) with respect to the quantity $\chi$, decreasing in $p_{LO}$, and obtain:

$$\frac{\partial (\alpha_1^{LO} + \alpha_2^{LO})}{\partial \chi} = \alpha_1^{LO} - (1 + \chi)p_L \frac{U'(W - d + \alpha_2^{LO}) + U''(W - d + \alpha_2^{LO})}{(1 - p_{LO})U''(W - \alpha_1^{LO})} + \chi^2 p_L U''(W - d + \alpha_2^{LO}).$$

This derivative is positive because $U'(W - d + \alpha_2^{LO}) + U''(W - d + \alpha_2^{LO}) \alpha_2^{LO} > 0$, which follows by the hypothesis that $-U''(w)w/U''(w) < (W - d)/d$.

By construction, the pair $(\alpha^{LO}, \alpha^H)$ is the (unique) perfectly-competitive equilibrium if and only if it does not admit any pooling, possibly large profitable deviation $\alpha$. Hence, it is necessary and sufficient that $V(W, d; p_L, \alpha^{LO}) \geq V(W, d; p_L, \beta)$, where $p_{LH} = \lambda p_L + \kappa p_O + (1 - \lambda - \kappa)p_H$ and

$$\beta = \arg \max_{\alpha} V(W, d; p_L, \alpha) \quad \text{s.t.} \quad p_{LH}\alpha_2 \leq (1 - p_{LH})\alpha_1, \quad \alpha \geq 0. \quad (A.8)$$

When $\kappa \geq \kappa_1(\lambda)$, condition (A.2) does not bind in equilibrium. When $p_{LO} \leq p_H$, by revealed preferences, $V(W, d; p_L, \alpha^{LO}) \geq V(W, d; p_L, \beta)$ because $p_{LH} \geq p_{LO}$. When $p_{LO} > p_H$, $V(W, d; p_L, \alpha^{LO}) > V(W, d; p_L, \alpha^H) > V(W, d; p_L, \beta)$. In both cases, $(\alpha^H, \alpha^{LO})$ is the perfectly-competitive equilibrium.

Suppose that $\kappa < \kappa_1(\lambda)$. The utility $V(W, d; p_L, \alpha^{LO})$ decreases in $\kappa$ and increases in $\lambda$ because $dp_{LO}/d\kappa > 0$, $dp_{LO}/d\lambda < 0$, and

$$\frac{\partial V(W, d; p_L, \alpha^{LO})}{\partial p_{LO}} = -(1 - p_L)U'(W - \alpha_1^{LO})\frac{\alpha_1^{LO} + \alpha_2^{LO}}{1 - p_{LO}} - p_L U''(w, d + \alpha_2^{LO})\frac{\alpha_1^{LO} + \alpha_2^{LO}}{p_{LO}} < 0,$$

after substituting in condition (A.7). By revealed preferences, $V(W, d; p_L, \beta)$ increases in $\lambda$ and decreases in $\kappa$. Hence, there is a unique strictly-increasing threshold $\lambda_0$, function of $\kappa$, such that $(\alpha^H, \alpha^{LO})$ is a perfectly-competitive equilibrium if and only if $\lambda > \lambda_0(\kappa)$.

**Perfect Competition – Positive Results** This section proves the results of section 4.

**Proof of Result 6.** For any $x$ such that $\kappa(x) < \kappa_2(\lambda(x))$, the equilibrium price of the high-risk agents’ contract $\alpha^H(x)$ is $P^H(x) = p_H$. The low-risk and overconfident individuals’ contract $\alpha^{LO}(x)$ satisfies the Zero-Profit condition $p_{LO}(x)\alpha^{LO}_2 = (1 - p_{LO}(x))\alpha^{LO}_1$; hence its price $P^{LO}(x)$ is $[\kappa(x)p_O + \lambda(x)p_L]/[\kappa(x) + \lambda(x)]$. Evidently $P^H(x) \neq P^{LO}(x)$ unless $[\kappa(x)p_O + \lambda(x)p_L]/[\kappa(x) + \lambda(x)] = p_H$. ■
Proof of Result 7. The previous analysis shows the insurance coverage is such that \( \alpha^H_1 + \alpha^H_2 > \alpha^L_1 + \alpha^L_2 \) and the prices are such that \( P^{LO} = [\kappa p_O + \lambda p_L]/[\kappa + \lambda] \), \( P^H = p_H \). So, the relationship between insurance coverage and risk is negative (positive) if and only \([\kappa p_O + \lambda p_L]/[\kappa + \lambda] < (>) p_H \); when \([\kappa p_O + \lambda p_L]/[\kappa + \lambda] < (>) p_H \), the pricing schedule displays quantity discounts.

Proof of Result 9. Differentiating \( C(a) \), we obtain:

\[
[k(\alpha) + \lambda(\alpha)] \frac{d}{da} [\alpha^L_1(a) + \alpha^L_2(a)] + \left[ (\alpha^L_1(a) + \alpha^L_2(a)) - (\alpha^H_1 + \alpha^H_2) \right] \frac{d}{da} [k(\alpha) + \lambda(\alpha)],
\]

by hypothesis \( \frac{d}{da} [k(\alpha) + \lambda(\alpha)] < 0 \) and \( (\alpha^L_1(a) + \alpha^L_2(a)) - (\alpha^H_1 + \alpha^H_2) < 0 \), following the analysis in the previous section.

To show that the first term is positive, let the average price of low-risk and overconfident agents be:

\[
p^{LO}(\alpha) = \frac{k(\alpha)p_O + \lambda(\alpha)p_L}{k(\alpha) + \lambda(\alpha)},
\]

and differentiating,

\[
p'^{LO}(\alpha) = \frac{\kappa'(\alpha)p_O + \lambda'(\alpha)p_L}{[\kappa(\alpha) + \lambda(\alpha)]^2} \times \left( \kappa'(\alpha) + \lambda'(\alpha) \right)
\]

where the last inequality follows by hypothesis. Following the analysis in the previous section, the insurance coverage \( \alpha^L_1 + \alpha^L_2 \) of low-risk and overconfident agents decreases as the average price \( p^{LO} \) increases.

Proof of Result 10. We have in Proposition A.2, for \( \kappa < \kappa_1(\lambda) \), the insurance coverage \( \alpha^L_1 + \alpha^L_2 \) increases in \( \kappa \) and decreases in \( \lambda \). Hence, introducing a small fraction of overconfident agents in the model by Rothschild and Stiglitz (1996), the insurance coverage increases.

We now introduce a fraction \( \phi \) of high-risk, moderately risk-averse (type-\( R \)) agents in the model by Rothschild and Stiglitz (1996). When purchasing a contract \( \alpha \) their utility is \( V(W, d; p_H, \alpha^H) = (1 - p_H) U(W - \alpha) + p_H U(W - d + \alpha_2) \), with \( U''(\cdot) < U''(\cdot) < 0 \).

In equilibrium, \( \alpha^H = (p_H d, (1 - p_H) d) \). When \( \phi = 0 \), \( \alpha^L \) solves \( p_L \alpha^L \) together with \( V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^L) \). When \( \phi > 0 \), \( \alpha^R = \alpha^H \) and, because \( U'' < U'' \), the incentive-compatibility constraint \( V(W, d; p_H, \alpha^R) > V(W, d; p_H, \alpha^L) \) is tighter than the constraint \( V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^L) \). Hence the coverage \( \alpha^L_1 + \alpha^L_2 \) is smaller when \( \phi > 0 \), and so is the average coverage \( (\alpha^L_1 + \alpha^L_2) \phi + (\alpha^L_1 + \alpha^L_2) \lambda + (\alpha^H_1 + \alpha^H_2)(1 - \lambda - \phi) \).

The monopoly solution We now prove the monopoly optimal solution characterization of section 5.
Proposition A.3 The monopoly solution is the pair of contracts \((\alpha^H, \alpha^{LO})\) that maximizes:

\[
(1 - \eta) \left[ (1 - p_{LO}) \alpha_1^{LO} - p_L \alpha_2^{LO} \right] + \eta \left[ (1 - p_H) \alpha_1^H - p_H \alpha_2^H \right]
\]

subject to

\[
V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^{LO})
\]
\[
V(W, d; p_L, \alpha^{LO}) = V(W, d; p_L, 0)
\]
\[
\alpha^H \geq 0, \alpha^{LO} \geq 0.
\]

In the optimal contract, \(\alpha_1^H + \alpha_2^H = d\).

**Proof.** The proof that all agents cannot pool on the same contract \(\alpha\) is a simple extension of the proof in Stiglitz (1976). Suppose by contradiction that the monopolist offers a contract \(\alpha\) such that \(\sigma_H^0 > 0\) and \(\sigma_L^0 + \sigma_O^0 > 0\). There are two cases. In the first one \(\alpha_1 + \alpha_2 < d\). The monopolist may increase its profit by offering a contract \(\alpha'\) with price \(P'\) equal to the price \(P\) of contract \(\alpha\) and such that \(\alpha_1' + \alpha_2' > \alpha_1 + \alpha_2\). The fraction \(\sigma_H^0\) of high-risk agent accept contract \(\alpha'\) and this determines a strict increment in profit, regardless of whether the fraction \(\sigma_L^0 + \sigma_O^0\) of low-risk and overconfident agents still purchase contract \(\alpha\) or change to contract \(\alpha'\). In the second case \(\alpha_1 + \alpha_2 = d\). Then, for all \(\varepsilon > 0\) small, the monopolist can increase its profit by offering a contract \(\alpha'\) such that \(V(W, d; p_H, \alpha') = V(W, d; p_H, \alpha)\) and \(\alpha_1' + \alpha_2' = d - \varepsilon\). In fact, by assumption, the fraction \(\sigma_H^0\) of high-risk agents purchase contract \(\alpha\), whereas because \(M(\alpha, p_L) > M(\alpha, p_H)\), the fraction \(\sigma_L^0 + \sigma_O^0\) of low-risk and overconfident agents purchase contract \(\alpha'\). This increases the profit of the monopolist because the price \(P'\) of contract \(\alpha'\) is larger than the price \(P\) of contract \(\alpha\).

Further, low-risk and overconfident agents must buy the same insurance contract \(\alpha^{LO}\) because they have the same beliefs, and, if indifferent between two contracts, they all must purchase the same contract with largest insurance coverage. Hence we conclude that in the monopoly solution, low-risk and overconfident agents pool on the contract \(\alpha^{LO}\) and high-risk agents separate on the contract \(\alpha^H\).

The proof that if the monopoly solution is a pair of separating contracts \((\alpha^H, \alpha^{LO})\), then \(\alpha_1^H + \alpha_2^H = d\), \(V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^{LO})\), \(V(W, d; p_L, \alpha^{LO}) = V(W, d; p_L, 0)\) are also a simple extension of the proofs in Stiglitz (1976). Suppose by contradiction that \(V(W, d; p_H, \alpha^H) > V(W, d; p_H, \alpha^{LO})\), then the monopolist can propose a different contract \(\alpha'\) such that \(V(W, d; p_H, \alpha^{LO}) < V(W, d; p_H, \alpha') < V(W, d; p_H, \alpha^H)\) and increase its profit. Suppose by contradiction that the constraint \(V(W, d; p_L, \alpha^{LO}) \geq V(W, d; p_L, \alpha^H)\) binds in the solution of the \(\alpha^H\)-maximization problem. Because \(M(\alpha, p_H) < M(\alpha, p_L)\) for all \(\alpha\) and \(V(W, d; p_H, \alpha^H) \geq V(W, d; p_H, \alpha^{LO})\), it follows that \(\alpha^H > \alpha^{LO}\). But this and \(V(W, d; p_L, \alpha^{LO}) = V(W, d; p_L, \alpha^H)\) are incompatible with the assumption that low-risk and overconfident agents buy the contract the highest coverage. Because \(V(W, d; p_L, \alpha^{LO}) > V(W, d; p_L, \alpha^H)\), the contract \(\alpha^H\) does not admit any local
profitable deviations $\alpha$. Because $U$ is twice differentiable and $U'' < 0$, the solution to the $\alpha^H$-maximization problem is $\alpha^H = (p_H d, (1 - p_H) d)$. A solution to the $\alpha^{LO}$-maximization problem exists and is unique because $U'' < 0$ and $M(\alpha, p_H) < M(\alpha, p_L)$ for all $\alpha'$. Finally, to show that $V(W, d; p_L, \alpha^{LO}) = V(W, d; p_L, 0)$, we notice that because $M(\alpha, p_H) < M(\alpha, p_L) < P^{LO}$, the monopoly profit increases as the low-risk agents’ rent $V(W, d; p_L, \alpha^{LO}) - V(W, d; p_L, 0)$ decreases; so the monopolist will choose to set $V(W, d; p_L, \alpha^{LO}) = V(W, d; p_L, 0)$. ■

Proposition A.4 Suppose that $\alpha^{LO} > 0$ in the optimal solution. As $\eta$ increases, or $\kappa$ increases, $\alpha_1^H$ increases and $\alpha_2^H$ decreases, whereas both $\alpha_1^{LO}$ and $\alpha_2^{LO}$ decrease; and both prices $P^{LO}$ and $P^H$ increase. When $\kappa \geq \bar{\kappa}(\eta)$ then $\alpha^{LO} = 0$ in the optimal solution. The function $\bar{\kappa}$ is decreasing in $\eta$. The high-risk unbiased agents’ utility $V(W, d; p_H, \alpha^H)$ and the overconfident agents’ utility $V(W, d; p_H, \alpha^{LO})$ decrease in $\kappa$ and $\eta$, the low-risk agents utility $V(W, d; p_L, \alpha^{LO})$ is constant in $\kappa$ and $\eta$.

Proof. The constraint set $V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^{LO}), V(W, d; p_L, \alpha^{LO}) = V(W, d; p_L, 0)$ and the equality $\alpha_1^H + \alpha_2^H = d$ allow to reduce the monopolist problem into:

$$\max_{\alpha^H} \Pi^{LO}(\alpha_1^H) + \Pi^H(\alpha_1^H),$$

where $\Pi^{LO}(\alpha_1^H) = (1 - \eta) [(1 - p_{LO}) \alpha_1^{LO}(\alpha_1^H) - p_{LO} \alpha_2^{LO}(\alpha_1^H)], \Pi^H(\alpha_1^H) = \eta [(1 - p_H) \alpha_1^H - p_H \alpha_2^H(\alpha_1^H)] = \eta [\alpha_1^H - p_H d].$ Note that $\partial \Pi^{LO} / \partial \alpha_1^H < 0$. To calculate $d\alpha_1^{LO} / d\alpha_1^H$ and $d\alpha_2^{LO} / d\alpha_1^H$, differentiate the constraints $V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^{LO})$ and $V(W, d; p_L, \alpha^{LO}) = V(W, d; p_L, 0)$, to obtain:

$$-U'(W - \alpha_1^H)d\alpha_1^H = - (1 - p_H) U'(W - \alpha_1^{LO})d\alpha_1^{LO} + p_H U'(W - d + \alpha_2^H)d\alpha_2^LO$$

$$- (1 - p_L) U'(W - \alpha_1^{LO})d\alpha_1^{LO} + p_L U'(W - d + \alpha_2^H)d\alpha_2^LO = 0.$$

Solving out,

$$\frac{d\alpha_1^{LO}}{d\alpha_1^H} = -\frac{p_L U'(W - \alpha_1^H)}{(p_H - p_L) U'(W - \alpha_1^{LO})} < 0$$

$$\frac{d\alpha_2^{LO}}{d\alpha_1^H} = -\frac{(1 - p_L) U'(W - \alpha_1^H)}{(p_H - p_L) U'(W - d + \alpha_2^H)} < 0$$

The first-order condition for an interior solution $\alpha^{LO}(\alpha_1^H) > 0$ is:

$$\partial \Pi^{LO}(\alpha_1^H) / \partial \alpha_1^H + \partial \Pi^H(\alpha_1^H) / \partial \alpha_1^H = 0,$$

and the second-order condition is:

$$\partial^2 \Pi^{LO} / \partial^2 \alpha_1^H + \partial^2 \Pi^H / \partial^2 \alpha_1^H < 0.$$

Applying the implicit function theorem,

$$\frac{\partial \alpha_1^H}{\partial \eta} = -\frac{\partial^2 \Pi^{LO} / \partial \alpha_1^H \partial \eta + \partial \Pi^H / \partial \alpha_1^H \partial \eta}{\partial^2 \Pi^{LO} / \partial^2 \alpha_1^H + \partial^2 \Pi^H / \partial^2 \alpha_1^H} \times \partial^2 \Pi^{LO} / \partial \alpha_1^H \partial \eta + \partial^2 \Pi^H / \partial \alpha_1^H \partial \eta$$

$$= -\partial [\alpha_1^{LO}(\alpha_1^H) - p_{LO} \alpha_2^{LO}(\alpha_1^H)] / \partial \alpha_1^H + \partial [\alpha_1^H - p_H d] / \partial \alpha_1^H > 0$$

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because \[ \partial [ (1 - p_{LO}) \alpha^L_1 (\alpha^H_1) - p_{LO} \alpha^L_2 (\alpha^H_1) ] / \partial \alpha^H_1 < 0 \] at the first-order condition, as \[ \partial \Pi^L (\alpha^H_1) / \partial \alpha^H_1 + \partial \Pi^H (\alpha^H_1) / \partial \alpha^H_1 = 0 \] and \[ \partial \Pi^L (\alpha^H_1) / \partial \alpha^H_1 > 0. \]

\[
\frac{\partial \alpha^H_1}{\partial \kappa} = -\frac{\partial^2 \Pi^L (\alpha^H_1) / \partial \alpha^H_1 \partial \kappa + \partial^2 \Pi^H (\alpha^H_1) / \partial \alpha^H_1 \partial \kappa}{\partial^2 \Pi^L (\alpha^H_1) / \partial \alpha^H_1 + \partial^2 \Pi^H (\alpha^H_1) / \partial \alpha^H_1} \propto \partial^2 \Pi^L (\alpha^H_1) / \partial \alpha^H_1 \partial \kappa + \partial^2 \Pi^H (\alpha^H_1) / \partial \alpha^H_1 \partial \kappa
\]

\[
\frac{\partial \alpha^H_1}{\partial \kappa} = \partial (1 - \eta) [\partial \alpha^L_1 (1 - p_{LO}) \partial \alpha^H_1 - p_{LO} \partial \alpha^L_2 (\partial \alpha^H_1) / \partial \kappa]
\]

\[
\frac{\partial \alpha^H_1}{\partial \kappa} = \partial (1 - \eta) [\partial \alpha^L_1 (1 - p_{LO}) \partial \alpha^H_1 - p_{LO} \partial \alpha^L_2 (\partial \alpha^H_1) / \partial \kappa / \partial \kappa]
\]

\[
(1 - \eta) \left[ -\partial \alpha^L_1 (\partial \alpha^H_1 - \partial \alpha^L_2 (\partial \alpha^H_1) / \partial \kappa / \partial \kappa) \right] / \partial \kappa > 0
\]

because \[ \partial p_{LO} / \partial \kappa = \frac{\partial H}{\partial \eta} > 0 \] and \[ \partial \alpha^L_1 / \partial \alpha^H_1 < 0, \partial \alpha^L_2 / \partial \alpha^H_1 < 0. \]

Whenever

\[
\partial \Pi^L (\alpha^H_1) / \partial \alpha^H_1 + \partial \Pi^H (\alpha^H_1) / \partial \alpha^H_1 \big|_{\alpha^L (\alpha^H_1)=0} > 0,
\]

the optimum is the corner solution \( \alpha^L (\alpha^H_1) = 0. \)

This allows to calculate the threshold \( \bar{k} (\eta) \): it is the solution of the following equation:

\[
\frac{\partial \Pi^L (\alpha^H_1) / \partial \alpha^H_1 + \partial \Pi^H (\alpha^H_1) / \partial \alpha^H_1}{\alpha^L (\alpha^H_1)=0} = 0 + \eta
\]

\[
= \left(1 - \eta\right) \left[ (1 - \kappa \rho v + (1 - \kappa - \eta) \rho L) - \frac{p_{LO} U'(W - \alpha^H_1)}{(p_H - p_L) U'(W)} \right] + \eta = 0.
\]

Finally, note that both prices \( P_L \) and \( P_H \) increase in \( \kappa \) and \( \eta \). The constraint that \( \alpha^H_1 + \alpha^H_2 = d \), then immediately implies that the high-risk unbiased agents’ utility \( V(W, d; p_H, \alpha^H) \) decreases in \( \kappa \) and \( \eta \). Because the coverage \( \alpha^L_1 + \alpha^L_2 = d \) also decreases in \( \kappa \) and \( \eta \), it follows that the overconfident agents’ utility \( V(W, d; p_H, \alpha^L) \) decreases in \( \kappa \) and \( \eta \). Because for any \( \kappa \) and \( \eta \), \( V(W, d; p_L, \alpha^L) = V(W, d; p_L, 0) \), the low-risk agents utility \( V(W, d; p_L, \alpha^L) \) is constant in \( \kappa \) and \( \eta \).

**Proof of Result 12.** Note that \( V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^L) \), \( V(W, d; p_L, \alpha^L) = V(W, d; p_L, 0) \) and \( \alpha^H_1 + \alpha^H_2 = d \). These constraints pin down \( (\alpha^H, \alpha^L) \) as a function of one and only one unknown, say \( \alpha^L_1 \). We already proved that the sign of \( d \alpha^L_1 / d \eta \) coincides with the sign of \( d \alpha^L_1 / d \kappa \). Hence, consider any \( \eta > 0 \) and \( \kappa \geq 0 \), such that \( \kappa + \eta \leq 1 \). For any small \( d \eta > 0 \), there exists a small \( d \kappa > 0 \) such that \( \eta - d \eta > 0, \kappa + d \kappa + \eta - d \eta \leq 1 \) and \( \alpha^L_1 [\kappa, \eta] = \alpha^L_1 [\kappa + d \kappa, \eta - d \eta] \). Because the constraints pin down \( (\alpha^H, \alpha^L) \) as a function of \( \alpha^L_1 \) only, this concludes the proof.

**Proof of Result 13.** A change in the fraction of overconfident agents \( \kappa \) does not change the constraints of the maximization problem (6), but only the objective function \( \pi(\alpha^H, \alpha^L) \). Suppose that \( \kappa < \bar{k} (\eta) \), and hence that \( \alpha^L_1 + \alpha^L_2 > 0 \). Because \( d p_{LO} / d \kappa = \kappa (p_{LO} - p_L) / (1 - \eta) > 0 \), and \( d \pi(\alpha^H, \alpha^L) / d p_{LO} = - (\alpha^L_1 + \alpha^L_2) < 0 \), it follows that \( d \pi(\alpha^H, \alpha^L) / d p_{LO} < 0 \). So,
the profit \( \pi(\alpha^H, \alpha^{LO}) \) for any pair of contracts \((\alpha^H, \alpha^{LO})\) that satisfies the maximization constraints decreases in \( \kappa \). Fix any \( \kappa \), let \((\alpha^H(\kappa), \alpha^{LO}(\kappa))\) be the solution of problem (6) when the fraction of overconfident agents equals \( \kappa \), let \( p_{LO}(\kappa) = [\kappa p_H + (1 - \kappa - \eta)p_L]/[1 - \eta] \) and let \( \pi(\alpha^H, \alpha^{LO}, \kappa) = [(1 - p_{LO}(\kappa))\alpha_1^{LO} - p_{LO}(\kappa)\alpha_2^{LO}] + \eta [(1 - p_H)\alpha_1^H - p_H\alpha_2^H] \). So, for any \( \kappa' > \kappa \),

\[
\pi(\kappa', \eta) = \pi(\alpha^H(\kappa'), \alpha^{LO}(\kappa'), \kappa') \geq \pi(\alpha^H(\kappa), \alpha^{LO}(\kappa), \kappa') > \pi(\alpha^H(\kappa), \alpha^{LO}(\kappa), \kappa) = \pi(\kappa, \eta),
\]

where the first inequality follows because \((\alpha^H(\kappa'), \alpha^{LO}(\kappa'))\) is the solution of problem (6) when the fraction of overconfident agents equals \( \kappa' \), and the second inequality follows because \( \pi(\alpha^H, \alpha^{LO}, \kappa) \) decreases in \( \kappa \). ■

**Monopoly – Positive Results** We here prove the positive results from section 6.

**Proof of Result 14.** The price of contract \( \alpha^H \) is simply: \( P^H = \alpha_1^H / (\alpha_1^H + \alpha_2^H) = \alpha_1^H / d \); because \( \partial \alpha^H / \partial \eta > 0 \) and \( \partial \alpha^H / \partial \kappa > 0 \) it follows that the price \( P^H \) increases in \( \eta \) and \( \kappa \).

The price of contract \( \alpha^L \) is \( P^{LO} = \alpha_1^{LO} / (\alpha_1^{LO} + \alpha_2^{LO}) = 1 / (1 + \alpha_2^{LO} / \alpha_1^{LO}) \), Hence

\[
dP^{LO}/d\alpha^H_1 = \frac{\partial}{\partial (\alpha_2^{LO}/\alpha_1^{LO})} [1 + \alpha_2^{LO}/\alpha_1^{LO}]^{-1} \frac{\partial (\alpha_2^{LO}/\alpha_1^{LO})}{\partial \alpha^H_1} = -\frac{1}{[1 + \alpha_2^{LO}/\alpha_1^{LO}]^2} \left[ \frac{\partial \alpha_2^{LO}}{\partial \alpha_1^{LO}} - \frac{\partial \alpha_1^{LO}}{\partial \alpha_1^{LO}} \right] = \frac{\partial \alpha_2^{LO}}{\partial \alpha_1^{LO}} \alpha_1^{LO} + \frac{\partial \alpha_1^{LO}}{\partial \alpha_1^{LO}} \alpha_2^{LO} = \frac{(1 - p_L)U'(W - \alpha_1^H)}{(p_H - p_L)U'(W - d + \alpha_2^{LO})} \alpha_1^{LO} - \frac{p_LU'(W - \alpha_1^H)}{(p_H - p_L)U'(W - \alpha_1^{LO})} \alpha_2^{LO} \propto (1 - p_L)\alpha_1^{LO}U'(W - \alpha_1^{LO}) - p_L\alpha_2^{LO}U'(W - d + \alpha_2^{LO}) > 0.
\]

The fact that \( P^{LO} < P^{HU} \) directly follows from \( V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^L) \) and \( V(W, d; p_L, \alpha^L) = V(W, d; p_L, 0) \). ■

**Proof of Result 15.** Previous analysis has shown that

\[
(1 - \eta) [(1 - p_{LO})\alpha_1^{LO} - p_L\alpha_2^{LO}] + \eta [(1 - p_H)\alpha_1^H - p_H\alpha_2^H]
\]

subject to

\[
V(W, d; p_H, \alpha^H) = V(W, d; p_H, \alpha^{LO})
\]
\[
V(W, d; p_L, \alpha^{LO} = V(W, d; p_L, 0)
\]
\[
\alpha^H \geq 0, \alpha^{LO} \geq 0.
\]

In the optimal contract, \( \alpha_1^H + \alpha_2^H = d \), whereas \( \alpha_1^{LO} + \alpha_2^{LO} < d \). Hence, The relationship between insurance coverage and risk is negative (positive) if and only if \( \kappa p_{LO} + (1 - \kappa - \eta)p_L < (>)p_H \). ■
Proof of Result 17. We have previously shown that as $\kappa$ or $\eta$ increase, $\alpha_1^{LO}$ and $\alpha_2^{LO}$ decrease. So, as $\kappa$ decreases and $\eta$ remains constant, $C = (\alpha_1^H + \alpha_2^H) \eta + [\alpha_1^{LO} + \alpha_2^{LO}] [1 - \eta] = d\eta + \left[\alpha_1^{LO} + \alpha_2^{LO}\right] [1 - \eta]$ increases.

References


