

Political Competition and Strategic Voting in Multi-Candidate Elections*

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Abstract

We develop a model of strategic voting in a spatial setting with multiple candidates. With multiple candidates, voting for another candidate against one's expressive interests provides a public good to other voters who similarly support this candidate. We borrow the concept of conditional cooperation from the experimental literature on public good provision, and integrate it into voters' utility functions. We show that this approach provides a more tractable alternative to group-based voting models as it permits general characterizations in settings with arbitrarily many candidates and multiple policy dimensions. Highlighting the value of our approach, we analyze a setting with two mainstream candidates and a spoiler candidate, showing that the spoiler can gain from entering, even though she has no chance of winning the election and entry reduces the winning probability of her preferred mainstream candidate.

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1 Introduction

In any election with even a moderate number of voters, the probability that the vote of any particular citizen matters is vanishingly small. Thus, “rational” citizens should simply vote for the candidates they like best, regardless of how others vote. However, in plurality elections with more than two candidates (e.g., in the UK or Canada), the literature finds that many voters willingly forsake their expressively most-preferred candidate in favor of a less-preferred candidate whom they believe has a better chance of winning (Fujiwara (2011); Pons and Tricaud (2018); Spenkuch (2018)); and for laboratory experiments see Forsythe et al. (1996); Duffy and Tavits (2008); Esponda and Vespa (2014); Bouton et al. (2015). In short, there is manifest evidence that voters both care about whom they vote for and about who ends up winning the election.

The standard approach to studying multi-candidate elections in the strategic voting literature (e.g., Myatt (2007); Bouton and Castanheira (2012); Bouton (2013)) is to focus on a finite number of strategic voter types who individually *only* care about how their voting decision affects a candidate’s probability of winning. Such pivotal voting models have a long tradition (e.g., Palfrey and Rosenthal (1985); Myerson and Weber (1993)), but do not apply to large elections if strategic voters have any expressive considerations at all; such considerations swamp the tiny probabilities that one’s vote is pivotal, and in the only Nash equilibrium all voters vote expressively.¹ And, indeed, there is extensive evidence that, while voters do care about electoral outcomes, they also care directly about whom they vote for. Illustrating this, Abramson et al. (2018) find that the extent of strategic voting varies with the perceived ability to sway the outcome: only 1.4% of voters with minimal strategic incentives reported an intention to vote for their second choice, but 27.1% of those with the strongest incentives did.

It is clear that these empirical findings cannot be reconciled by either the classical assumption of rational, purely selfish voters or by an assumption that voters only have instrumental preferences. The contribution of our paper is to develop a new formulation of voter behavior that gives rise to a tractable model of strategic voting in multi-candidate plurality elections within a canonical spatial environment. Our model considers voters who both directly derive utility from voting for a candidate and who are also more willing to vote strategically for a less favored candidate who has a better chance of winning, when other voters are also willing to incur the expressive cost of doing so. We characterize equilibria and detail how changes in candidate characteristics and policy platforms affect the incidence of strategic voting and electoral outcomes. We illustrate the tractability of our framework by using it to analyze political competition between policy-motivated candidates in the presence of strategic voting, and by showing that one can extend the concept to spatial settings with more than one dimension.

¹To skirt this issue, some authors assume that voters’ exogenously-perceived pivot probabilities are large (e.g., Chapter 1, Aldrich et al. (2018)).

To understand how we identify plausible foundations for voter behavior that can reconcile key features of the data, it helps to consider the simpler and better-understood problem of public good provision. Empirical and experimental evidence has consistently shown that individuals do not just follow the logic of Olson (1971) of solely considering their own private benefits and costs. Kelley and Stahelski (1970); Fischbacher and Gächter (2010); Fehr and Schurtenberger (2018); Andreozzi et al. (2020) and many others establish that individuals seem to free ride less than what purely “rational” behavior predicts. More subtly, as Kelley and Stahelski (1970) first observed, this deviation from classical behavior is not simply explained by an unconditional concern for the welfare of others. Instead, cooperation is conditional in the sense that if an individual believes others will cooperate more (e.g., by paying their taxes), then their willingness to cooperate also increases. The literature refers to this behavior as “conditional cooperation.” Importantly for our voting setting, conditional cooperation does not disappear when the number of subjects is large.² To the contrary, researchers find that conditional cooperation *increases* with group size (Isaac and Walker (1988); Zhang and Zhu (2011); Diederich et al. (2016); Pereda et al. (2019); Li and Noussair (2024)).

We adapt the concept of conditional cooperation from the public good setting to a spatial model of elections with multiple candidates. A key difference from the public good setting is that electoral contexts give rise to both positive and negative externalities: Voters who support a citizen’s preferred candidate provide a positive externality, while those who support rivals generate negative externalities. Our model features a continuum of two types of voters, non-partisans and partisans. Partisans for candidate i always vote for i , and are used to generate the uncertainty required to endogenize candidate platform choice à la Calvert-Witman.

Non-partisans are modeled as conditional cooperators. Their preferences have two components—a selfish component and a cooperative component. The selfish component is a standard quadratic spatial term—when a non-partisan votes for candidate i , this component decreases in the distance between the non-partisan’s ideal point and i ’s platform. The cooperative utility component is the utility a citizen derives from the externality generated by the willingness of other citizens to also vote for candidate i . To understand this component, consider again a public good provision problem. To measure the positive externality that an individual receives from the costly contributions of others, we would calculate the difference between the actual level of public good provided by those citizens versus the amount provided if everyone just contributes to maximize their own selfish utility. In our electoral setting, the analogue is the difference in utility a citizen derives from the election outcome given the actual level of support for candidate i compared to the case where all previous candidate i supporters vote for their expressively-preferred candidates. Non-partisans weight these two utility components by factors β and $1 - \beta$ respectively. Thus, $\beta = 1$ captures the classical setting with purely selfish voters.

²For example, Frey and Meier (2004) documents extensive conditional cooperation in a field experiment among *all* returning undergraduates at the University of Zurich.

The strategy of a non-partisan is a choice of which candidate to support. We consider Nash equilibria, i.e., each citizen maximizes utility taking the strategies of everyone else as given. At the time non-partisans make their voting decisions, the number of partisans supporting each candidate is public information. In our continuum environment, we can summarize strategies by a partition of the type space of non-partisans, $S = \{S_1, \dots, S_n\}$, where S_i is the set of all non-partisan types who vote for candidate i . S is a voting equilibrium, if it is a Nash equilibrium of this game. We say that strategic voting for candidate i occurs in equilibrium if and only if S_i does not solely consist of expressive candidate i supporters.

We first establish that all non-partisans voting for their expressively-preferred candidate is always a voting equilibrium. More importantly, we show that strategic voting can only occur for candidates who have positive probabilities of winning the election. Posed differently, in any voting equilibrium, spoilers only receive votes from expressive supporters. We then introduce a slight “no minimal expansion” refinement that eliminates ties as an equilibrium outcome. With this refinement we show that if strategic voting for a candidate i occurs, then all expressive candidate i supporters must also vote for i . Further, we prove that a candidate who has a plurality of expressive support does not receive strategic support in equilibrium. We also provide an algorithm that can be used to find all voting equilibria that satisfy our refinement.

To illustrate how our solution concept works, we provide a complete analysis of all equilibria in a three-candidate setting. This analysis provides two main insights. First, we identify that centrists have an inherent advantage due to their being at least the second choice of every non-partisan. We show that, as a result, if a centrist has a plurality of expressive and partisan support, then the centrist always wins and no candidate receives strategic votes. More generally, standard insights from two-candidate or purely-expressive voting models cease to hold. To show this, we consider a setting in which at the outset the candidate on the left wins in the unique voting equilibrium. In a standard setting given a symmetric and single-peaked distribution of voters, the centrist would lose even more support by moving to the right. In contrast, with strategic voting this need not be true. By becoming more extreme, the centrist can achieve two goals: (i) reducing the expressive cost to citizens on the right of voting for the centrist; and (ii) differentiating her platform by more from the leftist’s platform to increase the positive externality that voters on the right receive if the centrist can win with strategic support. These two forces make rightist citizens more willing to consider strategic support for the centrist, and can dominate the fact that moving to the right increases expressive support for the leftist.

We show how this logic extends when we endogenize candidate platform choice in a setting with a spoiler candidate who has no chance of winning. In the model, a citizen-candidate spoiler first makes an entry decision. Then both mainstream candidates simultaneously select their platforms, the number of partisans is realized, followed by non-partisans making their voting decisions.

If voters place little weight on their selfish utility, the spoiler’s entry has no impact on equilibrium outcomes, as even extreme expressive supporters of the spoiler always vote strategically for the centrist—it is

as if the spoiler were not there. However, when weights on the selfish component are increased further, the spoiler can attract votes, causing both of the main candidates to move their policy positions towards the spoiler—the spoiler’s preferred (more centrist) candidate does so to differentiate herself, thereby increasing strategic voting, while her mainstream rival to the left shifts to reduce that differentiation. This makes the spoiler strictly better off: While the spoiler’s preferred mainstream candidate is less likely to win due to the spoiler’s entry (consistent with Pons and Tricaud (2018)), this loss is outweighed from the spoiler’s perspective by the benefits of the policy shifts of the mainstream candidates’ platforms. Further increases of the weight on the selfish component of voter preferences eventually harm the spoiler, because the spoiler takes too many votes away from her preferred mainstream candidate. In particular, if citizens were never to act strategically, a spoiler would always find entry counterproductive from a pure policy perspective.

Our model can reconcile the entry of a spoiler candidate such as Ralph Nader who has no chance of winning, even though the spoiler does not receive ego rents. With strategic voting, spoilers can gain by inducing mainstream candidates to adopt some of their policies. Consistent with this, in an interview in 2019,³ Brunner writes that Ralph Nader had hoped to “push the Democrats toward a more progressive agenda,” understanding that his campaign could end up costing Al Gore the presidency.⁴ Of course, if the preferred mainstream candidate loses due to the spoiler’s entry, then the spoiler has ex-post regret. The entry of Nigel Farage’s UKIP party in the UK illustrates an outcome where the ex-post outcome benefitted the spoiler. Farage’s entry forced the Conservatives to adopt UKIP-like positions on Brexit (and induced Labour to moderate its opposition). It also resulted in a Conservative victory and the subsequent implementation of Brexit. Similarly, the entry of populist parties in Europe has encouraged mainstream parties to move to the right, especially on the issue of immigration, with center-right parties moving further.

While our main focus is on a unidimensional policy space, we conclude by showing that our concept remains sufficiently tractable that we can find all equilibria even when we go beyond one policy dimension.

2 Related Literature

Beginning with Duverger (1959), researchers have noted that plurality rule systems generate strong forces for only two parties to be competitive, as voters who care about electoral outcomes do not want to “waste” votes on candidates who are sure to lose. Cox (1997) documents consistent patterns of strategic voting across different electoral systems, and observes that the “reduction of parties” in single member districts reflects the coordination of voters on parties. Applied to the United Kingdom, supporters of the LDP and Labour some-

³see <https://www.washingtonian.com/2019/11/03/ralph-nader-is-opening-up-about-his-regrets/>, Ralph Nader Is Opening Up About His Regrets, Rob Brunner, November 3, 2019.

⁴Hillygus (2007) documents evidence of strategic voting by a subset of Nader voters. Nader supporters who were likely to vote for Gore were more concerned about policy outcomes and were in more competitive states.

times coordinate to defeat the Conservative candidate (see Aldrich et al. (2018), Figure 2.2). Similar coordination is found in Canada (Merolla and Stephenson, 2007), and elsewhere (Blais et al., 2019). The extent of strategic voting can be substantial: Abramson et al. (2018) finds that the incidence of voting for a second-choice candidate was almost 40% in some constituencies for the 2010 British general election, while Daoust (2018) finds that 22.6% of voters selected their second-choice in the 2015 Canadian general election. Similarly, Spenkuch (2018) provides strong evidence of strategic voting in the election to the German Bundestag.

Fujiwara (2011) uses a RDD on registered-voter thresholds to show that the top two candidates in Brazilian elections secure more votes under simple plurality than runoff or proportional elections, and Pons and Tricaud (2018) find the presence of a third candidate reduces vote shares for the two leading candidates in French elections. Structural estimates of models where some voters only have instrumental concerns while the others are purely expressive also suggest that many citizens vote strategically (see Kawai and Watanabe (2013) for Japan and Fisher and Myatt (2017) for Britain). Conversely, Degan and Merlo (2006) conclude that in U.S. national elections, which are dominated by two parties, the hypothesis that voters vote sincerely cannot typically be rejected. Ujhelyi et al. (2021) document extensive none-of-the-above voting in India, indicating that expressive voting considerations dominate for some voters.

The observation that voting decisions are not solely driven by selfish preferences dates back to Riker and Ordeshook (1968). An early formalization is Harsanyi (1977), who postulates that some voters are “rule utilitarians,” which means that according to the Kantian imperative, they follow those actions that, if adopted by everyone else, are beneficial to society. Based on this idea, Feddersen and Sandroni (2006) formulate a model of “ethical voters” who feel obligated to do their civic duty by following rules that maximize the welfare of their group. In their costly voting framework the sets of citizens who vote in equilibrium are determined endogenously, as citizens with low voting costs follow the Kantian imperative to do what is right for their group, while citizens with high voting costs can abstain. Coate and Conlin (2004) provide empirical support for this approach. Bouton and Ogden (2021) extend Feddersen and Sandroni (2006) to a three candidate spatial setting where candidate supporters share the exact policy preferences of their respective candidates. They characterize pure strategy Nash equilibria when each group acts as a single decision maker, identifying when equilibrium features sincere voting or coordination on two candidates in plurality elections. Levine and Mattozzi (2020) model group voting that complements ethical voting by stipulating that social norms of voting participation be enforced via costly peer monitoring and punishment. Uhlaner (1989) proposes that foundations for group formation can stem from leaders such as unions, churches, or environmental organizations who mobilize groups. Morton (1987, 1991) formalizes these ideas, while Herrera and Martinelli (2006) provide foundations for group mobilization when both leaders and groups are endogenous.

Unlike all of these group-based models of strategic behavior, we develop an individual-based model. In our model, an individual behaves as a standard player in a non-cooperative game, taking the voting choices

of all other players as given, and maximizing their own utility. A voter does not realize any direct instrumental benefits. Instead, the utility derived from conditional cooperation only indirectly reflects the outcome of the election: A citizen derives utility from cooperating with others who similarly provide externalities by voting for a less preferred candidate, and this utility itself depends on the election outcome.

Our approach offers several advantages over group-based approaches in multi-candidate settings. Existing group-based models naturally give rise to maximizing the social welfare of citizens who support a particular candidate as the appropriate rule-based utilitarian approach. In our setting the key choice is which candidate to support, where standard group-based approaches provide little guidance. In particular, which group does a citizen internalize the welfare of—concretely, consider a citizen who expressively supports a candidate D who is currently in a group that supports candidate A but considers switching to vote for candidate B to prevent C from winning. What “moral penalty” should that citizen incur from leaving the group supporting A , and how should this penalty be weighed against the “moral benefit” of supporting the group of B voters? Our approach does not run into these issues because citizens just maximize their own utility.

Group-based approaches do not lend themselves to rich analytical characterizations with more than three candidates, where voters do not share common spatial locations. Analyzing equilibria becomes intractable, as mixing may be required and entail the random formation of more than two groups. As a result, general characterizations with more than three candidates or more than one policy dimension become infeasible. In ongoing work, Bernhardt et al. (2026) adapt a related individual-based approach in a two-candidate setting with costly voting and abstention. They show that it can deliver multiple empirical regularities regarding turnout and turnout rates (e.g., bandwagon effects, win margins, who votes) that group-based and purely selfish expressive voting models struggle to explain.

Our analysis of strategic candidate platform selection finds that a fixed-platform extremist third party candidate may gain from entering an election between policy-motivated candidates. A related literature has modeled platform choices by existing “mainstream” office-motivated parties that face the threat of entry by a third party (e.g., Palfrey (1984); Weber (1992); Callander (2005b)). There are several differences between these models and ours. First, the mainstream parties separate to induce the entrant to locate at the center and hence steal votes equally from both parties. Second, voters in these models are not strategic. In particular, applying our model, as long as voters are not too selfish, central entry would result in an equilibrium where the centrist wins with strategic support from either the right or the left, an observation first made with purely instrumental voters by Callander (2005a). Of note, our spoiler does not care about vote shares or winning, but instead benefits by moving mainstream party policies in her direction.

3 Model

We first develop a model of strategic voting when candidate positions are fixed and citizens choose which candidates to support. We initially formulate the model for an arbitrary number of candidates to illustrate the general formulation of strategic voting. We then focus on a setting with three candidates to provide an exhaustive characterization of voting equilibria. Following this, we endogenize candidate location by embedding this three-candidate formulation into a model of political competition in platforms as in Wittman (1983). We conclude by showing that our approach extends to settings with more than one policy dimension.

There are $i = 1, \dots, n$ parties/candidates with policies ordered by $x_1 < x_2 < \dots < x_n$ and a continuum of citizens. The voting game features two types of citizen voters—partisans and non-partisans. A partisan for candidate i always votes for i regardless of the policy positions taken. Let $\rho_i \geq 0$ be the number of partisan voters for candidate i . Partisans serve two roles. First, they provide a simple way to parametrize heterogeneity in a party’s base support. Second, they provide a source for the uncertainty required when we endogenize candidate policy choices. Before the vote takes place, each candidate i ’s partisan support ρ_i is realized and observed by all non-partisans. That is, to highlight how strategic voting can generate endogenous uncertainty about who wins, we assume away all extrinsic sources of uncertainty at the voting stage.⁵

Each non-partisan voter is described by an ideal policy $\theta \in \mathbb{R}$. The ideal policies θ of non-partisans are distributed according to $\Phi(\cdot)$, with associated strictly positive density ϕ on a connected support that includes $[x_1, x_n]$. With a common notational abuse, we use $\Phi(S)$ to denote the probability of any set $S \subset \mathbb{R}$. Non-partisan θ ’s utility consists of two parts. One part measures the expressive utility derived from voting for a given candidate, taking the form $-(\theta - x)^2$, where x is the candidate’s policy. Each non-partisan places a weight $\beta \in [0, 1]$ on this selfish utility component. The other part, which a non-partisan weights by $1 - \beta$, reflects the utility from “doing the right thing,” by not free riding on the efforts of other non-partisans, whose joint voting decisions positively affect θ ’s utility via their effect on who wins the election, which we now define.

The strategy set of each non-partisan is given by $\{1, \dots, n\}$, i.e., by the set of possible candidates (abstracting from abstention). Consider some non-partisan θ . The strategy profile of all other non-partisans is summarized by a partition $\mathcal{S} = \{S_1, S_2, \dots, S_n\}$ of the type space $\mathbb{R}$, where S_i is the set of all non-partisan types who vote for candidate i .⁶ The social benefit that θ derives from other non-partisans voting for i is given by the difference in θ ’s utility when everyone in S_i votes for candidate i rather than selfishly vote for their expressively-preferred candidates. Let $E_i = \{\theta \mid -(\theta - x_i)^2 \geq -(\theta - x_j)^2, \text{ for all } j \neq i\}$ be the set of non-partisans who expressively prefer candidate i . Then if citizens in S_i vote for their expressively-preferred

⁵Bernhardt et al. (2026) analyze a related two-candidate costly voting setting and show how analysis extends when the timing of non-partisan and partisan voting is reversed so that non-partisans face extrinsic uncertainty when making their voting decisions.

⁶With a continuum of non-partisans it does not matter whether we consider a partition of $\mathbb{R}$ or $\mathbb{R} \setminus \{\theta\}$.

candidates, the set of non-partisans voting for candidate i is reduced to $S_i \cap E_i$, while voters in $S_i \cap E_k$ now vote for candidate $k \neq i$. We use

$$\mathcal{S}_{-S_i} = \{S_i \cap E_i, S_k \cup (S_i \cap E_k) \text{ for } k \neq i\} \quad (1)$$

to denote the partition that describes who votes for which candidate when all previous members of S_i vote according to their own selfish interests.

Candidate i receives $V_i = \Phi(S_i) + \rho_i$ votes. Candidate i wins if $V_i > V_j$, for all $j \neq i$. In the event of a tie, each of the candidates with the most votes wins with equal probability. Denote candidate i 's winning probability given strategy profile $\mathcal{S}$ by $P_i(\mathcal{S})$. The social benefit to θ when all non-partisans in S_i vote for i is

$$B_\theta(i, \mathcal{S}) = \sum_{j=1}^n P_j(\mathcal{S}_{-S_i})(\theta - x_j)^2 - \sum_{j=1}^n P_j(\mathcal{S})(\theta - x_j)^2. \quad (2)$$

Thus, non-partisan θ 's utility from voting for candidate i is

$$u_\theta(i, \mathcal{S}) = (1 - \beta)B_\theta(i, \mathcal{S}) - \beta(\theta - x_i)^2, \quad (3)$$

where β is the weight that a non-partisan places on his own selfish utility.

Definition 1 $\mathcal{S} = (S_1, \dots, S_n)$ is a voting equilibrium if and only if the voting choices of all non-partisans maximize their utility given $\mathcal{S}$, i.e.,

$$i \in \arg \max_{k=1, \dots, n} u_\theta(k, \mathcal{S}), \quad (4)$$

for all $\theta \in S_i$ and for all $i = 1, \dots, n$. In the case of indifference between multiple candidates, non-partisans vote for their expressively-preferred candidate among that set.

Definition 2 There is strategic voting for candidate i if and only if $S_i \setminus E_i \neq \emptyset$, i.e., there exist non-partisans who expressively prefer other candidates but vote for i .⁷

4 Characterization of Voting Equilibria

4.1 General Results

We begin by showing that citizens do not derive social benefits from voting for i unless candidate i receives some strategic votes. In particular, if $S_i \subset E_i$ then $\mathcal{S}_{-S_i} = \mathcal{S}$. The definition of $B_\theta(i, \mathcal{S})$ in (2) then implies the following result:

⁷Note that if θ is indifferent between candidates i and j , then by definition $\theta \in E_i \cap E_j$. Hence, if θ is a strategic voter for i then θ must expressively strictly prefer a different candidate.

Lemma 1 *Let S be the strategy profile of non-partisans. If $S_i \subset E_i$, then $B_\theta(i, S) = 0$ for all $\theta \in S_i$.*

When only non-partisans who expressively prefer candidate i vote for i , someone who votes for i is not free-riding on the costly efforts of others—as no one is providing a public good to i supporters by voting for i when i is their less-preferred candidate. Hence, if $S_i \subset E_i$, then $B_\theta(i, S) = 0$ for all $\theta \in S_i$.

An immediate implication of Lemma 1 is that equilibria always exist.

Proposition 1 *A voting equilibrium always exists.*

To see this, let $S = (E_1, \dots, E_n)$. Then by Lemma 1, $B_\theta(i, S) = 0$ for all θ . It follows that for each θ voting for their most expressively-preferred candidate solves (4).

Lemma 2 *$B_\theta(i, S)$ is affine linear in θ . $u_\theta(i, S)$ is strictly concave in θ for $\beta > 0$ and affine linear for $\beta = 0$.*

This result follows because, as inspection of equation (2) reveals, the quadratic terms in the definition of $B_\theta(i, S)$ cancel.

Lemma 3 *Let $S = (S_1, \dots, S_n)$ be a voting equilibrium. Then all $S_i \in S$ are intervals.*

We now illustrate the logic behind Lemma 3 via an example.

Example 1 Non-partisan ideal policies are uniformly distributed on $[-1, 1]$, and there are four candidates, located at $x_1 = -0.5$, $x_2 = 0.1$, $x_3 = 0.2$, and $x_4 = 1$. Let $\rho_i = 0$ for all candidates, and $\beta = 0.8$. Then $E_1 = [-1, -0.2]$, $E_2 = [-0.2, 0.15]$, $E_3 = [0.15, 0.6]$, and $E_4 = [0.6, 1]$. Thus, in the voting equilibrium where all non-partisans vote expressively, $S_i = E_i$, vote shares are $V_1 = 0.4$, $V_2 = 0.175$, $V_3 = 0.225$, $V_4 = 0.2$, and hence candidate 1 wins. However, there also exists an equilibrium with strategic voting, where candidate 2 wins. We now determine who votes for candidate 2 and who votes purely expressively.

Thus, $S_i = E_i \setminus S_2$, for $i \neq 2$. It is therefore sufficient to determine the endpoints of S_2 . Lemma 1 then yields that $B_\theta(i, S) = 0$ for $i \neq 2$, and hence the utilities derived from voting for a candidate other than 2 are $u_\theta(i, S) = -0.8(x_i - \theta)^2$. Candidate 2 wins if and only if he receives strategic votes, else candidate 1 wins. Thus, $B_\theta(2, S) = -(\theta - x_2)^2 + (\theta - x_1)^2$. It is immediate that $B_\theta(2, S)$ is strictly increasing in θ . Further, $B_\theta(2, S) < 0$ for $\theta < -0.2$, and is strictly positive for $\theta > -0.2$.

Figure 1 displays non-partisan utilities that each voter θ derives from each candidate i . Consistent with Lemma 2, the utility derived from a given candidate is strictly concave in θ and the payoffs from voting for different candidates satisfy a single-crossing property. The left endpoint of S_2 is $(x_1 + x_2)/2 = -0.2$, while the right endpoint, $\bar{y}$ satisfies $u_{\bar{y}}(2, S) = u_{\bar{y}}(4, S)$, which yields $\bar{y} = 0.7$. Thus, candidate 2 wins with a vote

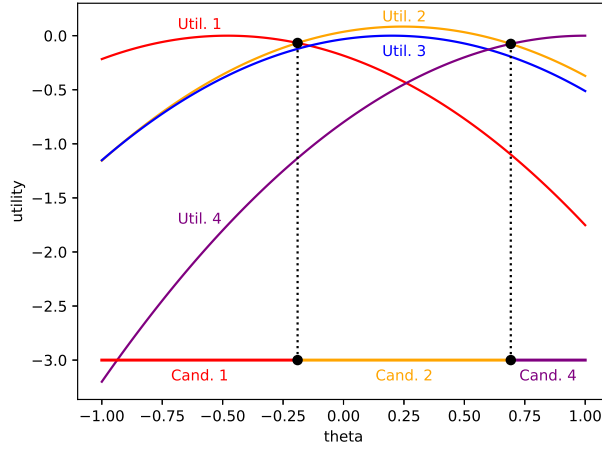


Figure 1: Citizen utilities of voting for each of the candidates.

share of 0.45 (versus 0.4 for candidate 1). Further, voting for candidate 3 is dominated by different voting choices for all types θ . In particular, all voters in E_3 support candidate 2, as do some non-partisans who expressively prefer candidate 4 and are sufficiently close to candidate 2. ■

To obtain tighter predictions, we next refine the set of Nash equilibria by excluding equilibria that are not robust with respect to an arbitrarily small amount of coordination by non-partisans.

Definition 3 *A voting equilibrium S violates no minimal expansion if and only if for all $\varepsilon > 0$ there exists a non-empty set of voters T with $\Phi(T) < \varepsilon$, and a candidate i with $T \cap S_i = \emptyset$ such that all members of T are strictly better off voting for candidate i , i.e., $u_\theta(i, S') > u_\theta(k(\theta), S)$ for all $\theta \in T$ where $S' = \{S_i \cup T, S_j \setminus T, j \neq i\}$ and $k(\theta)$ is the candidate that θ votes for in equilibrium S .*

The refinement addresses the observation that adding a single voter for some candidate never affects the outcome, as there is a continuum of voters. As a result, we can have a tied election, even if there are citizens who would be willing to shift their allegiance, because changing the vote of any single person does not alter the outcome. The refinement allows for an arbitrarily small group of voters to coordinate to break a tie if they would all benefit from doing so. The refinement lets us interpret our model as a limit of a polity with a large but finite number of citizens. Using the refinement, we prove the following result.

Proposition 2 *Let $\beta < 1$, and suppose that all candidate positions differ. Let S be a voting equilibrium that satisfies no minimal expansion. Then one of the following holds:*

1. *Exactly two candidates tie and all non-partisans in the support of Φ vote for these two candidates. Denoting these candidates by j and k where, without loss of generality $x_j < x_k$, all non-partisans to the left of $\frac{x_j+x_k}{2}$ vote for j while all non-partisans to the right vote for k .*
2. *Exactly one candidate wins with probability 1.*

To see the caveat required when only two candidates receive non-partisan support, suppose that voters are uniformly distributed on $[-1, 1]$ and the candidates are located at $x_1 = -1$ and $x_2 = 1$. When $\rho_1 = \rho_2$, $E_1 = [-1, 0]$, $E_2 = (0, 1]$ is a voting equilibrium in which candidates 1 and 2 tie. Voters $\theta < 0$ strictly prefer candidate 1, and voters $\theta > 0$ strictly prefer candidate 2. It is not possible to induce some voters $\theta > 0$ to strategically support candidate 1, because these voters would be made strictly worse off as a result. This would also be the equilibrium outcome if there is say a third candidate located at 0.9 who does not have sufficient partisan support to have a chance of winning—that candidate’s expressive supporters would all vote strategically for candidate 2 as long as β is not too large, again resulting in a tie. Further, all non-partisans must at least weakly prefer the candidate for whom they vote to the rival candidate receiving non-partisan support.

We next prove that spoiler candidates who have no chance of winning never receive strategic votes.

Proposition 3 *Let $S = (S_1, \dots, S_n)$ be a voting equilibrium satisfying no minimal expansion, and suppose that S_i receives strategic votes. Then candidate i wins with positive probability. Further, if candidate i receives strategic votes, then all expressive i supporters vote for candidate i , i.e., $S_i \supset E_i$.*

The challenge in the proof is ruling out the possibility of voting for a spoiler in order to take votes away from less-preferred candidates when there are many candidates. In particular, one might posit that if such strategic voting were not to occur, then it might have the unintended consequence of changing who wins the election in adverse ways, e.g., two extremist candidates on either side may now have a positive probability of winning. The proof shows that this cannot happen.

4.2 Comparison with Group-Based Voting Models

An alternative to our individual, Nash-based approach is to consider voter coalitions. These coalitions are supported by citizens’ desires to act ethically in support of their respective group. In Feddersen and Sandroni (2006), this amounts to citizens in each group following a rule that maximizes their group’s total expected utility, given the rules used by other groups. In Feddersen and Sandroni (2006) the groups are given exogenously. To extend their model to our spatial setting with more than two candidates, one would first have to figure out how to assign ex-ante citizens to different groups. Supposing that such an assignment is

determined, each group would then have to maximize the social welfare of their members. A necessary consequence of this is that in any pure strategy equilibrium between the coalitions, the winning coalition must be the minimal coalition to win whenever there is strategic voting. For example, applied to our Example 1 the coalition supporting candidate 2 must have exactly 40% of the vote, and one would have to include a tie-breaking rule in candidate 2's favor. However, in other settings pure strategy equilibria will no longer exist.

For example, suppose that there is an additional candidate 0 located at $x_0 = -1.46$. Then, $E_0 = [-1, -0.98]$. With our concept the nature of equilibrium does not change. In contrast, with an ethical voting model, the coalition for candidate 2 would have to encompass 39% of the votes, again breaking the tie in candidate 2's favor. However, the group representing candidate 0 can then do better by having members in $[-0.98 - \varepsilon, -0.98]$ vote for candidate 1, where ε can be arbitrarily small, so that candidate 1 now wins. It follows that an equilibrium, if it exists, must involve mixing.

An alternative, but related approach to the ethical voter model is to define “internal” and “external” stability properties that any group of voters must satisfy. Internal stability would mean that each coalition member is at least as well off in the coalition relative to some status quo, e.g., where the coalition does not form. Consider again Example 1, then $S_2 = [-0.2, 0.7] \setminus A$ would be an internally stable coalition for candidate 2 for any set $A \in [0.15, 0.7]$ with a measure less than 0.5. Thus, with internal stability as the only criterion, it is immediate that there are 2^c such equilibria. External stability would mean that outsiders should not want to join the coalition. If these outsiders take the existence of coalition S_2 as the status quo, then they do not have an incentive to join, as candidate 2 already wins when S_2 forms. To reduce the number of equilibria, one would have to choose a more extreme status quo, e.g., that the coalition disbands if the outsiders do not join. In this case we again get the same outcome as predicted by our equilibrium concept in Example 1. However, complications still arise with more than three candidates. In particular, assuming that an existing coalition changes dramatically depending on whether or not a particular individual citizen joins generates the problem that it is no longer clear what a status quo should be for a citizen who must choose between several coalitions.

To see this, suppose that voter θ expressively prefers candidate i but needs to choose whether to join a coalition S_j or S_k to defeat a fourth candidate. External stability may require θ to join one of these coalitions, but the question is which one. For example, if θ chooses to join S_j , what is the status quo? Does S_k form even if θ does not join it? The problem is that we no longer have a setting in which players take the actions of everyone else as given. In other words, a notion of external stability necessarily implies that some other players react to a deviation by a single player. This generates not just conceptual but also existence issues when attempting to use a group-based formulation with more than three candidates.⁸

⁸With three candidates this issue does not arise, because a citizen chooses from at most two options: (a) voting expressively, (b) strategically voting for the second choice to defeat the third choice.

4.3 Algorithm to Determine all Voting Equilibria that Satisfy no Minimal Expansion

The results of section 4.1 make it easy to determine all voting equilibria that satisfy no minimal expansion. In particular, we now describe an algorithm for finding these equilibria. With two candidates expressive voting is the only equilibrium. Thus, we now focus on the case with at least three candidates.

First, one has to check whether or not it is possible to generate two winning coalitions S_i and S_j that contain all non-partisan voters, such that $\Phi(S_i) + \rho_i = \Phi(S_j) + \rho_j$. Then let W be the set of candidates who tie for the win under purely expressive voting. Choose any candidate i and define

$$B_\theta(i, S_{-i}) = -(x_i - \theta)^2 + \frac{1}{\#W} \sum_{j \in W} (x_j - \theta)^2, \text{ and } B_\theta(j, S) = 0, j \neq i. \quad (5)$$

Recall that utility is defined by (3). Let $S_i = \{\theta | u_\theta(i, S) \geq u_\theta(j, S), \text{ for all } j = 1, \dots, n\}$. Let $S_j = E_j \setminus S_i$ for $j \neq i$. If $\Phi(S_i) + \rho_i > \Phi(S_j) + \rho_j$ for all $j \neq i$ then we have an equilibrium in which candidate i wins. Because i gets strictly more votes than any other candidate, the no-minimal expansion property is satisfied. If $\Phi(S_i) + \rho_i \leq \Phi(S_j) + \rho_j$ then candidate i cannot win in an equilibrium that satisfies no-minimal expansion.

This construction finds at least one equilibrium. This is immediate if W is a singleton. Now suppose that there is a tie under expressive voting. Let i be the least element of W . If $i > 1$, then voter $\theta = (x_{i-1} + x_i)/2$ and some voters to the left of θ must be in S_i as they strictly prefer candidate i winning to a tie between all candidates in W . Thus, $\Phi(S_i) + \rho_i > \Phi(S_j) + \rho_j$ for $j \neq i$. If $i = 1$, and $2, 3 \notin W$, then S_i as defined above would contain $\theta = (x_1 + x_2)/2$ as well as voters marginally to the right of θ , because they are strictly better off if candidate 1 wins with probability 1. If $1, 2 \in W$ and $3 \notin W$ then the above construction would imply that $\Phi(S_2) + \rho_2 > \Phi(S_j) + \rho_j, j \neq 2$. The remaining cases are similar.

4.4 The case of three candidates

We now focus on a setting with three candidates in order to characterize all voting equilibria that satisfy no minimal expansion. The case of three candidates is an interesting benchmark, as, in practice, strategic voting usually involves choosing among three main candidates or parties.⁹

⁹For example, in the UK, voters typically choose from the Conservatives, Liberal Democrats, and Labour, while the main federal parties in Canada are the Liberal Party, New Democratic Party (NDP), and Conservatives, and the main parties of Mexico have long been the Institutional Revolutionary Party (PRI), the Party of the Democratic Revolution (PRD), and National Action Party (PAN) until the rise of MORENA, which supplanted PRI and PRD. Taiwan's 2024 Presidential elections also followed a three-way pattern, with voters choosing from the Democratic Progressive Party (DPP), Kuomintang (KMT), and Taiwan People's Party (TPP). In Puerto Rico, strategic voting by Independentista supporters for the Populares often led to the defeat of the Estadistas. In the 2000 U.S. Presidential Election some left-leaning voters supported Ralph Nader rather than Al Gore, seemingly costing Gore the election.

Proposition 4 *Let $N_i = \Phi(E_i) + \rho_i$ be the number of votes that candidate i receives under purely expressive voting. Suppose that centrist candidate 2 has the most base support, $N_2 > N_1, N_3$. Then in the unique voting equilibrium that satisfies no minimal expansion, everyone votes expressively and candidate 2 wins.*

Proposition 2 yields that candidate 2 never receives strategic votes. Suppose there exists an equilibrium with strategic voting for say candidate 1—this is without loss of generality. Then candidate 1 must win with probability 1. Because S_1 is an interval it must contain some voters in E_2 . However, for these voters $B_\theta(2, S) < 0$ because a less-preferred candidate wins. In addition, they have an expressive cost of voting for candidate 1, a contradiction.

Further, recall that $S = \{E_1, E_2, E_3\}$ is always a voting equilibrium. In this equilibrium, candidate 2 wins with a strict plurality. Thus, an arbitrarily small amount of strategic voting for candidate 1 or 3 cannot change the outcome.

We next provide conditions under which candidate 1 or 3 wins in a voting equilibrium. Suppose that $N_1 > N_2, N_3$; the case where $N_3 > N_1, N_2$ is analogous. First observe that $S = \{E_1, E_2, E_3\}$ is always a voting equilibrium and it satisfies no minimal expansion. This is the unique equilibrium in which candidate 1 wins, because Proposition 2 shows that there cannot be strategic voting for candidate 1.

Proposition 2 shows that strategic voting for candidate 2 can only take place if candidate 2 wins. Further, $S_2 \supset E_2$. Thus, S_2 takes the form $S_2 = [(x_1 + x_2)/2, \bar{\theta}]$ for some $\bar{\theta} > (x_2 + x_3)/2$. Further, $\bar{\theta} \rightarrow \infty$ as $\beta \rightarrow 0$. If $1 - \Phi((x_1 + x_3)/2) + \rho_2 > N_1$, then for all sufficiently small β , there exists a voting equilibrium in which candidate 2 wins.

Alternatively, a voting equilibrium may exist in which some right-of-center expressive candidate 2 supporters vote strategically for candidate 3. In particular, because $S_3 \supset E_3$, S_3 must take the form $S_3 = [\bar{\theta}, \infty)$, where $\bar{\theta} > (x_1 + x_3)/2$. Voters $\theta < (x_1 + x_3)/2$ prefer that candidate 1 wins, and hence would not vote strategically for candidate 3. Suppose that $1 - \Phi((x_1 + x_3)/2) + \rho_3 > N_1$. Then $\bar{\theta} \rightarrow (x_1 + x_3)/2$ as $\beta \rightarrow 0$. Thus, for sufficiently small β , there exists a voting equilibrium in which candidate 3 wins.

We now summarize these results.

Proposition 5 *Suppose that $N_1 > N_2, N_3$. Then there exist at most three equilibria that satisfy no minimal expansion. There always exists an equilibrium where everyone votes expressively and candidate 1 wins. Further, there exist $\bar{\beta}_2, \bar{\beta}_3 < 1$ such that:*

1. *If $1 - \Phi((x_1 + x_2)/2) + \rho_2 > N_1$ and $\beta < \bar{\beta}_2$ then there exists an equilibrium in which some expressive candidate 3 supporters vote strategically for candidate 2 and candidate 2 wins.*
2. *If $1 - \Phi((x_1 + x_3)/2) + \rho_3 > N_1$ and $\beta < \bar{\beta}_3$ then there exists an equilibrium in which some expressive candidate 2 supporters vote strategically for candidate 3 and candidate 3 wins.*

A key implication of this proposition is that it is easier for a centrist to gain strategic support compared to a non-centrist, unless the non-centrist has differentially higher partisan support. To see this, suppose that $\rho_3 \approx \rho_2$. Because $\Phi((x_1 + x_2)/2) < \Phi((x_1 + x_3)/2)$, Proposition 5 implies that the condition in 2. is harder to satisfy than that in 1. Conversely, there are equilibria, for some β , where candidate 2 can gain enough strategic support to win, but candidate 3 cannot. The case $1 - \Phi((x_1 + x_3)/2) + \rho_3 < N_1$ would occur if x_3 is sufficiently extreme, or if candidate 3 has fewer partisans than candidate 1. In this case we refer to candidate 3 as a spoiler.¹⁰

We next illustrate how candidate 2's winning probability in the presence of a spoiler candidate 3 can be a non-monotone function of its platform x_2 . We focus on equilibria where strategic voting for candidate 2 occurs if possible. It turns out that with strategic voting, a centrist candidate may be able to increase its probability of winning by moving further away from the stronger candidate even though this increases the stronger candidate's expressive support, and does not increase its own.

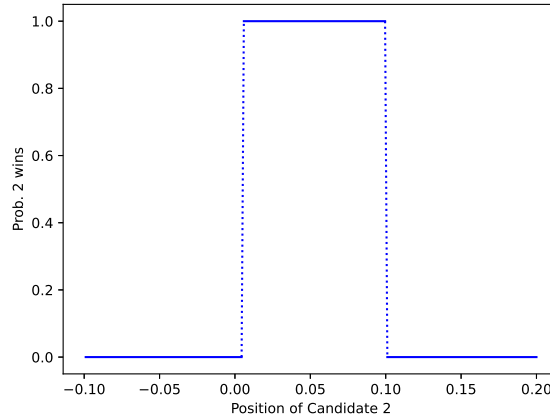


Figure 2: Candidate 2's winning probability as a function of x_2 .

Figure 2 illustrates this in a setting where voters are uniformly distributed on $[-1, 1]$. The candidate positions are $x_1 = -0.1$, $x_3 = 1$, the weight on expressive preferences is $\beta = 0.2$ and there are no partisans, i.e., all $\rho_i = 0$. Now as x_2 is shifted to the right, away from candidate 1, N_1 rises, N_2 stays unchanged, and N_3 falls. However, with strategic voting, this does not imply that candidate 2's winning probability falls.

With expressive voting, candidate 1 would always win whenever $x_1 < x_2 < x_3$. Now suppose that voters are not purely selfish, i.e., $\beta < 1$. As candidate 2 shifts x_2 to the right, it has three effects. First, it increases N_1 , increasing the amount of strategic voting needed to defeat candidate 1. Second, x_2 increasingly differ-

¹⁰Relatedly, Callander (2005a) observes that strategic voters may coalesce around a centrist in a setting where strategic voters only care about electoral outcomes.

entiate itself from x_1 by locating closer to expressive candidate 3 supporters. This starker contrast with candidate 1 makes strategic voting more attractive for expressive supporters of candidate 3 if it can deliver a victory for candidate 2. Third, locating closer to candidate 3 also facilitates strategic voting by reducing the expressive loss to strategic voters. At $x_2 < 0$, candidate 2 fails to attract strategic voters, both due to insufficient differentiation from candidate 1 and because the expressive voting cost is too high. For somewhat larger x_2 , strategic voting occurs, until $x_2 = 0.1$. At $x_2 = 0.1$ both candidates 1 and 2 receive 50% of the votes. For $x_2 > 0.1$, candidate 1's expressive vote share is large enough so that candidate 2 could never attract sufficient strategic votes to win.

5 Political Competition with an Extreme Spoiler Party

5.1 Overview

In this section, we illustrate how one can endogenize the platform choices of two policy-motivated parties when a spoiler candidate is also present. We consider an otherwise symmetric setting in which a third, extreme spoiler “citizen-candidate” party chooses whether to enter before the other candidates select their platforms. We focus on a setting with two mainstream parties that have symmetrically opposing ideal policies $\theta_1 = -\theta_2$. The position of candidate 3 is fixed at its ideal policy $x_3 = \theta_3 > \theta_2$. We assume that ρ_3 is sufficiently small relative to ρ_1 and ρ_2 that candidate 3 cannot win.

We now assume that when candidates 1 and 2 simultaneously choose policy positions x_1 and x_2 there is uncertainty about the extent of partisan support ρ_i for each party i . In a standard two-candidate model with policy motivation, candidates face a basic tradeoff between moving away from the other candidate's policy by locating closer to their own ideal point versus increasing their chance of winning by moving closer to their rival. This calculus can change when voters are strategic. In particular, recall the example depicted in Figure 2. The spoiler siphons off voters on the far right, disadvantaging candidate 2. By moving towards the spoiler and away from candidate 1, candidate 2 increases candidate 1's expressive support, hurting herself. However, this shift and resulting differentiation from candidate 1 raises candidate 2's appeal to the spoiler's supporters, more than offsetting the first effect. In response, candidate 1 also moves to the right to reduce differentiation with candidate 2, and hence the likelihood of strategic support for candidate 2.

To summarize, the possibility of strategic voting induces candidates 1 and 2 to move toward the spoiler: Candidate 2 does it to increase strategic voting, while candidate 1 moves to reduce it. We will show that in equilibrium candidate 2 moves further to the right than candidate 1, implying that candidate 1 becomes more likely to win. Thus, whether the spoiler benefits from these shifted positions depends on the relative magnitudes of the rightward shift versus the change in winning probabilities. We show that the first effect

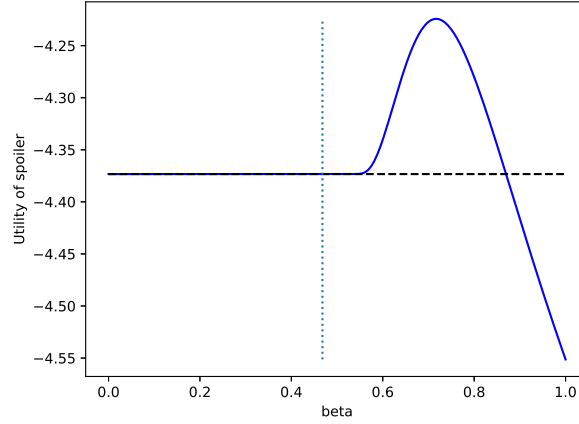


Figure 3: The spoiler's expected utility as a function of β for $-\theta_1 = \theta_2 = 1$, Φ and G are $N(0, 1)$ distributions.

dominates the second whenever voters have both sufficient expressive and strategic concerns.

Figure 3 illustrates the spoiler's utility as a function of β when $-\theta_1 = \theta_2 = 1$, $\theta_3 = x_3 = 2$, Φ and G are standard normal distributions. The dotted vertical line indicates the critical value $\bar{\beta}$ at which the spoiler begins to draw votes. For $\beta \leq \bar{\beta}$, non-partisan voters care so much about who wins the election that do not ever vote for the spoiler—even if their ideal position is arbitrarily far to the right. Increasing β past $\bar{\beta}$, first causes candidates 1 and 2 to move their platforms to the right in order to alter the incidence of strategic voting for candidate 2. As a result, the spoiler's utility first rises as voters care more and more about their expressive component of preferences before falling sharply as β approaches 1. That is, candidate 3 is best off when β is intermediate, neither too low nor too high.

Figure 4 provides intuition. For intermediate values of β the spoiler's entry induces both mainstream candidates to move toward the spoiler, and this “closer location” effect more than offsets the reduction in candidate 2's winning probability¹¹ in the spoiler's expected utility. In contrast, when selfish concerns are very low, then mainstream parties ignore the spoiler; and when voters are very selfish, the spoiler is hurt in multiple ways. First, once β is sufficiently high, further increases in β increasingly disadvantage candidate 2, because right wing citizens are more likely to vote for the spoiler and less likely to vote strategically to defeat candidate 1. In turn, reflecting this difficulty, candidate 2 starts to retreat away from the spoiler to reduce candidate 1's base support. Further, candidate 1 also moves away from the spoiler due to the reduced risk of strategic voting. This also underlies the non-monotonicity of candidate 2's winning probability in Figure 4.

¹¹Although this example is just illustrative, Pons and Tricaud (2018) use a regression discontinuity design to show that the presence of a spoiler in French parliamentary and local elections reduces the chances of the ideologically-closest candidate by about 20 percentage points. Our model can reconcile why the spoiler may want to enter despite the impact on winning probabilities.

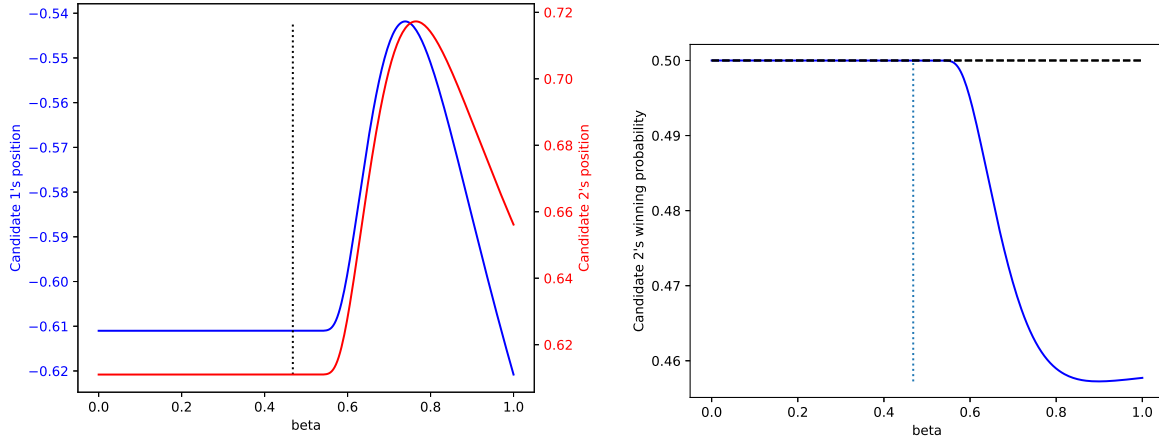


Figure 4: Candidate positions and candidate 2's winning probability as a function of β for $-\theta_1 = \theta_2 = 1$, where Φ and G are $N(0, 1)$ distributed.

In particular, once β becomes sufficiently large, reducing strategic voting matters less to candidate 1, causing her to shift her platform to the left by enough that 2's winning probability actually rises.

5.2 General Analysis

We next formally describe the model. Because candidate 3 is a spoiler with zero probability of winning, only the net-partisan difference $\rho = \rho_2 - \rho_1$ matters. First candidates choose policy positions x_1 and x_2 , and then ρ is realized. Let G be the cdf of ρ .

Assumption 1

1. G is twice continuously differentiable, with a single-peaked density g that is symmetric around 0.
2. The distribution of voter types Φ is three times continuously differentiable. Its density is symmetric around 0, strictly positive on $\mathbb{R}$, and convex in the tails, i.e., there exists N such that $\phi''(x) > 0$ for all $|x| > N$.
3. The density ϕ has exponential tails: There exists $b \in (0, 1)$ such that $\liminf_{x \rightarrow \infty} x^b |\phi'(x)| / \phi(x) > 0$.
4. The fourth moment of Φ is finite.

The third assumption is satisfied for distributions whose tails decrease exponentially, i.e., are of the form $e^{-\lambda x^\gamma}$ where $\gamma > 0$. Clearly, all assumptions are satisfied by normal distributions.

Let y_{32} be the upper boundary of the set S_2 of non-partisans voting for candidate 2. There is strategic voting if and only if $y_{32} > (x_2 + x_3)/2$, and strategic voting can occur if and only if ρ_2 is sufficiently large that candidate 2 wins. The value of y_{32} is finite if $\beta > \bar{\beta}$, where $\bar{\beta} = \frac{x_2 - x_1}{x_3 - x_1}$. In this case

$$y_{32} = \frac{1}{2} \frac{(1 - \beta)x_1^2 + \beta x_3^2 - x_2^2}{(1 - \beta)x_1 + \beta x_3 - x_2}. \quad (6)$$

Note that y_{32} strictly decreases in x_1 , i.e., M_{32} decreases: By shifting x_1 to the right, candidate 1 can reduce the ex-ante probability that voters strategically coordinate on candidate 2 to defeat 1. Conversely, increasing x_2 causes y_{32} and M_{32} to increase. That is, just as candidate 1 can reduce strategic voting for candidate 2 by making her policy more attractive to right-wing voters, candidate 2 can increase her strategic support from right-wing voters by making her policy more attractive to them. Thus, $\partial y_{32}/\partial x_1 < 0$ and $\partial y_{32}/\partial x_2 > 0$.

If there is strategic voting, then the votes for candidates 1 and 2 are $V_1 = \Phi((x_1 + x_2)/2)$, and $V_2 = \Phi(y_{32}) - \Phi((x_1 + x_2)/2) + \rho$, respectively. In the following, we focus on the equilibrium with strategic voting for candidate 2, if it exists. No other candidate can receive strategic votes in equilibrium. Let $\bar{\rho}$ be the value of ρ at which $V_1 = V_2$, i.e.,

$$\bar{\rho} = 2\Phi((x_1 + x_2)/2) - \Phi(y_{32}). \quad (7)$$

Candidate 1's winning probability is $G(\bar{\rho})$.

The candidates' optimization problems are therefore given by

$$\max_{x_1} -G(\bar{\rho})(x_1 - \theta_1)^2 - (1 - G(\bar{\rho}))(x_2 - \theta_1)^2, \text{ and} \quad (8)$$

$$\max_{x_2} -G(\bar{\rho})(x_1 - \theta_2)^2 - (1 - G(\bar{\rho}))(x_2 - \theta_2)^2. \quad (9)$$

In our theoretical analysis below we focus on local equilibria, i.e., the second-order conditions hold locally in a neighborhood of an equilibrium. This assumption is standard when working with the Wittman (1983) model in asymmetric settings, where there are no general global existence results. Numerical analysis using normal or uniform distributions indicates that all best response functions are single peaked, i.e., the solutions are standard (global) Nash equilibria of the platform choice game.

Proposition 6 *Let $\theta_2 = -\theta_1$, $x_3 \geq \theta_2$, and Assumption 1 hold. Then there exists a $\bar{\beta}$ such that in equilibrium:*

- *If $\beta \leq \bar{\beta}$ then there exists a symmetric (local) equilibrium. This equilibrium is the same as when candidate 3 is not present. The equilibrium platforms of candidates 1 and 2 are*

$$x_2 = -x_1 = \frac{\theta_2}{1 + 4\theta_2 g(0)\phi(0)}. \quad (10)$$

- *A sufficiently small increase in β above $\bar{\beta}$ causes both candidates 1 and 2 to shift x_1 and x_2 to the right in the local equilibrium, with x_2 shifting by more than x_1 , reducing candidate 2's chance of winning but raising candidate 3's expected utility.*

In the proof we show that $\bar{\beta} = (x_2 - x_1)/(x_3 - x_1)$, where x_1 and x_2 are given by (10). When β is small enough—where small enough depends on the spoiler’s location—non-partisans care so much about who wins that they all vote for either candidate 1 or 2. As a result, political competition reduces to the classical two-candidate Wittman (1983) setting, with associated symmetric locations. A slight increase in β above $\bar{\beta}$ now means that the spoiler can steal votes from extreme right-wing voters away from candidate 2. In the proof, we use the implicit function theorem at $\bar{\beta}$ to show that this induces both mainstream candidates to shift their policies to the right, with candidate 2 moving further because rightward shifts move toward candidate 2’s ideal policy and away from candidate 1’s. It follows that candidate 1’s probability of winning rises. A second application of the implicit function theorem reveals that the spoiler gains more from the rightward policy shifts of the two mainstream candidates than the spoiler loses from the increased probability that the spoiler’s least preferred candidate wins. As a result, even though the spoiler steals votes away from her preferred mainstream candidate, she still gains from entry.

6 The Multidimensional Case

Our definition of voting equilibria as well as the algorithm from Section 4.3 to find all equilibria immediately extends to multi-dimensional policy spaces. In particular, in (5) replace $(x_i - \theta)^2$ by $\|x_i - \theta\|^2$, and $(x_j - \theta)^2$ by $\|x_j - \theta\|^2$, where $\|\cdot\|$ is the Euclidean norm. The utility of voter θ is $u_\theta(i, S) = -\beta\|x_i - \theta\|^2 + (1 - \beta)B_\theta(i, S)$. We can then proceed as in Section 4.3. To see this, consider the following example.

Suppose that voter types are uniformly distributed on $[-1, 1] \times [-1, 1]$. Candidate positions are $x_1 = (-0.2, 0)$, $x_2 = (0, 0)$, and $x_3 = (0.5, 1)$. Suppose that $\beta = 0.3$.

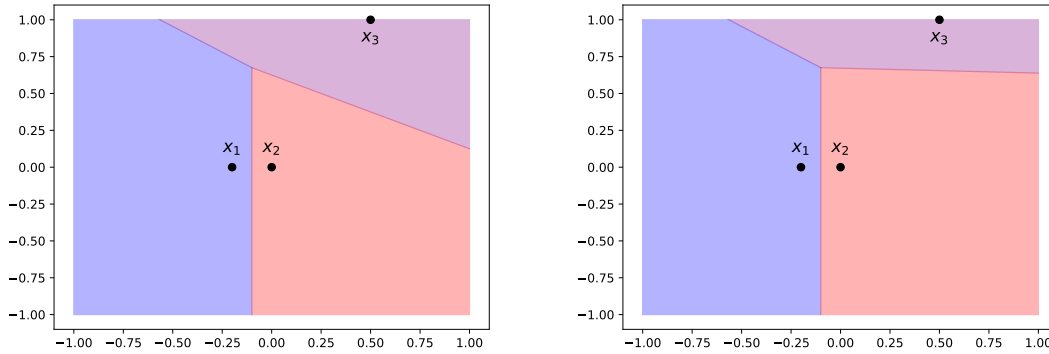


Figure 5: Voting equilibria for $\beta = 1$ (left) and strategic voting for candidate 2 for $\beta = 0.3$ (right).

The left panel of Figure 5 shows the equilibrium under expressive voting. The vote shares are 43.1%,

38.5%, and 18.4%, respectively, and hence candidate 1 wins. In the right panel we compute an equilibrium with strategic voting for candidate 2. To find this equilibrium, we follow the algorithm from Section 4.3 and assume that candidate 2 wins under strategic voting. Thus, define

$$B_\theta(1, \mathcal{S}) = B_\theta(3, \mathcal{S}) = 0, B_\theta(2, \mathcal{S}) = -\|y - x_2\|^2 + \|y - x_1\|^2, \quad (11)$$

where the expression follows because candidate 1 wins absent strategic voting. Thus, each set of voters $\mathcal{S}_i$ is described by exactly two inequalities $u_\theta(i, \mathcal{S}) \geq u_\theta(j, \mathcal{S})$, $j \neq i$. Because the quadratic terms cancel, these are linear inequalities. For $\beta = 0.3$ one can check that vote share are 43.1%, 45.6%, and 11.3%, and hence candidate 2 wins, which in turn means that we found an equilibrium with strategic voting.

One can also check that there is no equilibrium in which candidate 3 can win. In particular, if we assume that candidate 3 can win, and compute the resulting sets $\mathcal{S}_i$ then as $\beta \rightarrow 0$, candidate 3's vote share is bounded from above by 35.6%, whereas candidate 1's vote share is 43.1%, i.e., candidate 3 cannot win.

7 Conclusion

There is extensive evidence that voters care both about which candidate they vote for, and which candidate wins. In a two candidate setting, this distinction is irrelevant because these concerns coincide. However, as polling data over potential 2024 Republican presidential primary candidates illustrate, this distinction matters with more than two candidates—51 percent preferred a candidate with the best chance of winning versus 44 percent who wanted to agree with the candidate on everything even if the candidate would have a tougher time winning in November.¹² The observation that voters abandon their most preferred candidate to increase the probability that a less-favored candidate wins is difficult to reconcile with pivotal voting considerations in large elections. This paper argues that it does make sense in a setting where citizens are willing to sacrifice by voting for a candidate who has a better chance of winning if they know and care that other voters are doing the same. This behavior corresponds to well documented conditional cooperation found by researchers in public good provision experiments.

In addition to caring directly about the candidate for whom they vote, citizens in our model also care about the sacrifices other citizens make by voting for a candidate who is better situated to win compared to their favorite candidate, which we refer to as strategic voting. We show that strategic voting can only occur for candidates who have a positive probability of winning. Further, if a candidate attracts any strategic voting, our model predicts that all expressive supporters of that candidate will also vote for that candidate. We also provide an algorithm to determine all equilibria, which also works when there are multiple policy dimensions.

¹²See, FiveThirtyEight, “Which Republican Candidate Should Biden Be Most Afraid Of?” <https://fivethirtyeight.com/features/which-republican-candidate-should-biden-be-most-afraid-of/>

We fully characterize all equilibria in a three-candidate setting. We provide several insights. First, even though elections may be close, one candidate may be systematically more likely to win, indicating that close elections may not be a good natural experiment.¹³ Second, strategic voting does not only have to occur in close elections. Third, a virtue of our formulation of strategic voting is that it is simple enough to incorporate into a standard model of political competition with policy motivated candidates. To illustrate this, we endogenize candidate policy choices with the two mainstream candidates and a spoiler who understand that citizens may vote strategically. We show that the spoiler can gain from entering, even though she has no chance of winning the election and reduces the winning probability of her preferred mainstream candidate. This occurs because both mainstream candidates partially incorporate the spoiler's platform by moving toward the spoiler.

¹³See Levine and Martinelli (2022) who make a similar point in a setting with endogenous campaign spending.

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8 Appendix

Proof of Lemma 1. See text. ■

Proof of Proposition 1. See text. ■

Proof of Lemma 2. We first show that $B_\theta(i, \mathcal{S})$ is affine linear in θ . Let A be the set of all candidates that tie if all citizens in S_i vote expressively, and A' be the set of all candidates who tie if i receives strategic votes. Then

$$B_\theta(i, \mathcal{S}) = -\left(\frac{1}{\#A'} \sum_{j \in A'} (\theta - x_j)^2 - \frac{1}{\#A} \sum_{j \in A} (\theta - x_j)^2 \right). \quad (12)$$

As the coefficients of θ^2 sum to zero it follows that $B_\theta(i, \mathcal{S})$ is affine linear in θ . As $-\beta(x_i - \theta)^2$ is strictly concave in θ for $\beta > 0$, it follows that so is $u_\theta(i, \mathcal{S})$. ■

Proof of Lemma 3. Lemma 2 implies that the utility to θ of voting for a candidate i takes the form $u_\theta(i, \mathcal{S}) = -\beta(x_i - \theta)^2 + a_i + b_i\theta$. Let $j \neq k$. Then the equation $u_\theta(j, \mathcal{S}) = u_\theta(k, \mathcal{S})$ has at most one solution, unless $b_k - b_j = 2\beta(x_j - x_k)$ and $a_k - a_j = \beta(x_k^2 - x_j^2)$. In the former case, it immediately follows that $\{\theta | u_\theta(i, \mathcal{S}) \geq u_\theta(j, \mathcal{S}), \text{ for all } j \neq i\}$ is an interval, and hence S_i is an interval. In the latter case, $u_\theta(i, \mathcal{S}) = u_\theta(j, \mathcal{S})$ for all $i, j \in K$. The argument for the former case then implies that $\{\theta | u_\theta(i, \mathcal{S}) \geq u_\theta(j, \mathcal{S}), \text{ for all } j \notin K\}$ comprises an interval, and hence S_K is an interval. Because all citizens in S_K are indifferent, they vote for their expressively-preferred candidate in K , and hence each S_k is also an interval. ■

Proof of Proposition 2. Let $[a, b]$ be the support of Φ , where a and b can be infinite. Suppose that candidates $i \in I$ tie. Let $j = \min I$ and $k = \max I$ be the candidates in I who are the furthest to the left and right, respectively. We first rule out the possibility where one of these two candidates ties while only receiving partisan support. Suppose, without loss of generality, that candidate k only receives partisan support. Let $T_\varepsilon = [x_k - \varepsilon, x_k]$. Because x_k is in the support of Φ , this set has positive measure. By continuity, for ε sufficiently small, if these voters in the neighborhood of x_k switch to vote for k , they receive a strictly higher selfish payoff from voting for k and it shifts the electoral outcome so that their most-preferred candidate k wins with probability one, making them strictly better off, a contradiction of no minimal expansion.

We next show that $\text{cl } S_j = [a, \bar{s}_j]$ and $\text{cl } S_k = [\bar{s}_k, b]$. Suppose by way of contradiction that $\inf S_j = \underline{s}_j > a$. Then citizen $\underline{s}_j$ must be indifferent between voting for j and some other candidate, ℓ . Let $T_\varepsilon = [\underline{s}_j - \varepsilon, \underline{s}_j]$.

As $\underline{s}_j$ is in the interior of the support of Φ then candidate j wins with probability 1 if all citizens in T_ε vote for candidate j , because any arbitrarily small number of additional voters would swing the election to candidate j . Citizen $\underline{s}_j$ is strictly better off if j wins with probability 1 compared to the lottery over candidates in I because $\beta < 1$. By continuity, there exists $\varepsilon > 0$ such that all members of T_ε are also strictly better off, a contradiction of no minimal expansion. The argument that $\text{cl } S_k = [\bar{s}_k, b]$ is analogous.

If $\bar{s}_j = \bar{s}_k$ then no third party gets a positive number of non-partisan votes, and this must also cause the election to end in a tie. Further, suppose $\bar{s}_j = \bar{s}_k = \bar{s} \neq \frac{x_j + x_k}{2}$. Without loss of generality suppose $\bar{s} > \frac{x_j + x_k}{2}$. Then if types $[\bar{s} - \epsilon, \bar{s}]$, ϵ vanishingly small, jointly vote for k , then again this yields them higher selfish utility and candidate k now wins with probability 1 rather than with probability 0.5, making those types strictly better off, violating no minimal expansion.

Next, suppose that $\bar{s}_j < \bar{s}_k$. If citizen $\bar{s}_j$ is strictly better off when candidate j wins with probability 1 vis-à-vis the lottery over candidates in I then we can conclude as above that no minimal expansion would be violated, because any arbitrarily small group $T_\varepsilon = [\bar{s}_j, \bar{s}_j + \varepsilon]$ could strictly improve by jointly voting for candidate j . Thus, $\bar{s}_j$ must be indifferent between j winning with probability 1 and the lottery over candidates in I . Similarly, citizen $\bar{s}_k$ must be indifferent between candidate k winning with probability 1 and the lottery over I . Thus, $\bar{s}_i$ is the certainty equivalent of the lottery for $\bar{s}_i$ for $i = j, k$. Because x_j and x_k are the most extreme candidates who win with positive probability, this implies that $x_j < \bar{s}_j < \bar{s}_k < x_k$, else if $\bar{s}_j \geq x_j$ or $x_k \leq \bar{s}_k$ either $\bar{s}_j$ or $\bar{s}_k$ would strictly prefer that candidate j or k , respectively, win with probability 1.

If only j and k win with positive probability and $\bar{s}_j$ is indifferent between j winning with probability 1 and the lottery of j and k , then j and k 's ideologies must be equidistant from $\bar{s}_j$, i.e., $\bar{s}_j = (x_j + x_k)/2$. Similarly, it follows that $\bar{s}_k = (x_j + x_k)/2$, contradicting $\bar{s}_j < \bar{s}_k$.

Now suppose that at least three candidates win with positive probability. Let x_ℓ be the position of the candidate with a chance of winning closest to candidate j . For $\bar{s}_j$ to be indifferent between the lottery and getting x_j with certainty it must be the case that x_ℓ is closer to $\bar{s}_j$ than x_j . However, this implies that getting x_ℓ with certainty is strictly better for $\bar{s}_j$ than getting the lottery. By continuity, there exists a set $T_\varepsilon = (\bar{s}_j - \varepsilon, \bar{s}_j]$ of citizens that would be strictly better off voting jointly for ℓ , as ℓ would win the election and x_ℓ is closer than x_i for all citizens in T_ε . This contradicts no minimal expansion, i.e., not more than two candidates can win. ■

Proof of Proposition 3. If, in equilibrium, only two candidates i and j receive votes from non-partisans, then the result is immediate, i.e., there is no strategic voting for a losing candidate. Excluding this case, Proposition 2 implies that there exists a candidate w who wins with probability 1.

Suppose by way of contradiction that there is strategic voting for some candidates $i \neq w$. Without loss of generality suppose that S_i is to the right of S_w . Moreover, let S_i be the set of strategic voters that is furthest

to the right. Strategic voting for i could only generate benefits to voters in S_i if it prevents some other set of candidates, K , from winning or tying candidate w , and hence $\Phi(S_i \cap E_k) > 0$ for all $k \in K$. Recall that S_i and S_w are intervals.

Next, each x_k , $k \in K$ must be strictly to the right of S_w as $S_i \cap E_k \neq \emptyset$. Let $\bar{k} = \max K$, and $\theta \in E_{\bar{k}} \cap S_i$. Then θ is strictly better off with a lottery where candidates in K or $K \cup \{w\}$ win with equal probability vis-à-vis candidate w winning with probability 1. This follows because $\theta \in E_{\bar{k}}$ implies that $x_{\bar{k}}$ is closer to θ than x_k , $k \in K \setminus \{\bar{k}\}$. Further, all candidates in K are closer to θ than candidate w . Thus, $B_\theta(i, \cdot) < 0$.

Next, note that $S_{\bar{k}}$ is either empty or $S_{\bar{k}} \subset E_{\bar{k}}$. In particular, If $S_{\bar{k}} \setminus E_{\bar{k}} \neq \emptyset$ then there must be strategic voting for candidate $\bar{k}$. However, by construction S_i is the set of strategic voters that is furthest to the right. Lemma 1 therefore implies that $B_\theta(\bar{k}, \cdot) = 0$ and hence given that θ is also closer to $x_{\bar{k}}$ than x_i , it follows that θ is strictly better off voting for $\bar{k}$, a contradiction. ■

Proofs of Propositions 4 and 5. See text. ■

Proof of Proposition 6. The first-order conditions for the candidates' optimization problems are given by:

$$g(\bar{\rho}) \left(\phi \left(\frac{x_1 + x_2}{2} \right) - \phi(y_{32}) \frac{\partial y_{32}}{\partial x_1} \right) \left(\frac{x_1 + x_2}{2} - \theta_1 \right) (x_2 - x_1) - (x_1 - \theta_1) G(\bar{\rho}) = 0 \quad (13)$$

and

$$-g(\bar{\rho}) \left(\phi \left(\frac{x_1 + x_2}{2} \right) - \phi(y_{32}) \frac{\partial y_{32}}{\partial x_2} \right) \left(\theta_2 - \frac{x_1 + x_2}{2} \right) (x_2 - x_1) + (\theta_2 - x_2)(1 - G(\bar{\rho})) = 0. \quad (14)$$

If $\beta \leq \bar{\beta}$, then $y_{32} = \infty$, so $\bar{\rho} = 2\Phi((x_1 + x_2)/2) - 1$. Hence, the terms $\phi(y_{32}) \frac{\partial y_{32}}{\partial x_1}$ in the first-order conditions (13) and (14) disappear, which yields

$$g(\bar{\rho}) \left(\phi \left(\frac{x_1 + x_2}{2} \right) \right) \left(\frac{x_1 + x_2}{2} - \theta_1 \right) (x_2 - x_1) - (x_1 - \theta_1) G(\bar{\rho}) = 0$$

and

$$-g(\bar{\rho}) \left(\phi \left(\frac{x_1 + x_2}{2} \right) \right) \left(\theta_2 - \frac{x_1 + x_2}{2} \right) (x_2 - x_1) + (\theta_2 - x_2)(1 - G(\bar{\rho})) = 0.$$

Substitute $\theta_1 = -\theta_2$ and impose symmetry to solve the first-order conditions for the equilibrium platforms:

$$x_2 = -x_1 = \frac{\theta_2}{1 + 4\theta_2 g(0) \phi(0)}, \quad (15)$$

which also implies $\bar{\rho} = 0$. Verifying the second order conditions at the symmetric equilibrium is standard, as symmetry of the densities implies that $g'(0) = \phi'(0) = 0$.

We now show that we have enough structure to apply the implicit function theorem to characterize the equilibrium candidate location responses to slight increases in β above $\bar{\beta}$. Define

$$f(\beta) = \phi(y_{32}) \frac{\partial y_{32}}{\partial x_1}.$$

Clearly, f is continuously differentiable for $\beta \neq \bar{\beta}$. We next show that f is also continuously differentiable at $\bar{\beta}$. Because the left-derivative of f at $\bar{\beta}$ is trivially zero, it is sufficient to show that $\lim_{\beta \downarrow \bar{\beta}} f'(\beta) = 0$.

For $\beta > \bar{\beta}$

$$\frac{\partial y_{32}}{\partial \beta} = -\frac{1}{2} \frac{(x_2 - x_1)(x_3 - x_1)(x_3 - x_2)}{((1 - \beta)x_1 + \beta x_3 - x_2)^2}. \quad (16)$$

and

$$\frac{\partial^2 y_{32}}{\partial \beta \partial x_1} = \frac{1}{2} \frac{(x_3 - x_2)^2 (x_2 + \beta x_3 - (1 + \beta)x_1)}{((1 - \beta)x_1 + \beta x_3 - x_2)^3}. \quad (17)$$

Because $\frac{\partial y_{32}}{\partial \beta}$ and $\frac{\partial y_{32}}{\partial x_1}$ both go to infinity at rate $\frac{1}{(\beta - \bar{\beta})^2}$ as $\beta \downarrow \bar{\beta}$, for $\beta > \bar{\beta}$ there exists $K_1, K_2 > 0$ such that

$$|f'(\beta)| = \left| \phi'(y_{32}) \frac{\partial y_{32}}{\partial \beta} \frac{\partial y_{32}}{\partial x_1} + \phi(y_{32}) \frac{\partial^2 y_{32}}{\partial \beta \partial x_1} \right| \leq |\phi'(y_{32})| \frac{K_1}{(\beta - \bar{\beta})^4} + \phi(y_{32}) \frac{K_2}{(\beta - \bar{\beta})^3}. \quad (18)$$

Further, y_{32} goes to infinity at the rate $1/(\beta - \bar{\beta})$ as $\beta \downarrow \bar{\beta}$. Because the fourth moment of Φ is finite, it follows that $\lim_{x \rightarrow \infty} x^4 \phi(x) = 0$. In particular, suppose that $\limsup_{x \rightarrow \infty} x^4 \phi(x) = a > 0$. Because ϕ is a density that is convex in the tail, $x^4 \phi(x) \geq a/2$ for all sufficiently large x and hence the integral of $x^4 f(x)$ would not converge.

Integration by parts yields that $\int_0^\infty x^4 \phi'(x) dx = x^4 \phi(x)|_0^\infty - 4 \int_0^\infty x^3 \phi(x) dx$. Hence, $\int_0^\infty x^4 \phi'(x) dx$ is finite. Convexity in the tail implies that $\phi'(x)$ is strictly increasing for large x . Hence, $\lim_{x \rightarrow \infty} x^4 \phi'(x) = 0$. This and (18) yield $\lim_{\beta \downarrow \bar{\beta}} f'(\beta) = 0$.

To show differentiability at $\bar{\beta}$, it is sufficient to prove that

$$\lim_{\beta \downarrow \bar{\beta}} \frac{f(\beta) - f(\bar{\beta})}{\beta - \bar{\beta}} = 0. \quad (19)$$

The argument is similar to above. Note that $f(\beta) < \hat{K}/(\beta - \bar{\beta})^2$ for β marginally larger than $\bar{\beta}$ and some $\hat{K} > 0$. Further, we have shown that $\phi(y_{32}) < \varepsilon(\beta - \bar{\beta})^4$, for β near $\bar{\beta}$ because $\lim_{x \rightarrow \infty} x^4 \phi(x) = 0$. Thus, the limit in (19) exists and is zero. Hence, $f(\beta)$ is continuously differentiable, and $f'(\bar{\beta}) = 0$.

An analogous argument shows that $\phi(y_{32}) \frac{\partial y_{32}}{\partial x_2}$ is continuously differentiable, and that the first derivative with respect to β at $\bar{\beta}$ is zero.

Next differentiate candidate 1's first-order condition (13) with respect to β to obtain:

$$\begin{aligned} \frac{\partial \text{FOC}_1}{\partial \beta} = & \phi(y_{32}) \frac{\partial y_{32}}{\partial \beta} \left(-g'(\bar{\rho}) \left(\phi\left(\frac{x_1 + x_2}{2}\right) - \phi(y_{32}) \frac{\partial y_{32}}{\partial x_1} \right) \left(\frac{x_1 + x_2}{2} - \theta_1 \right) (x_2 - x_1) \right) \\ & - \phi(y_{32}) \frac{\partial y_{32}}{\partial \beta} g(\bar{\rho}) \left(\left(\frac{\phi'(y_{32})}{\phi(y_{32})} \frac{\partial y_{32}}{\partial x_1} + \frac{\partial^2 y_{32}}{\partial x_1 \partial \beta} \right) \left(\frac{x_1 + x_2}{2} - \theta_1 \right) (x_2 - x_1) - (x_1 - \theta_1) \right). \end{aligned} \quad (20)$$

Similarly, differentiating candidate 2's first-order condition (14) with respect to β yields

$$\begin{aligned} \frac{\partial \text{FOC}_2}{\partial \beta} = & \phi(y_{32}) \frac{\partial y_{32}}{\partial \beta} \left(g'(\bar{\rho}) \left(\phi \left(\frac{x_1 + x_2}{2} \right) - \phi(y_{32}) \frac{\partial y_{32}}{\partial x_2} \right) \left(\theta_2 - \frac{x_1 + x_2}{2} \right) (x_2 - x_1) \right) \\ & + \phi(y_{32}) \frac{\partial y_{32}}{\partial \beta} g(\bar{\rho}) \left(\left(\frac{\phi'(y_{32})}{\phi(y_{32})} \frac{\partial y_{32}}{\partial x_2} + \frac{\frac{\partial^2 y_{32}}{\partial x_2 \partial \beta}}{\frac{\partial y_{32}}{\partial \beta}} \right) \left(\theta_2 - \frac{x_1 + x_2}{2} \right) (x_2 - x_1) + (\theta_2 - x_2) \right). \end{aligned} \quad (21)$$

Note that the term in the large parentheses on the first lines of (20) and (21) are zero, respectively at $\beta = \bar{\beta}$. In contrast, the terms inside the large parentheses on the second lines of (20) and (21) go to infinity. Thus,

$$\lim_{\beta \downarrow \bar{\beta}} \frac{\frac{\partial \text{FOC}_1}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial \beta}} = - \lim_{\beta \downarrow \bar{\beta}} \frac{\frac{\phi'(y_{32})}{\phi(y_{32})} \frac{\partial y_{32}}{\partial x_1} + \frac{\frac{\partial^2 y_{32}}{\partial x_1 \partial \beta}}{\frac{\partial y_{32}}{\partial \beta}}}{\frac{\phi'(y_{32})}{\phi(y_{32})} \frac{\partial y_{32}}{\partial x_2} + \frac{\frac{\partial^2 y_{32}}{\partial x_2 \partial \beta}}{\frac{\partial y_{32}}{\partial \beta}}} = \frac{x_3 - x_2}{x_3 - x_1}. \quad (22)$$

Next, differentiate the first-order conditions with respect to x_1 and x_2 at $\beta = \bar{\beta}$. Note that $\bar{\rho} = 0$ at $\bar{\beta}$, and hence $g'(\bar{\rho}) = 0$. Similarly, because $x_1 = -x_2$ in equilibrium at $\bar{\beta}$, $\phi'((x_1 + x_2)/2) = 0$. Further, $\phi(y_{32}) \frac{\partial y_{32}}{\partial x_1} = 0$ and $\frac{\partial G(\bar{\rho})}{\partial x_i} = g(0)\phi(0)$. Therefore,

$$\left. \frac{\partial \text{FOC}_1}{\partial x_1} \right|_{\beta=\bar{\beta}} = -2g(0)\phi(0)(x_1 - \theta_1) - \frac{1}{2} < 0, \quad \left. \frac{\partial \text{FOC}_1}{\partial x_2} \right|_{\beta=\bar{\beta}} = g(0)\phi(0)(x_2 - x_1) > 0, \quad (23)$$

and

$$\left. \frac{\partial \text{FOC}_2}{\partial x_1} \right|_{\beta=\bar{\beta}} = g(0)\phi(0)(x_2 - x_1) > 0, \quad \left. \frac{\partial \text{FOC}_2}{\partial x_2} \right|_{\beta=\bar{\beta}} = -2g(0)\phi(0)(\theta_2 - x_2) - \frac{1}{2} < 0. \quad (24)$$

Let $x_1(\beta)$ and $x_2(\beta)$ be the optimal policy of candidates 1 and 2 given β . Further, let $\tilde{x}_1(\beta, x_2)$ be the solution to the first-order conditions (13) with respect to x_1 . Similarly, let $\tilde{x}_2(\beta, x_1)$ be the solution to (14) with respect to x_2 . Then $x_1(\beta) = \tilde{x}_1(\beta, x_2(\beta))$ and $x_2(\beta) = \tilde{x}_2(\beta, x_1(\beta))$. Differentiating with respect to β yields

$$x'_1(\beta) = \frac{\partial \tilde{x}_1(\beta, x_2)}{\partial \beta} + \frac{\partial \tilde{x}_1(\beta, x_2)}{\partial x_2} x'_2(\beta); \quad (25)$$

$$x'_2(\beta) = \frac{\partial \tilde{x}_2(\beta, x_1)}{\partial \beta} + \frac{\partial \tilde{x}_2(\beta, x_1)}{\partial x_1} x'_1(\beta). \quad (26)$$

Solving these equations for $x'_1(\beta)$ and $x'_2(\beta)$ yields

$$x'_1(\beta) = \frac{\frac{\partial \tilde{x}_1(\beta, x_2)}{\partial x_2} \frac{\partial \tilde{x}_2(\beta, x_1)}{\partial \beta} + \frac{\partial \tilde{x}_1(\beta, x_2)}{\partial \beta}}{1 - \frac{\partial \tilde{x}_1(\beta, x_2)}{\partial x_2} \frac{\partial \tilde{x}_2(\beta, x_1)}{\partial x_1}}; \quad (27)$$

$$x'_2(\beta) = \frac{\frac{\partial \tilde{x}_2(\beta, x_1)}{\partial x_1} \frac{\partial \tilde{x}_1(\beta, x_2)}{\partial \beta} + \frac{\partial \tilde{x}_2(\beta, x_1)}{\partial \beta}}{1 - \frac{\partial \tilde{x}_1(\beta, x_2)}{\partial x_2} \frac{\partial \tilde{x}_2(\beta, x_1)}{\partial x_1}}. \quad (28)$$

The implicit function theorem implies that for $i \neq j$,

$$\frac{\partial \tilde{x}_i(\beta, x_j)}{\partial \beta} = - \frac{\frac{\partial \text{FOC}_i}{\partial \beta}}{\frac{\partial \text{FOC}_i}{\partial x_i}}, \text{ and } \frac{\partial \tilde{x}_i(\beta, x_j)}{\partial x_j} = - \frac{\frac{\partial \text{FOC}_i}{\partial x_j}}{\frac{\partial \text{FOC}_i}{\partial x_i}} \quad (29)$$

Let D be the denominator term in equations (27) and (28). Substituting (23), (24), and (29) into D yields

$$\lim_{\beta \rightarrow \bar{\beta}} D = 1 - \frac{\frac{\partial \text{FOC}_1}{\partial x_2} \frac{\partial \text{FOC}_2}{\partial x_1}}{\frac{\partial \text{FOC}_1}{\partial x_1} \frac{\partial \text{FOC}_2}{\partial x_2}} = 1 - \frac{4g(0)^2 \phi(0)^2 (x_2 - x_1)^2}{(1 + 4g(0)\phi(0)(\theta_2 - x_2))(1 + 4g(0)\phi(0)(x_1 - \theta_1))}. \quad (30)$$

Using symmetry, i.e., $\theta_2 = -\theta_1$ and x_1 and x_2 from the symmetric solution (15) implies

$$\lim_{\beta \rightarrow \bar{\beta}} D = \frac{(1 + 4g(0)\phi(0)\theta_2)^2 (1 + 16g(0)^2 \phi(0)^2 \theta_2^2)}{(1 + 4g(0)\phi(0)\theta_2 + 16g(0)^2 \phi(0)^2 \theta_2^2)^2} > 0. \quad (31)$$

Equations (27) and (29) imply

$$D \frac{\partial x_1(\beta)}{\partial \beta} = \frac{\frac{\partial \text{FOC}_1}{\partial x_2} \frac{\partial \text{FOC}_2}{\partial \beta}}{\frac{\partial \text{FOC}_1}{\partial x_1} \frac{\partial \text{FOC}_2}{\partial x_2}} - \frac{\frac{\partial \text{FOC}_1}{\partial \beta}}{\frac{\partial \text{FOC}_1}{\partial x_1}} = \frac{\frac{\partial \text{FOC}_2}{\partial \beta}}{\frac{\partial \text{FOC}_1}{\partial x_1}} \left(\frac{\frac{\partial \text{FOC}_1}{\partial x_2}}{\frac{\partial \text{FOC}_2}{\partial x_2}} - \frac{\frac{\partial \text{FOC}_1}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial \beta}} \right) \quad (32)$$

$$D \frac{\partial x_2(\beta)}{\partial \beta} = \frac{\frac{\partial \text{FOC}_2}{\partial x_1} \frac{\partial \text{FOC}_1}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial x_2} \frac{\partial \text{FOC}_1}{\partial x_1}} - \frac{\frac{\partial \text{FOC}_2}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial x_2}} = \frac{\frac{\partial \text{FOC}_2}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial x_2}} \left(\frac{\frac{\partial \text{FOC}_2}{\partial x_1}}{\frac{\partial \text{FOC}_1}{\partial x_1}} \frac{\frac{\partial \text{FOC}_1}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial \beta}} - 1 \right) \quad (33)$$

Substituting the above derivatives we get

$$\lim_{\beta \downarrow \bar{\beta}} D \frac{\frac{\partial x_1(\beta)}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial \beta}} = \frac{2(x_3 - x_2) + 8g(0)\phi(0)(\theta_2(x_3 - x_2) + 2x_2^2)}{(x_2 + x_3)(1 + 4g(0)\phi(0)(\theta_2 - x_2))^2}. \quad (34)$$

Thus, $\frac{\partial x_1(\beta)}{\partial \beta} > 0$ for β that are marginally larger than $\bar{\beta}$, because $D > 0$. Similarly,

$$\lim_{\beta \downarrow \bar{\beta}} D \frac{\frac{\partial x_2(\beta)}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial \beta}} = \frac{2(x_2 + x_3) + 8g(0)\phi(0)(\theta_2(x_2 + x_3) - 2x_2^2)}{(x_2 + x_3)(1 + 4g(0)\phi(0)(\theta_2 - x_2))^2}, \quad (35)$$

which implies $\frac{\partial x_2(\beta)}{\partial \beta} > 0$ for β marginally larger than $\bar{\beta}$, because $\theta_2, x_3 > x_2$.

Next, we prove that candidate 2 moves her policy by more to the right than candidate 1 moves her policy:

$$\lim_{\beta \downarrow \bar{\beta}} D \frac{\frac{\partial x_2(\beta)}{\partial \beta} - \frac{\partial x_1(\beta)}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial \beta}} = \frac{4x_2(1 + 4g(0)\phi(0)\theta_2 - 8g(0)\phi(0)x_2)}{(x_2 + x_3)(1 + 4g(0)\phi(0)(\theta_2 - x_2))^2}, \quad (36)$$

which is strictly positive because (10) implies

$$1 + 4g(0)\phi(0)\theta_2 - 8g(0)\phi(0)x_2 = \frac{1 + 16g(0)^2 \phi(0)^2 \theta_2^2}{1 + 4g(0)\phi(0)\theta_2} > 0. \quad (37)$$

Finally, we prove that the spoiler is better off for β marginally larger than $\bar{\beta}$. The spoiler's utility is

$$U_3(\beta) = -G(\bar{\rho})(x_1(\beta) - x_3)^2 - (1 - G(\bar{\rho}))(x_2(\beta) - x_3)^2. \quad (38)$$

Differentiating with respect to β yields

$$\begin{aligned}
U'_3(\beta) = & -g(\bar{\rho}) \left(\phi \left(\frac{x_1(\beta) + x_2(\beta)}{2} \right) (x'_1(\beta) + x'_2(\beta)) - \phi(y_{32}) \left(\frac{\partial y_{32}}{\partial \beta} + \frac{\partial y_{32}}{\partial x_1} x'_1(\beta) + \frac{\partial y_{32}}{\partial x_2} x'_2(\beta) \right) \right) \\
& \cdot ((x_1(\beta) - x_3)^2 - (x_2(\beta) - x_3)^2) \\
& + 2G(\bar{\rho})(x_3 - x_1(\beta))x'_1(\beta) + 2(1 - G(\bar{\rho}))(x_3 - x_2(\beta))x'_2(\beta).
\end{aligned} \tag{39}$$

Note that

$$\lim_{\beta \downarrow \bar{\beta}} \left| \frac{\frac{\partial y_{32}}{\partial \beta} \phi(y_{32})}{\frac{\partial \text{FOC}_2}{\partial \beta}} \right| = \lim_{\beta \downarrow \bar{\beta}} \left| \frac{\phi(y_{32})}{\phi'(y_{32})g(\bar{\rho}) \frac{\partial y_{32}}{\partial \beta} \theta_2(x_2 - x_1)} \right| \leq \lim_{\beta \downarrow \bar{\beta}} C(\beta - \bar{\beta})^{2-b} = 0, \tag{40}$$

where $C > 0$ is some constant. The last inequality follows because $|\phi'(x)/\phi(x)| > \varepsilon/x^b$ for $0 < b < 1$ and $\frac{\partial y_{32}}{\partial \beta}$ goes to infinity at the rate $1/(\beta - \bar{\beta})^2$. Further,

$$\lim_{\beta \downarrow \bar{\beta}} \frac{\frac{\partial y_{32}}{\partial x_i} \phi(y_{32}) x'_i(\beta)}{\frac{\partial \text{FOC}_2}{\partial \beta}} = 0. \tag{41}$$

Thus,

$$\lim_{\beta \downarrow \bar{\beta}} \frac{U'_3(\beta)}{\frac{\partial \text{FOC}_2}{\partial \beta}} = \lim_{\beta \downarrow \bar{\beta}} -2g(0)\phi(0) \left(\frac{\frac{\partial x_1(\beta)}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial \beta}} + \frac{\frac{\partial x_2(\beta)}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial \beta}} \right) x_2(\bar{\beta})x_3 + (x_3 + x_2(\bar{\beta})) \frac{\frac{\partial x_1(\beta)}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial \beta}} + (x_3 - x_2(\bar{\beta})) \frac{\frac{\partial x_2(\beta)}{\partial \beta}}{\frac{\partial \text{FOC}_2}{\partial \beta}}. \tag{42}$$

Substituting (34) and (35) and writing x_2 for $x_2(\bar{\beta})$ yields

$$\lim_{\beta \downarrow \bar{\beta}} D \frac{U'_3(\beta)}{\frac{\partial \text{FOC}_2}{\partial \beta}} = \frac{4(1 + 4g(0)\phi(0)\theta_2)(x_3^2 - x_2^2) + 8g(0)\phi(0)x_2(4x_2^2 - x_3^2(1 + 4g(0)\phi(0)\theta_2))}{(x_2 + x_3)(1 + 4g(0)\phi(0)(\theta_2 - x_2))^2}. \tag{43}$$

The denominator of (43) is strictly positive.

To verify that the numerator is strictly positive for $x_3 \geq \theta_2$ substitute the solutions for x_1 and x_2 from (15) into the numerator and evaluate at $x_3 = \theta_2$ to get

$$8\theta_2^3\phi(0)g(0) \frac{80\theta_2^2\phi(0)^2g(0)^2 + 64\theta_2^3\phi(0)^3g(0)^3 + 28\theta_2\phi(0)g(0) + 7}{(4\theta_2\phi(0)g(0) + 1)^3} > 0. \tag{44}$$

Differentiating the numerator of (43) with respect to x_3 and again using (15) yields

$$-8x_3(4\theta_2\phi(0)g(0) + 1)(2\phi(0)g(0)x_2 - 1) = 8x_3(2\theta_2\phi(0)g(0) + 1) > 0. \tag{45}$$

Thus, (43) is strictly positive for $x_3 \geq \theta_2$. Hence, candidate 3's utility from entry is increasing in β for β slightly larger than $\bar{\beta}$. ■