UNIVERSITY OF WARWICK

January Examinations 2019/20

Economic Analysis: Microeconomics

Time Allowed: 3 Hours plus 15 minutes reading time during which notes may be made (on the question paper only) **BUT NO ANSWERS MAY BE BEGUN**.

Answer **FOUR** questions: **TWO** questions must be from Section A and **TWO** questions must be from Section B. Answer Section A questions in one booklet and Section B questions in a separate booklet. All questions carry equal weight.

Approved pocket calculators are allowed.

Read carefully the instructions on the answer book provided and make sure that the particulars required are entered on each answer book. If you answer more questions than are required and do not indicate which answers should be ignored, we will mark the requisite number of answers in the order in which they appear in the answer book(s): answers beyond that number will not be considered.

Section A: Answer TWO questions

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1. A consumer chooses consumption levels of three goods $1, 2, 3$ and faces prices p_1, p_2, p_3 and fixed income m. She has utility function:

$$u = (x_1 - y_1)^{\alpha_1}(x_2 - y_2)^{\alpha_2}(x_3 - y_3)^{\alpha_3},\ \alpha_i > 0,\ \sum_{i=1}^{3} \alpha_i = 1$$

where the y_i are fixed constants.

(a) Assume $y_i \geq 0,\ i = 1, 2, 3,\ \sum_{i=1}^{3} p_i y_i \equiv z < m$. What is the economic interpretation of these assumptions? **(5 marks)**

(b) Derive the Marshallian demands for the three goods. Are the goods Marshallian complements or substitutes? **(5 marks)**

(c) Derive the indirect utility function, and show that it is homogenous of degree zero in prices and income. **(5 marks)**

(Question 1 continued overleaf)

(d) Derive the expenditure function. How does it differ from the standard Cobb-Douglas case where $y_i = 0,\ i = 1, 2, 3$? Explain the economic significance of the difference. **(5 marks)**

(e) Find a general formula for the second cross- partial of the expenditure function i.e.

$$\frac{\partial^2 e}{\partial p_i \partial p_j},\ j \neq i$$

Using this result, state whether the goods are Hicksian substitutes or complements. Explain your reasoning. **(5 marks)**

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2. Answer **either** part (a) **or** part (b).

(a) Daniel is deciding whether or not to buy a new car. He has preferences over a car (good 1) and other consumption of the form:

$$u(c, x; r_c, r_x) = \alpha c + x + v(c - r_c) + v(x - r_x)$$

where $c = 1$ if the car is bought and $c = 0$ otherwise, and x is the level of consumption over other goods. Here:

$$v(c - r_c) = \begin{cases} c - r_c, & c \geq r_c \\ \lambda(c - r_c), & c < r_c \end{cases}$$

$$v(x - r_x) = \begin{cases} x - r_x, & x \geq r_x \\ \lambda(x - r_x), & x < r_x \end{cases}$$

where $\lambda > 1$. Here, r_c, r_x are reference levels of consumption. Finally, Daniel faces a budget constraint $pc + x = m$, where p is the price of a car.

(i) Provide an interpretation of the utility function and the assumption that $\lambda > 1$. **(4 marks)**

(ii) Define a personal equilibrium for Daniel in the sense of Koszegi and Rabin. **(4 marks)**

(iii) Find conditions on a, λ for which a personal equilibrium exists where Daniel buys the car. **(4 marks)**

(iv) Find conditions on a, λ for which a personal equilibrium exists where Daniel does not buy the car. **(4 marks)**

(v) Under what conditions do multiple personal equilibria exist? How do the number of equilibria vary with α and λ? **(4 marks)**

(Question 2 continued overleaf)

(vi) How would Daniel behave in this situation if his reference vector r_c, r_x was given by past consumption, rather than rational expectations? **(5 marks)**

(b) What is hyperbolic discounting? Show, using example(s), how hyperbolic discounting can give rise to a self-control problem. Under what circumstances might individuals use pre-commitment devices to overcome this problem? **(25 marks)**

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3. Consider an investor who has initial wealth w to invest, and can choose between a safe and a risky asset. Specifically, he can put a share s of his initial wealth in the risky asset. The investor has a strictly increasing and strictly concave utility function over final wealth y. The safe asset has a return of zero. The risky asset has a random return R, which can take on values $\{r, 0, -r\}$, with $r > 0$. The probabilities of these returns are $qp, 1-q$, and $q(1-p)$ respectively, with $1 \geq p, q > 0$.

(a) Under what conditions on p, q will the investor invest a positive share s in the risky asset? **(5 marks)**

(b) If the investor puts a share s of his wealth in the risky asset, write down expressions for final wealth y for each of the three possible returns on the risky asset. **(5 marks)**

(c) If utility of final wealth $u(y) = \ln y$, write down an expression for the expected utility of final wealth, using your answer to (b). **(5 marks)**

(d) If $q = 1, p > 0.5,\ w = 1$, find the optimal value of s for the investor (Hint: it can be an interior or a corner solution.) **(5 marks)**

(e) How does your answer to (d) change if $0 < q < 1$? Explain. **(5 marks)**

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4. Consider the following two-person, two-good exchange economy. Amy has an endowment of e units of good 1, and $1-e$ units of good 2, and utility function:

$$u_A = \alpha \ln x_1^A + (1-\alpha) \ln x_2^A,\ 0 < \alpha < 1$$

Bob has an endowment of 1 unit of good 2, and utility function:

$$u_B = \min\{x_1^B, x_2^B/\beta\}$$

Also, we assume $\beta < \frac{1}{1-\alpha}$.

(Question 4 continued overleaf)

(i) Assume that $e = 1$.

(a) Find the Marshallian demands for the two goods by Amy and Bob as functions only of prices p_1, p_2 of the two goods. **(5 marks)**

(b) Using your answers in (a), find the Walrasian equilibrium prices. Is there a unique Walrasian equilibrium? [Hint: take good 2 as the numeraire.] **(5 marks)**

(c) Using your answers in (b), find the Walrasian allocation for each of Amy and Bob. How does this allocation depend on the preference parameters? Comment on what you find. **(5 marks)**

(ii) Assume that $e = 0.5$.

(a) Find the Marshallian demands for the two goods by Amy and Bob as functions only of prices p_1, p_2 of the two goods. **(5 marks)**

(b) Using your answers in (a), find the Walrasian equilibrium prices. Is there a unique Walrasian equilibrium? [Hint: take good 2 as the numeraire.] **(5 marks)**

Note: in your answer to part (ii), you may use the formulae for the Marshallian demands that you derived in part (i), with the appropriate modifications.

Section B: Answer TWO questions
Please use a separate booklet

5. (a) Consider the following normal form game:

Column Player

Row Player	L	C	R
T	$(4, 0)$	$(4, 10)$	$(10, 4)$
M	$(10, 5)$	$(0, 5)$	$(0, 4)$
B	$(0, 10)$	$(10, 0)$	$(0, 4)$

Solve the game by iterated dominance (strict or weak). (Hint: start by examining the strategies of the column player.) **(7 marks)**

(b) Consider the following normal form game:

Column Player

Row Player	F	R
F	$(1, 1)$	$(0, c)$
R	$(c, 0)$	$(1, 1)$

For which values of $c > 0$ does the game have a symmetric mixed-strategy equilibrium? **(6 marks)**

(Question 5 continued overleaf)

(c) N symmetric countries each generate greenhouse emissions equal to $\bar{e} - a_i$, with $\bar{e} > 0$, where $a_i \in [0, \bar{e}]$ is emissions abatement in country i, for which country i must incur a cost equal to $c\,a_i$, $c \in (0, N)$. Assume $\bar{e} = 1$. The total amount of greenhouse emissions is thus $E = \sum_j (1 - a_j) = N - \sum_j a_j$, which results in a level of environmental cost to each country equal to $-(1/2)E^2 = -(1/2)(N - \sum_j a_j)^2$. Country i's payoff, as a function of its abatement choice, a_i, and of the abatement choices of all other countries, a_{-i}, is then:

$$u_i(a_i, a_{-i}) = -\frac{1}{2}\left(N - a_i - \textstyle\sum_{j \neq i} a_j\right)^2 - c\,a_i.$$

(i) Derive a symmetric Nash Equilibrium in abatement choices, a_i. **(5 marks)**

(ii) Derive an expression for the efficient symmetric abatement level, a^*, that maximises the level of payoff, $u(a_j = a^*, \forall j) = -(1/2)(N(1 - a^*))^2 - c\,a^*$, for a representative country. **(3 marks)**

(iii) Now suppose that emissions, $1 - a_i$, are taxed at a rate t, the same in all countries, with the revenue being shared equally across all countries. Country i's payoff is now equal to:

$$u_i(a_i, a_{-i}) = -\frac{1}{2}\left(N - a_i - \textstyle\sum_{j \neq i} a_j\right)^2 - c\,a_i - t(1 - a_i) + \frac{t}{N}\left(N - a_i - \textstyle\sum_{j \neq i} a_j\right).$$

Derive the symmetric Nash Equilibrium abatement choice for this case, and compute the level of t for which the symmetric N.E. abatement choice equals a^*. **(4 marks)**

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6. (a) Describe the differences between a First-Price and a Second-Price Auction, in relation to the auctioning rules used in each, the equilibrium bidding strategies and their interpretation, and the expected levels of revenues that they yield to the seller. **(6 marks)**

(b) Two players, 1 and 2, must each choose to move left (L) or right (R). If they both choose L they both get a payoff of 100. If they both choose R they both get a payoff of 60. If 1 chooses L and 2 chooses R, payoffs for 1 and 2 are respectively 0 and 60. If 1 chooses R and 2 chooses L, payoffs for 1 and 2 are respectively 60 and 0.

(i) Describe all pure-strategy Nash equilibria for this game. **(2 marks)**

(ii) Suppose that each of the two players believes that with probability $1/2$ her opponent is a rational player whose strategies are chosen as best responses to her opponent's choices, but with probability $1/2$ her opponent is a machine that selects L or R by tossing a coin, L and R being selected with equal probabilities. What are the pure-strategy equilibria in this scenario? Explain. **(5 marks)**

(Question 6 continued overleaf)

(c) Two profit-maximising firms, A and B, producing differentiated goods are competing in the same market, simultaneously selecting prices p_A and p_B (Bertrand competition). Profits for each firm equal the quantity it sells times the difference between the price it charges and marginal cost. The demand schedule faced by A is $q_A = 1 - p_A + p_B/2$ and the demand schedule faced by B is $q_B = 1 - p_B + p_A/2$. Firm A produces the good at a marginal cost of 1/2. Firm A's costs are known to both competitors.

(i) Suppose that firm B's marginal cost is 1/2, and that this is also known to both competitors. Derive equilibrium prices. **(3 marks)**

(ii) Next suppose that firm B's marginal cost can be either 0 or 1, with equal probabilities, and that B observes its cost before A and B select their prices, while A does not observe firm B's marginal cost but knows that it is either zero or 1 with equal probabilities. Derive equilibrium prices. **(5 marks)**

(iii) Now suppose that, as in (ii), B's marginal cost can be either 0 or 1 with equal probabilities, but A can fully observe B's costs. Derive equilibrium prices and expected equilibrium profits for this scenario and compare them with those obtained under (ii). **(4 marks)**

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7. (a) Two ice-cream makers, A and B, can both produce ice cream at zero marginal cost. Ice cream is sold at a price of one pound per ice-cream cone. There is a single location (a park) where they can sell their ice cream, and the maximum total number of ice-cream cones that can be sold at that location is 100. Entering the market costs 30 to each potential seller.

(i) Consider the following game between the two potential sellers:

I. A chooses whether or not to enter the market.

II. Having observed A's move, B decides whether or not to enter; if A has entered and B does not enter, then the game ends and A's payoffs are $100 - 30 = 70$ and B's payoff are zero. If A has not entered and B enters, then the game ends, B's payoffs are $100 - 30 = 70$ and A's payoff are zero. If neither enters the game ends with zero payoffs for both.

III. If both A and B have entered, then A has two choices: it can passively accept to share the market with B, in which case they both obtain a revenue of 50 and a payoff of $50 - 30 = 20$; or it can undermine B's sales through negative advertising, incurring a cost of 20 and causing a reduction in sales for B from 50 to 20, while A's sales remain at 50; in this case A's payoff is $50 - 20 - 30 = 0$ and B's payoff is $20 - 30 = -10$.

Characterise a subgame perfect equilibrium of this game. Explain your answer. **(6 marks)**

(Question 7 continued overleaf)

(ii) Now consider a game as in (i) but where producing ice an ice-cream cone involves a marginal cost of $1/2$. Characterise a subgame perfect equilibrium of this game. Explain your answer. **(5 marks)**

(iii) With reference to (ii), suppose that before stage I of the game (at a stage 0), ice-cream maker A can incur a cost c to learn about a new process that reduces marginal cost from $1/2$ to zero. If A adopts this process, however, B can copy it at zero cost. For what levels of c would A choose to learn about the new process? Explain. **(2 marks)**

(b) A firm and its union bargain as follows. First the union makes a wage demand, w. The firm observes w and chooses whether or not to accept it. If the firm accepts, then it chooses a level of employment, L. If it does not accept, then L is zero. The firm's payoff is $\pi = 6\,L^{\frac{1}{2}} - wL$, and the union's payoff is $u = (w - \beta)\,L$, where $\beta > 0$. (Assume that if the firm is indifferent between accepting and not-accepting, it will choose to accept.)

Derive the unique subgame perfect equilibrium outcome. **(6 marks)**

(c) Two countries, A and B, independently select trade policies. If they both select low tariffs, they each get a payoff of 5. If A selects high tariffs and B selects low tariffs, A gets 6 and B gets 3, while if A selects low tariffs and B selects high tariffs, A gets 3 and B gets 6. If they both select high tariffs, they both get 4.

Derive conditions on the period discount factor for which an equilibrium featuring low tariffs in every period can be sustained under infinite repetition by the threat of indefinite reversion to high tariffs in response to any deviation from low tariffs (*grim trigger strategy*). Explain. **(6 marks)**

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8. (a) Consider the following version of Spence's signalling model. The productivity of a worker in a given job is θ. A worker can be of high ability ($\theta = 2$) or low ability ($\theta = 1$), with equal probability. Each worker chooses a level of education $e \geq 0$. The total cost of obtaining education level e is $e^2/\sqrt{\theta}$. The worker's wage is equal to her expected productivity.

(i) Find the the lowest education level supporting a separating perfect Bayesian equilibrium. **(7 marks)**

(ii) Are high-ability workers (of type $\theta = 2$) better off in the separating equilibrium described in part (i) or in the pooling perfect Bayesian equilibrium in which no one gets education? **(3 marks)**

(Question 8 continued overleaf)

(b) A worker's job consists of operating a machine. If the worker operates the machine as instructed ($e = 1$), then with probability $1/2$ the level of output of 1 ($s = H$) and the firm's revenue is $\bar{v} > 0$, and with probability $1/2$ output and revenue are zero ($s = L$). Alternatively, the worker can choose to use the machine for her own private purposes ($e = 0$), obtaining a private consumption value of $\gamma = 1/2$ from doing so. If she does so, however, output and revenue are zero with certainty. The worker has utility $u(y) = y - (1/2)\,y^2$, where y is her disposable income, inclusive of any private value she obtain from the machine, i.e. $y(w, e) = w + (1 - e)/2$, where w is the wage received from her employer and $e \in \{0, 1\}$ is an indicator of how she uses the machine ($e = 1$: as intended; $e = 0$, for private purposes).

Employers operate under conditions of perfect competition, meaning that they must break even in expectations (zero expected profit). So, if their expected revenue is $E(v)$ and the cost of using the machine is r, the expected wage paid, $E(w)$, must be such that $E(v) - r - E(w) = 0$. Assume $\bar{v} = 2\,(1 + r)$. When $e = 1$, this implies $E(v) - r = \bar{v}/2 - r = 1$, making the zero-profit condition:

$$E(w) = 1.$$

(i) Suppose that firms can observe how a worker uses the machine, and offer a contract $(w(e = 1), w(e = 0))$ where the wage paid is conditioned on how the machine is used. Assuming $w(e = 0) = 0$ (firms pay a zero wage if they see improper use of the machine), what is equilibrium the level of $w(e = 1)$ (the level for which firms break even)? **(4 marks)**

(ii) Suppose now that firms cannot observe how machines are used but that a project's outcome is verifiable, implying that the firm can offer a wage contract (w_H, w_L) where the wage paid depends on the outcome of the project, $s \in \{H, L\}$. Characterise wage contracts that induce workers to use the machines as intended and for which firms break even. **(7 marks)**

(iii) What is the level of w_L that makes the consumer indifferent between choosing $e = 0$ and $e = 1$ in such a contract? **(4 marks)**

(End)