UNIVERSITY OF WARWICK

January Examinations 2018/19

Economic Analysis: Microeconomics

Time Allowed: 3 hours.

Answer **FOUR** questions: **TWO** questions must be from Section A and **TWO** questions must be from Section B. Answer Section A questions in one booklet and Section B questions in a separate booklet. All questions carry equal weight.

Approved pocket calculators are allowed.

Read carefully the instructions on the answer book provided and make sure that the particulars required are entered on each answer book. If you answer more questions than are required and do not indicate which answers should be ignored, we will mark the requisite number of answers in the order in which they appear in the answer book(s): answers beyond that number will not be considered.

Section A: Answer TWO questions

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1. (a) A consumer chooses non-negative consumption levels of two goods x_1, x_2, to maximise utility subject to the budget constraint $p_1x_1 + p_2x_2 = m$. The utility function is:

$$u = \min\left\{\frac{x_1}{a}, \frac{x_2}{1-a}\right\}, \quad 0 < a < 1.$$

(i) Find the Marshallian demands for this consumer. Are the goods Marshallian complements or substitutes? **(4 marks)**

(ii) Find the indirect utility function. Show that it is homogenous of degree zero in prices and income. **(4 marks)**

(iii) Find the expenditure function and show that it is homogenous of degree 1 in prices. **(4 marks)**

(iv) Find Hicksian demands for goods 1 and 2. Are the goods Hicksian complements or substitutes? **(4 marks)**

(Question 1 continued overleaf)

(b) Show that in the case of three goods, whatever the utility function of the consumer, at least one of goods 2 and 3 is a Hicksian substitute for good 1. [You may assume that Hicksian demands are differentiable in prices and that $\frac{\partial h_1}{\partial p_1} < 0$.] **(9 marks)**

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2. Either:

(a) Explain the difference between naïve and sophisticated expectations in decision problems with hyperbolic discounting by the decision-maker. What difference, if any, do these two different forms of expectations have on behaviour in this problem? **(25 marks)**

Or:

(b) What is the Generalized Axiom of Revealed Preference? How can it be used to empirically test consumer theory? **(25 marks)**

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3. Consider the following portfolio design problem. An investor has initial wealth of w, and invests a fraction s of this wealth in a risky asset, and a fraction $1 - s$ of this wealth in a safe asset. The safe asset pays a return of zero. The risky asset pays a return of $r > 0$ with probability $1 > p > 0.5$, and a return of $-r$ with probability $1 - p$.

(a) Assume that the investor has a utility function $\ln(c)$, where c is the final wealth (and consumption) of the investor.

(i) Derive the expected utility of the investor, V, as a function of s, and the parameters of the problem p and r. **(5 marks)**

(ii) Assuming that the investor invests a share s strictly between 0 and 1 in the risky asset, find the s that maximizes the investor's expected utility. **(6 marks)**

(iii) How does this value of s change with p, w, and r? Give an economic intuition for your findings. **(5 marks)**

(b) Consider a general version of the above problem, where a risk-averse investor facing the problem of investing a fixed wealth in one safe asset and one risky asset. Show that if the utility function is of the constant relative risk aversion form, the share invested in the risky asset is independent of wealth. **(9 marks)**

4. Farmer A grows rice, and Farmer B grows wheat. Both farmers consume both wheat and rice. The amounts of rice and wheat consumed by Farmer A are denoted x_1^A, x_2^A, and for Farmer B, x_1^B, x_2^B. The amounts of rice and wheat grown are g_1, g_2 respectively. The utility functions of the farmers are:

$$u^A = \left(x_1^A\right)^a \left(x_2^A\right)^{1-a}, u^B = \left(x_1^B\right)^\beta \left(x_2^B\right)^{1-\beta},$$

with $0 < \alpha, \beta < 1$.

(a) (i) Find the Marshallian demands for rice and wheat by the two farmers as functions only of prices p_1, p_2 of rice and wheat. **(6 marks)**

(ii) Using your answers in (a), find the Arrow-Debreu equilibrium prices (take good 2 as the numeraire). Explain how and why the equilibrium price ratio depends on are g_1,g_2. If the harvest of both rice and wheat doubles, what happens to equilibrium prices? **(5 marks)**

(iii) Using your answers in (b), find the Walrasian allocation for each household, i.e. the levels of consumption of the two goods at Walrasian equilibrium, as functions of g_1, g_2, and the resulting equilibrium levels of utility for each household. If Farmer A could pay to give Farmer B a stronger preference for rice, would she want to do so? **(5 marks)**

[**Important note:** In your answer to part (a), it is acceptable to use the following mathematical result without proof: the solution to the problem of maximizing α ln $x_1 + (1-\alpha)$ ln x_2 subject to $p_1 x_1 + p_2 x_2 = m$ is $x_1 = am/p_1$, $x_2 = (1-\alpha)m/p_2$.]

(b) What does it mean for goods to be gross substitutes? If goods are gross substitutes, what does that imply about the number of Walrasian equilibria? In the example in part (a), are the two good gross substitutes? **(9 marks)**

Section B: Answer TWO questions

5. (a) Explain what is meant by a dominated strategy. **(3 marks)**

(b) Explain why no player can play a strictly dominated strategy in a Nash Equilibrium. Is the same true of a weakly dominated strategy? **(3 marks)**

(c) Consider the following normal-form game:

		Column Player		
		L	C	R
Row Player	T	$(0,-1)$	$(1,0)$	$(2,1/3)$
	B	$(1,1)$	$(0,2/3)$	$(0,1/3)$

Find all the Nash Equilibria of this game (in pure and mixed strategies). **(8 marks)**

(d) Consider the following game of complete information. N individuals each simultaneously choose whether or not to provide a public good. An individual who provides the good incurs a cost $c > 0$. If the good is provided by one or more individuals, each individual obtains $v > c$. Denoting i's pure-strategy choice with $s_i \in \{0, 1\}$ ($s_i = 0$ meaning "do not provide", $s_i = 1$ meaning "provide"), i's payoff can thus be written as $u_i = \min\left\{\sum_j s_j, 1\right\} v - s_i\, c$; i.e., i's payoff is $v - c$ if i provides the good, v if i does not provide the good but at least one of the other individuals does, and zero if nobody provides the good.

(i) Derive a symmetric mixed-strategy equilibrium of this game. **(4 marks)**

(ii) How does the equilibrium probability that the good is provided by at least one individual vary with N? **(4 marks)**

(iii) Suppose that $N = 2$, $c = 1/2$, $v = 1$; and suppose that each individual (wrongly) thought that their opponent was a machine providing the good with probability $1/2 - \epsilon$, with $\epsilon > 0$. How would the probability of the good being provided by at least one individual change in this case? **(3 marks)**

(Continued overleaf)

6. (a) Write down the conditions for a Bayesian Nash Equilibrium in pure strategies, and briefly comment on them. **(4 marks)**

(b) Sotheby's is auctioning a painting by Turner. There are two buyers who wish to buy it. Their valuations are independent and private, and lie between 1 and 2. Furthermore, these are drawn from a uniform distribution.

(i) What is the symmetric equilibrium bidding strategy in a sealed-bid second-price auction? Explain. **(4 marks)**

(ii) What is the seller's expected revenue for (i)? **(3 marks)**

(iii) Suppose the seller runs a sealed-bid first-price auction instead. Derive an expression for the symmetric equilibrium bidding strategy and discuss how it compares with the equilibrium bidding strategy under (i). What is the seller's expected revenue in this case? **(4 marks)**

(c) Consider the following game between two ice-cream sellers, 1 and 2, both producing ice-cream at zero cost and each having to choose a location at which to park their ice-cream vans. There are two possible locations, location A and location B. Total revenues from ice-cream sales at location A are $R_A = 100$, whereas total revenues from ice-cream sales at location B are $R_B = 60$. If the two sellers choose different locations, they each get the total revenues for the location they have chosen. If they choose the same location, however, they must split the revenues for that location equally between them.

(i) Describe all the pure-strategy Nash equilibria of this game. **(3 marks)**

(ii) Now suppose that, with probability 1/2, seller 2 is able to create a new flavour of ice-cream which, when two sellers are present at any given location, allows seller 2 to take 7/10 of the total revenues at any location (rather than 1/2 of the total revenues). Before the sellers choose their location, seller 2 can observe whether or not she has succeeded in creating the new flavour, but seller 1 only knows that with probability 1/2 player 2 has succeeded. Describe all the pure-strategy Bayesian Nash equilibria of this game. **(7 marks)**

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7. (a) Draw the extensive form of the following game. Two players, 1 and 2, play a game that involves three choice stages.

* In the first stage, player 1 can choose either L or R. If she chooses L, the game ends, with player 1 obtaining a payoff of 6 and player 2 obtaining a payoff of 50; if she chooses R, the game continues to the next stage.
* In the second stage, the two players simultaneously and independently must choose A or B. If they both choose A, their payoff is 2; if 1 chooses A and 2 chooses B, 1's payoff is 3 and 2's payoff is 100; if 1 chooses B and 2 chooses A, 1's payoff is

(Question 7 continued overleaf)

100 and 2's payoff is 3; if they both choose B, the game continues to the next stage.

* In the last stage, player 1 again chooses L or R; if she chooses L, players 1 gets 4 and player 2 gets 0; if player 1 chooses R, player 1 gets V and player 2 gets 4. **(4 marks)**

(b) With reference to part (a):

(i) Find a pure-strategy subgame perfect equilibrium of the game for $V = 3$. **(4 marks)**

(ii) Find a pure-strategy subgame perfect equilibrium of the game for $V = 5$. **(4 marks)**

(c) Two profit-maximising firms, L and F, producing differentiated goods at zero cost compete in prices in a given market with demands $q_L = 1 - p_L + p_F/2$, $q_F = 1 - p_F + p_L/2$, where p_F, p_L are the prices charged by the each firm and q_F, q_L are the corresponding quantities.

(i) Derive an expression for F's profit maximising price as a function of the price charged by L. **(2 marks)**

(ii) Derive symmetric (Bertrand) Nash equilibrium prices, and compare them with the corresponding symmetric price level that maximises joint profits. **(3 marks)**

(iii) Now suppose that F selects its price after L has selected its price, being able to observe the price that has been selected by L. What are the firms' prices in a subgame perfect Nash equilibrium? **(3 marks)**

(iv) Does L have a first-mover advantage under (iii)? Explain. **(3 marks)**

(v) Would your answer to (iii) change if F chose its price second but were unable to observe L's price choice before making its choice? Draw an extensive-form representation of the game to explain your answer. **(2 marks)**

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8. (a) With reference to a signalling model, explain, in a short paragraph, what a pooling equilibrium and what a separating equilibrium are. **(4 marks)**

(b) In the market for used cars, there are good cars and bad cars for sale. A value of a working car to a buyer is unity. Good cars break down with probability 1/4, bad cars break down with probability 3/4. So the expected value of a good car to a buyer is $v_H = 3/4$ and the expected value of a bad car to a buyer is $v_L = 1/4$. The value of used cars to sellers is zero, independently of whether they are good or bad. The proportion of used cars that are good is known to buyers and is equal to $1/2$. The number of potentially willing buyers is greater than the number of used cars for sale.

(i) If buyers cannot observe the quality of used cars, at what price will used cars be sold? **(1 mark)**

(Question 8 continued overleaf)

(ii) Now suppose that the owners of used cars can offer a warranty whereby a car that is sold and breaks down is replaced by the seller with a working car at no cost to the buyer and at a cost c to the seller. Derive conditions on c under which there exists a separating equilibrium where only sellers of good cars offer a warranty. **(3 marks)**

(iii) Next suppose that the owners of used cars have a positive valuation for their car equal to three-quarters of the corresponding valuations of the buyers, i.e. $z_H = 9/16$ for a good car and $z_L = 3/16$ for a bad car. If a warranty cannot be offered and buyers cannot observe quality, will good cars be sold? What will the market equilibrium look like? Explain. **(2 marks)**

(iv) With reference to the scenario described under (iii) suppose now that sellers can offer a replacement warranty as in (ii). Can both good cars and bad cars be sold in equilibrium in this case? Explain. **(2 marks)**

(c) A risk-averse individual with income $y = 1$ faces the possibility of getting into a car accident. If the accident occurs, the individual incurs a monetary damage equal to $D = 1/2$, which lowers her disposable income to $1 - 1/2 = 1/2$. If the individual exerts no effort ($e = 0$), the accident occurs with certainty (with probability one), but if the individual exerts effort to try to avoid getting into an accident ($e = 1$), the accident occurs with probability $1/4$.

A risk-neutral monopoly insurer can offer the individual an insurance contract that reimburses a fraction $k\,D = k/2$ of the damage incurred in case of accident, at a premium m to be paid in all states of the world.

The individual's realised payoff in state $s \in \{\mathit{Accident}, \mathit{No\_Accident}\}$, as a function of the effort exerted, $e \in \{0, 1\}$, and of disposable income in that state, x_s, is $u(x_s, e) = x_s - (1/2)(x_s)^2 - e/12$. In the absence of an insurance contract, disposable income is $x_s = 1$ with no accident and $x_s = 1 - D = 1/2$ in case of accident. If the individual has entered into an insurance contract, disposable income is $x_s = 1 - m$ with no accident and $x_s = 1 - m - (1 - k)D = 1 - m - (1 - k)/2$ in case of accident.

(i) Is exerting effort socially efficient; i.e. is the utility cost of effort less than the expected reduction in damage from effort? Explain. **(2 marks)**

(ii) Would the individual exert effort in the absence of insurance? Explain. **(2 marks)**

(iii) The insurer cannot observe effort, e. Derive the insurance contract (as characterised by a combination of m and k) that maximises the insurer's profits, inducing the individual to enter into the contract and exert positive effort ($e = 1$). **(9 marks)**

(End)