Warwick University Department of Economics

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EC9D3 Advanced Microeconomics

Review Session - 2024/25

Question 1

Alice chooses consumption levels of two goods X and Y and faces prices P_X and P_Y with a fixed income M.
Suppose Alice's utility function is given by:

$$U(X, Y) = X^{\alpha} Y^{1-\alpha}, \quad 0 < \alpha < 1.$$

(a) Using the Lagrangean method derive the Marshallian demands for the two goods. Interpret the parameter α. Are the goods Marshallian complements or substitutes? **(7 marks)**

(b) Derive the indirect utility function, and show that it is homogeneous of degree zero in prices and income. **(5 marks)**

(c) Derive the expenditure function. **(3 marks)**

(d) Assume that the initial prices of goods X and Y are $P_X = 4$ and $P_Y = 2$, $\alpha = 0.5$ and also that Alice is endowed with an income $M = 40$. What is the optimal bundle of X and Y purchased by Alice and its associated level of utility? **(3 marks)**

(e) Suppose the price of good X decreases to $P'_X = 2$, while the price of Y and Alice's income remain constant.
Using Slutsky decomposition and keeping the consumer on the same indifference curve, calculate the substitution and income effects on the consumer's demand for good X due to the price change? **(7 marks)**

Answers to Question 1

(a) To derive the Marshallian demand functions, we set up the Lagrangian function as follows:

$$\mathcal{L}(X, Y, \lambda) = X^{\alpha} Y^{1-\alpha} + \lambda(M - P_X X - P_Y Y)$$

First Order Conditions (FOCs):

$$\frac{\partial \mathcal{L}}{\partial X} = \alpha X^{\alpha-1} Y^{1-\alpha} - \lambda P_X = 0,$$
$$\frac{\partial \mathcal{L}}{\partial Y} = (1-\alpha) X^{\alpha} Y^{-\alpha} - \lambda P_Y = 0,$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = M - P_X X - P_Y Y = 0.$$

Solve for λ: From the first two FOCs, we can express λ as:

$$\lambda = \frac{\alpha X^{\alpha-1} Y^{1-\alpha}}{P_X} = \frac{(1-\alpha) X^{\alpha} Y^{-\alpha}}{P_Y}$$

Equating these expressions and solving for Y in terms of X:

$$\frac{\alpha Y}{P_X X} = \frac{(1-\alpha) X}{P_Y Y} \Rightarrow Y = X \cdot \frac{(1-\alpha) P_X}{\alpha P_Y}$$

Substitute Y **back into the Budget Constraint**: Using $P_X X + P_Y Y = M$:

$$P_X X + P_Y \left(X \cdot \frac{(1-\alpha) P_X}{\alpha P_Y} \right) = M$$

Solving for X, we get:

$$X^* = \frac{\alpha M}{P_X}$$

Similarly, for Y:

$$Y^* = \frac{(1-\alpha)M}{P_Y}$$

Interpretation of α: The parameter α represents the proportion of income Alice allocates to good X; a higher α indicates a stronger preference for X.

Are the Goods Complements or Substitutes? Since each demand depends only on its own price, the goods are *neither* Marshallian complements nor substitutes.

(b) The indirect utility function is obtained by substituting X^* and Y^* into the utility function:

$$U^* = (X^*)^\alpha (Y^*)^{1-\alpha} = \left(\frac{\alpha M}{P_X}\right)^\alpha \left(\frac{(1-\alpha)M}{P_Y}\right)^{1-\alpha}$$

Simplifying:

$$U^* = M \left(\frac{\alpha}{P_X}\right)^\alpha \left(\frac{1-\alpha}{P_Y}\right)^{1-\alpha}$$

Homogeneity of Degree Zero: The indirect utility function $U^* = M \left(\frac{\alpha}{P_X}\right)^\alpha \left(\frac{1-\alpha}{P_Y}\right)^{1-\alpha}$ is homogeneous of degree zero in prices and income, as scaling M, P_X, and P_Y by a common factor does not change U^*.

(c) The expenditure function represents the minimum income needed to reach a utility level U_0:

$$U_0 = M \left(\frac{\alpha}{P_X} \right)^{\alpha} \left(\frac{1-\alpha}{P_Y} \right)^{1-\alpha}$$

Solving for M:

$$M = U_0 \cdot \left(\frac{P_X}{\alpha} \right)^{\alpha} \left(\frac{P_Y}{1-\alpha} \right)^{1-\alpha}$$

(d) Given $P_X = 4$, $P_Y = 2$, $M = 40$, and $\alpha = 0.5$:

$$X^* = \frac{0.5 \cdot 40}{4} = 5,$$
$$Y^* = \frac{(1 - 0.5) \cdot 40}{2} = 10.$$

The optimal bundle is $(X^*, Y^*) = (5, 10)$.

Associated Utility Level:

$$U(X^*, Y^*) = (X^*)^\alpha (Y^*)^{1-\alpha} = (5)^{0.5}(10)^{0.5} = \sqrt{50} \approx 7.07.$$

(e) With the price of X decreasing to $P'_X = 2$:

New Optimal Bundle:

$$X' = \frac{0.5 \times 40}{2} = 10, \quad Y' = \frac{(1 - 0.5) \cdot 40}{2} = 10$$

The new bundle is $(X', Y') = (10, 10)$.

Compensated Income M^c to Maintain Original Utility:

$$M^c = U(X^*, Y^*) \cdot \left(\frac{P'_X}{\alpha}\right)^{\alpha} \cdot \left(\frac{P_Y}{1-\alpha}\right)^{1-\alpha}$$

Substituting $U(X^*, Y^*) = \sqrt{50}$:

$$M^c = \sqrt{50} \cdot \left(\frac{2}{0.5}\right)^{0.5} \cdot \left(\frac{2}{0.5}\right)^{0.5} = 4\sqrt{50} \approx 28.28.$$

Compensated Demand for X (Substitution Effect):

$$X^c = \frac{\alpha M^c}{P'_X} = \frac{0.5 \cdot 28.28}{2} = 7.07.$$

Substitution Effect on X:

$$X^c - X^* = 7.07 - 5 = 2.07.$$

Income Effect on X:

$$X' - X^c = 10 - 7.07 = 2.93.$$

Summary of Effects:

$$\text{Substitution Effect on } X : 2.07,$$
$$\text{Income Effect on } X : 2.93.$$

Question 2

Consider the following two scenarios that illustrate the Allais Paradox. Assume you are an individual making decisions based on potential monetary gains.

Scenario 1

You are given two options:

Option A: A guaranteed payoff of £1,000,000.

Option B: A 10% chance of winning £5,000,000, an 89% chance of winning £1,000,000, and a 1% chance of winning nothing.

Scenario 2

You are given a different set of options:

Option C: A 10% chance of winning £5,000,000, with a 90% chance of winning nothing.

Option D: An 11% chance of winning £1,000,000, with an 89% chance of winning nothing.

(a) Calculate the expected value (EV) of each option in **Scenario 1** and **Scenario 2**. Show all calculations. **(8 marks)**

(b) Based on the expected values (EV) calculated in part (a), indicate which option would be chosen in each scenario if the individual were an expected utility maximiser. Explain your answer. **(5 marks)**

(c) Describe the likely choices for an individual exhibiting behavior consistent with the **Allais Paradox** in both scenarios. Explain how these choices demonstrate the **certainty effect** and violate the **independence axiom** of expected utility theory. **(7 marks)**

(d) Explain how the choices in each scenario can be understood through **Prospect Theory**. Specifically, discuss the role of **reference points**, **loss aversion**, and **probability weighting**. **(10 marks)**

Answers to Question 1

(a) Let's calculate the expected monetary value (EV) for each option in both scenarios.

Scenario 1

Option A: Since this option guarantees £1,000,000, the EMV is simply:

$$\text{EV}_A = 1,000,000 \times 1 = 1,000,000$$

Option B: This option has three possible outcomes, with probabilities 10%, 89%, and 1%. Calculating the EV:

$$\text{EV}_B = (5,000,000 \times 0.10) + (1,000,000 \times 0.89) + (0 \times 0.01)$$

$$= 500,000 + 890,000 + 0 = 1,390,000$$

Scenario 2

Option C: This option has a 10% chance of winning £5,000,000 and a 90% chance of winning nothing:

$$\text{EV}_C = 5,000,000 \times 0.10 = 500,000$$

Option D: This option has an 11% chance of winning $1,000,000 and an 89% chance of winning nothing:

$$\text{EV}_D = 1,000,000 \times 0.11 = 110,000$$

(b): An expected utility maximiser would choose the option with the highest EV in each scenario.

From the calculations in part (a):

In **Scenario 1**, an expected utility maximiser would choose **Option B**, with an EV of £1,390,000, which is greater than the £1,000,000 EMV of **Option A**.

In **Scenario 2**, an expected utility maximiser would choose **Option C**, with an EV of £500,000, which is higher than the £110,000 EV of **Option D**.

(c) An individual showing the **Allais Paradox** would likely choose:

Option A in Scenario 1, preferring the certain $1,000,000 over the higher EV of Option B — illustrating the **certainty effect**.

Option C in Scenario 2, as it has the higher EV and no certain alternative.

This violates the **independence axiom**, which states that preferences should remain consistent when identical outcomes are added or removed. The inconsistent choices reflect an overvaluation of certainty.

(d) Prospect Theory explains this behavior through:

Reference Points: In Scenario 1, the guaranteed £1,000,000 in Option A acts as a reference point, making the riskier Option B less attractive.

Loss Aversion: The chance of getting nothing in Option B is seen as a loss relative to the certain £1,000,000, leading people to prefer Option A.

Probability Weighting: People tend to overweight small probabilities. In Scenario 2, the low chance of winning £5,000,000 in Option C seems more appealing than the safer Option D, despite its lower expected value.