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Solutions to Assignment 2
EC9D3 Advanced Microeconomics

1. Denote $p^1 = (100, 100)$, $p^2 = (100, 80)$, $x^1 = \begin{pmatrix} 100 \\ 100 \end{pmatrix}$, $x^2 = \begin{pmatrix} 120 \\ y \end{pmatrix}$, $m^1 = p^1 x^1 = 20,000$.

(i) By Weak Axiom for the consumer to be better off in Year 1 the following two conditions need to hold:

$$p^1 x^2 = 12,000 + 100\, y \le m^1 = 20,000$$

and

$$m^2 = p^2 x^2 = 12,000 + 80\, y < p^2 x^1 = 18,000.$$

We get $y \le 80$ from the former condition and $y < 75$ from the latter. Therefore we conclude $y < 75$.

(ii) By Weak Axiom for the consumer to be better off in Year 2 the following two conditions need to hold:

$$m^2 = p^2 x^2 = 12,000 + 80\, y \ge p^2 x^1 = 18,000.$$

and

$$p^1 x^2 = 12,000 + 100\, y > m^1 = 20,000$$

We get $y \ge 75$ from the former condition and $y > 80$ from the latter. Therefore we conclude $y > 80$.

(iii) The behaviour is inconsistent if the following two conditions both hold:

$$m^2 = p^2 x^2 = 12,000 + 80\, y \ge p^2 x^1 = 18,000.$$

and

$$m^1 = 20,000 \geq p^1 x^2 = 12,000 + 100\ y$$

We get $y \geq 75$ from the former condition and $y \leq 80$ from the latter. Therefore we conclude $75 \leq y \leq 80$.

(iv) We know that $\Delta x = 20$ for the good to be inferior we need $\Delta m < 0$, restricting attention to the income effect of the price change.

To be able to buy x^1 in Year 2 it is enough that the consumer has income level $m^* = p^2 x^1 = 18,000$ (compensated income change). Therefore we need

$$m^2 = p^2 x^2 = 12,000 + 80\ y < m^* = p^2 x^1 = 18,000$$

which gives $y < 75$.

3. Let p_T is the price of tea, p_C is the price of coffee. Define $\bar{p}_i(p_j, x_i, x_j)$ for $i \in \{T, C\}$ as the maximum price that the consumer is willing to pay for a fixed amount $x_i > 0$ of beverage i, when the price and demand of the other beverage j are p_j and x_j.

For any pair $\mathbf{x} = (x_C, x_T)$, let $(p_T^*(\mathbf{x}), p_C^*(\mathbf{x}))$ be the solution of the system

$$p_T = \bar{p}_T(p_C, \mathbf{x}), \qquad p_C = \bar{p}_C(p_T, \mathbf{x}).$$

Note that by definition:

$$h_T(\bar{p}_T(p_C, \mathbf{x}), p_C) = x_T,$$

Applying the implicit function theorem we get

$$\frac{d\bar{p}_T}{dp_C} = -\frac{\partial h_T / \partial p_C}{\partial h_T / \partial \bar{p}_T} > 0,$$

where the inequality sign follows from the negative sign of the substitution effect and the fact that tea and coffee are substitutes. We conclude that for any pair of quantities x the function $\bar{p}_T$ increases in p_C.

Fix any pair of beverages $x > 0$, and for any $\varepsilon \geq 0$ small, let x_ε be such that $x_{T,\varepsilon} = x_T$ and $x_{C,\varepsilon} = \varepsilon$. Let $p_C^*(x_0) = \lim_{\varepsilon \to 0} p_C^*(x_\varepsilon)$ be the smallest price of coffee that makes the consumer unwilling to consume coffee when consuming x_T amount of tea.

Note that by the law of demand, $\partial h_C/\partial p_C < 0$, the price $p_C^*(x_\varepsilon)$ strictly increases in ε and hence $p_C^*(x_0) > p_C^*(x)$. The consumer stops consuming coffee if it costs more than the price she is willing to pay to consume quantity x_C, holding the consumption of tea x_T and price of tea p_T constant.

Because the the function $\bar{p}_T$ increases in p_C, this implies that $p_T^*(x_0) > p_T^*(x)$. The consumer is willing to pay more not to be deprived of tea when the price of coffee is so high that she cannot afford it, than when coffee is affordable.

3. The profit maximization problem is:

$$\max_{x_1,x_2} p\, x_1^\alpha x_2^\beta - w_1 x_1 - w_2 x_2.$$

From the solutions to this problem we obtain the following functions.

(i) The factor demands are:

$$x_1(p, w_1, w_2) = \left(\frac{\alpha}{w_1}\right)^{\frac{1-\beta}{\gamma}} \left(\frac{\beta}{w_2}\right)^{\frac{\beta}{\gamma}} p^{\frac{1}{\gamma}}$$

and

$$x_2(p, w_1, w_2) = \left(\frac{\alpha}{w_1}\right)^{\frac{\alpha}{\gamma}} \left(\frac{\beta}{w_2}\right)^{\frac{1-\alpha}{\gamma}} p^{\frac{1}{\gamma}}$$

where $\gamma = 1 - \alpha - \beta$.

(ii) The supply function is:

$$y(p, w_1, w_2) = \left(\frac{\alpha}{w_1}\right)^{\frac{\alpha}{\gamma}} \left(\frac{\beta}{w_2}\right)^{\frac{\beta}{\gamma}} p^{\frac{\alpha+\beta}{\gamma}}$$

(iii) The own price effects are:

$$\frac{\partial x_1}{\partial w_1} = -\frac{(1-\beta)}{\gamma\, w_1} x_1 < 0,$$

$$\frac{\partial x_2}{\partial w_2} = -\frac{(1-\alpha)}{\gamma\, w_2} x_2 < 0$$

and

$$\frac{\partial y}{\partial p} = \frac{(\alpha+\beta)}{\gamma\, p} y > 0.$$

The cross price effects are instead:

$$\frac{\partial x_1}{\partial w_2} = \frac{\partial x_2}{\partial w_1} = -\frac{\alpha}{\gamma\, w_1} x_2,$$

$$\frac{\partial y}{\partial w_1} = -\frac{\partial x_1}{\partial p} = -\frac{1}{\gamma\, p} x_1$$

and

$$\frac{\partial y}{\partial w_2} = -\frac{\partial x_2}{\partial p} = -\frac{1}{\gamma\, p} x_2.$$

(iv) The profit function is:

$$\pi(p, w_1, w_2) = \gamma p^{\frac{1}{\gamma}} \left(\frac{\alpha}{w_1}\right)^{\frac{\alpha}{\gamma}} \left(\frac{\beta}{w_2}\right)^{\frac{\beta}{\gamma}}.$$

(v) Simple differentiation of the profit function gives:

$$\frac{\partial \pi}{\partial w_1} = -x_1 \quad \frac{\partial \pi}{\partial w_2} = -x_2 \quad \frac{\partial \pi}{\partial p} = y.$$

4. The definition of profit function is:

$$\pi(p, w) = pf(x^*) - wx^*$$

where x^* denotes the solution to the producer's profit maximizing problem. The first order conditions of this problem in the case $x^* >> 0$ are:

$$w = p\nabla f(x^*).$$

By substitution we obtain:

$$\pi(p, w) = pf(x^*) - p\nabla f(x^*)x^*.$$

Euler theorem implies for a constant returns to scale technology:

$$f(x) = \nabla f(x)\ x$$

which gives:

$$\pi(p, w) = pf(x^*) - p\nabla f(x^*)x^* = pf(x^*) - pf(x^*) = 0$$

which proves the result.