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Economic Theory

Mistaken self-perception and equilibrium*

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Summary. Motivated by real-world information economics problems and by experimental findings on overconfidence, this paper introduces a general epistemic construction to model strategic interaction with incomplete information, where the players' self-perception may be mistaken. This allows us to rigorously describe equilibrium play, by formulating appropriate equilibrium concepts. We show that there always exist "objective" equilibria, where the players correctly anticipate each other's strategies without attempting to make sense of them, and that these outcomes coincide with the equilibria of an associated Bayesian game with subjective priors. In population games, these equilibria can also be always introspectively rationalized by the players, despite their possibly mistaken self-perception.

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> "...and that's the news from Lake Wobegon, Minnesota, where all the women are strong, and all the men are good-looking, and all the children are above average!"
>
> Garrison Keillor, "A Prairie Home Companion"

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1 Introduction

In Bayesian games of private information that model adverse-selection problems, some individuals' unobservable characteristics (such as ability and intelligence) influence the players' payoff. It is customary to assume that informed players precisely assess their own characteristics. Well-established experimental evidence in psychology contradicts this assumption: on average, individuals overestimate their own personal characteristics.[1] This paper introduces a general epistemic construction to model games of incomplete information, where the players' self-perception may be mistaken. This allows us to rigorously describe equilibrium play, by formulating equilibrium concepts based on the players' high-order beliefs with respect to overconfidence and to each others' strategies.

The need for such a rigorous construction naturally arises in simple signalling problems à-la Spence (1974).[2] We shall consider for concreteness two simple scenarios. Suppose that (i) an overconfident junior employee of a company or institution may attempt to signal her (perceived) ability by volunteering for the heaviest and most difficult tasks; alternatively, consider (ii) a start up entrepreneur applying for credit to a bank or to a venture capitalist: if overconfident, she may be tempted to present a very bold company plan, in order to signal that her (perceived) skills are better than the skills of the average applicant.[3]

By construction, the "informed" player cannot be aware that her self-perception is mistaken, or else she would revise her self-judgement until correctly assessing her characteristics. But her counterpart is likely aware of the possibility that her opponent is overconfident. After all, the experimental evidence on overconfidence is readily available to any company and credit institution manager. Since the overconfident player is unaware of being overconfident, she is also likely to be unaware that her counterpart knows that she is overconfident. It is an established experimen-

[1] Among the numerous accounts of the so-called "illusion of absolute overconfidence," Fischoff, Slovic and Lichtenstein (1977) found that subjects' hit rate when answering quizzes is typically 60% when they are 90% certain; Buehler, Griffin and Ross (1994) found that people expect to complete projects in less time than it actually takes; Radhakrishnan, Arrow and Sniezek (1996) found that students expect to receive higher scores on exams than they actually receive; Hoch (1985) found that MBA students overestimate the number of job offers they will receive, and the magnitude of their salary. According to DeBondt and Thaler (1995): "Perhaps the most robust finding in the psychology of judgment is that people are overconfident."

[2] Signalling problems have primal prominence in economics since the Nobel-prize winning piece by Spence (1974). A large literature in social psychology addresses the set of strategies (self–promotion, excuse–making, supplication, intimidation, ingratiation, etc.) adopted to manipulate others people's beliefs about oneself (see Wortman and Linsenmeier, 1977; Baumeister, 1998; for a review). The need for formal modeling of signaling in relation to self-confidence is also stressed by Benabou and Tirole (2003).

[3] Entrepreneurs' overconfidence is studied, for example, by Camerer and Lovallo (1999), Busenitz and Barney (1997), and Cooper, Woo, and Dunkelberg (1988). To our knowledge, the effect of overconfidence on entrepreneurs' credit applications has not been explored.

tal finding that normally healthy subjects systematically overestimate what other subjects think about them.[4]

Describing the equilibrium of such an environment is not a conceptually easy task. The overconfident individual is tempted to try and signal her perceived ability. But suspecting that the individual may be overconfident, the counterpart is likely to discount this signal. A venture capitalist would most likely dismiss a very bold company plan, and infer that the applicant is likely to be self-deluded. Similarly, an experienced manager would likely be skeptical of an overenthusiastic junior employee who always volunteers for the most difficult tasks. But this response should be anticipated in equilibrium. Our overconfident start up entrepreneur should strategically present a more humble and conservative plan, whereas our junior employee should strategically only volunteer for tasks that she feels she can easily accomplish. Still, it is hard to see how an overconfident individual can correctly anticipate that her costly signalling choices will not convince her counterpart of her ability. This response is based on the knowledge that she may be overconfident, an eventuality that the individual is not aware about.

Our formal epistemic construction superimposes to any incomplete-information game, a layer of uncertainty about the players' understanding of the game itself, represented by appropriate beliefs operators. This allows us to provide two rigorous definitions of equilibrium. We stipulate that the play is in a *naive equilibrium* when, despite their possibly mistaken self-perceptions, the players are rational (i.e. utility-maximizing) and correctly anticipate each other's strategies, without attempting to make sense of them. The play is in a *sophisticated equilibrium,* when the players' anticipations of the opponent's equilibrium strategies are correct and furthermore they are rationalized by introspective reasoning. Specifically, we require that equilibrium strategies and rationality are common knowledge among the players. Clearly the set of sophisticated equilibria is a subset of the naive equilibrium set.

We show that naive equilibrium always exists and that it is equivalent to the equilibrium of a reduced Bayesian game with multiple priors, generated by the description of higher-order beliefs of overconfidence where the players agree to disagree on the informed player's ability.[5] In the context of our motivating signalling examples, however, a sophisticated equilibrium does not exist, because the objective requirement that strategies are in equilibrium is in conflict with the informed player's introspective rationalization of her opponent's strategy.[6]

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[4] Lewinsohn, Mischel, Chaplin, and Barton (1980) had subjects rate each other along personality dimensions, and found self-ratings significantly more positive than other subjects' ratings. Mentally depressed individuals displayed greater congruence between self-evaluations and evaluations of others (see also Brown, 1986; Taylor and Brown, 1988).

[5] Games without common priors are fairly common in epistemic game theory and in behavioral economics (see for instance, Brandeburger and Dekel, 1987; Brandeburger et al., 1993, Eyster and Rabin, 2003; Yildiz, 2003) and they are reviewed by Dekel and Gul (1997). One of the contributions of this paper is to make the role of multiple priors games precise in relation to overconfidence.

[6] This conflict has been recognized as a theoretical possibility also by Mertens and Zamir (1985), who introduced the so-called "consistency" requirement on their universal type space to rule it out. Their consistency requirement makes it impossible to study mistaken self perception, because it rules out high-order beliefs that naturally arise in this context.

We then turn from standard Bayesian games to population games. We imagine a large replica population of informed players. The distribution of ability in the population is common knowledge, and each informed player is aware that the other players' perception may be mistaken, while she thinks that her own self-perception is correct. These assumptions are consistent with the overwhelming experimental evidence that subjects overestimate their own characteristics relative to other subjects.[7] Before the game is played, an informed player is anonymously and randomly chosen. Her opponent's strategy does not depend on the player's individual characteristics, but only on the distribution of characteristics in the population. Hence there is no conflict in the mind of the player between her understanding of the game and the anticipation of her opponent's strategy. Our key result is that a sophisticated equilibrium always exists in these population games, and that the sophisticated equilibrium set and the naive equilibrium set coincide.

We conclude the analysis by studying the strategic value of mistaken self-perception. As long as players cannot directly observe each other's states of mind, and may only try to infer whether their opponents are overconfident or unbiased by observing their play, we show that they cannot be better-off by being overconfident. While ex-post this result is fairly intuitive, it is natural to investigate the value of mistaken beliefs in the context of behavioral economics. For example, experimental studies (such as Busenitz and Barney, 1997) finding that successful entrepreneurs are typically overconfident naturally suggest that overconfidence may be beneficial. Our results suggest the contrary: over all overconfident start up entrepreneurs should be less likely to be successful than unbiased ones. In fact, it is a well documented fact that entry of new companies is excessive, and that a large number of (overconfident) start up entrepreneurs are unsuccessful.[8]

This paper is presented as follows. Related literature is reviewed in the next section. The third section lays down basic notation and introduces a simple signalling example. The fourth and fifth sections give a precise account of the players' high-order beliefs and of equilibrium play, respectively in Bayesian and in population games. The sixth section determines the strategic value of overconfidence, and is followed by the conclusion. The Appendix presents omitted proofs, generalizes the analysis, and reformulates the problem in the language of universal types à-la Mertens and Zamir (1985).

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[7] Many experiments find that well over half of subjects judge themselves in possession of more desirable attributes than fifty percent of other individuals. For instance, people routinely overestimate themselves relative to others in IQ (Larwood and Whittaker, 1977), driving (Svenson, 1981), and the likelihood that they will have higher salaries and fewer health problems than others in the future (Weinstein, 1980).

[8] For example, in a sample of 2,994 entrepreneurs, Cooper, Woo, and Dunkelberg (1988) find that 81% believe their chances of success are at least 70%, and 33% believe their chances are a certain 100%. But in reality, about 75% of new businesses no longer exist after five years. The experimental study by Camerer and Lovallo (1999) suggests a relationship between excess entry of new companies and entrepreneurs' overconfidence.

2 Literature review

In the recent wave of research on behavioral economics, some contributions have explored the economic consequences of overconfidence. Camerer and Lovallo (1999) conduct an experimental study that suggests a relationship between excess entry of new companies and entrepreneurs' optimism with respect to their own ability, relative to the ability of competitors. Babcock and Loewenstein (1997) review experimental work that suggests that parties to legal disputes are reluctant to settle out of court because they hold overly optimistic beliefs about the merits of their case. In a model where individuals repeatedly choose whether to take informative tests and may override failed tests, Flam and Risa (1998) show that overconfident decision-makers eventually hold a higher status than unbiased ones, but because of longer periods of testing their ex-ante discounted utility is smaller. Benabou and Tirole (2002) and Koszegy (2000) show that an overconfident time-inconsistent individual may strategically choose to ignore information about her uncertain payoff.

Benabou and Tirole (2003) characterize some incentive schemes that an individual may use to manipulate her opponent's self-confidence to her own benefit. They further study some self-representation problems that are close in spirit to our motivating signalling environments. Specifically, they show that (i) depressed individuals may adopt a self-deprecation strategy, that (ii) in a game of common interest poor students may choose to call for leniency from an advisor who would like to impose high standards, and that (iii) individuals who have a piece of favorable verifiable information on their work may adopt a humble profile in the expectation that this information will be later independently verified. In the general equilibrium literature, Sandroni (2000) examines whether and when over-optimistic investors survive in the long run. Moscarini and Fang (2003) interpret workers self-confidence as their morale level, and assume that it is affected by wage contracts, which reveal firms' private information on the worker skills. They show that non-differentiation wage policies may arise, so as to preserve some workers morale. Yildiz (2003) analyzes a sequential bargaining game with multiple priors about the recognition process, the value of outside offers, or the break-down probability of bargaining, and shows that excessive optimism can cause delays in agreement.

3 The basic model

This section lays down basic notation and informally introduces the relevance of mistaken self-perception in a simple adverse-selection, signalling interaction. For simplicity, we consider only two players: it is straightforward to extend our analysis to the case with arbitrarily many players. Player 1 has private information about her own individual characteristics, summarized as $\theta \in \Theta$ (which we will denote for short as *ability),* where Θ is finite for simplicity. We may think that player 1 is a junior employee whose performance is being evaluated by the employer, or an entrepreneur who is applying for a credit line.

Her counterpart, player 2, is not informed about θ. It is common knowledge among the players that θ is distributed according to the distribution $\phi \in \Delta(\Theta)$. Each player j's plan of action is denoted by $s_j \in S_j$, and we specify nature's actions s_0 in the game, to allow for the possibility that the players learn about θ while playing the game. Nature's choice is denoted by $\mu \in \Delta(S_0)$, and it is common knowledge.[9] The strategy space is $S = S_0 \times S_1 \times S_2$, and for simplicity we assume it finite. The players' payoffs $u : S \times \Theta \to \mathbb{R}^2$ depend on the players' choices s_1 and s_2, on nature's choice s_0 and on player 1's ability. In the case where player 1 knows the value of θ, this situation is represented by the 2-player Bayesian game with common prior $G = (\Theta, \phi, S, \mu, u)$, and the associated equilibrium concept of Bayesian Equilibrium is well understood.

This game-theoretical formulation is quite general. The ability θ need not be a number, it may be a vector or a distribution over personal characteristics. Each player j's strategy s_j need not be a single action, but may be a complicated strategy, or even an infinite horizon policy. This framework thus applies both to one-shot and to dynamic interactions. By explicitly including nature's moves in the framework, we allow for the possibility of experimentation and learning by the players. Since our results hold for any equilibrium of a given Bayesian game, they hold a fortiori for any equilibrium refinement motivated by robustness, or by a particular sequence of choices in an underlying extensive form game.

In order to represent the case where player 1's perception may be mistaken, we distinguish between player 1's actual ability, denoted by θ, and her perception, denoted by $\hat{\theta}$.[10] Let $\phi \in \Delta(\boldsymbol{\Theta})$, where $\boldsymbol{\Theta} = \Theta^2$, denote the (full-support) distribution over the pairs $(\theta, \hat{\theta})$. We denote the players' utility by $u : S \times \Theta_1 \to \mathbb{R}^2$, where Θ_1 denotes the first component of space $\boldsymbol{\Theta}$. The distribution ϕ is commonly known, and whenever player 1's personal characteristics are $(\theta, \hat{\theta})$, she is informed only of $\hat{\theta}$. From the Bayesian game $G = (\Theta, \phi, S, \mu, u)$, we have obtained the augmented Bayesian game $\mathbf{G} = (\boldsymbol{\Theta}, \phi, S, \mu, u)$. Let ϕ_1 denote the marginal of ϕ on the ability component, and ϕ_2 the marginal of ϕ on the perception component.

In the next section we will formally represent the players' beliefs about the game they are playing. Here it suffices to say that, whenever the game $\mathbf{G} = (\boldsymbol{\Theta}, \phi, S, \mu, u)$ is played, player 1 believes that she is instead playing the game $\mathbf{G}^0 = (\boldsymbol{\Theta}, \phi^0, S, \mu, u)$, where the distribution $\phi^0 \in \Delta(\boldsymbol{\Theta})$ is derived from ϕ according to the rule that for any $\hat{\theta} \in \Theta$, $\phi^0(\hat{\theta}, \hat{\theta}) = \phi_2(\hat{\theta})$.[11] By construction, in game $\mathbf{G}^0$ player 1's ability coincides with her perception. It is immediate to see that the operator $(\cdot)^0$ maps Bayesian games into Bayesian games, and that $(\mathbf{G}^0)^0 = \mathbf{G}^0$. Our construction is appropriate to represent general games with mistaken beliefs. Overconfidence may be represented by considering games $\mathbf{G}$ such that $\phi(\{(\theta, \hat{\theta}) : \theta \leq \hat{\theta}\}) = 1$.

The next example studies a simplified version of the signalling problems discussed in the introduction. It shows that the possibility of mistaken self-perception

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[9] We want to allow for the possibility that players learn about their abilities if presented with clear evidence by nature. Thus we require that they are not mistaken about the move of nature μ.

[10] For simplicity, the perception θ_2 belongs to the same space as the ability θ_1. Alternatively, we could say that perception is a measure that belongs to $\Delta(\Theta)$. It is easy to see that all our results can be extended under this alternative formulation, but that they would be more difficult to state and interpret.

[11] It is not enough to say that each player with characteristics (θ_1, θ_2) believes that $\theta_1 = \theta_2$, because this allows for the possibility that she may think that, if her perception had been $\theta_2' \neq \theta_2$, then she would have believed that her ability would have been $\theta_1' \neq \theta_2'$.

may overturn our understanding of even one of the simplest information economics problems. The standard prediction of separating equilibrium may be upset when player 2 is aware that player 1 may be overconfident. This instance is formalized by assuming that, while player 1 thinks that she is playing game $\mathbf{G}^0$, player 2 knows that she is playing game $\mathbf{G}$.

Example 1. Player 1 's ability θ may be either high (θ_H) or low (θ_L). Player 2 prefers a high-profile policy y_H (such as promoting player 1 or granting her a line of credit) if the opponent's ability is high, and a low-profile policy (y_H) if the opponent's ability is low. Before player 2 chooses her policy, player 1 may either send a costly signal s_H (such as volunteering for a difficult task, or preparing a bold company plan) or a default signal s_L. The value of the policy y_H makes up for the cost of the signal if and only if player 1's ability is high.[12] As is well known, the key result of this model is that there is a separating equilibrium (the so-called Riley outcome) where type θ_H signals her ability by sending the costly signal s_H, and player 2 responds by implementing policy y_H if and only if she is sent the signal s_H.

In order to introduce mistaken beliefs, we say that player 1 may be overconfident (but not underconfident): $\phi(\theta_L, \theta_H) > 0$, and $\phi(\theta_H, \theta_L) = 0$. To make the issue relevant we say that player 2 prefers the low-profile policy y_L when she cannot tell if player 1 has high ability or is just overconfident:

$$\{y_L\} = \arg\max_y \phi(\theta_H, \theta_H) u_2(s, y, \theta_H) + \phi(\theta_L, \theta_H) u_2(s, y, \theta_L), \quad \text{for any } s. \tag{1}$$

Whenever player 2 is aware that player 1 may be overconfident, it is intuitive to see that the Riley outcome is disrupted. By contradiction, suppose that player 2 plays y_H if and only if she is sent s_H. Then the overconfident sender would play s_H, as she mistakenly thinks that this choice maximizes her utility. However, Condition (1) implies that player 2 plays y_L upon receiving s_H. We shall formalize this intuition in the next section. □

4 The players' understanding of the game

This section formally represent the players' understanding of the game they are playing. We fix any Bayesian game $\mathbf{G}$ and superimpose a layer of uncertainty about the players' understanding of the game itself, represented by appropriate beliefs operators.[13] Since the players' perception of their own ability may be mistaken, the players' beliefs on each other's understanding of the game cannot be common knowledge. As a result, the original Bayesian game augmented with this layer of uncertainty does not constitute itself a game. But our augmented model provides a precise framework to describe equilibrium play, and determine whether high-order

[12] While this game is sequential in essence, we can use the framework presented above, because this game may be represented as a Bayesian game with a standard transformation.

[13] An introduction to the formal representation of knowledge and beliefs may be found in Dekel and Gul (1997).

beliefs and mistaken self-perception preclude the players from making sense of each other's strategies. Obviously this can only be assessed from the standpoint of an external observer: players whose self-perception is mistaken are not aware of their own mistakes and cannot be aware that this induces a conflict with equilibrium conjectures.[14]

Formally, we introduce an underlying (compact metric) state space Ω, an associated Borel σ-algebra on Ω, denoted by $\mathcal{B}(\Omega)$, and a nature's choice p on the probability space $(\Omega, \mathcal{B}(\Omega))$. We say that the nature selects the Bayesian game $\mathbf{G}(\omega)$ as a function of the state of the world. We thus introduce the measurable relation $\tilde{\mathbf{G}} : \omega \mapsto \mathbf{G}$, and the event $[\mathbf{G}] = \{\omega | \tilde{\mathbf{G}}(\omega) = \mathbf{G}\}$. We require that Ω and $\tilde{\mathbf{G}}$ are picked so that $\tilde{\mathbf{G}}$ is surjective: for any game $\mathbf{G}$, there must exist a state ω such that $\tilde{\mathbf{G}}(\omega) = \mathbf{G}$. Given the game $\mathbf{G}(\omega)$, the nature then selects the individual characteristics $(\theta, \hat{\theta})$ according to the distribution $\phi(\omega)$, and the strategy s_0 according to the distribution $\mu(\omega)$.

In practice, for each state of the world, the nature determines what game $\mathbf{G}$ the players will be playing, this description includes not only a distribution of possible abilities of the informed player, but also the relation between her perceived ability and her actual ability.[15]

The simplest way to represent players' knowledge of the game that they are playing in any different state of the world is by means of the information structures $P_j : \Omega \to \mathcal{B}(\Omega), j = 1, 2$.[16] In order to capture mistaken self-perception, we allow for the possibility that j's information is mistaken, i.e. $\omega \notin P_j(\omega)$. We denote by *information model* the collection $\mathcal{I} = (\Omega, P_1, P_2, p)$. It is useful to introduce the certainty operators $C_j : \mathcal{B}(\Omega) \to \mathcal{B}(\Omega)$, such that, for any $E \in \mathcal{B}(\Omega)$, $C_j E = \{\omega | P_j(\omega) \subseteq E\}$ for $j = 1, 2$.[17] The operator C_j associates to each event $E \subseteq \Omega$ the set of states in which player j believed that the event E occurred with certainty.

[14] Our belief construction is non-standard as it separates the subjective understanding of players from the account of an omniscient external observer. This is unavoidable if one wants to study mistaken self perception in a knowledge-based framework, because common knowledge of information structures prevents mistaken self-perception. Since the Mertens and Zamir (1985) model closes the belief system without explicitly assuming common knowledge of beliefs, it does not prevent the study of mistaken self-perception, but only at the cost of making indirect arguments that make a precise account much less transparent. Our analysis is reformulated in the language of Mertens and Zamir (1985) in the last section of the Appendix.

[15] While this implies that the description of the state of the world Ω is incomplete, as it does not capture all uncertainty in the game, it is easy to see how to expand the state space to account for nature's choice of θ_2 and s_0. We adopt this "reduced" formulation of the state space to simplify the analysis, and to focus our attention on the players' understanding of the game.

[16] When Ω is a finite set, information structures are defined as $P_j : 2^{\Omega} \to 2^{\Omega}$. This definition is an appropriate extension to transfinite spaces because it is straightforward to show that if $P_j(\omega) \in \mathcal{B}$, then the restriction of $\mathcal{B}$ onto $P_j(\omega)$ is a σ-algebra (in fact the Borel σ-algebra on $P_j(\omega)$).

[17] Unlike the more well-known knowledge operators, certainty operators do not need to satisfy the "truth" axiom $C(E) \subseteq E$, for any E. Instead of deriving certainty operators from information structures, one could also present certainty operators as a primitive concept. However this would make the exposition more cumbersome, as the representation $P_j : \Omega \to \mathcal{B}(\Omega)$ embeds consistency requirements that would need to be explicitly spelled out if C_j were to be treated as primitive (see Dekel and Gul, 1997).

While certainty operators differ from knowledge operators because they allow that one's information is mistaken, the construction of the common certainty operator $CC : \mathcal{B}(\Omega) \to \mathcal{B}(\Omega)$ is analogous to the construction of the common knowledge operator (see Dekel and Gul, 1997; Monderer and Samet, 1989). We define the sequence of operators $\{C^n\}_{n\geq 0}$, where for any n, $C^n : \mathcal{B}(\Omega) \to \mathcal{B}(\Omega)$, and specifically, $C^0 E = C_1 E \cap C_2 E$, and $C^n E = C_1(C^{n-1}E) \cap C_2(C^{n-1}E)$ for any $n \geq 1$. Let the event "E is common certainty" be defined as $CCE = \cap_{n\geq 0} C^n E$.

When considering high-order beliefs of overconfidence, for ease of exposition, we focus on the crucial case that, as anticipated in the introduction, player 2 is aware that player 1's perception may be mistaken, and player 1 is not aware of this.[18] This event is described by means of an iterative construction similar to the construction of common certainty operator.

Suppose that the players are playing an arbitrary Bayesian game $\mathbf{G}$. We define the events $\kappa_1^0[\mathbf{G}] = C_1[\mathbf{G}^0] \cap C_2[\mathbf{G}^0]$, $\kappa_2^0[\mathbf{G}] = C_1[\mathbf{G}^0] \cap C_2[\mathbf{G}]$, and iteratively, for any $n \geq 1$, $\kappa_2^n[\mathbf{G}] = C_2\kappa_2^{n-1}[\mathbf{G}]$, and $\kappa_1^n[\mathbf{G}] = C_1\kappa_1^{n-1}[\mathbf{G}]$. The set of states of the world where player 2 knows that she is playing game $\mathbf{G}$, that player 1 thinks that player 2 thinks that player 1's perception is correct (i.e. player 1 thinks that player 2 thinks that she is playing game $\mathbf{G}^0$), that player 2 knows that player 1 believes that her perception is correct, (and so on...) is described by the event

$$E[\mathbf{G}] = [\mathbf{G}] \cap C_1[\mathbf{G}^0] \cap C_2[\mathbf{G}] \cap [\cap_{n\geq 1} (\kappa_1^n[\mathbf{G}] \cap \kappa_2^n[\mathbf{G}])].$$

If the event $E[\mathbf{G}]$ were empty for all information models, the task of describing the play when the event $E[\mathbf{G}]$ takes place would be meaningless. The following lemma exploits the iterated construction of the event $E[\mathbf{G}]$ to show that there exist information models such that it is non-empty.

Lemma 1. *There exist information models $\mathcal{I} = (\Omega, P_1, P_2, p)$ such that, for any game $\mathbf{G}$, the event $E[\mathbf{G}]$ is non-empty.*

The players' strategies also depend on the underlying state of the world. Player 2's strategy consists of the function $\tilde{\sigma}_2 : \Omega \to \Delta(S_2)$, measurable with respect to the information structure P_2. When choosing her action, player 1 is informed of $\hat{\theta} \in \Theta$. Her strategy is thus expressed by the function $\tilde{\sigma}_1 : \Omega \to \Delta(S_1)^\Theta$, measurable with respect to the information structure P_1. We denote by σ_1 any arbitrary element of $\Delta(S_1)^\Theta$ and by σ_2 any arbitrary element of $\Delta(S_2)$. The event that player j plays a strategy σ_j is denoted by the notation $[\sigma_j] = \{\omega | \tilde{\sigma}_j(\omega) = \sigma_j\}$.[19]

Player j acts rational in a given state of the world if she maximizes her utility on the basis of her information. Formally, we let the notation $Supp(\tilde{\sigma}_j(\cdot, \omega))$ denote

[18] In the Appendix we consider general descriptions of high-order beliefs.

[19] Unlike Auman and Brandeburger (1995), in this formulation player i does not know the specific action a_i she takes at a certain state ω, but only the mixed strategy σ_i. It is assumed that after choosing the state ω (which identifies which game $\mathbf{G}$ is played, and which mixed strategies σ are taken by the players), nature moves again in the game $\mathbf{G}$, operating the randomizing device identified by σ. It will be seen that this formulation greatly simplifies our analysis.

the support of strategy $\tilde{\sigma}_j(\cdot,\omega)$, and we define the events:

$$[R_2] = \left\{\omega \left| Supp(\tilde{\sigma}_2(\cdot,\omega)) \subseteq \arg\max_{s_2'} E\left[u_2(s_1, s_2', \theta)|P_2(\omega)\right]\right.\right\} \quad (2)$$

$$[R_1] = \left\{\omega \left| Supp(\tilde{\sigma}_1(\cdot,\hat{\theta},\omega)) \subseteq \arg\max_{s_1'} E\left[u_1(s_1', s_2, \theta)|\hat{\theta}, P_1(\omega)\right]\right.\right\}. \quad (3)$$

We say that the play is *in equilibrium* if the players are rational and correctly anticipate each other's strategies in the game. Without imposing further requirements, one remains agnostic regarding the way in which these anticipations are formed. Alternatively, one may require that the players formulate these anticipations by introspective reasoning. To capture this distinction we introduce two concepts of equilibrium.

First, given the description of knowledge $E[\mathbf{G}]$, we define as *naive equilibrium* any profile $\sigma = (\sigma_1, \sigma_2)$ such that, upon knowing that player 2 plays σ_2, player 1 rationally chooses σ_1, and vice versa. Define the events $[\sigma] = [\sigma_1] \cap [\sigma_2]$, and $[R] = [R_1] \cap [R_2]$.

Definition 1. *For any arbitrary information model $\mathcal{I}$ and game $\mathbf{G}$, the profile σ is a naive equilibrium for $E[\mathbf{G}]$ if the event $E[\mathbf{G}] \cap [R] \cap [\sigma] \cap C^0[\sigma]$ is non-empty.*

In practice, when the event $E[\mathbf{G}] \cap [R] \cap [\sigma] \cap C^0[\sigma]$ takes place, the players play a naive equilibrium σ of the game $\mathbf{G}$, where they correctly anticipate the opponent's strategies and maximize their utility, but do not necessarily rationalize the choice of their opponents on the basis of their perception of the game that they are playing.

Second, we define as "sophisticated" an equilibrium play that is consistent with introspective reasoning. For any game $\mathbf{G}$, and description of knowledge of the game $E[\mathbf{G}]$, we say that the profile σ is a *sophisticated equilibrium* if it is possible that the players' understanding of the game is described by $E[\mathbf{G}]$, while at the same time it is common certainty that the players are rational, and that the play is σ. For any event $E \in \mathcal{B}(\Omega)$, we introduce the notation $CC^*[E] = [E] \cap CC[E]$.

Definition 2. *For any arbitrary information model $\mathcal{I}$ and game $\mathbf{G}$, the profile σ is a sophisticated equilibrium for $E[\mathbf{G}]$ if the event $E[\mathbf{G}] \cap CC^*[[R] \cap [\sigma]]$ is non-empty.*

In practice, when the event $E[\mathbf{G}] \cap CC^*[[R] \cap [\sigma]]$ occurs, the players play a sophisticate equilibrium σ of the game $\mathbf{G}$. They do not only correctly anticipate the opponent's strategies and maximize their utility. They also rationalize the choice of their opponents on the basis of their perception of the game that they are playing. Since $CC^*[[R] \cap [\sigma]] \subseteq [R] \cap [\sigma] \cap C^0[\sigma]$ for any profile σ, the set of sophisticated equilibria is a subset of the naive equilibrium set.

We now proceed with our first characterization result. We show that naive equilibrium is equivalent to the equilibrium of a reduced Bayesian game with multiple priors, generated by the description of higher-order beliefs of overconfidence where the players agree to disagree on the informed player's characteristics.

Formally, we show that it is possible to construct an information model $\mathcal{I}$ such that, for any game $\mathbf{G}$, the naive equilibria for $E[\mathbf{G}]$ coincide with the equilibria of the Bayesian game with subjective priors $\mathbf{G}' = (\Theta, \phi, \phi^0, S, \mu, u)$, where ϕ identifies both the move of nature and the prior of player 2, and ϕ^0 identifies the prior of player 1. The definition of Bayesian equilibrium in games of subjective priors is omitted as it is well understood.

Proposition 1. *There is an information model $\mathcal{I} = (\Omega, P_1, P_2)$, such that for any game $\mathbf{G} = (\Theta, \phi, S, \mu, u)$, any strategy profile σ is a naive equilibrium for $E[\mathbf{G}]$ if and only if σ is an equilibrium of the associated (subjective-prior) game $\mathbf{G}' = (\Theta, \phi, \phi^0, S, \mu, u)$.*[20]

It is well known that equilibrium exists in all finite Bayesian games with subjective priors (see, for example, Dekel and Gul, 1997). It follows that there is an information model $\mathcal{I}$ such that for any finite game $\mathbf{G}$, there exists a naive equilibrium for $E[\mathbf{G}]$.

We conclude this section by showing that one can construct games G such that for any information model $I = (\Omega, P_1, P_2)$, there does not exist any sophisticated equilibrium for $E[\mathbf{G}]$. Specifically, we consider a simplified version of the signalling games discussed in the introduction, and show that an overconfident sender cannot correctly anticipate the receiver's equilibrium strategy.

Example 2. Consider a signalling game $\mathbf{G} = (\Theta, \phi, S, \mu, u)$ as described in Example 1. Let player 2's payoff be such that for any s, $u_2(s, y_H, \theta_H) > u_2(s, y_L, \theta_H)$, $u_2(s, y_L, \theta_L) > u_2(s, y_H, \theta_L)$, $\{y_L\} = \arg\max \phi(\theta_H, \theta_H) u_2(s, y, \theta_H) + \phi(\theta_L, \theta_H) u_2(s, y, \theta_L)$, and $\{y_H\} = \arg\max(\phi(\theta_H, \theta_H) + \phi(\theta_L, \theta_H)) u_2(s, y, \theta_H) + \phi(\theta_L, \theta_L) u_2(s, y, \theta_L)$. Let player 1's payoff be such that for any y, $u_1(s_L, y, \theta_L) > u_1(s_H, y, \theta_L)$, and that $u_1(s_L, y_H, \theta_H) < u_1(s_H, y_L, \theta_H) < u_1(s_L, y_L, \theta_H) < u_1(s_H, y_H, \theta_H)$.

Player 1 believes that the signalling game $\mathbf{G}^0$ is played, where player 1's ability is high with probability $\phi(\theta_H, \theta_H) + \phi(\theta_L, \theta_H)$ and low with probability $\phi(\theta_L, \theta_L)$, and that this is common knowledge. Simple calculations show that the unique Bayesian equilibrium of this signalling game is the Riley separating outcome: if player's ability is high, she signals this by sending the signal s_H, whereas if it is low, she sends the signal s_L. Therefore in any sophisticated equilibrium, it must be the case that player 1 plays s_H when overconfident. But then, as in Example 1, the unique best response of player 2 is to play y_L. However, if player 1 anticipates that player 2 play y_L, then she should play s_L. This concludes by contradiction that there is no sophisticated equilibrium for this signalling game.

Formally, first note that for any $\omega \in C_2[\mathbf{G}] \cap [R_2]$ and any distribution $\sigma_1'(\omega) \in \Delta(S_1)^{\Theta}$, under our assumptions, player 2 plays y_L for any s and hence $\tilde{\sigma}_2(y_L|s, \omega) = 1$ for any s. Letting $\sigma_2(y_L|s) = 1$, we obtain that

[20] If the underlying game $\mathbf{G}$ has complete information, it follows that σ is a naive equilibrium if and only if it is a Nash Equilibrium of $\mathbf{G}$. Aumann and Brandenburger (1995) show that, in 2-player games, Nash Equilibrium conjectures follow from public knowledge of payoffs, rationality, and conjectures, where a player j's conjecture is a conditional distribution on the actions of her opponent, player l. Our requirement that each player j knows that player l surely plays σ_l is stronger than just requiring that player j's belief over player l's action coincides with σ_l. This allows to obtain Nash Equilibrium without requiring public knowledge of rationality (see also Aumann and Brandenburger, 1995, p. 1167).

$C_2[\mathbf{G}] \cap [R_2] \subseteq [\sigma_2]$, and because $E[\mathbf{G}] \subseteq C_2[\mathbf{G}]$, that $E[\mathbf{G}] \cap [R_2] \subseteq [\sigma_2]$, so that $E[\mathbf{G}] \cap [R_2] \cap [\sigma_2'] = \emptyset$ for any strategy $\sigma_2' \in \Delta(S_2)$, $\sigma_2' \neq \sigma_2$. □

Second, for any $\omega \in C_1[\mathbf{G}^0] \cap [R_1] \cap C_1[\sigma_2]$, because player 1 best-responds to y_L by playing s_L, $\tilde{\sigma}_1(s_L|\hat{\theta}, \omega) = 1$ for any $\hat{\theta} \in \{\theta_L, \theta_H\}$. So letting $\sigma_1(s_L|\hat{\theta}) = 1$, for any $\hat{\theta} \in \{\theta_L, \theta_H\}$, we obtain that $C_1[\mathbf{G}^0] \cap [R_1] \cap C_1[\sigma_2] \subseteq [\sigma_1]$, and since $E[\mathbf{G}] \subseteq C_1[\mathbf{G}^0]$, that $[\mathbf{G}] \cap [R_1] \cap C_1[\sigma_2] \subseteq [\sigma_1]$. So for any strategy $\sigma_1' \in \Delta(S)^{\Theta}$, $\sigma_1' \neq \sigma_1$, it follows that $E[\mathbf{G}] \cap [R_1] \cap C_1[\sigma_2] \cap [\sigma_1'] = \emptyset$.

Third, for any $\omega \in C_2[\mathbf{G}^0] \cap [R_2] \cap C_2[\sigma_1]$, under our assumptions, player 2 plays y_H for any s, and hence $\tilde{\sigma}_2(y_L|s, \omega) = 0$. This implies that $C_1C_2[\mathbf{G}^0] \cap C_1[R_2] \cap C_1C_2[\sigma_1] \cap C_1[\sigma_2] = \emptyset$, and since $E[\mathbf{G}] \subseteq C_1C_2[\mathbf{G}^0]$, that $E[\mathbf{G}] \cap C_1[R_2] \cap C_1C_2[\sigma_1] \cap C_1[\sigma_2] = \emptyset$.

The above three conclusions imply that for any strategy pair $\sigma' = (\sigma_1', \sigma_2')$,

$$E[\mathbf{G}] \cap [R] \cap [\sigma'] \cap C_1[\sigma_2'] \cap C_1[R_2] \cap C_1C_2[\sigma_1'] = \emptyset,$$

and hence $E[\mathbf{G}] \cap CC^*[[R] \cap [\sigma']] = \emptyset$: a sophisticated equilibrium does not exist. □

We conclude the section by briefly discussing this result in the context of the signalling problem of an overconfident junior employee discussed in the introduction. On the one hand, the employee is tempted to try and signal her perceived ability by volunteering for the most challenging tasks, in the expectation of being promoted by the company. On the other hand, suspecting that the individual may be overconfident, the management is likely to discount the value of this risky conduct, and choose not to promote the employee. But in equilibrium, the employee should anticipate this and choose a more humble course of action. Still, it is hard to see how the junior employee can make sense of the management equilibrium policy. This is policy is based on the knowledge that she may be overconfident, an eventuality that the individual is not aware about. A conflict is generated between the objective requirement that strategies are in equilibrium, and the worker's introspective rationalization of the management's strategy. Hence a sophisticated equilibrium does not exist. In practice, if the overconfident employee somehow manages to figure out the company policy, she would not be able to make any sense of it.

5 Population games

In this section we turn from standard Bayesian games to population games. We imagine a continuous population of informed players indexed by $i \in I = [0, 1]$. Each pair $(\theta, \hat{\theta})$ is interpreted as the actual characteristics of any individual i in the population I. The characteristics are assigned by the (measurable) function ζ: $I \to \Theta$, where $\zeta(i)$ denotes the ability of sender i, and $\hat{\zeta}(i)$ denotes her perception. Given the assignment ζ, the distribution $\phi(\zeta) : \Theta \to [0, 1]$ over the pairs $(\theta, \hat{\theta})$ is derived according to the rule:

$$\phi(\zeta)(\theta, \hat{\theta}) = \nu\{i : \zeta(i) = (\theta, \hat{\theta})\}, \tag{4}$$

where ν denotes the Lebesgue measure. Intuitively, we imagine a large replica population of informed players where the relative fractions of individuals with different abilities and perceptions are the same as the distribution described by ϕ.

The sequence of moves in population games is as follows. At the first period, a single informed player $i \in I$ is randomly chosen by nature according to the uniform distribution on $[0,1]$. At the second period, player i plays against nature and player 2 a Bayesian game $\mathbf{G} = (\Theta, \phi, S, \mu, u)$. The strategy space of each player i coincides with S_1, her utility is denoted by $u^i(s, \zeta)$ and coincides with $u_1(s, \zeta(i))$ for each strategy profile $s \in S$. We have associated a population game $\Gamma = (\Theta, I, \zeta, \mu, S, u)$ to each Bayesian game $\mathbf{G}$.

As in the previous section, we represent the players' knowledge of the game by means of an information model $\mathcal{I}$, and a measurable surjective relation $\tilde{\Gamma} : \omega \mapsto \Gamma$, where the relation $\tilde{\Gamma}$ includes the relation $\tilde{\zeta}$: $\omega \mapsto \zeta$. For any Γ and ζ, we define the events $[\Gamma] = \{\omega | \tilde{\Gamma}(\omega) = \Gamma\}$, and $[\phi(\zeta)] = \{\omega | \phi(\tilde{\zeta}(\omega)) = \phi(\zeta)\}$. Nature first chooses the population game Γ, then selects player i according to ν, and finally she takes μ in the game $\mathbf{G}$. The certainty and common certainty operators are constructed as in the previous section. For any game $\Gamma = (\Theta, I, \zeta, \mu, S, u)$, we restrict attention to information models $\mathcal{I}$ for which the collection (Θ, I, μ, S, u) is common certainty on $[\Gamma]$.

We want to represent instances where player 2 is not able to distinguish the identity of the players in the pool I, but knows the aggregate distribution of the individual characteristics $(\theta, \hat{\theta})$. This makes sure that player 2's equilibrium strategy does not depend on the player's individual characteristics, but only on the distribution of characteristics in the population. We thus make the following assumption on player 2's information.

Assumption 1. *The information model* $\mathcal{I}= (\Omega, (P^i)_{i \in I}, , P_2, p)$ *is such that* $P_2(\omega) \subseteq [\phi(\zeta)]$ *for any game* Γ *and any state* $\omega \in [\Gamma]$; *and such that* $p(\zeta(\iota(B)) \in \Theta' | P_2(\omega)) = p(\zeta(B) \in \Theta' | P_2(\omega))$ *for any* ν*-preserving isomorphism* $\iota : \mathcal{B}[0,1] \to \mathcal{B}[0,1]$, *any set* $B \in \mathcal{B}[0,1]$, *and any set* $\Theta' \subseteq \Theta$.

The key assumption of our construction is that each informed player acknowledges that the other informed players are on average overconfident, while she thinks that her own self-perception is correct. Formally, while each informed player i believes that her ability $\zeta(i)$ coincides with perception $\hat{\zeta}(i)$.

Assumption 2. *For any game* Γ, *the information model* $\mathcal{I}= (\Omega, (P^i)_{i \in I}, P_2, p)$ *is such that for any* $\omega \in [\Gamma]$, *and any* $i \in I$, $P^i(\omega) \subseteq \{\omega' : \tilde{\zeta}(\omega')(i) = (\hat{\zeta}(i), \hat{\zeta}(i))\} \cap [\phi(\zeta^i)]$, *where the function* $\zeta^i : [0,1] \to \Theta$ *is such that* $\zeta^i(j) = \zeta(j)$ *for any* $j \neq i$, *and* $\zeta^i(i) = (\hat{\zeta}(i), \hat{\zeta}(i))$.

Under these assumptions, by construction, any player i is overconfident (and unaware of this) whenever $\zeta(i) \neq \hat{\zeta}(i)$. It is immediate to see, in fact, that for any perception $\hat{\theta}$, at any state ω such that $\tilde{\zeta}(\omega')(i) = (\theta, \hat{\theta})$, it is the case that $\omega \in C_i\{\omega | \tilde{\zeta}(\omega')(i) = (\hat{\theta}, \hat{\theta})\}$, regardless of player i's actual ability θ. Nevertheless, the following Lemma verifies that there are information models satisfying our assumptions, where the players always share common knowledge of the aggregate distribution of individual characteristics.

Lemma 2. *There is an information model* $\mathcal{I}= (\Omega, (P^i)_{i\in I}, P_2, p)$ *satisfying Assumptions 1 and 2 such that for any game* $\Gamma = (\Theta, I, \zeta, \mu, S, u)$, *it is the case that* $[\Gamma] \subseteq CC^*[\phi(\zeta)]$.

For any $i \in I$, we let the function $\tilde{\sigma}^i : \Omega \to \Delta(S_1)$, measurable with respect to the information structure P^i, be the strategy of player i. As in the previous section, player 2's strategy is described by $\tilde{\sigma}_2 : \Omega \to \Delta(S_2)$ measurable with respect to P_2. We denote by σ_1 any arbitrary element of $\Delta(S_1)^I$ and by σ_2 any arbitrary element of $\Delta(S_2)$. Since the players in population I and we have assumed that player 2's information satisfies an anonymity requirement, it is natural to restrict attention to symmetric strategy profiles, where all players with the same ability and assessment choose the same strategy.

Definition 3. *A strategy profile* $\sigma \in \Delta(S_1)^I \times \Delta(S_2)$ *is symmetric if for any pair* $(i, j) \in I^2$, *it is the case that* $\sigma^i = \sigma^j$ *whenever* $\hat{\zeta}(i) = \hat{\zeta}(j)$.

The definitions of the events $[\sigma_1]$, $[\sigma_2]$, $[\sigma]$, $[R_2]$, $[R_s^i]$ for any i, and $[R]$ are immediately extended from the analogous definition in the previous section, and so are the definitions of naive and sophisticated equilibrium. Because of Lemma 2, for each game Γ, we are interested in the naive and sophisticated equilibria associated with the event $[\Gamma]$, under information models satisfying Assumptions 1 and 2.

Our key result is that a sophisticated equilibrium always exists in these population games, and that the sophisticated equilibrium set and the naive equilibrium set coincide. In order to derive this result, we first associate, to any population game $\Gamma = (\Theta, I, \zeta, \mu, S, u)$, a 2-player Bayesian game $\mathbf{G}' = (\Theta, \phi, \phi^0, S, \mu, u)$, where for any pair $(\theta, \hat{\theta})$, nature's choice (and player 2's prior) is $\phi(\theta, \hat{\theta}) = \nu\{i : \zeta(i) = (\theta, \hat{\theta})\}$, and the prior of player 1 is $\phi^0(\hat{\theta}, \hat{\theta}) = \nu\{i : \hat{\zeta}(i) = \hat{\theta}\}$. Each symmetric strategy profile σ of a game Γ identifies a unique strategy profile σ' of $\mathbf{G}'$ according to the rule $\sigma_2' = \sigma_2$, and $\sigma_1'(\cdot|\hat{\theta}) = \sigma^i$ if $\hat{\zeta}(i) = \hat{\theta}$. Up to equivalence classes, each strategy profile σ' of $\mathbf{G}'$ identifies a symmetric strategy profile σ of Γ.

Proposition 2 below shows that, within the restrictions imposed by Assumptions 1 and 2, it is possible to construct information models such that the symmetric naive equilibria of any game Γ coincide with the subjective Bayesian equilibria of the associated game $\mathbf{G}'$. Moreover, the symmetric naive equilibria of any game Γ coincide with the symmetric sophisticated equilibria of Γ. This immediately implies the existence of sophisticated symmetric equilibrium in all population games with finite characteristics and strategy spaces.

Proposition 2. *There is an information model* $\mathcal{I}= (\Omega, (P^i)_{i\in I}, P_2, p)$ *satisfying Assumptions 1 and 2 such that, for any game* $\Gamma = (\Theta, I, \zeta, S, \mu, u)$, *the set of symmetric sophisticated equilibria for* $[\Gamma]$ *coincides with the set of symmetric naive equilibria for* $[\Gamma]$, *which is isomorphic (up to equivalence classes) to the set of equilibria of the subjective-prior game* $\mathbf{G}' = (\Theta, \phi(\zeta), (\phi(\zeta))^0, S, \mu, u)$.

Unlike the case studied in the previous section, no conflict arises in the mind of the player between her understanding of the game and the anticipation of her opponent's strategy. In practice, she rationalizes the equilibrium choice of her opponent with the following consideration: "My opponent is acting as if I were not as good as I am, because there are many overconfident individuals in my population, and my opponent does not know that I am not one of them..."

This construction is apt to represent our signalling and overconfidence examples discussed in the introduction.

Example 3. Reconsider the signalling problem described in Example 1, and formalize it as a population game $\Gamma = (\Theta, I, \zeta, \mu, S, u)$ such that $\phi(\zeta)(\theta_L, \theta_H) > 0$ and $\phi(\zeta)(\theta_H, \theta_L) = 0$. Each player i in population 1 perceives that her ability is $\hat{\zeta}(i)$, while in fact it is $\zeta(i)$. Because of anonymity, however, she also knows that player 2's strategy depends only on the distribution of abilities and perceptions $\phi(\zeta)$, and not on her own specific individual ability. The distribution $\phi(\zeta)$ is common knowledge among all players.

We shall now show that in the unique symmetric sophisticated and naive equilibrium of this game, all players i in population 1 play s_L, and player 2 responds by playing y_L. First, note that for all players i such that $\hat{\zeta}(i) = \theta_L$, playing s_L is strictly dominant. Second, suppose that all players i such that $\hat{\zeta}(i) = \theta_H$ played s_H with positive probability. Then because $\{y_L\} = \arg\max \phi(\zeta)(\theta_H, \theta_H)u_2(s, y, \theta_H) + \phi(\zeta)(\theta_L, \theta_H)u_2(s, y, \theta_L)$, player 2 would respond to the signal s_H by playing y_L. Because the distribution $\phi(\zeta)$ is common knowledge among all players, each player i would anticipate player 2's best response and conclude that it is optimal play s_L. Hence there is no symmetric equilibrium where any player i in population 1 with $\hat{\zeta}(i) = \theta_H$ plays s_H with positive probability.

Third, suppose that all players i in population 1 play s_L. Then player 2 responds to the signal s_L by playing y_L. Say that player 2 plays y_L upon receiving the signal s_H off the equilibrium path: for this to be the case, it is enough that her off-path beliefs are induced by symmetric strategy σ_1, because then σ^i is constant for all player i with the same perception $\hat{\zeta}(i)$ and $\{y_L\} = \arg\max_y \phi(\theta_H, \theta_H)u_2(s_H, y, \theta_H) + \phi(\theta_L, \theta_H)u_2(s_H, y, \theta_L)$. Then, because the distribution $\phi(\zeta)$ is common knowledge, the players in population 1 correctly anticipate player 2's response, and hence have no reason to deviate to s_H. Hence the strategy profile such that all players in population i play s_L and player 2 responds by playing y_L is the unique equilibrium. □

We conclude this section by discussing our findings in relation to the signalling problems discussed in the introduction. Consider the scenario where a start up entrepreneur applies for credit. The analysis of Example 3 suggests that a sophisticated overconfident entrepreneur will strategically choose to present a conservative and sound company plan. She does not question her own skills, but she is aware that many applicants for credit are self-deluded in their ambitions, and she does not want to be confused with them. Thus our analysis provides a rigorous theory for strategic humbleness (on this, see also Benabou and Tirole, 2003).

While the problem of a start up entrepreneur applying for credit can be appropriately modelled as a population game, this is less plausible for a scenario where an overconfident employee would like to signal her exceptional abilities to the management, especially in relatively small companies. Whenever this scenario is more appropriately represented by the construction in Example 2, our analysis suggests that an overconfident employee would not be able to make any sense of company policy, even if she manages to figure it out. By comparing the analyses of Example 2 and 3, we have therefore uncovered a key difference between our two motivating problems. Previous work that did not include the possibility of overconfidence deemed these two problems as equivalent manifestations of the same adverse-selection model.

6 Utility comparisons

This section studies the strategic value of mistaken beliefs. It is not difficult to conceive environments where *prima facie* intuition suggests that overconfidence is beneficial. Experimental findings suggest that successful entrepreneurs are overconfident with respect to the general population (see for instance, Busenitz and Barney, 1997). It is tempting to conclude that such overconfidence is beneficial as it allows these individuals to work hard, overcome obstacles and eventually make their companies successful. But our analysis yields a different result. As long as players cannot directly observe each other's states of mind, and may only try to infer whether their opponents are overconfident or unbiased by observing their play, we show that they cannot be better-off by being overconfident. Over all, overconfident start up entrepreneurs should be less likely to be successful than unbiased ones. This suggests that the observed relation between overconfidence and entrepreneurial success is largely due to a selection bias: also unsuccessful entrepreneurs are likely overconfident. In fact, it is a well documented fact that entry of new companies is excessive and that a large number of (overconfident) start up entrepreneurs are unsuccessful (see for instance Cooper et al., 1988; Camerer and Lovallo, 1999).

The intuition for our result is simple and compelling. In equilibrium, each player correctly anticipates her opponent's strategy, which must be the same regardless of the player's state of mind. While the player's choice depends on her perceived characteristics, her actual utility depends on her actual characteristics. If overconfident, the player mistakenly plays a strategy that would be optimal if her own characteristics were better than they actually are. Hence her actual utility cannot be larger than the utility of an unbiased player with the same characteristics, correctly playing her optimal strategy.

For any population game $\Gamma = (\Theta, I, \zeta, S, \mu, u)$, and any symmetric equilibrium σ, we introduce the notation $u^i(\sigma)$ which identifies player i's actual payoff (in ex-ante terms) at the equilibrium σ:

$$\text{for any } i,\ u^i(\sigma) = \sum_{s_2 \in S_2} \sum_{s_1 \in S_1} \sum_{s_0 \in S_0} u^i(s_0, s_1, s_2, \zeta)\mu(s_0)\sigma^i(s_1)\sigma_2(s_2).$$

Proposition 3. *In any symmetric equilibrium σ of any population game $\Gamma = (\Theta, I, \zeta, S, \mu, u)$, for any level of ability $\theta \in \Theta$, and any pair of players (i, j) such* *that* $\zeta(i) = (\theta, \theta)$ *and* $\zeta(j) = (\theta, \theta')$ *with* $\theta' \neq \theta$*, it must be the case that* $u^i(\sigma) \geq u^j(\sigma)$.[21]

Proof. Consider any symmetric equilibrium σ of any population game $\Gamma = (\Theta, I, \zeta, S, \mu, u)$. By Proposition 2, σ identifies a naive equilibrium σ for $E[\mathbf{G}]$. For any $\omega \in [\Gamma] \cap [R_1] \cap C^i[\sigma_2]$, player i plays strategy $\tilde{\sigma}^i(\cdot|\omega)$, such that $\tilde{\sigma}^i(s_1|\omega) > 0$ only if

$$s_1 \in \arg\max_{s_1' \in S_1} \sum_{s_2 \in S_2} \sum_{s_0 \in S_0} u_1(s_0, s_1', s_2, \hat{\theta}) \mu(s_0) \sigma_2(s_2). \tag{5}$$

It follows that any $s_1 \in Supp(\sigma^i)$ must satisfy Condition (5).

For any arbitrary level of activity θ, pick any pair of players $(i, j) \in I^2$ such that

$$\zeta(i) = \hat{\zeta}(i) = \zeta(j) = \theta, \text{ and } \hat{\zeta}(j) = \theta', \text{ where } \theta' \neq \theta.$$

Since $\zeta(i) = \zeta(j) = \theta$, it follows that for any profile of pure strategies s, $u^i(s, \zeta) = u^j(s, \zeta) = u_1(s, \theta)$.

Condition (5) implies that for any $s_1 \in Supp(\sigma^i)$,

$$\sum_{s_2 \in S_2} \sum_{s_0 \in S_0} u_1(s_0, s_1, s_2, \theta) \mu(s_0) \sigma_2(s_2)$$
$$\geq \sum_{s_2 \in S_2} \sum_{s_0 \in S_0} u_1(s_0, s_1', s_2, \theta) \mu(s_0) \sigma_2(s_2), \text{ for any } s_1' \in S_1,$$

this condition holds a fortiori for any $s_1' \in Supp(\sigma^j)$. It follows that

$$u^i(\sigma) = \sum_{s_1 \in Supp(\sigma^i)} \sigma^i(s_1) \sum_{s_2 \in S_2} \sum_{s_0 \in S_0} u_1(s_0, s_1, s_2, \theta) \mu(s_0) \sigma_2(s_2)$$
$$\geq \sum_{s_1' \in Supp(\sigma^j)} \sigma^j(s_1') \sum_{s_2 \in S_2} \sum_{s_0 \in S_0} u_1(s_0, s_1', s_2, \theta) \mu(s_0) \sigma_2(s_2) = u^j(\sigma).$$

□

Proposition 3 compares the utility of overconfident and unbiased players for any fixed game and equilibrium. This result would not be valid if a player could directly observe her opponent's state of mind. Equivalently put, the result is not valid if one compares the players' payoffs across different games, where the frequency of overconfidence in the informed players' population may differ. One may specify a game where informed players are likely to be unbiased, and a game where they are likely to be overconfident. It may then be possible that an overconfident player in the second game fares better than unbiased players in the first game, because the opponent's strategy is modified by the knowledge that the informed player is more

[21] Proposition 3 may be equivalently restated for naive equilibrium of Bayesian games involving only a single "informed" player (see Proposition 7, in the Appendix). Since sophisticated equilibrium may fail to exist in that environment, we choose to present this result for population games, where sophisticated equilibrium always exists and coincides with naive equilibrium.

likely to be overconfident. Such a comparison across different games, however, does not allow us to conclude that overconfidence is beneficial, because any unbiased player playing in the second game would fare at least as well as this overconfident player. To substantiate this point, we present the following example.

Example 4 (Take-It-or-Leave-It Offer). Consider a family of population games Γ_α indexed in $\alpha \in (0,1)$. An employer (player 2) makes a take-it-or-leave-it offer w to a worker i randomly selected from a large pool I. For example, imagine that a large company has opened a new plant in town I. The company is now hiring local employees, and its high bargaining power is due to being the only large employer in town. If the worker accepts the offer, she produces the output $\pi > 0$. If she rejects, she will take an outside option of value $\theta_L < \pi$. A fraction α of the workers overestimates the value of the outside option: $\hat{\zeta}(i) = \theta_H \in (\theta_L, \pi)$ for any $i \leq \alpha$, and $\hat{\zeta}(i) = \theta_L$ for any $i > \alpha$.

Each game Γ_α allows for a plethora of equilibria, however only one of them is consistent with sequential rationality, and we will focus on that equilibrium.[22] Player i rejects the offer w if $w < \hat{\zeta}(i)$, and hence player 2 offers

$$w = \begin{cases} \theta_L \ \text{ if } \alpha \leq \dfrac{\theta_H - \theta_L}{\pi - \theta_L} \\ \theta_H \ \text{ if } \alpha \geq \dfrac{\theta_H - \theta_L}{\pi - \theta_L}. \end{cases}$$

For any fixed α, each player in pool I achieves the same payoff, regardless of whether she is overconfident or unbiased. However, their payoff is larger if $\alpha \geq \frac{\theta_H - \theta_L}{\pi - \theta_L}$.

The workers in a largely overconfident pool cut a better deal than the workers in a mostly unbiased pool, because the employer's offer strategy is modified by the knowledge that the randomly selected worker is more likely to be overconfident. In this sense, overconfidence acts as a collective commitment device. This suggests that it is in the best interest of an employer to set up a plant in an area where prospective local workers are, on average, less overconfident. □

Our results on the strategic value of self-perception in games may be related to the literature on the value of information initiated by Hirshleifer (1971): while the value of information is positive in any decision problem, less informed players are better off in certain games, as their opponents' strategy is modified by the knowledge that the players are less informed.[23] The key difference with the case of mistaken self-perception lies in the plausibility of the assumption that players know each other's information quality. While it is easy to identify instances where an agent's

[22] One can also describe games where the equilibrium is unique and again the player's payoff increases in the population's aggregate overconfidence. This analysis is available upon request to the author.

[23] Neyman (1991) however underlines that such a result depends on the assumption that the information structure is common knowledge at the beginning of each game, and hence it cannot be said that a player's information is modified without changing her opponent's information. This makes the exercise in Hirshleifer (1971) logically equivalent to comparing the equilibrium of different games. Neyman (1991) shows that if one compares equilibria of interactions embedded in the same fully-specified game, a player whose information is unilaterally refined cannot be worse off in equilibrium.

informational advantage is common knowledge among the players (e.g. it may be known that an agent has access to superior sources of information), it is less plausible to think that a player may read the mind of her opponents and directly observe whether their self-perception is correct or mistaken. As a consequence, while less information can plausibly make agents better off, it is less likely that overconfidence makes a player better off.

A second comment is in place. Proposition 3 presumes that self-perception does not directly influence the player's payoff, and can only have an indirect effect on utility by modifying the player's behavior. Conceivably, overconfidence may also have a direct psychological effect on a player's utility: common wisdom deems that confident people feel better about themselves. We rule this out from our analysis to clearly identify the role of self-perception, which is a payoff-irrelevant property of beliefs, and distinguish it from the direct effect that self-esteem may possibly have on one's welfare.[24]

7 Conclusion

Motivated by information economics problems of adverse selection and by experimental findings on overconfidence, we have formalized in this paper a general epistemic model to represent strategic interaction with incomplete information, where the players' self-perception may be mistaken, and specifically overoptimistic. Our construction allows us to rigorously describe equilibrium play. Specifically, we have introduced formal equilibrium concepts based on the players' high-order beliefs of overconfidence, and on the knowledge of each others' strategies.

We have determined that there always exist "objective" equilibria, where the players correctly anticipate each other's strategies without attempting to make sense of them, and that these outcomes coincide with the equilibria of an associated Bayesian game with subjective priors. However, we have described simple signalling scenarios where the players cannot introspectively rationalizing each other's strategies. For instance, this occurs if the informed player is overconfident, and her counterpart is aware of this, but the informed player is unaware of her counterpart's assessment. In population games, instead, the objective equilibrium outcome can be always rationalized by the players, despite their possibly mistaken self-perception.

Finally, we have shown that the players cannot be made better-off in equilibrium by overestimating their characteristics, unless their opponents can recognize overconfident players prior to observing their equilibrium actions. This suggests that the strategic value of mistaken self-perception is less preponderant than the strategic value of incomplete information.

[24] Note, however, that if one understands the beneficial effect of overconfidence on welfare as the result of incorporating high expectations on future achievements, then such an effect is captured in our framework. If instead self-confidence is assumed to directly improve one's proficiency in a task, then Compte and Postlewaite (2003) show that it is optimal to hold mistaken, overconfident beliefs.

A Omitted proofs

Proof of Lemma 1. Pick an information model $\mathbf{I} = (\Omega, P_1, P_2, p)$ such that for any game $\mathbf{G}$, and any $\omega \in [\mathbf{G}]$, $P_2(\omega) \subseteq [\mathbf{G}]$, and $P_1(\omega) \subseteq [\mathbf{G}^0]$; hence $[\mathbf{G}] \subseteq C_2[\mathbf{G}] \cap C_1[\mathbf{G}^0]$. Notice that $[\mathbf{G}] \subseteq C_2[\mathbf{G}]$ and $[\mathbf{G}] \subseteq C_1[\mathbf{G}^0]$ imply $[\mathbf{G}] \subseteq C_2[C_1[\mathbf{G}^0] \cap C_2[\mathbf{G}]] = \kappa_2^1[\mathbf{G}]$; while $[\mathbf{G}] \subseteq C_1[\mathbf{G}^0]$ and $[\mathbf{G}^0] \subseteq C_2[\mathbf{G}^0]$ imply $[\mathbf{G}] \subseteq C_1[C_1[\mathbf{G}^0] \cap C_2[\mathbf{G}^0]] = \kappa_1^1[\mathbf{G}]$. Since $[\mathbf{G}^0] \subseteq C_1[\mathbf{G}^0]$, for any $n \geq 1$, $[\mathbf{G}] \subseteq \kappa_1^{n-1}[\mathbf{G}]$ implies $[\mathbf{G}] \subseteq \kappa_1^n[\mathbf{G}]$. Since $[\mathbf{G}^0] \subseteq C_2[\mathbf{G}^0]$ and $[\mathbf{G}] \subseteq C_2[\mathbf{G}]$, for any $n \geq 1$, $[\mathbf{G}] \subseteq \kappa_2^{n-1}[\mathbf{G}^0]$ implies $[\mathbf{G}] \subseteq \kappa_2^n[\mathbf{G}^0]$. The result is then obtained by induction. □

Proof of Proposition 1. The profile σ is an equilibrium of $\mathbf{G}' = (\Theta, \phi, \phi^0, S, \mu, u)$ if for any $\hat{\theta}$,

$$Supp(\sigma_1|\hat{\theta}) \subseteq \arg\max_{s_1' \in S_1} \sum_{\theta \in \Theta} \sum_{s_2 \in S_2} \sum_{s_0 \in S_0} u_1(s_0, s_1', s_2, \theta)\mu(s_0)\sigma_2(s_2)\frac{\phi^0(\theta, \hat{\theta})}{\phi_2(\hat{\theta})} \quad (6)$$

$$Supp(\sigma_2) \subseteq \arg\max_{s_2' \in S_2} \sum_{(\theta,\hat{\theta}) \in \Theta} \sum_{s_1 \in S_1} \sum_{s_0 \in S_0} u_2(s_0, s_1, s_2', \theta)\mu(s_0)\sigma_1(s_1|\hat{\theta})\phi(\theta, \hat{\theta}) \quad (7)$$

Take the information model $\mathcal{I}$ such that $[\mathbf{G}] \cap [\sigma] \neq \varnothing$ for any game $\mathbf{G}$ and any strategy profile σ of $\mathbf{G}$, and such that $P_1(\omega) = [\sigma] \cap [\mathbf{G}^0]$ and $P_2(\omega) = [\sigma] \cap [\mathbf{G}]$ for any $\omega \in [\mathbf{G}] \cap [\sigma]$. By Lemma 1, $E[\mathbf{G}] \neq \varnothing$, and since $[\mathbf{G}] \cap [\sigma] \subseteq C^0[\sigma]$, the profile σ is a naive equilibrium if and only if $[\mathbf{G}] \cap [\sigma] \cap [R] \neq \varnothing$. Suppose that this is the case: there is a ω such that $\mathbf{G}(\omega) = \mathbf{G}$, $\tilde{\sigma}(\omega) = \sigma$, and that for every $\hat{\theta}$,

$$Supp(\tilde{\sigma}_1(\hat{\theta}, \omega)) \subseteq \arg\max_{s_1'} E\left[u_1(s_1', s_2, \theta)|\hat{\theta}, P_1(\omega)\right], \quad (8)$$

$$Supp(\tilde{\sigma}_2(\omega)) \subseteq \arg\max_{s_2'} E\left[u_2(s_1, s_2', \theta)|P_2(\omega)\right]. \quad (9)$$

Since $\tilde{\sigma}(\omega) = \sigma$, by plugging the expressions $P_2(\omega) = [\sigma] \cap [\mathbf{G}]$ and $P_1(\omega) = [\sigma] \cap [\mathbf{G}^0]$ in Conditions (8) and (9), we obtain that σ satisfies Conditions (6) and (7), i.e. σ is an equilibrium of $\mathbf{G}'$. Conversely, if σ is a subjective equilibrium of $\mathbf{G}'$, then it must satisfy Conditions (6) and (7), and hence $\omega \in [R]$ for any $\omega \in [\mathbf{G}] \cap [\sigma]$; thus σ is a naive equilibrium of $\mathbf{G}$. □

Proof of Lemma 2. By Condition (4) and Assumption 2, for any pair $(\theta, \hat{\theta})$,

$$\begin{aligned}\phi(\zeta^i)(\theta, \hat{\theta}) &= \nu\{j \in [0,1] : \zeta^i(j) = (\theta, \hat{\theta})\} = \nu\{j \in [0,i) \cup (i,1] : \zeta^i(j) = (\theta, \hat{\theta})\} \\ &= \nu\{j \in [0,i) \cup (i,1] : \zeta(j) = (\theta, \hat{\theta})\} = \phi(\zeta)(\theta, \hat{\theta}).\end{aligned}$$

Pick an information model $\mathcal{I}$ such that P_2 satisfies Assumption 1, and such that $P^i(\omega) = P_2(\omega) \cap \{\omega' : \tilde{\zeta}(\omega')(i) = (\hat{\zeta}(i), \hat{\zeta}(i))\}$ for any $i \in I$; since $P^i(\omega)$ is non-empty, $\mathcal{I}$ is well-defined and satisfies Assumption 2. We have shown that for any $\omega \in [\Gamma]$, for any i, $P^i(\omega) \subseteq P_2(\omega) \subseteq [\phi(\zeta)]$, where the latter relation follows by Assumption 1. Since $P^i(\omega) \subseteq [\phi(\zeta)]$ for any i, and $P_2(\omega) \subseteq [\phi(\zeta)]$, it follows that $\omega \in CC[\phi(\zeta)]$ for any $\omega \in [\Gamma]$. □

Proof of Proposition 2. Take a model $\mathcal{I}$ such that $[\Gamma] \cap [\sigma] \neq \varnothing$ and $P_2(\omega) = [\sigma] \cap [\phi(\zeta)]$ for any $\omega \in [\Gamma] \cap [\sigma]$, any game Γ, and any symmetric profile σ of Γ, such that $p(\zeta(\iota(B)) \in \Theta' | P_2(\omega)) = p(\zeta(B) \in \Theta' | P_2(\omega))$ for any isomorphism $\iota : \mathcal{B}[0,1] \to \mathcal{B}[0,1]$, any set $B \in \mathcal{B}[0,1]$, and any set $\Theta' \subseteq \Theta$, and such that $P_s^i(\omega) = \{\omega' : \tilde{\zeta}(\omega')(i) = (\hat{\zeta}(i), \hat{\zeta}(i))\} \cap P_2(\omega)$ for any i. Because $[\phi(\zeta)] = [\phi(\zeta^i)]$ for any i, $\mathcal{I}$ satisfies Assumptions (1), and (2).

Take a game $\Gamma = (\Theta, I, \zeta, S, \mu, u)$, and say that σ' is an equilibrium of $\mathbf{G}' = (\Theta, \phi(\zeta), (\phi(\zeta))^0, S, \mu, u)$. Up to equivalence classes σ' identifies a symmetric profile σ of Γ. Say that $\omega \in [\Gamma] \cap [\sigma]$: hence $\omega \in [R_2]$ if and only if

$$\begin{aligned} &Supp(\tilde{\sigma}_2(\cdot, \omega)) \\ &\subseteq \arg\max_{s_2' \in S_2} E_\zeta \left[\int_I \sum_{s_1 \in S_1} \sum_{s_0 \in S_0} u_2(s_0, s_1, s_2', \zeta(i)) \mu(s_0) \sigma^i(s_1|\omega) d\nu(i) \middle| P_2(\omega) \right]. \end{aligned} \tag{10}$$

By construction, $\sigma_1'(s_1|\hat{\theta}) = \sigma^i(s_1|\omega)$ whenever $\hat{\zeta}(i) = \hat{\theta}$. Since $P_2(\omega) \subseteq [\phi(\zeta)]$ and $p(\zeta(\iota(B)) \in \Theta' | P_2(\omega)) = p(\zeta(B) \in \Theta' | P_2(\omega))$ for any $B \in \mathcal{B}[0,1]$, any $\Theta' \subseteq \Theta$, and any isometry $\iota : \mathcal{B}[0,1] \to \mathcal{B}[0,1]$, it follows that $\phi(\zeta)$ is a sufficient statistic of player 2's information on the assignment ζ. Thus the expression (11) can be summarized by aggregating the players in I across the characteristics $(\theta, \hat{\theta})$. Substituting the expressions for $\phi(\zeta)$ and for σ_1', we thus obtain:

$$\begin{aligned} &Supp(\tilde{\sigma}_2(\cdot, \omega)) \\ &\subseteq \arg\max_{s_2' \in S_2} \sum_{(\theta, \hat{\theta}) \in \Theta} \sum_{s_1 \in S_1} \sum_{s_0 \in S_0} u_2(s_0, s_1, s_2', \theta) \mu(s_0) \sigma_1'(s_1|\hat{\theta}) \phi(\theta, \hat{\theta}). \end{aligned} \tag{11}$$

This condition coincides with Condition (6), which σ' satisfies by definition. It follows that $[\Gamma] \cap [\sigma] \subseteq [R_2]$.

For any i, and any $\omega \in [\Gamma] \cap [\sigma]$, it is the case that $\omega \in [R^i]$ if and only if

$$\begin{aligned} Supp\left(\tilde{\sigma}^i(\cdot|\omega)\right) &\subseteq \arg\max_{s_1' \in S_1} \sum_{\theta \in \Theta} \sum_{s_2 \in S_2} \sum_{s_0 \in S_0} u^i(s_0, s_1', s_2, \theta) \mu(s_0) \sigma_2(s_2|\omega) p(\zeta(i) \\ &= \theta | \hat{\zeta}(i)) \\ &= \arg\max_{s_1' \in S_1} \sum_{s_2 \in S_2} \sum_{s_0 \in S_0} u_1(s_0, s_1', s_2, \hat{\zeta}(i)) \mu(s_0) \sigma_2(s_2|\omega). \end{aligned} \tag{12}$$

By construction, $\tilde{\sigma}^i(\cdot|\omega) = \sigma^i(\cdot) = \sigma'(\cdot|\hat{\zeta}(i))$, thus σ_1 satisfies Condition (12) for every i if and only if σ' satisfies Condition (7) for every $\hat{\theta}$, which is the case by definition. It follows that $[\Gamma] \cap [\sigma] \subseteq [R_1]$.

The above arguments have shown that $[\Gamma] \cap [\sigma] \subseteq [R]$. Since $[\Gamma] \subseteq [\phi(\zeta)]$, it follows that $P_2(\omega) \cap [R] \neq \emptyset$ for any $\omega \in [\Gamma] \cap [\sigma]$. To show that for any i, it is also the case that $P_s^i(\omega) \cap [R] \neq \emptyset$, pick any state ω' such that $\tilde{\zeta}(\omega') = \zeta^i$. By construction, $\omega' \in P_s^i(\omega)$ and, moreover, the 2-player subjective-priors Bayesian game associated with $\tilde{\Gamma}(\omega')$, coincides with $\mathbf{G}'$. It follows that $[\tilde{\Gamma}(\omega')] \cap [\sigma] \subseteq [R]$; and hence by construction of $\mathcal{I}$, that $[\tilde{\Gamma}(\omega')] \cap [\sigma] \neq \varnothing$ and that $P_s^i(\omega) \cap [R] \neq \varnothing$. Thus we can refine $(P^i)_{i \in I}$ and P_2, by defining $\hat{P}_2(\omega) = P_2(\omega) \cap [R]$, and $\hat{P}_s^i(\omega) =$

$P_s^i(\omega) \cap [R]$, for every $i \in I$. Letting $[\hat{R}]$ be the event that the players are rational relative to $(\hat{P}^i)_{i \in I}$ and $\hat{P}_2$, we can show that $[\Gamma] \cap [\sigma] \cap [R] = [\Gamma] \cap [\sigma] \cap [\hat{R}]$: since $[\Gamma] \cap [\sigma] \subseteq [R_2]$, it is rational for player 2 to play σ_2 on the event $[\Gamma] \cap [\sigma]$. As a result, the information that player 2 is rational does not add anything to the belief that she plays σ_2, and so $E[u^i(s_1, s_2', \theta)|P^i(\omega)] = E[u^i(s_1, s_2', \theta)|\hat{P}^i(\omega)]$ for any $\omega \in [\Gamma] \cap [\sigma]$. Conversely, $E[u_2(s_1, s_2', \theta)|P_2(\omega)] = E[u_2(s_1, s_2', \theta)|\hat{P}_2(\omega)]$ for any $\omega \in [\Gamma] \cap [\sigma]$, because $[\Gamma] \cap [\sigma] \subseteq [R^i]$ for any i.

Since, by construction, $\hat{P}^i(\omega) \subseteq \hat{P}_2(\omega) \subseteq [\hat{R}] \cap [\sigma]$ for any i, it follows that $\omega \in C\hat{C}[[\hat{R}] \cap [\sigma]]$. In summary, we have shown that $[\Gamma] \cap [\sigma] \subseteq [\hat{R}] \cap C\hat{C}[[\hat{R}] \cap [\sigma]]$. Because, by construction, $[\Gamma] \cap [\sigma]$ is non-empty, we conclude that if σ' is an equilibrium of G', then σ is a sophisticated equilibrium of Γ.

If σ is a sophisticated equilibrium of Γ, then it is also a naive equilibrium of Γ, because $[\Gamma] \cap CC^*([R] \cap [\sigma]) \subseteq [\Gamma] \cap [R] \cap [\sigma] \cap C^0[\sigma]$.

We are left to show that if σ is a naive equilibrium of Γ (under the information model $\hat{\mathcal{I}}$), then σ' is an equilibrium of the associated game $\mathbf{G}'$. Since $[\Gamma] \cap [\sigma] \subseteq \hat{C}^0[\sigma]$, σ is a naive equilibrium if and only if $[\Gamma] \cap [\sigma] \cap [\hat{R}] \neq \varnothing$; i.e. there is a ω such that $\tilde{\Gamma}(\omega) = \Gamma, \tilde{\sigma}(\omega) = \sigma, Supp(\tilde{\sigma}_2(\cdot,\omega)) \subseteq \arg\max_{s_2'} E[u_2(s_1, s_2', \theta)|\hat{P}_2(\omega)]$ $= E[u_2(s_1, s_2', \theta)|P_2(\omega)]$, and $Supp(\sigma^i(s_1,\omega)) \subseteq \arg\max_{s_1'} E[u^i(s_1', s_2, \theta)|\hat{P}^i(\omega)]$ $= E[u^i(s_1', s_2, \theta)|P^i(\omega)]$, for any i. Since $\tilde{\sigma}(\omega) = \sigma$, we obtain Conditions (12) and (12) by plugging in the expressions for $P_2(\omega)$ and $P^i(\omega)$. This implies that σ' satisfies Conditions (6) and (7), i.e. σ' is a subjective equilibrium of $\mathbf{G}'$. □

B General high-order beliefs of self-perception

This part of the Appendix studies self-perception and equilibrium, for general descriptions of the players' understanding of the game. The players play a game $\mathbf{G} = (\Theta, \phi, S, \mu, u)$, but player 1 believes that she is playing game $\mathbf{G}^0$: her perception may be mistaken. A description of the players' understanding of the game is generated by the events $[\mathbf{G}]$, and $[\mathbf{G}^0]$, and by the iterated application of the operators C_1, C_2, as well as complementation and intersection. Given the space Ω, and the relation $\tilde{\mathbf{G}} : \omega \mapsto \mathbf{G}$, for any game $\mathbf{G} = (\Theta, \phi, S, \mu, u)$, we consider the space $\Omega_{\mathbf{G}} = [\mathbf{G}] \cup [\mathbf{G}^0]$. We introduce the algebra $\mathcal{A}^1_{\mathbf{G}} = \{\emptyset, [\mathbf{G}], [\mathbf{G}^0], \Omega_{\mathbf{G}}\}$, and for any $n \geq 1$, the algebra $\mathcal{A}^n_{\mathbf{G}}$ generated by $\mathcal{A}^{n-1}_{\mathbf{G}} \cup \{C_i E | E \in \mathcal{A}^{n-1}_{\mathbf{G}}, i = 1, 2\}$. The algebra that includes all the descriptions of players' knowledge of the game $\mathbf{G}$ is $\mathcal{A}_{\mathbf{G}} = \cup_{n=1}^{\infty} \mathcal{A}^n_{\mathbf{G}}$.[25] It is known (see Aumann, 1999; Hart Heifetz and Samet, 1996) that not all the lists in $\mathcal{A}_{\mathbf{G}}$ are consistent: there are lists of events $l_{\mathbf{G}}$ whose intersection is empty for all information model $\mathbf{I}$, these lists are ruled out of the analysis, to avoid triviality.[26]

[25] An algebra of Ω is a collection of subsets of Ω that contains Ω, that is closed under complementation and finite intersection. An example of a list of events in $\mathcal{A}_{\mathbf{G}}$ is the list $l_{\mathbf{G}} = \{[\mathbf{G}], C_1[\mathbf{G}^0], C_2[\mathbf{G}], (\kappa_1^2[\mathbf{G}], \kappa_2^2[\mathbf{G}]), ..., (\kappa_1^n[\mathbf{G}], \kappa_2^n[\mathbf{G}]), ...\}$, which represent the instance studied in the fourth section.

[26] Whether a list $l_{\mathbf{G}}$ generated by the event $[\mathbf{G}]$ is consistent or not depends only on the combinations of certainty and logic operators, and is independent of the generating event. Hence we shall drop the subscript from the notation $l_{\mathbf{G}}$, with the understanding that the notation l identifies the list $l_{\mathbf{G}}$ when in conjunction with a specific game $\mathbf{G}$.

First, we extend Proposition 1 to any instance where the informed player is overconfident, and her opponent is aware of this, regardless of the players' high-order beliefs of overconfidence. We introduce the collection of lists $A = \{l \in \mathcal{A}|$ for any $\mathbf{G}$, $\emptyset \neq E_l(\mathbf{G}) \subseteq [\mathbf{G}] \cap C_1[\mathbf{G}^0] \cap C_2[\mathbf{G}]\}$. Recall that for any game $\mathbf{G}$, the game $\mathbf{G}'$ denotes the associated game with subjective priors.[27]

Proposition 4. *For any list $l \in A$, there is an information model $\mathbf{I}$ such that for any game $\mathbf{G}$, the profile σ is a naive equilibrium for $E_l(\mathbf{G})$ if and only if σ is an equilibrium of the subjective-prior game $\mathbf{G}'$.*

Second, we show that if player 2 is unaware that player 1 may be overconfident, then the naive equilibria of any game $\mathbf{G}$ coincide with the Bayesian equilibria of the game $G = (\Theta, \phi_2, S, \mu, u)$. We let $U = \{l \in \mathcal{A}|$ for any $\mathbf{G}$, $\emptyset \neq E_l(\mathbf{G}) \subseteq [\mathbf{G}] \cap C_1[\mathbf{G}^0] \cap C_2[\mathbf{G}^0]\}$.

Proposition 5. *For any list $l \in U$, there is an information model $\mathbf{I}$ such that for any game $\mathbf{G}$, the profile σ is a naive equilibrium for $E_l(\mathbf{G})$ if and only if σ is a Bayesian equilibrium of G.*

The final and most important result of this section identifies the conditions under which sophisticated and naive equilibrium coincide. This may occur in two instances. First, it may be the case that, while they are truly playing game $\mathbf{G}$, the players share common certainty that they are playing game $\mathbf{G}^0$, so that not only is the informed player unaware of being overconfident, but also her opponent is unaware that she could be overconfident. In this case, sophisticated, naive and Bayesian equilibrium all coincide. Second, it may be the case that the players "agree to disagree" on the game they play. Player 1 is overconfident and unaware of it, player 2 knows that player 1 is overconfident, player 1 thinks that player 2 thinks that player 1 is overconfident, and so on. In this case, naive and sophisticated equilibria of game $\mathbf{G}$ coincide with the subjective equilibria of the associated game with subjective priors $\mathbf{G}'$. For any other description of the players' knowledge of overconfidence, there are games that do not have any sophisticated equilibrium.

We denote by l^0 the (consistent) list $l \in \mathcal{A}$ such that for any game $\mathbf{G}$, $E_l(\mathbf{G}) = [\mathbf{G}] \cap CC[\mathbf{G}^0]$, and by l^* the (consistent) list l such that for any game $\mathbf{G}$, $E_l(\mathbf{G}) =[\mathbf{G}] \cap (\cap_{n \geq 0} \bar{C}^n[\mathbf{G}])$, where $\bar{C}^0[\mathbf{G}] = C_1[\mathbf{G}^0] \cap C_2[\mathbf{G}]$, and for any $n > 0$, $\bar{C}^n[\mathbf{G}] = C_1\bar{C}^{n-1}[\mathbf{G}] \cap C_2\bar{C}^{n-1}[\mathbf{G}]$.

Proposition 6. *For $l \in \{l^0, l^*\}$ there is a model $\mathbf{I}$ such that for any game $\mathbf{G}$, the profile σ is a sophisticated equilibrium for $E_l(\mathbf{G})$ if and only if σ is a naive equilibrium for $E_l(\mathbf{G})$. For any other list l, and model $\mathbf{I}$, there exist games $\mathbf{G}$ where a sophisticate equilibrium does not exist for $E_l(\mathbf{G})$.*

It is immediate to extend Proposition 3 to description of high-order beliefs of overconfidence and naive equilibrium. For any strategy profile σ, any ability θ and

[27] The proofs of Proposition 4 and 5 are analogous to the proof of Proposition 1. Similarly the proof of Proposition 7 (resp. 6) are easily derived from the proofs of Proposition 3 (resp. Proposition 2 and Example 1). These derivations are omitted for brevity, but are available upon request to the author.

any perception $\hat{\theta}$, let the ex-ante actual utility be:

$$u_1(\sigma, \theta, \hat{\theta}) = \sum_{s_2 \in S_2} \sum_{s_1 \in S_1} \sum_{s_0 \in S_0} u_1(s_0, s_1, s_2, \theta)\mu(s_0)\sigma_1(s_1|\hat{\theta})\sigma_2(s_2).$$

Proposition 7. *For any list $l \in A$, any information model $\mathbf{I}$, and any game $\mathbf{G}$, in any naive equilibrium σ for $E_l(\mathbf{G})$, for any level of ability θ, and any perception $\hat{\theta}$, it must be the case that $u_1(\sigma, \theta, \theta) \geq u_1(\sigma, \theta, \hat{\theta})$.*

C Games with universal types

This part of the Appendix shows how universal types can be used to describe self-perception.[28] We propose a straightforward extension of the construction in Brandenburger and Dekel (1993), so as to includes in a player's type also individual objective characteristics (i.e. ability), as well as high-order beliefs. Since this construction is well understood, proofs and unnecessary calculations are omitted, and made available upon request.

For any player $j = 1, 2$, let the space of j's ability be a complete separable metric space Θ_j. Iteratively set $X_1 = \Theta_1 \times \Theta_2$, and for any $n \geq 1$, $X_{n+1} = X_n \times [\Delta(X_n)]^2$. Let a type t_j be a hierarchy $(\delta_{0j}, \delta_{1j}, \delta_{2j}...) \in \Theta_j \times (\times_{n=1}^{\infty} \Delta(X_n))$, and define $T_{j0} = \Theta_j \times (\times_{n=1}^{\infty} \Delta(X_n))$. A type is coherent if for any $n \geq 1$, the marginal distribution projected by δ_{n+1j} on X_n coincides with δ_{nj}, let the set of coherent types be T_{j1}.[29] Because of Kolmogoroff Extension Theorem, each coherent type uniquely identifies a system of beliefs with respect to her own and the opponent's type, through the (unique) homeomorphism $f_j : T_{j1} \to \Theta_j \times \Delta(T_{j0} \times T_{-j0})$.[30] Since this permits a coherent type to identify a belief that she or her opponent is not coherent, we impose "common certainty of coherency:" For any $n \geq 1$, let $T_{jn+1} = \{t \in T_{j1} : f_j(t)(T_{jn} \times T_{-jn}) = 1\}$. The universal type space of player j is $T_j = \cap_{n=1}^{\infty} T_{jn}$. Each universal type identifies a unique belief over the state of nature and an opponent's universal type, through the (unique) homeomorphism $g_j : T_j \to \Theta_j \times \Delta(T_j \times T_{-j})$, generated by f_j.

Given the abilities space $\Theta = \Theta_1 \times \Theta_2$, and the universal types space $T = T_1 \times T_2$, we specify a nature's prior $p \in \Delta(T)$, a strategy space $S = S_0 \times S_1 \times S_2$, a move of nature μ, and payoffs $u : S \times \Theta \to \mathbb{R}^2$, and obtain a fully specified game $\mathcal{G} = \{\Theta, T, p, S, u\}$. A strategy in $\mathcal{G}$ is a profile $\sigma = (\sigma_1, \sigma_2)$, where for each j, the function $\sigma_j : T_j \to \Delta(S_j)$ is measurable. The players' actual utility is expressed by $u_j : \Theta \times S \to \mathbb{R}$; for any j and σ, the actual utility of type t_j when playing against t_{-j} is:

$$u_j(t, \sigma) = \sum_{s_1 \in S_1} \sum_{s_2 \in S_2} \sum_{s_0 \in S_0} u_j(\delta_0, s)\sigma_1(s_1|t_1)\sigma_2(s_2|t_2)\mu(s_0).$$

[28] The concept of universal types has been first introduced by Mertens and Zamir (1985).

[29] If a type is not coherent, its beliefs are not well-defines because the type includes more than one possible specification for each level of high-order beliefs X_n.

[30] This result is an extension of Proposition 1 in Brandenburger and Dekel (1993), where the reader can find additional details. For the Kolmogoroff Extension Theorem, see for instance Dudley (1999, p. 201).

Some types t_j include a mistaken belief about their ability, and their perceived payoff may differ from their actual payoff. For any mixed strategy σ, any player j of type t_j perceives that her utility is:

$$\tilde{u}_j(t_j,\sigma) = \int \sum_{s_1\in S_1} \sum_{s_2\in S_2} \sum_{s_0\in S_0} u_j(g_j(t_j)(\theta), s)\sigma_1(s_1|t_1)\sigma_2(s_2|t_2)\mu(s_0)dg_j(t).$$

For this construction, the most appropriate definition of equilibrium requires that all types choose a payoff-maximizing strategy, and not only those that are selected with positive prior probability.[31]

Definition 4. *An equilibrium of game $\mathcal{G}$ is a profile $\sigma = (\sigma_1, \sigma_2)$, where for each player $j \in \{1, 2\}$, and any type $t_j \in T_j$,*

$$\begin{aligned} &Supp\,(\sigma_j\,(\cdot|t_j)) \\ &\subseteq \arg\max_{s'_j\in S_j} \int \sum_{s_{-j}\in S_{-j}} \sum_{s_0\in S_0} u_j(g_j(t_j)(\theta), s'_j, s_{-j}, s_0)\sigma_{-j}(s_{-j}|t_{-j})\mu(s_0)dg_j(t). \end{aligned}$$

In order to show how this construction relates to our analysis of overconfidence, we present a simplification of Example 1.

Example 5. Recall that in the game $G = (\Theta, \phi, S, \mu, u)$ of Example 1, player 1's ability belongs to the set $\Theta = \{\theta_L, \theta_H\}$, that $\phi_1(\theta_L) = 1$ and that $\phi_2(\theta_H) = 1$. Letting $S_1 = \{L, H\}$ and $S_2 = \{l, h\}$, the players' payoffs are:

θ_L	l	h
L	1,1	0,0
H	0,1	1,0

θ_H	l	h
L	1,1	0,2
H	0,1	2,2

We are interested in the equilibrium play associated with the knowledge description identified by $E[\mathbf{G}] = [\mathbf{G}] \cap C_1[\mathbf{G}^0] \cap C_2[\mathbf{G}] \cap [\cap_{n\geq 2}(\kappa_1^n[\mathbf{G}] \cap \kappa_2^n[\mathbf{G}])]$.

In the language of universal types, one can show that the event $E[\mathbf{G}]$ identifies a type-distribution $p = (p_1, p_2) \in \Delta(T)$ such that for any $(\theta, \hat{\theta}) \in \Theta$, p assigns probability $\phi(\theta, \hat{\theta})$ to the pair of types (t_1, t_2), such that $\delta_{01} = \theta$, $\delta_{11} = \delta(\hat{\theta})$, $\delta_{12} = \phi_1$ (where the notation $\delta(\cdot)$ identifies the distribution degenerate on $\cdot$), and such that the high-order beliefs are recursively defined as follows. Let $\delta_{21} = \delta_{11} \cdot \delta(\delta_{11}) \cdot \delta(\phi_2)$, $\delta_{22} = \delta(\delta_{12}) \cdot \phi(\theta, \delta_{11})$, where the last term assigns probability $\phi(\theta, \hat{\theta})$ to the state $\{\theta, \delta(\hat{\theta})\}$, and for any $n \geq 2$, $\delta_{n+1,1} = \delta_{n1} \cdot \delta(\delta_{n1}) \cdot \delta(\delta_{n2} \cdot \phi^0(\hat{\theta}, \delta_{11}, ..., \delta_{n1}))$, $\delta_{n+1,2} = \delta(\delta_{n2}) \cdot \phi(\theta, \delta_{11}, ..., \delta_{n-1,1})$, where the terms ϕ^0 and ϕ are derived as before. For any given $\hat{\theta} \in \Theta$, let $t_1[\hat{\theta}]$ denote the type $t_1 \in Supp(p_1)$ such that $\delta_{11} = \delta(\hat{\theta})$. Note that p_2 is degenerate.

The key observation is that the event $E[\mathbf{G}]$ identifies a type distribution incompatible with the consistency assumption of Mertens and Zamir (1985). Hence, player

[31] In the context of correlated equilibrium, Brandenburger and Dekel 1987 introduce the distinction between ex-ante equilibrium (which requires that each player maximizes ex-ante payoff), and a-posteriori equilibrium (which requires that also null- probability types choose payoff-maximizing strategies).

1 cannot anticipate player 2's choice because she believes to play against a fictitious type of player 2. Formally, we observe that for any $\hat{\theta} \in \Theta$, type $t_1[\hat{\theta}]$ identifies through g_1 the belief that player 1 is of type $t_1'[\hat{\theta}]$ and that player 2 is of type t_2', where $\delta_{01}' = \hat{\theta}, \delta_{11}' = \delta(\hat{\theta}), \delta_{12}' = \phi_2, \delta_{21}' = \delta_{11} \cdot \delta(\delta_{11}) \cdot \delta(\phi_2), \delta_{22}' = \delta(\delta_{12}) \cdot \phi_2(\hat{\theta}, \delta_{11})$, and for any $n \geq 2$, $\delta_{n+1,1}' = \delta_{n1}' \cdot \delta(\delta_{n1}) \cdot \delta(\delta_{n2}' \cdot \phi_2(\hat{\theta}, \delta_{11}', ..., \delta_{n1}'))$, $\delta_{n+1,2}' = \delta(\delta_{n2}') \cdot \phi_2(\hat{\theta}, \delta_{11}', ..., \delta_{n-1,1}')$. Since the types $t_1'[\theta_L]$, $t_1'[\theta_H]$, and t_2' identify a Bayesian game with common prior $\mathbf{G} = (\Theta, \phi_2, S, u)$, any equilibrium σ of game $\mathcal{G}$ must be such that $\sigma(t_2')(h) = 1$, that $\sigma_1(t_1'[\theta_L])(L) = 1$, and that $\sigma_1(t_1'[\theta_H])(H) = 1$. Player 2, on the other hand, is of type t_2, which identifies the belief that player 1 is of type $t_1[\theta_L]$ with probability $\phi(\theta_L, \theta_L) = 1$. Since this type of player 2 believes that $\theta = \theta_L$ with probability 1, in any equilibrium she must play $\sigma(t_2)(l) = 1$. □

The formulation of the game with universal types allows us to construct a Bayesian equilibrium that predicts that at any state $\omega \in E[\mathbf{G}]$, player 1 plays s_H when believing her ability to be high, regardless of the fact that player 2 will respond to that choice by playing y_L. This occurs because state ω identifies a type of player 1 that believes to be playing against a type of player 2 which is different from the type of player 2 identified by state ω. Hence, player 1 cannot anticipate the strategy played by player 2. In this sense, this reformulation does not change the message of Example 1. While it is true that in any Bayesian equilibrium of the game with universal types, the assignment of strategies to types is common knowledge among the players, it is also the case that player 1 cannot anticipate player 2's choice because she believes that she is playing against a completely fictitious type of player 2.

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