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Assignment 4
EC9D3 Advanced Microeconomics

1. Consider the following economy with production, two consumption goods, labelled 1 and 2 and two inputs labour and capital, labelled L and K. A large number of firms have access to a technology which comprises two activities:

$$\begin{pmatrix} a_{K1} \\ a_{L1} \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \qquad \begin{pmatrix} a_{K2} \\ a_{L2} \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

where a_{Kh}, a_{Lh} are the inputs of capital and labour required to produce 1 unit of commodity h, $(h = 1, 2)$. Capitalists own the total stock $\bar{K} = 22$, and workers supply total labour $\bar{L} = 40$, that are sold to firms at prices r, respectively w.

Assume that the capitalists and workers have preferences respectively represented by the utility functions:

$$u^K(x_1, x_2) = 8 \ln x_1 + 3 \ln x_2 \qquad u^L(x_1, x_2) = 2 \ln x_1 + 3 \ln x_2$$

Consider the equilibrium in which there is no surplus labour or capital.

Let the price of the consumption commodities 1 and 2 be p and $(1 - p)$ respectively: we normalize the prices of the two commodities x_1 and x_2 to sum to one.

(i) Find the equilibrium values of p, w and r.

(ii) Find the equilibrium allocations of commodities x_1 and x_2.

2. A competitive industry consisting of a large number of firms faces total demand for output given by

$$Q = 10 - p,$$

where p is the price. A representative firm, parameterized by α, requires q_α^2/α units of labour to produce q_α units of output.

(i) Given that α is uniformly distributed on the interval $[1, 3]$ and the total mass of firms is 1, solve for the competitive equilibrium price in terms of the wage w.

(ii) Suppose now that w increases slightly. Show that if the initial value of w is smaller than 1, each firm's profit increases in equilibrium.

(iii) What is the economic intuition behind this result?

3. Once upon a time students in economics received no grades and worked only for the pleasure of learning. In those days of yore, each student received cardinal utility $G(x)$ from working at the level of effort of x hours per day.

(i) At what level of effort did each student choose to work?

Then one dark day, grades were introduced. Now each student cares not only about learning, but also about his relative standing in the class. If y is the average of hours worked per day by all the students, each student now has a utility function of the form:

$$G(x) + F\left(\frac{x}{y}\right),$$

where $F'(\cdot) > 0$ and $F(1) = 0$.

In this modern era, each student maximizes his utility with respect to the variable under his own control, x, for a given level of y. Assume from now on $G''(x) < 0$ for all $x \in [0, 24]$.

(ii) What are the symmetric equilibrium values of x, y and utility? Focus on the case in which each student maximization problem has an interior solution.

Suppose now that all students get together and hire a consultant to plan for the socially optimal level of work.

(iii) What is the symmetric Pareto optimal value of x, and how does this differ from the decentralized case above?

(iv) If hours of work were publicly measurable, how could the symmetric Pareto optimal optimal solution be achieved by quota, or a tax/subsidy scheme?