Signaling Quality by Delaying Sales*

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Abstract

This paper studies the problem of a monopolist who has private information about the quality of its product, and faces forward-looking buyers who learn about quality over time. We show that if the monopolist prefers to sell sooner rather than later, then in the unique equilibrium satisfying a standard refinement criterion, high-quality types postpone sales in order to separate themselves from lower-quality types. Also, an increase in the precision of monopolist's information has a negative effect on economic efficiency. Several testable implications are derived in the comparative static analysis.

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1 Introduction

This paper studies the problem of a monopolist with private information about the quality of her product facing forward-looking buyers who learn quality over time. We show that, if the monopolist prefers to sell sooner rather than later, the unique equilibrium satisfying Universal Divinity (Banks and Sobel 1987) is such that higher quality types postpone sales in order to separate themselves from lower quality ones. Separation occurs because buyers are willing to pay progressively less for low-quality products as their informational disadvantage shrinks, thus the monopolist's expected loss from selling in later periods is lower when her initial information is better.

The monopolist's initial informational advantage is modeled by assuming that she privately observes the realization of a binomial experiment of given size on the underlying parameter which determines the product's true quality. This structure allows for a detailed comparative statics analysis. We show that the expected social surplus is inversely related to the monopolist's informational advantage. Intuitively, this is because an increase in the size of the binomial experiment shifts mass onto higher extractions, hence we have a larger average delay of sales and larger welfare losses.

As an application, consider a sport team where the management must decide how to allocate sales between season tickets and game tickets, and is privately informed about the quality of the team. In this model, the fans make inferences based on the quantity of season tickets on sale, and learn progressively about the true quality of the team, as the season goes on. Under the assumption that the management prefers to sell sooner than later, we show that in equilibrium, high-quality teams sell fewer season tickets than lower-quality teams.

As another application, consider a real estate company who is developing a new neighborhood. The company can either sell properties to private owners, or rent units over time. Initially the developer has better information than any potential buyer on the quality of houses in the new neighborhood. In this case, 'quality' may mean future market value: the developer knows what the city really plans to do with a lot nearby, whether a fancy shopping mall, or an unpleasant recycling center. Over time this information will become public (possibly incrementally), i.e. the city will send signals about its plans. We may thus expect that a developer with better information will sell fewer houses in advance.

Our results are also related to the general issue of delay in bargaining, on which clear consensus has not been reached, despite extensive research. In a durable-good monopoly model, Coase (1972) formulated the conjecture, verified by Gul, Sonnenschein and Wilson (1986), that prices converge to marginal cost, and all trade occurs with arbitrarily small delay, as the frequency of offers becomes arbitrarily large, because the monopolist cannot credibly commit not to serve residual consumers after selling those willing to accept earlier offers. Gul and Sonnenschein (1988) have pointed out that, while under complete information delay can only be imputed to the passage of time between offers, "with incomplete information during the bargaining process, agents might be expected to signal their valuations with their offers, and this takes time." It is known that Coase's conjecture does not extend to the case of two-sided private information. Ausubel and Deneckere (1992) have shown that trade may be delayed for very long periods of time as neither party is willing to signal its weakness by settling for unfavorable terms. Moreover, Ausubel and Deneckere (1989) have established a folk-theorem result (in the case with one-sided private information), as the possibility of reputation allows for any outcome in between monopolistic and competitive prices. In our paper delay is due to the fact that the seller's informational advantage deteriorates over time. In the *unique* equilibrium, high-quality sellers will find it advantageous to partially postpone trade.

This result is consistent with the marketing practice of launching innovations with small initial quantity offers and high prices, so as to induce the perception of high quality. Bagwell and Riordan (1991) also have a model in which high price signal high quality. Their results however hinge on both the presence of informed consumers and on the assumption that high quality entails higher production costs. (See also Kihlstrom and Riordan, 1984). In our model all consumers learn at the same pace, and the separating equilibrium is sustained without restricting the analysis to industries where higher quality translates into higher variable costs. Thus incentives for cost reduction and technological innovation are preserved, even though in equilibrium high prices signal high costs which in turn translate into high quality (see also Judd and Riordan, 1994, and Shieh, 1993).

While our result predicts low initial sales by high-quality monopolists, others have pointed out that an oligopolist may try increase the quantity sold in order to signal quality (e.g. see the work Caminal and Vives, 1996, on market shares). Similarly, models based on word-of-mouth communication among consumers, while not explicitly modeling sales as a signal of quality, suggest that high quality sellers may try to increase sales in order to have more consumers informed about their quality (see for example Rogerson, 1983, and Vettas, 1997). In our model, a high quality seller has low initial sales to distinguish herself from low-quality types who have more to lose from waiting until the consumers' informational disadvantage becomes small.

The rest of the paper is organized as follows. Section 2 presents the key ideas in a simple two period model. Section 3 contains the analysis of the equilibrium in the infinite horizon model, with comparative statics and welfare implications. Sections 4 concludes. All formal proofs are in the appendix.

2 The Two-Period Case

There are two periods, indexed by their initial dates $t = 0, 1$, and a continuum of buyers, identified with the interval $[0, 1]$. The product's quality is a random variable $\theta \in \{\theta_H, \theta_L\}$, with $0 < \theta_L < \theta_H < 1$ and $\Pr[\theta = \theta_L] := \lambda \in (0, 1)$. The seller knows the realization of θ at date 0, while the buyers learn θ only at date 1. The good is produced and delivered at date 1, but the monopolist can sell on both dates. Let $q \in [0, 1]$ denote the quantity sold in advance, i.e. at date 0. This quantity is observed by all buyers, who use it to update their expectations about θ. We want to introduce intertemporal gains from trade in the model: as it is often the case, the monopolist is impatient to sell to the buyers, and the buyers are impatient to buy from the sellers. A simple way to model this feature is by assuming that the buyers do not discount future utility from enjoying the good, whereas the seller's discount factor for profits is $\delta < 1$.[1]

To focus on the effect that is generated by the information disclosure, we assume that there are

[1] For example, the seller may be impatient to collect revenues from sales, because she may be able to lend in capital markets at a premium interest rate relative to buyers.

An alternative model with intertemporal gains from trade would be one where the good is durable, and consumers may enjoy it at both periods 0 and 1, with the same discount factor as (or even lower than) the sellers. It is easy to appreciate that our analysis covers also this case, with minor modifications.

no costs of production, and all buyers have value θ for the good. Thus each buyer is willing to pay θ at $t = 1$, and $E[\theta|q]$ at $t = 0$. Since $\theta_L > 0$, the quantity sold at date 1 is always $1 - q$. Thus the monopolist's profit, given her type θ and her sales at $t = 0$, is:

$$\pi(q, \theta) = q\ E[\theta|q] + \delta(1 - q)\,\theta. \tag{1}$$

We begin the analysis by characterizing the set of all separating equilibrium profiles $\mathbf{q} = (q_L, q_H)$. Since the buyers' beliefs must be correct on the equilibrium path, we have $E[\theta|q_H] = \theta_H$ and $E[\theta|q_L] = \theta_L$. The low-quality monopolist's profit is $q_L\theta_L + \delta\theta_L(1 - q_L)$. Clearly the only equilibrium value for q_L is 1.

As is typical of signaling games, the freedom of choosing any belief off the equilibrium path generates a large multiplicity of equilibria. In this case the set of separating equilibria is parametrized by q_H, the quantity sold in advance by the high-quality seller.

Proposition 1 *In any separating equilibrium, $q_L = 1$ and $q_H = x$, where $0 \le x \le \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}$. The only separating equilibrium that satisfies the Intuitive Criterion*[2] *has $q_H = \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}$.*

The formal proof of Proposition 1 is in the Appendix. Here we point out that, in any separating equilibrium, the following incentive compatibility constraint for the low-quality monopolist must hold

$$q_L\theta_L + \delta(1 - q_L)\,\theta_L \ge q_H\theta_H + \delta(1 - q_H)\,\theta_L. \tag{IC$_L$}$$

Since $q_L = 1$, this is equivalent to

$$q_H \le \frac{(1 - \delta)\,\theta_L}{\theta_H - \delta\theta_L}.$$

These equilibria are supported by off-path beliefs such as: $E[\theta|q] = \theta_L$ for all $q < q_H$, and $E[\theta|q] = \theta_H$ for all $q \ge q_H$. The proof shows that incentive compatibility is also satisfied for the high-quality monopolist, whenever $q_H \in \left[0, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right]$.

[2] A Perfect Bayesian Equilibrium fails the Intuitive Criterion if any type is willing to unilaterally deviate, once the buyers' beliefs off-path are restricted to assign zero probability to any type θ taking actions that are equilibrium dominated. (Cho and Kreps, 1987)

Clearly, the Pareto-optimal equilibrium in this class is the one where the above incentive constraint is binding:

$$q_H = \frac{(1-\delta)\,\theta_L}{\theta_H - \delta\theta_L}.$$

The proof in the appendix establishes that this equilibrium is the only one that satisfies the Intuitive Criterion.

Pooling equilibria, where both types sell the same amount in advance, i.e. $q_H = q_L = q$, may also exist depending of the values of λ, $\frac{\theta_L}{\theta_H}$, and δ. These equilibria are supported by 'pessimistic' off-path beliefs, such as $E[\theta|q'] = \theta_L$, for all $q' \neq q$. The next proposition establishes that: i) no pooling equilibria, if the monopolist is sufficiently patient; and ii) no pooling equilibrium satisfies the Intuitive Criterion.

Proposition 2 *There is no pooling Perfect Bayesian Equilibrium if* $1 - \lambda + \lambda\frac{\theta_L}{\theta_H} < \delta < 1$. *If* $0 < \delta \leq 1 - \lambda + \lambda\frac{\theta_L}{\theta_H}$, *there is a pooling equilibrium for any* $q \in \left[\frac{(1-\delta)\theta_L}{(1-\lambda)(\theta_H-\theta_L)+(1-\delta)\theta_L}, 1\right] : \theta_L$. *No pooling equilibrium satisfies the Intuitive Criterion.*

We conclude this section with some comparative statics and an examination of the welfare properties of the unique intuitive equilibrium $(q_L, q_H) = \left(1, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right)$. First, since

$$\frac{\partial q_H}{\partial \delta} = -\theta_L \frac{\theta_H - \theta_L}{(\theta_H - \delta\theta_L)^2} < 0,$$

high-quality forward sales are decreasing in δ. As the monopolist becomes more patient, the loss of postponing sales becomes smaller, hence the high-quality type is willing to restrict advance sales more to distinguish herself from the low-quality type.

By the same token, since

$$\frac{\partial q_H}{\partial\left(\theta_H - \theta_L\right)} = -\frac{\theta_H(1-\delta)}{(\theta_H - \delta\theta_L)^2} < 0,$$

if the gap in quality increases, the high-quality monopolist sells less in advance. This is because when the gap in quality is higher, the low-quality monopolist has a stronger incentive to pool with the high-quality one, hence the high-quality must contract advance sales to offset this.

The ex ante expected social surplus is given by the net present value of total trade:

$$W = \lambda\theta_L + (1-\lambda)\left[\frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L} + \delta\left(1 - \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right)\right]\theta_H.$$

It is straightforward to see that an increase in the probability of high quality increases ex-ante welfare. Less obvious is the fact that, *when the monopolist becomes more patient, ex-ante welfare increases,* in fact $\partial W/\partial\delta = \frac{\lambda\theta_H(\theta_H-\theta_L)^2}{(\theta_H-\delta\theta_L)^2} > 0$. In order to appreciate this result, note that the discount factor enters the loss function in two separate ways. It enters indirectly by reducing the equilibrium period-0 sales of the high-quality monopolist, and thus increasing the undiscounted loss, but it also enters directly by discounting (and thus reducing) the loss. It turns out that the direct effect dominate the indirect one, so that the net effect is a loss reduction.

It is also possible to show that the parameters θ_H and θ_L have a positive impact on the social welfare. In fact,

$$\frac{\partial W}{\partial\theta_H} = \lambda\delta\frac{(\theta_H - \delta\theta_L)^2 - (1-\delta)^2\theta_L^2}{(\theta_H - \delta\theta_L)^2} > 0,$$

$$\frac{\partial W}{\partial\theta_L} = \frac{(\theta_H - \delta\theta_L)^2 - \lambda\delta(\theta_H - \theta_L)[(1-\delta)\theta_H + (\theta_H - \delta\theta_L)]}{(\theta_H - \delta\theta_L)^2} > 0,$$

since $\delta < 1$, and $\lambda \leq 1$.

The efficiency loss with respect to the perfect information case equals the value of high quality transactions that are postponed to period 1, times the deterioration rate, multiplied by the probability of a high quality product:

$$L = \lambda(1-\delta)\left[1 - \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right]\theta_H = \lambda\frac{\theta_H(\theta_H - \theta_L)(1-\delta)}{\theta_H - \delta\theta_L}.$$

It is straightforward to see that the loss increases with the probability of a high-quality product. In fact, the low-quality monopolist does not impose any direct social cost, as she does not postpone sales. When the monopolist becomes less impatient to sell, the loss decreases, in fact $\partial L/\partial\delta = -\theta_H\frac{(\theta_H-\theta_L)^2}{(\delta\theta_L-\theta_H)^2} < 0$. While an improvement of the low-quality product reduces the efficiency loss ($\partial L/\partial\theta_L = -\frac{\lambda(1-\delta)^2\theta_H^2}{(\theta_H-\delta\theta_L)^2} < 0$), *an increase of the high quality product makes the efficiency loss larger:* $\partial L/\partial\theta_H = \lambda(1-\delta)\frac{(\theta_H-\delta\theta_L)^2+\delta(1-\delta)\theta_L^2}{(\theta_H-\delta\theta_L)^2} > 0$. When the high-quality product is more valuable, in fact, the low-quality monopolist has a stronger incentive to copy the high-quality one. This results in a larger quantity of high-quality product withdrawn from sale at period 0, and thus more trade inefficiently delayed.

3 The Infinite-Period Case

3.1 The Model

Time is discrete, and indexed by $t = 1, 2, ...$. The quality x_t of the good at time t is either good ($x_t = 1$) or bad ($x_t = 0$), and is extracted from a Bernoulli distribution of unknown parameter $\theta := \Pr[x_t = 1]$. Thus θ represents the average quality over time, and is extracted from the Beta distribution[3] with parameters (α, β). At time $t = 0$, the monopolist has some private information on the product's quality. In order to use the Beta-binomial conjugate model, we represent this information as an extraction $y = \sum_{j=1}^{m} x_j$ from an experiment of size m. Since the probability of success of each trial is θ, the sum y is distributed according to a binomial distribution of parameters (θ, m). The statistic y/m is an estimate of the quality, as $E[y/m] = \theta$. The size m is a measure of the precision of the monopolist's information, as the variance of $\frac{1}{m}\theta(1-\theta)$ is inversely proportional to the size of the experiment.

After observing y, the monopolist decides the quantity $q_y \in [0, 1]$ to offer as forward sales. The residual quantity $1 - q_y$ will be sold in the spot markets at each time t. The monopolist discount factor is $\delta \in (0, 1)$.

As in the tow-period model there is a continuum of buyers of unit mass, each willing to pay,

$$u_i(t|\Omega_\tau) = \gamma^{t-\tau}\ E[x_t|\Omega_\tau],$$

at time τ, for the delivery of a unit of the good at time t, where γ is the buyers' discount factor, and Ω_τ represents the information available at time τ, consisting of both q_y and the sum $\mathbf{x}_{\tau-1} :=$

[3] The Beta distribution with integer parameters α and β has density $f(x|\alpha, \beta) = \frac{(\alpha+\beta-1)!}{(\alpha-1)!(\beta-1)!} x^{\alpha-1} (1-x)^{\beta-1}$ on the interval $[0, 1]$. It is the standard prior used to model updating of Bernoulli extractions, and allows great flexibility with respect to the first two moments. See for example Mood, Graybill and Boes (1988).

$\sum_{s=1}^{\tau-1} x_s$ of past quality realizations (which is a sufficient statistic for the history $x_1, ..., x_{\tau-1}$). The buyers use the quantity q_y to infer the monopolist's extraction y. That, together with $\mathbf{x}_{\tau-1}$, allows them to form the expectation $E[x_t|\Omega_\tau]$, according the Beta-binomial updating model introduced above.

To isolate signalling as the sole force behind the presence of delay in equilibrium, we take $\delta \to 1$, while holding the ratio $k := \frac{1-\delta}{1-\gamma}$ fixed. We assume that $\gamma > \delta$, hence $k > 1$. These assumptions guarantees that, even without any Coasian dynamics, the monopolist would never choose to delay sales, in the absence of private information. Thus any delay in our model is entirely attributable to signaling.

3.2 The Equilibrium

Our first Lemma pertains to any Perfect Bayesian equilibrium.

Lemma 1 *In any PBE, we have:*

- *for any amount q of forward sales, the monopolist sells $Q_t = 1 - q$, at price $p_t = E\left[\theta|q, \mathbf{x}_{t-1}\right]$, in any period $t > 0$.*
- *the price for any amount q of forward sales is $P_0 = \frac{\gamma}{1-\gamma} E\left[\theta|q\right]$;*
- *for any size m of the seller's initial information, and for any type $y \in \{0, 1, ..., m\}$, the expected profit function of $q \in [0,1]$ tends to*

$$u_y(q) \equiv q\ k\ E[\theta|q] + [1-q]\ E[\theta|y] = q\ k\ E[\theta|q] + [1-q]\frac{\alpha + y}{\alpha + \beta + m}, \tag{2}$$

  as $\delta \to 1$.

Lemma 1 derives the equilibrium price P_0 of advance sales as a function of the quantity offered q. The buyers' expectations on quality include the information contained in q. The monopolist's profit function can be represented as the sum of two components: one from advance sales and one from future spot sales.

As is typical of signaling games, many Perfect Bayesian Equilibria exists, due to the indeterminacy of off-equilibrium beliefs. Following Banks and Sobel (1987), we restrict attention to Perfect Bayesian Equilibria satisfying Universal Divinity, a refinement of the Intuitive Criterion.[4] Essentially, this refinement imposes the following restrictions on off-equilibrium beliefs. Consider any action off the equilibrium path, and for each type y of the monopolist, determine the set of buyers' strategies that improve the monopolist's payoff relative to the equilibrium. Whenever the set associated with one type strictly contains the set associated with another, the equilibrium beliefs are required to assign at most infinitesimal mass on the second type.

The following Proposition characterizes the unique Perfect Bayesian Equilibrium satisfying Universal Divinity. It is a fully-separating equilibrium, as the monopolist reveals all her information with her forward sales offer.

Proposition 3 *For any size* m*, and* δ *close to* 1*, there exists a unique PBE satisfying Universal Divinity. In this equilibrium, upon observing signal* y*, the monopolist offers for forward sale a quantity close to* q_y*, where* $q_0 := 1$*, and*

$$q_y = \prod_{s=1}^{y} \frac{(\alpha + s - 1)(k-1)}{(\alpha + s - 1)(k-1) + k}, \qquad y = 1, ..., m.$$

The forward sales price P_0 *is*

$$P_0 = \frac{\gamma}{1-\gamma} \cdot \frac{\alpha + y}{\alpha + \beta + m},$$

and in each period $t \geq 0$*, the unit price at time* t *is*

$$p_{t+1} = \frac{\alpha + y + \mathbf{x}_t}{\alpha + \beta + m + t}.$$

The key insight of Proposition 3 is that q_y is strictly decreasing in y, so that the monopolist chooses to postpone sales in order to signal higher quality. Specifically, for any $y \geq 1$, we have $q_y = z_y \; q_{y-1}$, where $z_y \equiv \frac{(\alpha+y-1)(k-1)}{(\alpha+y-1)(k-1)+k}$. Since $z_y \in (0,1)$, it follows that q_y is decreasing in y. The coefficient z_y provides a measure of the loss incurred by the y-th lowest quality type due to the presence of the $(y-1)$-th lowest quality type. To distinguish herself from the $(y-1)$-th type,

[4] In the Appendix, we show that the Intuitive Criterion is ineffective in refining equilibrium, when $m > 2$.

the y-th type cannot sell more than fraction $z_y < 1$ of the amount sold by the $(y-1)$-th type. Since the $(y-1)$-th type must also distinguish herself from the $(y-2)$-th type, the latter type imposes an additional indirect loss on the y-th type. Iteratively, all types $s = 0, 1, ..., y-1$ impose an informational loss on type y, as $q_y = \prod_{s=1}^{y} z_s$.

Since the equilibrium is separating, all information contained in y is revealed by the initial sale offer q_y. Thus the forward price P_0 must be equal to the discounted value of a stream of purchases with expected quality equal to the Bayesian estimate of θ given the draw y from a binomial experiment of size m. The spot prices will incorporate the public information which is disclosed over time, i.e. the period $t+1$ spot price p_{t+1} consist of the Bayesian estimate of θ given the extraction $y + \mathbf{x}_t$ from a binomial experiment of size $m+t$.

3.3 Comparative Statics

We have already shown that q_y is strictly decreasing in y. In this section we establish further testable implications with respect to forwards sales.

First, it is worth noticing that while the monopolist's estimate of quality coincide with the statistic y/m, the forward sale offer q_y depends only on y, the (inverse) order of quality, regardless of m, the size of the experiment. This is because the informational constraint of postponing sales affects high-quality types only because they need to differentiate themselves from lower-quality types, regardless of the actual quality.

It is also interesting to note that while q_y is decreasing in y, the absolute value of the decrement $q_{y+1} - q_y$ is also decreasing in y, since $|q_{y+1} - q_y| = |(z_{y+1} - 1)q_y| = \frac{k}{(\alpha+y-1)(k-1)} q_y$. Thus the informational loss imposed by the y-th lowest type on the $(y+1)$-th type becomes smaller as y increases. The biggest share of sale postponement suffered by a monopolist with a high quality y/m is thus imposed by the lowest-quality type, rather than by the types whose quality is closer to y/m.

Let us focus now on the effect of a change in the "trade impatience" ratio k on the quantity q_y. We will show by induction that $\partial q_y / \partial k > 0$, for any $y \geq 1$. Notice first that $\partial q_1 / \partial k = \partial z_1 / \partial k =$

$\frac{\alpha}{(\alpha k-\alpha+k)^2} > 0$. For any $y \geq 1$, $\partial q_y/\partial k = q_{y-1}\partial z_y/\partial k + z_y \partial q_{y-1}/\partial k$. By the induction hypothesis, $\partial q_{y-1}/\partial k > 0$. Since, $q_{y-1} > 0$, and $z_y > 0$, it suffices to show that $\partial z_y/\partial k > 0$, for any y. In fact, $\partial z_y/\partial k = \frac{\alpha+(y-1)}{((\alpha+y)(k-1)+1)^2} > 0$. Thus, *when k increases, forward sales increase.* High-quality monopolists are required to postpone sales to avoid being copied by lower-quality types. Since a larger trade impatience increases the disincentive for sale postponements, it allows separation with the delay of a smaller amount of sales.

Considering the limit cases, we first see that for $k \to 1^+$, even the monopolist with the second-lowest experiment extraction will offer a negligible quantity of forwards, i.e. $q_1 \to 0^+$. This does not imply that the separating equilibrium unravels however. Regardless of how large m is, each $y \in \{1, ..., m\}$ will sell smaller and smaller quantities q_y, and these apparently negligible differences will be sufficient to separate out the types. When $k \to +\infty$, agents are infinitely impatient to trade. For any given m, even the 'best-quality' monopolist delays only negligible sales, i.e. $q_m \to 1^-$. Separation occurs with very little constraint on forwards sales, because the immediate revenue from selling forward is much higher than the present value of the stream of income obtained by sales in spot markets.

Finally, we consider the effects that changes in the mean and in the variance of the prior distribution over quality. Since $\theta \sim Beta(\alpha, \beta)$, its mean is $\mu = \frac{\alpha}{\alpha+\beta}$, and standard deviation is $s = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$. Solving out for α as a function of μ, and s, we obtain $\alpha = -\mu\frac{\mu^2+s^2-\mu}{s^2}$. Since $\partial z_y/\partial \alpha = \frac{k(k-1)}{((\alpha+y)(k-1)+1)^2} > 0$, it can be shown by induction that $\partial q_y/\partial \alpha > 0$, for all $y \geq 1$. Since $\partial \alpha/\partial \mu = -\frac{(\mu-s)^2+2\mu^2}{s^2} < 0$, and $\partial \alpha/\partial s = -\mu^2\frac{1-\mu}{s^2} < 0$, it turns out that *an increase in either the mean or in the variance of the distribution of quality results in less forward sales.*

3.4 Welfare Implications

The welfare and informational loss associated with the equilibrium derived in Proposition 3 display a very simple functional form.

Proposition 4 *In the equilibrium derived in Proposition 3, the ex-post welfare and loss, conditional* *on θ and y, are approximately*

$$W(\theta, y) = \theta[q_y k + (1 - q_y)] \qquad\qquad L(\theta, y) = \theta k(1 - q_y).$$

For a close to 0 and δ close to 1, both the expected welfare and the informational loss depend only on the good's average quality θ, the trade-impatience ratio k, and the level of forward sales q_y. The trade impatience ratio k enters the ex-post welfare function W both directly and indirectly in the formula for q_y. Since $\partial q_y/\partial k > 0$, we have that ex-post welfare is increasing in k. Clearly this result extends to both interim and ex-ante expected welfare.

The effect of k on the loss function is in general ambiguous. For instance, the derivative $\partial L(\theta, 1)/\partial k = \theta(1 - q_1 - k\partial q_1/\partial k) = \theta k \frac{\alpha(k-1)+k-\alpha}{(\alpha k - \alpha + k)^2}$ is positive for $k > \frac{2\alpha}{\alpha+1}$, and negative otherwise.

It is easy to see that *an increase in θ increases the informational loss L*, hence it also increases the interim expected loss $E\left[L(\theta, y)|\theta\right]$. Since y is an extraction from a binomial distribution of parameters θ and m, an increase in θ shifts mass onto higher values of y, and thus it results in fewer forward sales q_y. Since $E\left[L(\theta, y)|\alpha, \beta\right] = E\left[E[L(\theta, y)|\theta]|\alpha, \beta\right]$, an increase in the mean of the distribution of θ increases the ex-ante loss $E\left[L(\theta, y)|\alpha, \beta\right]$.

The effect of a change in θ on interim welfare, for fixed m, is seen by analyzing the relation

$$E\left[W(\theta, y)|\theta\right] = \theta \sum_{y=0}^{m} [q_y k + (1 - q_y)] \binom{m}{y} \theta^y (1 - \theta)^{m-y}.$$

The parameter θ enters the interim expected welfare function both as a positive multiplier and as the control for $E[q_y k + (1 - q_y) \mid \theta]$, where it has a negative effect because it shifts mass onto lower q_y. Since we know that the quantity $q_y - q_{y+1}$ is decreasing in y, and that all the quantities q_y are increasing in k, we can conclude that the effect of θ on interim welfare is always positive if it is so for the worst-case scenario, where $k \to 1^+$. In fact, since $\lim_{k=1^+} q_y = 0$ for $y > 0$, and $q_0 = 1$, we have $\lim_{k=1^+} E\left[W(\theta, y)|\theta\right] = \theta(1 - \theta)^m + \theta \sum_{y=1}^{m} \binom{m}{y} \theta^y (1 - \theta)^{m-y} = \theta > 0$. Again, the result is the same in the ex-ante sense, that is, an increase in the mean of the random variable θ increases expected welfare.

The Beta-binomial model allows us to determine the impact of the monopolist's information on economic efficiency. It turns out that, first, *fixing the unknown parameter θ, and the size of the* *experiment m, bad news improves efficiency.* Second, *the effect of the monopolist's information precision m on economic efficiency is also negative.* This second result holds both in the interim case, when the quality θ is fixed and unknown, and in the ex-ante case, in which θ is a random extraction of a Beta distribution parametrized by α and β.

Proposition 5 *In the equilibrium of Proposition 3, both the interim welfare* $E\left[W(\theta,y)|\theta,m\right]$ *and the ex-ante welfare* $E\left[W(\theta,y)|\alpha,\beta,m\right]$ *are strictly decreasing in* q_y *and* m; *both the interim loss* $E\left[L(\theta,y)|\theta,m\right]$ *and the ex-ante loss* $E\left[L(\theta,y)|\alpha,\beta\right]$ *are strictly increasing in* q_y *and in* m.

4 Conclusions

We have shown that, if the quality of a new product becomes observable over time, a monopolist can signal high quality by limiting forward sales. This result can be applied to situations in which quantity and price signalling take place in low-competition industries characterized by the coexistence of long-term contracting and spot markets. In financial markets, for instance, there is a common perception that large sales disclose insider information about the performance of a company, and the timing of trade is extremely important. While in this paper we only consider the case in which the seller is allowed to trade on a single forward market, the results would remain qualitatively similar if we had forward markets open in every period. The equilibrium in this case would involve mixed strategies as in Noldeke and Van Damme (1990).

5 Appendix

Proof of Proposition 1. For any profile $\mathbf{q}=(1,q_H)$, with $q_H\in\left[0,\frac{(1-\delta)\theta_L}{\theta_H-\delta\theta_L}\right]$ to be a separating equilibrium, it must be the case that incentive compatibility is satisfied for the *high-quality* monopolist. This requires

$$q_H\theta_H+\delta(1-q_H)\theta_H\geq q\theta_L+\delta(1-q)\theta_H,\forall q>q_H \tag{IC$_H$}$$

For $\theta_L<\delta\theta_H$, this translates into $q_H\theta_H+\delta(1-q_H)\theta_H\geq\delta\theta_H$, which is satisfied, since $q_H(1-\delta)\theta_H\geq 0$. For $\theta_L>\delta\theta_H$, it must be that $q_H\theta_H+\delta(1-q_H)\theta_H\geq\theta_L$, i.e. $q_H\geq-\frac{\theta_L-\delta\theta_H}{\theta_H(1-\delta)}$. This always holds, as the last quantity is negative. We conclude that any $q_H \in \left[0, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right]$ characterizes a separating equilibrium.

Recall that a set of Perfect Bayesian Equilibrium *beliefs* conforms with equilibrium dominance, whenever they assign zero probability to any type θ that takes equilibrium dominated actions. A PBE fails to satisfy the Intuitive Criterion if a type is willing to unilaterally deviate, once the buyers' beliefs are adjusted to conform with equilibrium dominance.

Consider any separating equilibrium with $q_H \in \left[0, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right]$. By incentive compatibility, any strategy $q \in \left(q_H, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right)$ is equilibrium dominated for type θ_L. Once the buyers' beliefs are adjusted to follow $E\left[\theta|q\right] = \theta_H$ for all $q \in \left(q_H, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right)$, Equation IC$_H$ implies that all actions $q \in \left(q_H, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right)$ are unilateral deviations that make type θ_H better off. Thus the only separating PBE satisfying the Intuitive Criterion has $q_H = \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}$. □

Proof of Proposition 2. All pooling equilibria, with $q_H = q_L = q$, can be supported by 'pessimistic' off-path beliefs such as $E\left[\theta|q'\right] = \theta_L$ for all $q' \neq q$. The necessary conditions for q to be a PBE outcome are that neither type of monopolist wants to deviate to any $q' \neq q$, i.e.

$$q\overline{\theta} + \delta(1-q)\theta_H \geq q'\theta_L + (1-q')\delta\theta_H, \tag{ICP$_H$}$$

and

$$q\overline{\theta} + \delta(1-q)\theta_L \geq q'\theta_L + (1-q')\delta\theta_L, \tag{ICP$_L$}$$

where $\overline{\theta} := \lambda\theta_L + (1-\lambda)\,\theta_H$. Since its RHS is increasing in q', the inequality in (ICP$_L$) holds if and only if it holds for $q' = 1$, i.e.

$$q\overline{\theta} + \delta(1-q)\theta_L \geq \theta_L. \tag{3}$$

Since $\overline{\theta} - \delta\theta_L = (1-\lambda)\left(\theta_H - \theta_L\right) + (1-\delta)\,\theta_L > 0$, solving for q yields we have

$$q \geq \frac{(1-\delta)\,\theta_L}{\overline{\theta} - \delta\theta_L} > 0. \tag{4}$$

Suppose first that $\overline{\theta} < \delta\theta_H$. Then $\theta_L < \delta\theta_H$, and the RHS in (ICP$_H$) is decreasing in q'. Thus (ICP$_H$) holds if and only if it holds for $q' = 0$, i.e.

$$q\overline{\theta} + \delta(1-q)\theta_H \geq \delta\theta_H \quad \Leftrightarrow \quad q\left(\overline{\theta} - \delta\theta_H\right) \geq 0. \tag{5}$$

Since $\overline{\theta} - \delta\theta_H < 0$, we have $q = 0$, which contradicts (4). We conclude that if $\overline{\theta} < \delta\theta_H$, there is no pooling equilibrium.

Now suppose that $\theta_L < \delta\theta_H \leq \overline{\theta}$. In this case any $q \geq 0$ satisfies (5), which is still equivalent to (ICP$_H$). Thus there exists a pooling equilibrium for any q satisfying (4).

Finally, if $\delta\theta_H < \theta_L$, then (ICP$_H$) holds if and only if it holds for $q' = 1$, i.e.

$$\left(\overline{\theta} - \delta\theta_H\right) q \geq \theta_L - \delta\theta_H,$$

which holds for any $q \geq 0$. Thus again any q satisfying (4) can be supported in a pooling equilibrium.

To summarize, we have established that: i) if $\overline{\theta} < \delta\theta_H$, there is no pooling equilibrium; and ii) if $\delta\theta_H \leq \overline{\theta}$, there is a pooling equilibrium for any $q \in \left[\frac{(1-\delta)\theta_L}{\overline{\theta}-\delta\theta_L}, 1\right]$.

To see that, when $\delta\theta_H \leq \overline{\theta}$ i.e. $\delta \leq 1 - \lambda + \lambda\frac{\theta_L}{\theta_H}$, all pooling PBE profiles q fail to satisfy the Intuitive Criterion, note that any action $q' \in [0, q'')$ is equilibrium dominated for the low-quality monopolist, when $q'' = q\frac{\lambda(\theta_L-\theta_H)+\theta_H-\delta\theta_L}{\theta_H-\delta\theta_L}$. In fact, the best that the low-quality monopolist can achieve by taking q' is $q'\theta_H + (1-q')\delta\theta_L$, which is less than the equilibrium payoff $q\left[(1-\lambda)\theta_L + \lambda\theta_H\right] + (1-q)\delta\theta_L$ for the specified q'. At the same time, $\frac{(1-\lambda)(\theta_L-\theta_H)+\theta_H-\delta\theta_L}{\theta_H-\delta\theta_L} \in (0,1)$, because $\delta < 1 < \frac{\lambda\theta_L+(1-\lambda)\theta_H}{\theta_L}$, $\theta_H - \delta\theta_L > 0$, and $\lambda(\theta_L - \theta_H) < 0$. Since $q \geq \frac{(1-\delta)\theta_L}{\lambda(\theta_H-\theta_L)+(1-\delta)\theta_L} > 0$, it follows that $q'' > 0$.

Once the off-path beliefs have been adjusted to conform with equilibrium dominance, the PBE profile q fails the Intuitive Criterion test because there exists an ε small enough, for which the high-quality monopolist prefers to play $q' = q'' - \varepsilon$, rather than taking the equilibrium action q. In fact,

$$\begin{aligned}
& q''\theta_H + (1-q'')\delta\theta_H = q''\theta_H + (1-q'')\delta\theta_L + (1-q'')\delta(\theta_H - \theta_L) \\
= \;& q(\lambda\theta_L + (1-\lambda)\theta_H) + (1-q)\delta\theta_L + (1-q'')\delta(\theta_H - \theta_L) \\
= \;& q(\lambda\theta_L + (1-\lambda)\theta_H) + (1-q)\delta\theta_H + \delta(\theta_H - \theta_L)(q - q'') \\
> \;& q(\lambda\theta_L + (1-\lambda)\theta_H) + (1-q)\delta\theta_H.
\end{aligned}$$

This concludes the proof. □

Proof of Lemma 1. i) Since all buyers have identical preferences and information at any time t, it is optimal to set $p_t = E\left[x_t|\Omega_t\right]$. At this price it is optimal for any buyer who has not bought in advance to buy in period t. Thus $Q_t = 1 - q$.

ii) Consider the expected value of x_t conditional on $\Omega_t = (q, \mathbf{x}_{t-1})$, the information held at time t. Since the buyers have rational expectations, for any $t \geq 1$, we have $E\left[\theta|q\right] = E\left[E\left[\theta|q, \mathbf{x}_{t-1}\right] \mid q\right]$. Therefore, each buyer i forecasts that the utility of a unit held in perpetuity is

$$\sum_{t=1}^{\infty} \gamma^t E\left[x_t|q\right] = \sum_{t=1}^{\infty} \gamma^t E\left[\theta|q\right] = \frac{\gamma}{1-\gamma} E\left[\theta|q\right].$$

The optimal price is $P_0 = \frac{\gamma}{1-\gamma} E\left[\theta|q\right]$.

iii) In light of i) and ii), the monopolist's problem can be written as

$$\max_{q\in[0,1]} q\frac{1-\delta}{1-\gamma}\gamma E\left[\theta|q\right] + [1-q](1-\delta)\delta E\left[\sum_{t=0}^{\infty}\delta^t E[\theta|q,\mathbf{x}_t]\middle| y\right]. \tag{6}$$

Given that $\theta \sim Beta(\alpha,\beta)$, and $y \sim bin(\theta,m)$, it follows that $\theta|y \sim Beta(\alpha+y,\beta+m-y)$, hence

$$E(\theta|y) = \frac{\alpha+y}{\alpha+\beta+m}.$$

In particular,

$$E[\theta|q,\mathbf{x}_t] = \frac{\alpha+E(y|q)+\mathbf{x}_t}{\alpha+\beta+m+t}.$$

Since $\mathbf{x}_t|y \sim bin(t,\theta|y)$, it is also the case that

$$E(\mathbf{x}_t|y) = \frac{\alpha+y}{\alpha+\beta+m}t,$$

so that

$$(1-\delta)\delta E\left[\sum_{t=0}^{\infty}\delta^t E[\theta|q,\mathbf{x}_t]\middle| y\right] = \delta\frac{\alpha+E(y|q)+\frac{\alpha+y}{\alpha+\beta+m}t}{\alpha+\beta+m+t}.$$

Therefore

$$\lim_{\delta\to 1}\left((1-\delta)\delta E\left[\sum_{t=0}^{\infty}\delta^t E[\theta|q,\mathbf{x}_t]\middle| y\right]\right) = \frac{\alpha+y}{\alpha+\beta+m}.$$

This establishes the equality in (2) and concludes the proof. □

Proof of Proposition 3. The proof consists of two separate Lemmata.

Lemma A.1 *There exists a PBE such that for any* $y \in \{1,...,m\}$, $q_y = \prod_{t=1}^{y}\frac{(\alpha+t-1)(k-1)}{(\alpha+t-1)(k-1)+k}$.

Proof. At any fully separating equilibrium,

$$E[\theta|q_y] = E(\theta|y) = \frac{\alpha+y}{\alpha+\beta+m},$$

and, the Incentive Compatibility constraints

$$u_y(q_y) \geq u_y(q),\ q\in[0,1], \tag{7}$$

must be satisfied. Thus, since type 0's payoff is

$$u_0(q_0) = q_0\ k\frac{\alpha}{\alpha+\beta+m} + (1-q_0)\frac{\alpha}{\alpha+\beta+m},$$

it must also be the case that $q_0 = 1$, as $k > 1$.

We propose an equilibrium where q_y is strictly decreasing in y, $q_0 = 1$, the supporting beliefs off-path are a step function, defined as

$$\Pr[y|q] = 1 \text{ iff } q \in (q_{y+1}, q_y], \text{ where } q_{m+1} = 0, \text{ and } \Pr[m|0] = 1$$

and all the constraints

$$u_y(q_y) \geq u_y(q_{y+1}), \quad y \in \{0, 1, ..., m-1\}, \tag{8}$$

are binding.

Given that $q_0 = 1$, the binding constraints (8) uniquely pin down all the equilibrium quantities q_y, for $y \in \{1, 2, ..., m\}$, in a recursive fashion. To show that the profile that we propose is an equilibrium, we will only be left to show that the q_y pinned down are admissible, i.e. $q_y \in [0, 1]$, and that the remaining Incentive Compatibility requirements are satisfied.

To explicitly calculate q_1, observe that the constraint $u_0(q_0) \geq u_0(q_1)$ boils down to

$$k\frac{\alpha}{\alpha+\beta+m} = q_1 \; k\frac{\alpha+1}{\alpha+\beta+m} + [1-q_1]\frac{\alpha}{\alpha+\beta+m},$$

hence

$$q_1 = \frac{\alpha(k-1)}{\alpha(k-1)+k} \in (0, 1),$$

regardless of m.

Similarly, for any $y \in \{1, 2, ..., m-1\}$, the constraint $u_y(q_y) \geq u_y(q_{y+1})$, translates as

$$q_y k\frac{\alpha+y}{\alpha+\beta+m} + [1-q_y]\frac{\alpha+y}{\alpha+\beta+m} = q_{y+1}k\frac{\alpha+y+1}{\alpha+\beta+m} + [1-q_{y+1}]\frac{\alpha+y}{\alpha+\beta+m},$$

hence

$$q_{y+1} = q_y\frac{(\alpha+y)(k-1)}{(\alpha+y)(k-1)+k} \in (0, 1).$$

Solving the difference equation above yields

$$q_y = \prod_{t=1}^{y} \frac{(\alpha+t-1)(k-1)}{(\alpha+t-1)(k-1)+k}, \quad \text{for } y = \{1, ..., m\}.$$

For future reference we define

$$z_{y+1} = \frac{(\alpha+y)(k-1)}{(\alpha+y)(k-1)+k},$$

and rewrite the above solution as $q_{y+1} = z_{y+1}q_y$.

We now show that, for any $y \in \{0, 1, ..., m-1\}$ and any $h \in \{1, 2, ..., m-y\}$, the incentive constraints requiring any type $y+h$ not to adopt q_y are satisfied (with slack). That is, we want to show that:

$$u_{y+h}(q_{y+h}) - u_{y+h}(q_y) > 0, \; \forall y \in \{0, 1, ..., m-1\}, \; \forall h \in \{1, 2, ..., m-y\} \tag{9}$$

First consider $u_{y+1}(q_{y+1}) - u_{y+1}(q_y)$, for any $y > 0$

$$u_{y+1}(q_{y+1}) - u_{y+1}(q_y)$$

$$= q_y z_{y+1} k \tfrac{a+y+1}{a+b+m} + [1 - q_y z_{y+1}] \tfrac{a+y+1}{a+b+m} - q_y k \tfrac{a+y}{a+b+m} - [1 - q_y] \tfrac{a+y+1}{a+b+m}$$

$$= \tfrac{kq_y}{(a+b+m)[(k-1)(a+y)+k]} > 0.$$

Note that, for any $y \in \{0, 1, ..., m-1\}$, for any $l \in \{0, 1, ..., m-y-1\}$, and for any $h \in \{l+1, l+2, ..., m\}$,

$$u_{y+h}(q_{y+l+1}) - u_{y+h}(q_{y+l})$$

$$= z_{y+l+1} q_{y+l} k \tfrac{a+y+l+1}{a+b+m} + [1 - z_{y+l+1} q_{y+l}] \tfrac{a+y+h}{a+b+m} - q_{y+l} k \tfrac{a+y+l}{a+b+m} - [1 - q_{y+l}] \tfrac{a+y+h}{a+b+m}$$

with the substitution $y' = y + l$,

$$\begin{aligned}
&= z_{y'+1} q_{y'} k \frac{a+y'+1}{a+b+m} + [1 - z_{y'+1} q_{y'}] \frac{a+y'+h-l}{a+b+m} - q_{y'} k \frac{a+y'}{a+b+m} - [1 - q_{y'}] \frac{a+y'+h'-l}{a+b+m} \\
&= \left[u_{y'+1}(q_{y'+1}) - u_{y'+1}(q_{y'})\right] + [1 - z_{y'+1} q_{y'}] \frac{h-l-1}{a+b+m} - [1 - q_{y'}] \frac{h-l-1}{a+b+m} \\
&= \left[u_{y'+1}(q_{y'+1}) - u_{y'+1}(q_{y'})\right] + (h-l-1) \frac{(1 - z_{y'+1}) q_{y'}}{a+b+m}.
\end{aligned}$$

So, for any $y \in \{0, 1, ..., m-1\}$, and any $h \in \{1, 2, ..., m-y\}$,

$$\begin{aligned}
u_{y+h}(q_{y+h}) - u_{y+h}(q_y) &= \sum_{l=0}^{h-1} [u_{y+h}(q_{y+h-l}) - u_{y+h}(q_{y+h-l-1})] \\
&= \sum_{l=0}^{h-1} \left[\left[u_{y+h-l}(q_{y+h-l}) - u_{y+h-l}(q_{(y+h-l)-1})\right] + l \frac{(1 - z_{y+h-l}) q_{(y+h-l)-1}}{a+b+m} \right] \\
&> 0.
\end{aligned}$$

Finally, we show that the incentive constraints requiring any type $y \in \{0, ..., m-1\}$ not to adopt q_{y+h}, for any $h \in \{1, 2, ..., m-y\}$ are satisfied, and not binding. That is, we want to show that:

$$u_y(q_y) - u_y(q_{y+h}) > 0, \ \forall y \in \{1, ..., m-1\}, \ \forall h \in \{1, 2, ..., m-y\}. \tag{10}$$

By construction, $u_y(q_y) - u_y(q_{y+1}) = 0$, for any arbitrary $y \in \{0, 1, ..., m-1\}$, expanding the expression for further reference, we obtain

$$\begin{aligned}
0 &= u_y(q_y) - u_y(q_{y+1}) \\
&= q_y k \frac{a+y}{a+b+m} + [1 - q_y] \frac{a+y}{a+b+m} - z_{y+1} q_y k \frac{a+y+1}{a+b+m} - [1 - z_{y+1} q_y] \frac{a+y}{a+b+m}.
\end{aligned}$$

Notice that, for any $y \in \{0, 1, ..., m-1\}$, for any $l \in \{0, 1, ..., m-y-1\}$,

$$
\begin{aligned}
& u_y(q_{y+l}) - u_y(q_{y+l+1}) \\
= \quad & q_{y+l} k \frac{a+y+l}{a+b+m} + [1 - q_{y+l}] \frac{a+y}{a+b+m} \\
& - z_{y+l+1} q_{y+l} k \frac{a+y+l+1}{a+b+m} - [1 - z_{y+l+1} q_{y+l}] \frac{a+y}{a+b+m}
\end{aligned}
$$

with the substitution $y' = y + l$,

$$
\begin{aligned}
= \quad & q_{y'} k \frac{a+y'}{a+b+m} + [1 - q_{y'}] \frac{a+y'-l}{a+b+m} - z_{y'+1} q_{y'} k \frac{a+y'+1}{a+b+m} - [1 - z_{y'+1} q_{y'}] \frac{a+y'-l}{a+b+m} \\
= \quad & \left[u_{y'}(q_{y'}) - u_{y'}(q_{y'+1})\right] + [1 - q_{y'}] \frac{-l}{a+b+m} - [1 - z_{y'+1} q_{y'}] \frac{-l}{a+b+m} \\
= \quad & \left[u_{y'}(q_{y'}) - u_{y'}(q_{y'+1})\right] + [1 - q_{y'}] \frac{-l}{a+b+m} - [1 - z_{y'+1} q_{y'}] \frac{-l}{a+b+m} \\
= \quad & l \frac{q_{y'}(1 - z_{y'+1})}{a+b+m} > 0.
\end{aligned}
$$

Thus, for any $h \in \{1, 2, ..., m-y\}$,

$$
\begin{aligned}
u_y(q_y) - u_y(q_{y+h}) &= \sum_{l=0}^{h-1} \left[u_y(q_{y+l}) - u_y(q_{y+l+1})\right] \\
= \quad & \sum_{l=0}^{h-1} l \frac{q_{y+l}(1 - z_{y+l+1})}{a+b+m} > 0.
\end{aligned}
$$

This concludes the proof of the Lemma. $\square$

In order to refine the equilibrium, we invoke Banks and Sobel's Universal Divinity notion. Equilibrium beliefs satisfy criterion D1 if, whenever the set of consumer's best-response that makes a type y willing to deviate to q is strictly smaller than the set of responses that makes a type y' willing to deviate to q, then $\Pr(y|q) = o(\Pr(y'|q))$. A PBE satisfies Universal Divinity whenever its supporting beliefs satisfy D1.

Lemma A.2 *The unique PBE satisfying Universal Divinity is the one in Lemma A.1.*

Proof. Consider any fully-separating equilibrium $\mathbf{q}' \neq \mathbf{q}$. Since $\mathbf{q}$ satisfies the incentive compatibility constraints in Condition (8) without slack, for $\mathbf{q}'$ to be a separating equilibrium, it must be the case that there exists a type y such that $u_y(q'_y) > u_y(q'_{y+1})$. Suppose that the supporting beliefs are such that: $\Pr[y|q] = 1$ iff $q \in (q'_{y+1}, q'_y]$, where $q'_{m+1} = 0$, and $\Pr[m|0] = 1$.[5]

[5] In order to show that the Intuitive Criterion does not fully refine the PBE of this game, consider any quantity $q \in (q'_{y+1}, z_{y+1} q'_y)$. Condition (8) assures that the equilibrium payoff of type y is higher than the payoff obtained when

Consider now the quantities $q \in (q'_{y+1}, z_{y+1}q'_y)$, the set of prices that make type y willing to deviate from q'_y is $[(\alpha + y) \frac{q'_y(k-1)+q}{(\alpha+\beta+m)qk}, 1]$, whereas the set of prices that make type $y+1$ willing to deviate from q'_{y+1} is $[(\alpha + y + 1) \frac{q'_{y+1}(k-1)+q}{(\alpha+\beta+m)qk}, 1]$. So we need to check when the latter set is larger. Let $D = (\alpha + y + 1) \frac{q'_{y+1}(k-1)+q}{(\alpha+\beta+m)qk} - (\alpha + y) \frac{q'_y(k-1)+q}{(\alpha+\beta+m)qk}$. Since $q'_{y+1} < z_{y+1}q'_y$, it is the case that $\partial D/\partial q > z_{y+1}q'_y$. So we need to have q smaller than the threshold q'' solving $D = 0$. Since $q'_{y+1} < z_{y+1}q'_y$, it is the case that $q'' > z_{y+1}q'_y$. Thus, for any $q \in (q'_{y+1}, z_{y+1}q'_y)$, the only beliefs satisfying D1 must be such that $\Pr(y|q) = o(\Pr(y+1|q))$. But when that is the case, type $y+1$ may deviate from q'_{y+1} to $z_{y+1}q'_y - \varepsilon$, so the equilibrium $\mathbf{q}'$ fails Universal Divinity.

We now consider the separating equilibrium $\mathbf{q}$ such that, for any $y \in \{0, 1, ..., m-1\}$, it is the case that $q_{y+1} = \frac{(\alpha+y)(k-1)}{(\alpha+y)(k-1)+k} q_y$, and $\Pr[y|q] = 1$ iff $q \in (q_{y+1}, q_y]$, where $q_{m+1} = 0$, $\Pr[m|0] = 1$. Given any q, the set of prices that make monopolist y willing to deviate to q is $[(\alpha + y) \frac{q_y(k-1)+q}{(\alpha+\beta+m)qk}, 1]$. For any type y, and any $h \in \{-y, -y+1, ..., -2, -1, 1, 2, ..., m-y\}$, we introduce the following function of q

$$D^{yh}(q) = (\alpha + y + h)\,(q_{y+h}(k-1) + q) - (\alpha + y)\,(q_y(k-1) + q).$$

Following the previous derivations, the equilibrium $\mathbf{q}$ satisfies Universal Divinity whenever for any y, and any $q \in (q_{y+1}, q_y)$, $D^{yh}(q) \geq 0$ for any $h \in \{-y, -y+1, ..., -2, -1, 1, 2, ..., m-y\}$.

First note that for any y,

$$\begin{aligned}
(\alpha + y + 1)\, q_{y+1}(k-1) - (\alpha + y)\, q_y(k-1) &= -q_{y+1} \\
(\alpha + y - 1)\, q_{y-1}(k-1) - (\alpha + y)\, q_y(k-1) &= q_y.
\end{aligned}$$

For any $h > 0$, and $q \in (q_{y+1}, q_y)$,

$$\begin{aligned}
D^{yh}(q) &= (\alpha + y + h)\,(q_{y+h}(k-1) + q) - (\alpha + y)\,(q_y(k-1) + q) \\
&= (\alpha + y + h)\, q_{y+h}(k-1) - (\alpha + y)\, q_y(k-1) + hq \\
&> (\alpha + y + h)\, q_{y+h}(k-1) - (\alpha + y)\, q_y(k-1) + hq_{y+1} \\
&= \sum_{l=1}^{h} [(\alpha + y + l)\, q_{y+l}(k-1) - (\alpha + y + l - 1)\, q_{y+l-1}(k-1)] + hq_{y+1} \\
&= -\sum_{l=1}^{h} q_{y+l} + hq_{y+1} = \sum_{l=1}^{h} [q_{y+1} - q_{y+l}] \geq 0.
\end{aligned}$$

---

taking q if the buyers would believe that in that case the monopolist is of type $y+1$. However, q is not necessarily equilibrium dominated because Condition (8) does not imply that $u_y(q'_y)$ is higher than the payoff obtained by the y type when taking q if the buyers believe $\Pr(y+2|q) = 1$.Consider the following numerical example. Say that $\alpha = 1$, $\beta = 1$, $k = 2$, and $m = 3$. Thus $u_0(1) = 2/5$, $z_1 = 1/3$. Say that $q'_1 = 1/5$, simple calculations show that none of the quantity $q \in (1/3, 1/5)$ is equilibrium dominated. Since the only possible equilibrium dominance refinement of the stipulated supporting beliefs concerns the quantities $q \in (q'_{y+1}, z_{y+1}q'_y)$, it is concluded that the Intuitive Criterion fails to refine the fully separating equilibrium $\mathbf{q}'$.

And similarly, for any $h < 0$, and $q \in (q_{y+1}, q_y)$,

$$\begin{aligned}
D^{yh}(q) &= (\alpha + y + h)\, q_{y+h}(k-1) - (\alpha + y)\, q_y(k-1) + hq \\
&> (\alpha + y + h)\, q_{y+h}(k-1) - (\alpha + y)\, q_y(k-1) + hq_y \\
&= \sum_{l=0}^{-(h-1)} [(\alpha + y - l - 1)\, q_{y-l-1}(k-1) - (\alpha + y - l)\, q_{y-l}(k-1)] + hq_y \\
&= \sum_{l=0}^{-(h-1)} q_{y-l} + hq_y = \sum_{l=0}^{-(h-1)} [q_{y-l} - q_y] \geq 0.
\end{aligned}$$

Let us now consider pooling and semi-pooling equilibrium. Take any equilibrium $\mathbf{q}'$, where more than one type plays the quantity q' with positive probability. A necessary condition for q' to be an equilibrium is that y_-, the smallest type playing q', is unwilling to deviate. Since $k > 1$, that requires at least that $q' \frac{\alpha + E(y|q')}{\alpha+\beta+m} k + (1 - q') \frac{\alpha + y_-}{\alpha+\beta+m} = \frac{\alpha+y_-}{\alpha+\beta+m} k$, thus $q' \geq \frac{(k-1)(\alpha+y_-)}{k(\alpha+E(y|q'))-(\alpha+y_-)} =: \hat{q}$, and note that $\hat{q} > 0$.

In order to show that any pooling or semi-pooling equilibrium fails to satisfy Universal Divinity, consider the set of types that play q' with positive probability, and denote by y^+ the largest of such types. Note that for any small $\varepsilon > 0$, the set of prices that make y^+ willing to deviate to $q' - \varepsilon$ is strictly larger than the set of responses that makes any other type playing q' deviate. In fact the condition $q' \frac{\alpha+E(y|q')}{\alpha+\beta+m} k + (1 - q') \frac{\alpha+y}{\alpha+\beta+m} = (q' - \varepsilon) pk + (1 - q' + \varepsilon) \frac{\alpha+y}{\alpha+\beta+m}$ yields the solution $p = \frac{q' k(\alpha+E(y|q')) - \varepsilon(\alpha+y)}{(\alpha+\beta+m)(q'-\varepsilon)k}$ which is decreasing in y.

But once the beliefs have been fixed to conform with criterion D1, it is the case that for any $\varepsilon > 0$, $\Pr(y^+ | q' - \varepsilon)$ is arbitrarily close to 1. By deviating to $q' - \varepsilon$, thus, type y^+ will achieve payoff of almost $(q' - \varepsilon) \frac{\alpha+y^+}{\alpha+\beta+m} k + (1 - (q' - \varepsilon)) \frac{\alpha+y^+}{\alpha+\beta+m}$ which dominates $q' \frac{\alpha+E(y|q')}{\alpha+\beta+m} k + (1 - q') \frac{\alpha+y^+}{\alpha+\beta+m}$ for ε small enough. □

Proof of Proposition 4. The welfare is the sum of the seller profit and of the buyers' utility, with appropriate normalizations.

$$\begin{aligned}
W(\theta, y, \delta) &= (1-\delta) \left[\left(q_y P_0 + (1 - q_y) \sum_{t=1}^{\infty} \delta^t p_t \right) + \left(\sum_{t=1}^{\infty} \gamma^t \theta - \sum_{t=1}^{\infty} \gamma^t (1 - q_y) p_t - q_y P_0 \right) \right] \\
&= (1-\delta) \left[(1 - q_y) \sum_{t=1}^{\infty} \delta^t p_t + \sum_{t=1}^{\infty} \gamma^t (\theta - (1 - q_y) p_t) \right] \\
&= (1-\delta)(1-q_y) \sum_{t=1}^{\infty} \delta^t p_t + \gamma \frac{1-\delta}{1-\gamma} \theta - (1-\delta)(1-q_y) \sum_{t=1}^{\infty} \gamma^t p_t.
\end{aligned}$$

For $\delta \to 1$, thus,

$$\begin{aligned} W(\theta, y, \delta) \to W(\theta, y) &= (1-q_y)\theta + \frac{1-\delta}{1-\gamma}\theta - \frac{1-\delta}{1-\gamma}(1-q_y)\theta \\ &= (1-q_y)\theta - k(1-q_y)\theta + k\theta \\ &= \theta\left[kq_y + (1-q_y)\right]. \end{aligned}$$

With analogous derivations, the informational loss is approximately $L(\theta, y) = \theta k(1-q_y)$. □

Proof of Proposition 5. It has been previously shown that q_y is decreasing in y, and since $k > 1$, welfare $W(\theta, y)$ is increasing in q_y, and loss $L(\theta, y)$ is decreasing in q_y.

Since y is an extraction from a binomial distribution with parameters θ and m, $Pr(y|\theta, m) = \binom{m}{y}\theta^y(1-\theta)^{m-y}$. Thus, for a fixed θ, the function

$$\sum_{s=0}^{y} \Pr\left[s|\theta, m\right] = \sum_{s=0}^{y} \binom{m}{s}\theta^s(1-\theta)^{m-s}$$

is strictly decreasing in m. That is to say, if $m' > m$, then $y|\theta, m'$ first-order stochastically dominates $y|\theta, m$. Since q_y is a decreasing function of y, it follows that $E\left[q_y|\theta, m'\right] < E\left[q_y|\theta, m\right]$. Then, as

$$E\left[W(\theta, y)|\theta, m\right] = \theta[kE(q_y|\theta, m) + 1 - E(q_y|\theta, m)],$$

and $k > 1$, it follows that $E\left[W(\theta, y)|\theta, m'\right] < E\left[W(\theta, y)|\theta, m\right]$. Therefore the interim expected welfare $E\left[W(\theta, y)|\theta\right]$ is strictly decreasing in m, and an analogous argument shows that the interim expected loss $E\left[L(\theta, y)|\theta\right]$ is strictly increasing in m.

The result extends to the ex-ante case in which θ is a random extraction of a Beta distribution parametrized by α and β: since $E\left[W(\theta, y)|\alpha, \beta\right] = E\left[E[W(\theta, y)|\theta, m]|\alpha, \beta\right]$, an increase in m reduces $E\left[W(\theta, y)|\alpha, \beta\right]$, and increases $E\left[L(\theta, y)|\alpha, \beta\right]$. □

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