

Example: Estimating the demand for M1 using US Data

Ref: (i) Patterson, Kerry - An Introduction to Applied Econometrics - a time series approach, (2000), Macmillan.

The file US-money contains the following variables:

m= log of real money; y= log of real income; SR =short term interest rate; LR=long term interest rate;
p=log of Consumer price index

M1: Monetary aggregate usually associated with ‘transactions’ money and includes currency, traveller’s cheques, demand deposits at commercial banks, and other checkable deposits (OCDs).
Seasonally adjusted.

Income: Personal Income. **Seasonally adjusted.**

Price variable: US Consumer Price Index. **Seasonally adjusted.**

Interest rate: r: measures the opportunity cost of holding money

(i) short rate - 90 day Treasury Bill rate;

(ii) long rate - 3-year Treasury Bond rate. **Seasonally NOT adjusted.**

Sample Period: 1974m1 to 1993m12. Monthly data. 20 years. 240 monthly observations.

1. Analyse the trend properties of these variables.
2. Use the Engle and Granger approach to test for cointegration between real money, real income and interest rate (use the short term interest rate SR). Report the results of the long run regression amongst these three variables as well as the test for cointegration. Can you conclude that there exists an equilibrium money demand function?
3. Estimate an appropriate error correction model, and conduct the relevant tests to ensure that the residuals are white noise.
4. Now use the Johansen approach to cointegration. Report the results of the test(s) for reduced rank, the estimated cointegrating equation(s) and the Vector Error Correction Model. Compare the results with those obtained with the EG approach.
5. Test for weak exogeneity of income and interest rate. In the light of these results, what are the relative advantages of the two approaches for the analysis of cointegration?

Figure 1- Real Money



Figure 2 – Real Income

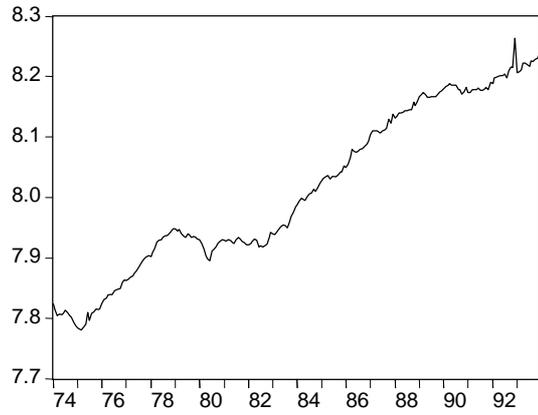


Figure 3 – Short Rate

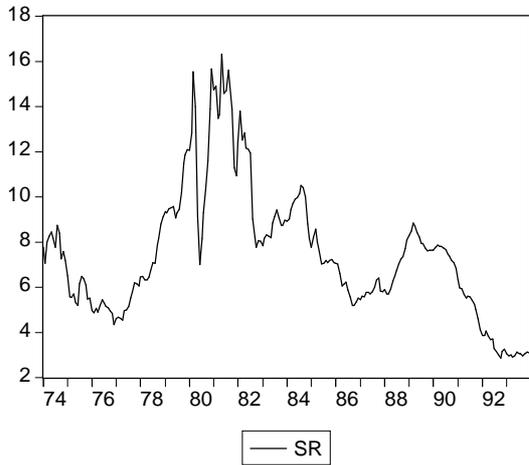
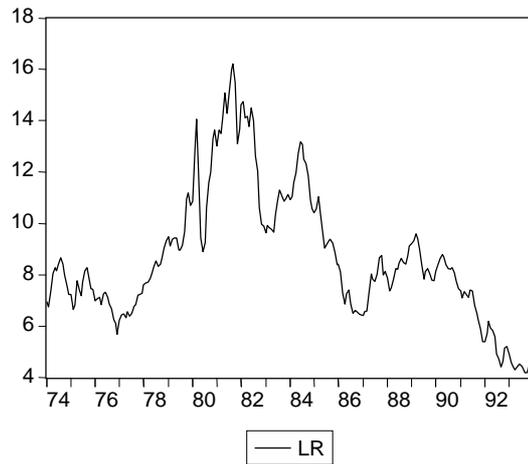


Figure 4 – Long Rate



ADF tests show that the following variables all have a unit root in levels but not in first differences. (i) $m - p$ (log of real money); (ii) $y - p$ (log of real income); (iii) r_1 & r_2 - short and long rate;

We now build an equilibrium demand for money equation.

$$m_t = \beta_1 + \beta_2 y_t + \beta_3 r_t + u_t \quad (1)$$

This will be a proper equilibrium relationship and not a spurious regression if u was $I(0)$. Therefore we have to check whether there is a unit root by carrying out an ADF test using the residuals from the OLS regression of (1). The number of variables in this equation are 3 and we use $N=3$ row from MacKinnon's table of critical values.

Dependent Variable: M (this is log of real money)				
Method: Least Squares				
Sample: 1974:01 1993:12				
Included observations: 240				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.557017	0.196785	7.912276	0.0000
Y	0.616609	0.024151	25.53183	0.0000
SR	-0.020984	0.001147	-18.29236	0.0000
R-squared	0.865104	Mean dependent var		6.341175
Adjusted R-squared	0.863965	S.D. dependent var		0.128826
S.E. of regression	0.047515	Akaike info criterion		-3.243128
Sum squared resid	0.535066	Schwarz criterion		-3.199620
Log likelihood	392.1753	F-statistic		759.9514
Durbin-Watson stat	0.105598	Prob(F-statistic)		0.000000

Residual from the above is saved as $ressr$. We can then carry out the unit root test on this residuals. This time we only want the intercept and not the trend. The result of the test, including 2 lags of the $\Delta ress_r$ series to get rid of any serial correlation is

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-3.3417	0.0141

According to MacKinnon's critical value table, the 5% critical value is:

$$5\% \text{ cv} = -3.7429 - 8.352/237 - 13.41/(237*237) = -3.778.$$

We cannot reject the null that there is a unit root in the residual series. This implies that the above is a spurious regression.

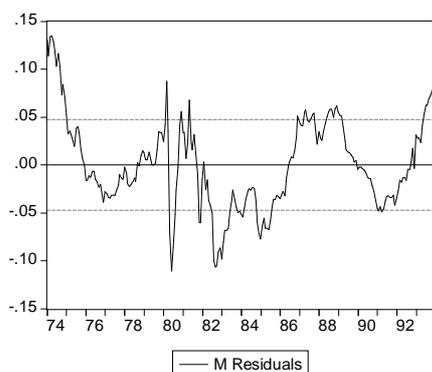
$$10\% \text{ cv} = -3.4518 - 6.241/237 - 2.79/(237*237) = -3.478.$$

The null is not rejected here either! But marginally, so perhaps this is a CI relationship.

The long run income elasticity is estimated to be 0.616609.

The long run effect of the short rate is -0.020984. That is, one percentage point increase in the short rate (one unit increase, as interest rates measured in percentage) will decrease the demand for money by about 2%.

CI Residuals (model with SR)



There might be some structural breaks that are causing the problem with the unit root test. Since dynamics is missing from the equation, there will be some persistence as shown by the residual plots.

The long-run equation using the long-rate looks very similar to the one using the short-rate. We will therefore concentrate on the short-rate equation for further analysis.

We now check to see whether we were right in assuming that there was only one cointegrating relationship or not. This is done via the VAR representation.

- First estimate an unrestricted VAR with various choice of lag length and choose the model in terms of some Information Criterion or by just looking at the t-ratios on the last lag coefficient and restrict it down.

Choosing in terms of AIC and SIC gives you two different lag length choices. Note that with 4 the trace statistic indicates that there is no cointegrating relationship. So we select 2. This is not entirely correct! Just shows you how difficult the whole thing is... Always use some economic theory and your judgment to arrive at the model as there is no 'correct' way!

Remember:

Trace statistic : $\lambda_{\text{trace}} = -T \sum_{i=r+1}^n \ln(1 - \hat{\lambda}_i)$, $r = 0, 1, \dots, n-1$. $H_0: \text{rank} \leq r$; $H_1: \text{rank} > r$.

The lambda max statistic : $\lambda_r^{\text{max}} = -T \ln(1 - \hat{\lambda}_{r+1})$, $r = 0, 1, \dots, n-1$. $H_0: \text{rank} \leq r$; $H_1: \text{rank} = r+1$.

For both tests the distribution is nonstandard and depends upon the deterministic terms included in the model. Approximate asymptotic critical values for both the LR and the Trace statistic are provided by Osterwald-Lenum (1992) (There are five tables of critical values, for VARs up to 11 variables). The most common case is an intercept in both cointegrating equation and the differenced form of the VAR (this implies a linear trend in the levels of the variables).

λ_{\max} test			Trace test		
H_0	H_1	λ_{\max} test	H_0	H_1	Trace test
$r=0$	$r=1$	$-T \ln(1 - \hat{\lambda}_1)$	$r=0$	$r \geq 1$	$-T \sum_{i=1}^n \ln(1 - \hat{\lambda}_i)$
$r \leq 1$	$r=2$	$-T \ln(1 - \hat{\lambda}_2)$	$r \leq 1$	$r \geq 2$	$-T \sum_{i=2}^n \ln(1 - \hat{\lambda}_i)$
$r \leq 2$	$r=3$	$-T \ln(1 - \hat{\lambda}_3)$	$r \leq 2$	$r \geq 3$	$-T \sum_{i=3}^n \ln(1 - \hat{\lambda}_i)$
\vdots			\vdots	\vdots	
$r \leq n-1$	$r=n$	$-T \ln(1 - \hat{\lambda}_n)$	$r \leq n-1$	$r=n$	$-T \ln(1 - \hat{\lambda}_k)$

Sample(adjusted): 1974:04 1993:12
 Included observations: 237 after adjusting endpoints
 Trend assumption: Linear deterministic trend
 Series: M Y SR
 Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.128067	39.31800	29.68	35.65
At most 1	0.028095	6.838957	15.41	20.04
At most 2	0.000359	0.085017	3.76	6.65

** denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates 1 cointegrating equation(s) at both 5% and 1% levels

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.128067	32.47904	20.97	25.52
At most 1	0.028095	6.753940	14.07	18.63
At most 2	0.000359	0.085017	3.76	6.65

** denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates 1 cointegrating equation(s) at both 5% and 1% levels

Unrestricted Cointegrating Coefficients (normalized by $b^*S_{11}^{-1}b=1$):

M	Y	SR	
23.02749	-16.34945	0.768196	This is the matrix beta transposed (rxp) with the CVs in the rows. As there is only one CV, we will only look at the first row.
0.802582	-8.021381	-0.235483	
14.34752	-6.411325	0.091799	

Unrestricted Adjustment Coefficients (alpha):

D(M)	-0.001898	-0.000112	-4.54E-06
D(Y)	-0.000812	0.000969	-3.86E-05
D(SR)	0.021483	0.040291	0.010419

1 Cointegrating Equation(s): Log likelihood 1571.668

Normalized cointegrating coefficients (std.err. in parentheses)

M	Y	SR	
1.000000	-0.709997 (0.06112)	0.033360 (0.00312)	First row of unrestricted beta above, divided by 23.02749

Adjustment coefficients (std.err. in parentheses)

D(M)	-0.043705 (0.00761)	This is the first column of unrestricted alfa multiplied by 23.02749.
D(Y)	-0.018691 (0.00987)	If the underlying coefficients here are zero, then this implies that y and sr are weakly exog.
D(SR)	0.494700 (0.91757)	

We have the following cointegrating relationship.

$$m = 0.7100 y - 0.0334 sr + \text{residual}$$

(0.61) (0.00312)

Since there is only one cointegrating vector that has been identified, we don't have to worry about imposing restrictions to identify the CVs. At most, there could have been two CVs, in which case we would have needed one restriction (r-1) on each CV on top of the normalisation.

Engle-Granger equation was: $m = 0.6166 y - 0.0201 sr + \text{residual}$

(std. errors are biased so not reported here.)

The estimated long-run elasticities are quite similar

The above VAR tells you that the equation for m is:

$$\Delta m_t = \dots - 0.043705 (m - 0.7100 y + 0.0334 sr)_{t-1} + \text{residual}$$

But note the above equation will contain two lags of all the variables in addition to the error-correction term. The ECM is given below. Important to note that this model only contains lagged values of other variables.

Vector Error Correction Estimates

Sample(adjusted): 1974:04 1993:12

Included observations: 237 after adjusting endpoints

Standard errors in () & t-statistics in []

Cointegrating Eq:	CointEq1		
M(-1)	1.000000		
Y(-1)	-0.709997 (0.06112) [-11.6164]		
SR(-1)	0.033360 (0.00312) [10.6996]		
C	-0.899905		
Error Correction:	D(M)	D(Y)	D(SR)
CointEq1	-0.043705 (0.00761) [-5.74126]	-0.018691 (0.00987) [-1.89414]	0.494700 (0.91757) [0.53914]
D(M(-1))	0.206268 (0.06540) [3.15397]	0.163155 (0.08477) [1.92460]	26.88125 (7.88293) [3.41006]
D(M(-2))	0.130220 (0.06491) [2.00602]	-0.018443 (0.08415) [-0.21918]	-13.29803 (7.82451) [-1.69954]
D(Y(-1))	-0.000916 (0.05250) [-0.01745]	-0.289618 (0.06806) [-4.25552]	10.89798 (6.32849) [1.72205]
D(Y(-2))	-0.062770 (0.05201) [-1.20685]	-0.018051 (0.06742) [-0.26773]	16.87150 (6.26923) [2.69116]
D(SR(-1))	-0.002031 (0.00058) [-3.49664]	0.000854 (0.00075) [1.13388]	0.350981 (0.07002) [5.01292]
D(SR(-2))	-0.000555 (0.00054) [-1.03000]	-4.83E-05 (0.00070) [-0.06923]	-0.207745 (0.06490) [-3.20116]
C	0.000957 (0.00037) [2.61628]	0.002184 (0.00047) [4.60540]	-0.085185 (0.04410) [-1.93162]

Weak Exogeneity of income and SR can be checked via testing whether the corresponding α coefficient is zero. Testing this restriction gives us the following:

Cointegration Restrictions:

A(2,1)=0,A(3,1)=0

Convergence achieved after 3 iterations.

Not all cointegrating vectors are identified

LR test for binding restrictions (rank = 1): **conditional on rank=1**

Chi-square(2) 3.565840

Probability 0.168146

So income and SR are weakly exogenous.

We now estimate the short-run elasticities using EG methodology

We already have the lagged residuals from the CI regression.

ECM

Equation using up to 4 lags of all the variables

Since the data are de-seasonalised, there is no reason to expect how many lags would be needed in the equation. This is subjective. Having up to the 4th difference implies that we include up to 5 lags in the equation. Anything less than 5, you will see, suffers from problems.

Dependent Variable: DM

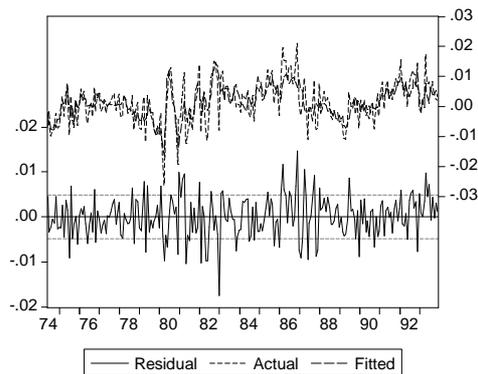
Method: Least Squares

Sample(adjusted): 1974:06 1993:12

Included observations: 235 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000122	0.000393	0.311353	0.7558
DY	0.169895	0.048460	3.505846	0.0006
DY(-1)	0.102587	0.052349	1.959650	0.0513
DY(-2)	0.001497	0.053690	0.027879	0.9778
DY(-3)	-0.012879	0.053304	-0.241609	0.8093
DY(-4)	-0.107905	0.050387	-2.141541	0.0333
DSR	-0.001846	0.000529	-3.490077	0.0006
DSR(-1)	-0.002324	0.000604	-3.844798	0.0002
DSR(-2)	-0.001964	0.000630	-3.116316	0.0021
DSR(-3)	-0.000664	0.000616	-1.078083	0.2822
DSR(-4)	-6.12E-05	0.000530	-0.115484	0.9082
DM(-1)	0.224803	0.068027	3.304609	0.0011
DM(-2)	0.051984	0.066342	0.783577	0.4341
DM(-3)	0.288710	0.066109	4.367152	0.0000
DM(-4)	0.034141	0.064097	0.532655	0.5948
RESSR(-1)	-0.023069	0.008635	-2.671385	0.0081
R-squared	0.549317	Mean dependent var		0.001387
Adjusted R-squared	0.518449	S.D. dependent var		0.006960
S.E. of regression	0.004830	Akaike info criterion		-7.762262
Sum squared resid	0.005109	Schwarz criterion		-7.526716
Log likelihood	928.0658	F-statistic		17.79529
Durbin-Watson stat	1.977095	Prob(F-statistic)		0.000000

DW is biased towards 2 because of the presence of lagged dependent variable among the regressors. The coefficient on the Error Correction Term (RESSR(-1)) which is the residual from the CI relationship, has the correct sign and also it is significant. This says that, 2.3% of last periods equilibrium error is corrected this period, ceteris paribus. But this is not the only correction to the DM variable this period since there are other lagged differenced DM variables in the equation.



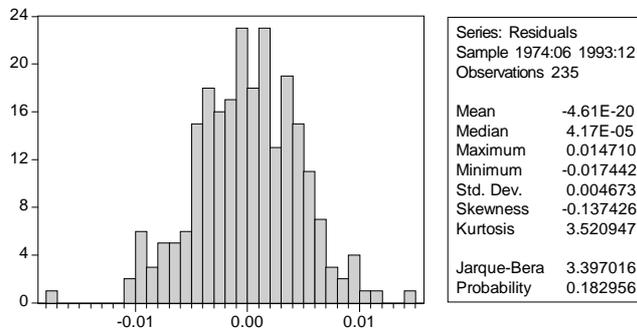
Breusch-Godfrey Serial Correlation LM Test 6 restrictions

F-statistic	1.819678	Probability	0.096527
Obs*R-squared	11.45842	Probability	0.075201

No serial correlation problems here.

Test for Normality

Jarque-Bera – Chi sq (2) = 3.397 [0.182] See below (there are some outliers on both sides). We cannot reject the null that the errors are normally distributed.



White Heteroskedasticity Test 15 restrictions

F-statistic	1.355946	Probability	0.113505
Obs*R-squared	39.06934	Probability	0.124183

Test Equation: We cannot reject the null of homoskedasticity. I only included the squared terms and not the cross products as well to avoid too many variables in the test equation.

Dependent Variable: DM **THIS IS THE RESTRICTED VERSION OF THE ECM**

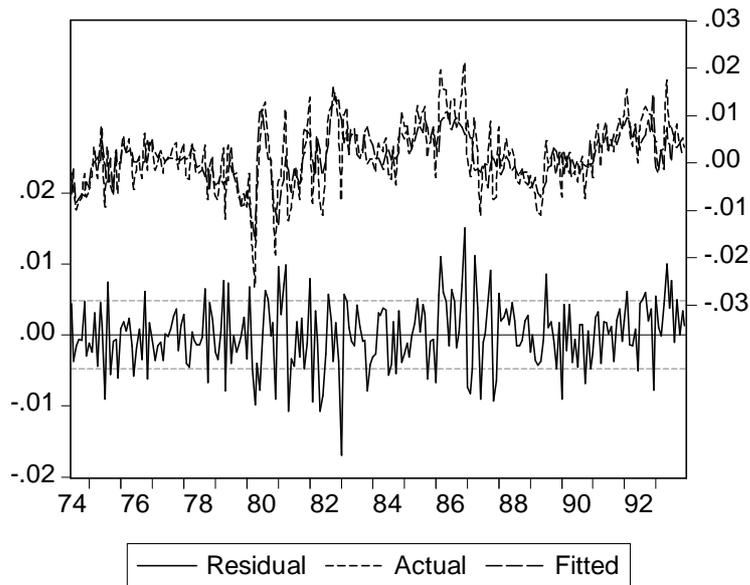
Method: Least Squares

Sample(adjusted): 1974:06 1993:12

Included observations: 235 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000124	0.000352	0.352074	0.7251
DM(-1)	0.270952	0.058118	4.662132	0.0000
DM(-3)	0.320228	0.054299	5.897455	0.0000
DY	0.168619	0.048034	3.510405	0.0005
DY(-1)	0.101781	0.049374	2.061412	0.0404
DY(-4)	-0.109566	0.046024	-2.380607	0.0181
DSR	-0.001975	0.000507	-3.894514	0.0001
DSR(-1)	-0.001930	0.000520	-3.712976	0.0003
DSR(-2)	-0.002154	0.000577	-3.731786	0.0002
RESSR(-1)	-0.025565	0.008229	-3.106899	0.0021

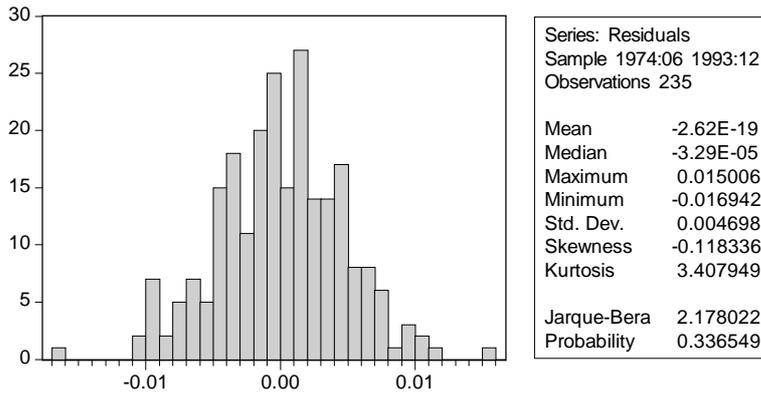
R-squared	0.544400	Mean dependent var	0.001387
Adjusted R-squared	0.526176	S.D. dependent var	0.006960
S.E. of regression	0.004791	Akaike info criterion	-7.802475
Sum squared resid	0.005165	Schwarz criterion	-7.655259
Log likelihood	926.7908	F-statistic	29.87273
Durbin-Watson stat	2.047112	Prob(F-statistic)	0.000000



Breusch-Godfrey Serial Correlation LM Test 6 restrictions

F-statistic	1.414799	Probability	0.209974
Obs*R-squared	8.769074	Probability	0.186988

We cannot reject the null of no serial correlation using the AR(6) specification.



We cannot reject the null that the errors are normally distributed.

White Heteroskedasticity Test: 9 restrictions

F-statistic	1.520505	Probability	0.084662
Obs*R-squared	26.42791	Probability	0.090369

Ramsey RESET Test: 2 restrictions

F-statistic	0.745065	Probability	0.475882
Log likelihood ratio	1.565092	Probability	0.457240
