Identity-Based Elections

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August 15, 2024

Abstract

We propose a model of elections in which partisans wish to convince themselves that their party is the better choice. They select information sources from a broad array of media outlets with different biases to achieve that goal, but they may not always succeed due to their rationality which acts as a constraint. We explore how asymmetries between the two political sides skew electoral outcomes despite rationality. Here we consider salient examples such as asymmetric exposure or asymmetric trust in media, as well as propaganda, but this notion of partisanship is easily applicable to a wide variety of electoral contexts.

JEL codes: D72, D83, D90

Keywords: Behavioral Voters, Belief-based Utility, Information Aggregation

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‡We thank conference and seminar participants at UC Berkeley, Caltech, Princeton, Harvard, Georgetown, Rutgers, SMU, Vanderbilt, Utah, LSE, LBS, PSE, QMUL, University of Venice, FGV - Rio de Janeiro, CREST, SITE, Stony Brook GT Conference, AMES, NASMES, AEA, U Penn, Ashoka University, and the Delhi School of Economics for useful comments and suggestions. All mistakes are our own.
1 Introduction

“More often than not, citizens do not choose which party to support based on policy opinion; they alter their policy opinion according to which party they support.”  

Lilliana Mason (2018)

In the era of the Internet, people can choose from a plethora of possible news sources. Although traditional mainstream news sources continue to exist, many new ones have emerged in just the past two decades. Figure 1, which is taken from Benkler et al. [2017], maps the sharing on Twitter of 1.25 million news articles from 25,000 outlets. This richness of the media landscape has allowed people to more precisely tailor their media choices to their preferences. However, the incredible diversity of viewpoints on offer, combined with new technologies, has also facilitated the formation of “echo chambers” which insulate people from possibly contrary perspectives offered by traditional media outlets.

Figure 1: Pattern of news sharing on Twitter

At the same time, there are stark asymmetries between the two political sides. For instance, while trust in traditional media has declined markedly over the past two decades, this distrust has developed along radically different paths on each side of the political spectrum, particularly during the last five years in the U.S. As can be seen in figure 2
below, the level of trust in mainstream media differs dramatically between Republicans and Democrats.

Figure 2: American’s Trust in Mass Media, by Political Party (Source: Gallup Polling)

Indeed, based on Pew Foundation opinion polls reported in Jurkowitz et al. [2020], “... one of the clearest differences between Americans on opposing sides of the political aisle is that large portions of Democrats express trust in a far greater number of news sources.”

These two phenomena — the emergence of a dense array of media outlets and heterogeneous partisan distrust of mainstream media — are likely to be having an impact on the formation and updating of political beliefs, and as a consequence, may be influencing voter decisions. But can this new information environment generate aggregate beliefs biased enough to swing an election?

In the U.S., the influence of the above-mentioned phenomena on electoral outcomes is compounded by the particularly polarized landscape, in which traditional ideological, religious, and racial affiliations are being replaced by overlapping meta-identities that align almost entirely along party lines. Citizens have become less responsive to new information about national issues, as if political affiliations determine what information

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1For instance, as noted in Benkler et al. [2017] and illustrated in figure 1, during the 2016 US presidential campaign, “Pro-Clinton audiences were highly attentive to traditional media outlets, which continued to be the most prominent outlets across the public sphere, alongside more left-oriented online sites. But pro-Trump audiences paid the majority of their attention to polarized outlets that have developed recently, many of them only since the 2008 election season. [...] Breitbart News became the center of a distinct right-wing media ecosystem, surrounded by Fox News, the Daily Caller, the Gateway Pundit, the Washington Examiner, Infowars, Conservative Treehouse, and Truthfeed.” Lee [2010] finds that trust in media is negatively correlated with conservatism and Republican-leaning views. Pennycook and Rand [2019] note that Democrats trust mainstream news more than Republicans, with the difference ranging between 11.5 to 14.7%. According to Jones [2004], “...only 16.5 percent of Democrats (including Democratic-leaning independents) can be classified as media skeptics compared with nearly 40 percent of Republicans and Republican-leaning independents.”
people absorb, rather than the other way around (see, for instance, Mason [2018], Stroud [2008], and Kahan [2017]).

Given this situation, we propose a model in which media choices are driven by political identity, which is a broader concept of political partisanship. Here, individuals gain utility if they are able to convince themselves, after all the information has been received and processed rationally by them, that their party is the better choice. We assume that an individual chooses specific media to follow, but allow for the possibility that she might also be exposed to information shocks from the outside world, namely the broader media environment which she does not explicitly choose. We refer to the latter as Outside media and the former, namely, media that individuals choose, in full awareness of their bias, as Inside media. In other words, individuals make their media choice in order to counter news that may be potentially unfavorable to their political identity. They then choose between the candidates according to their rational posterior beliefs. In sum, the election aggregates all votes, each of which is based on two conditionally independent signals about the candidates. Our goal is to determine whether information aggregation fails in such elections, and if so, exactly when and why.

Equivalently, one can think of agents as having two selves — an emotional self (heart) and a rational self (mind). The heart chooses Inside media in order to preserve the individual’s political identity, whereas the mind processes all the information it receives rationally and votes for the party it believes to be superior. The objective of the heart could alternatively be to persuade the mind to vote for the heart’s preferred party, while the mind simply prefers to vote for the correct party. Under our baseline preferences, agents do not want to learn directly the true state of the world, rather they wish to conclude that the true state more likely matches their heart’s disposition. This is a rational choice model in which rationality is the constraint and the objective is the value of the posterior. That is, there is motivated information acquisition, but no motivated reasoning. One can think of our agents as partisan voters, albeit in a weaker sense: they want to vote for their preferred party but they may not succeed in doing so because of their rational side.

Our goal is to propose a model of electoral outcomes based on certain features of mod-

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2 How we model the media is also a key innovation. Each Inside media choice (and there are 25000 of them at least in Fig. 1) is a particular known signal structure (under commitment).

3 Inside media choice can be viewed as a long-term loyalty to particular outlets rather than one that is instrumental to voting in a specific election (such that the utility is not derived from the voting itself but rather purely from ex-post beliefs). Alternatively, the choice of Inside media can be thought of as being made more in the context of a particular election, in which case we can think of agents as deriving explicit utility from being able to vote sincerely for the party they are aligned with.

4 We refer to the choice of the wrong candidate by voters as a failure of information aggregation, a failure arising from motivated information acquisition, rationally processed.
ern politics, namely a novel way of thinking about partisanship in a world with a rich media landscape and where individual choice of media could be motivated by political identity. This model is adaptable to studying winning margins and information aggregation in a variety of contexts with heterogeneity in several dimensions, e.g. distribution of identities and of heterogeneous priors, exposure or trust in mainstream/Outside media, misspecified beliefs, and alternative forms of democracy. In what follows, we chose to present only a few simple applications we believe are salient, but obviously many other heterogeneity-combinations can be explored in this framework, also using numerical methods.

In the example below, we highlight the electoral consequences of different exposure to mainstream media along partisan lines: we present our benchmark results for a setup in which the two sides are perfectly symmetric, except in the extent to which they are exposed to information from mainstream media.\(^5\)

**Illustrative example:** Assume a symmetric benchmark in which an equal proportion of countably infinite voters have partisan affiliations, referred to as left (L) and right (R). There are symmetric common priors\(^6\): two equally likely states of the world (\(\omega \in \{L, R\}\)), differentiated by which of the two candidates is superior. The only asymmetry between the two sides is that the left is more exposed to Outside media than the right. Assume, for instance, that the left-wing individuals receive i.i.d. symmetric binary signals from the Outside media with precision \(t_L = 0.75\), while right-wing individuals receive noisier Outside signals, with precision of only \(t_R = 0.51\). As a baseline, we consider electoral outcomes in the absence of Inside media — that is, when the agent is only exposed to an Outside signal. The winning margin and winning probability for the R-side are then as follows:

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<thead>
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<th>Ex-Ante</th>
<th>(\omega = R)</th>
<th>(\omega = L)</th>
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<tbody>
<tr>
<td><strong>R Win Margin</strong></td>
<td>0%</td>
<td>+26%</td>
<td>-26%</td>
</tr>
<tr>
<td><strong>R Win Prob</strong></td>
<td>50%</td>
<td>100%</td>
<td>0%</td>
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Thus, asymmetric exposure to mainstream media generates symmetric electoral outcomes. In this baseline case, the superior candidate is always elected, i.e., information is

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\(^5\)In the paper we show that the same results hold if we assume instead heterogeneous trust in mainstream media between the two sides, as depicted e.g. in Figure 2 Note that, to possibly obtain non-symmetric outcomes, some exogenous asymmetry between the two sides has to be assumed as a premise at some level. So if trust/exposure to media is deemed to be endogenous instead, then a more primitive asymmetry must be driving its heterogeneity and the model can analyze it. One general goal of this paper is to understand when asymmetries in the premises lead to asymmetries in the outcomes and when/how these may prevent information aggregation.

\(^6\)We assume common priors in our benchmark setup. Later, we show how results extend to biased priors.
perfectly aggregated. No personal media choice is made by citizens, and thus political identity, whether R or L, plays no role in decisions.

Now suppose that individuals can also optimally select Inside media sources. In this case, their voting decision is made after having rationally updated their beliefs based on two signals, rather than one. If the media is chosen in order to maximize the chance of preserving their political identity, then the outcome of the election is no longer symmetric. In fact, it may be so drastically skewed to overturn the election, even with common priors. In this example, which we solve in the paper, the winning margin and winning probability of the R-side turn out to be as follows:

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<th>Ex-Ante</th>
<th>$\omega = R$</th>
<th>$\omega = L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R Win Margin</td>
<td>+29%</td>
<td>+56%</td>
<td>+2%</td>
</tr>
<tr>
<td>R Win Prob</td>
<td>100%</td>
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So the R-side has not only an ex-ante winning margin advantage, but surprisingly, also an ex-post advantage: the R-side wins the election in either state of the world. Information is not aggregated, despite the fact that individuals vote solely on the basis of the information they received which was used to rationally update their beliefs. The message here is that the embedded ex-ante advantage of the R-side (who is less exposed) can be so large to always overturn even an election with symmetric common priors in which all signals are, on average, pointing towards the L-side.

Identity-R agents are less exposed to the information shock and optimally choose a qualitatively different Inside media source than Identity-L agents who must contend with a more precise information shock. Importantly, in a world without a rich set of signal structures (Inside media) to choose from — even if partisan biases still drive media choice — we would not see such stark aggregate electoral bias as in the example above.7

**Structure of the paper.** Subsection 1.1 describes the contribution of the model to various strands of literature. Section 2 presents the solution for a single agent. This allows us to highlight some novel aspects of the model, namely the preferences and the conceptualization of media.

The core of the analysis is the influence of aggregated individual media choices in determining electoral outcomes. Section 3 demonstrates the flexibility of the model by considering electoral outcomes in several distinct contexts. In Subsection 3.1, we sup-

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7We solve the general case in the paper and provide intuition for the result explaining when and why information aggregations fail. The example above is not knife-edge: party R would win regardless of the state of the world even if there were slightly more identity-L agents than identity-R, or when the common prior leans towards the L side.
pose that the two political sides are exposed unequally to the information shock. Here, we build on the example above in showing that the region of information aggregation failure is a salient subset of parameters. We also show that the result of information aggregation failure is robust to considering non-common priors, a gain from learning the truth, correlated signals, etc.

Subsection 3.2 looks at distrust of the mainstream media by introducing cognitive bias, thus a mis-specified model. Here, the Outside signal is unbiased and fairly precise, but individuals on one side believe (mistakenly) either that it is imprecise (Subsection 3.2.1) or that it is biased against their side (Subsection 3.2.2). This may be particularly relevant in the case of liberal democracies, such as the U.S. (see figure 2). We show that significant political advantage accrues to the side which believes (incorrectly) that the mainstream media, as a whole, is either imprecise or biased against them. This result is robust to our interpretation of distrust and may provide a rationale for the efforts of politicians to encourage distrust of the mainstream media.

The modeling strategy, which is based on the dichotomy between Inside media and Outside media, can be extended to consider other important questions. For example, in Section 3.3, we allow the Outside media to be biased as it is in the case of government propaganda. Subsection 3.3.1 looks at a media censorship regime, in which the biased Outside media is the only media available. In this case, propaganda can benefit the government. However, as shown in Subsection 3.3.2, if individuals are able to consume Inside media of their choice, then propaganda is not just ineffective, it may backfire completely in favor of the anti-government side. This reversal is striking and occurs because individuals from both political camps choose qualitatively different Inside media. Finally, in Subsection 3.3.3, we show that if individuals are unaware of the bias in the Outside signal, then propaganda may benefit the government. This provides a rationale for the massive investment by authoritarian governments in their propaganda machine. Propaganda can succeed if the electorate remains unaware of government influence on the media, or when it is paired with censorship.

The notion of partisanship we consider, i.e., political identity preservation, is fundamentally different from earlier conceptualizations of partisanship. Thus, instead of a bias in the utility function, location of ideal points, or behavioral types, we consider an individual who is partisan in the beliefs she wants to hold. In other words, she would like to believe that her party is the better choice. We highlight this contribution of the model in Subsection 3.4. Section 4 concludes. The proofs are presented in the Appendix.
1.1 Related Literature

Given the novelty of the model we consider, our analysis does not fit neatly into one subliterature. Rather, it lies at the intersection of various strands of literature. Below, we provide an overview of the papers and topics we build on and contribute to.

**Information biases and politics.** Biases in the processing of information influence electoral outcomes in Levy and Razin [2015] and Ortoleva and Snowberg [2015]. The source of bias in both their models is correlation neglect, according to which individuals underestimate the correlation between their information sources.\(^8\) Although we also use a simple decision-theoretic problem aggregated to derive expected electoral outcomes, the behavioral bias in our benchmark case is not cognitive but rather it resides in the preferences that drive information acquisition. Specifically, agents rationally update using all the information they receive.

**Belief-based utility.** The tradition of models with agents deriving utility from their beliefs goes back to Akerlof and Dickens [1982] who incorporate beliefs explicitly in the decision maker’s utility function. In their framework, beliefs are a choice variable, whereas, in our case, beliefs are a stochastic outcome of choosing a particular signal structure. This growing literature includes Caplin and Leahy [2001] and Brunnermeier and Parker [2005], in which belief-based utility drives agents’ actions, as well as Eliaz and Spiegler [2006] in which belief-based utility drives an agent’s choice of information source. Bénabou and Tirole [2016] provide a survey of the main findings that emerge from economic models of motivated beliefs. In our model agents’ identity determines which beliefs provide utility. In that respect, we build on Akerlof and Kranton [2000], in which the implications of introducing identity-based utility are considered in various contexts.

Though cast in a different context, our model has a similar flavor to the Köszegi [2006] model of overconfidence, in which agents are unbiased in their beliefs (since they start from a correct prior and update rationally) but end up with a systematic bias in their choice due to the bias in their information collection process. Intuitively, agents derive intrinsic utility from believing that “something” is the case (specifically, that they have superior ability in some task, as in Köszegi’s case, or that their party is superior, as in our case), and therefore they tend to collect information that preserves those beliefs as often

\(^8\)For a survey of the recent literature on the electoral outcomes under these and similar cognitive biases, see Levy and Razin [2019]. This literature continues to grow (see, for example, Little et al. [2020] on motivated reasoning cognitive bias). Gentzkow et al. [2021] find that small ideological differences in the extent of trust in information sources and in beliefs about the state of the world can result in a polarized electorate.
as possible.

**Information aggregation.** Like Feddersen and Pesendorfer [1996] and Feddersen and Pesendorfer [1997] and the rich literature in their wake, our goal is to understand the aggregation of information held by differently informed voters. We propose a broader notion of partisanship which blurs the dichotomy between partisan voters (who vote in one way regardless of their beliefs or the state of the world) and non-partisans (who just want the correct electoral outcome) which is typical of previous models. In our model, all voters are partisans, albeit in a weaker sense; that is, they have an allegiance to a specific candidate, but (depending on the realization of the state, their prior, and their exposure to the outside world) they may or may not succeed in convincing themselves to vote for her. While aggregate welfare measures are tricky given our preferences, the question of correct information aggregation in elections is well-posed since all information received is processed in a purely Bayesian manner by all voters.

**Media.** Our model assumes that citizens can choose their Inside media from a dense distribution of sources, spanning all possible biases. The innovation lies in the fact that media are passive and non-strategic information senders with known reputation/bias, and media consumption is demand-driven only. Motivated by the fast-changing media landscape, there is a burgeoning literature that examines media bias — albeit usually from the supply side — in which media behave strategically. This includes Gentzkow and Shapiro [2006], Mullainathan and Shleifer [2005], Bernhardt et al. [2008], Chan and Suen [2008], Hu et al. [2019], Gitmez and Molavi [2022], and Perego and Yuksel [2022]. In a similar vein, politicians can also influence the media or manipulate the information available to voters, as in for instance Gehlbach and Sonin [2014], Aköz and Arbatli [2016].

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Motivated information acquisition to preserve political identity is similar in spirit to minimizing cognitive dissonance, the electoral implications of which are considered in Acharya et al. [2018]. The objective of preserving one’s political identity is also consistent with findings in a large strand of the psychology literature, which primarily looks at cognitive biases (unlike our model). Sherman and Cohen [2006] note that biases in the assimilation of information can be generated from the motivation to maintain and protect political identity. Motivated reasoning as a way of preserving political identity is also noted by Kahan [2017] and Kahan and Braman [2006]. Furthermore, the ability to explicitly deliberate does not mitigate this phenomenon according to Kahan [2012] who finds that cognitive reflection exacerbates ideologically motivated reasoning. Kaanders et al. [2022] conduct an experiment with active information sampling and find that individuals are more likely to choose information that allows them to preserve their beliefs, which is consistent with the setup of our model. Similarly, Schwardmann et al. [2022] use data from international debating competitions to show that individuals can persuade themselves regarding the superiority of their position while Chen et al. [2021] find that individuals tend to continue searching for information until they are able to preserve their moral beliefs.

The literature on potential failure of information aggregation is vast and includes Razin [2003], Callander [2008], Acharya [2016], Ekmekci and Lauermann [2020], Aina [2021], and Barelli et al. [2022].
Our theoretical results are consistent with a number of empirical studies, such as DellaVigna and Kaplan [2007] and Martin and Yurukoglu [2017] which show that the introduction of new (and largely conservative) media outlets has resulted in a persistent and significant increase in Republican vote share. Sonin et al. [2023] show that being in isolated political echo chambers amplifies partisan beliefs, which is in line with our results. The absence of a systemic partisan bias in recognizing fake news, as shown by Angelucci and Prat [2021], indicates the absence of systemic partisan cognitive differences. Broockman and Kalla [2022] find that the political views of partisans are malleable, indicating that while voters have partisan preferences, they update beliefs based on the information they receive. An overview of the recent literature on populism and identity politics is provided by Guriev and Papaioannou [2022].

**Persuasion.** A key innovation is to model media as senders with known bias, namely with known reputation/commitment. So, as in recent work on this topic, we apply simple Bayesian persuasion techniques to political economy. Our focus is merely on aggregating individual decisions in a large population of individuals. We adapt and distill results developed by Kolotilin [2018] who builds on Kamenica and Gentzkow [2011]. Kolotilin requires the sender to choose an information structure while being uncertain about the receiver’s type. This is similar to our agent’s problem of choosing an information structure while bracing for an Outside signal. Lipnowski and Mathevet [2018] consider an information design problem with a benevolent sender who chooses a signal structure for a receiver with psychological preferences, an approach with similarities to our model of information choice by agents who derive belief-based utility.

## 2 The single agent model

Suppose there are two states of the world (R and L), which indicate which of the political parties (R or L) is superior. An agent is either identity-R or identity-L, which indicates her political affiliation. We begin by considering the problem for a single identity-R agent. The problem and solution for an identity-L agent are analogous.

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11 Voters themselves may cause polarization through the selective sharing of information if combined with misspecified learning, as seen in Bowen et al. [2023].

12 There has recently been a great deal of effort in applying information design to political environments including Edmond [2013], Cotton and Li [2018], Gratton and Lee [2022], Luo and Rozenas [2022], Innocenti [2022], Heese et al. [2019], Gradwohl et al. [2022], and Prato and Turner [2022].

13 Kolotilin et al. [2017] establish the equivalence of implementation by persuasion mechanisms, which can condition the signal structure on the type of the receiver, and implementation by experiments, which are unconditional on the receiver’s type.
Preferences. The identity-$R$ agent identifies with party $R$. She gains utility if she is able to believe that party $R$ is at least as good as party $L$. In this model, this is equivalent to a posterior belief that the state of the world is $R$ with a probability of at least one-half.\footnote{There is a vast literature on the importance of political identity for individuals. See, for instance, Mason [2018]. The notion of political identity preservation is closely related to ego utility which is studied in Köszegi [2006].} Specifically, her utility equals one ($U_R = 1$) if her posterior belief is weakly favorable to her party ($P[\omega = R | signals] \geq 0.5$) and zero otherwise.

Alternatively, we could suppose that the agent’s utility depends on an action, rather than a belief. That is, she receives positive utility if she votes for her preferred party, but she can only do that if she is able to convince herself that her preferred party is superior. If she can’t convince herself, she could either vote for the opposing party, leading to an equivalent isomorphic model, or abstain, leading to the same results as our model with the voting margins halved.

Timing. The agent is born with some priors regarding the state of the world. First, she chooses the information source from which she would like to receive a signal. Next, the state of the world is realized. She then receives two signals that update her beliefs. One is from the chosen source (Inside media), and the other is an information shock that she is exposed to (Outside media). Finally, she forms Bayesian posteriors and realizes utility. Her problem is to select the structure of the chosen source so as to maximize the chances of preserving her political identity given her prior and anticipating her exposure to an informational shock.\footnote{Kaanders et al. [2022] show that information sampling can be motivated by a desire to preserve beliefs. That political identity preservation can motivate reasoning is documented in Kahan and Braman [2006] and Kahan [2017], among others.} Her desire to preserve her political identity influences the way she acquires information and, as shown in Section 3, leads to interesting aggregate electoral outcomes.\footnote{An alternative but equivalent setup supposes that an identity-$R$ agent has two selves: a sender and a receiver. The sender-self selects the chosen media. The receiver-self observes the signal generated by the chosen media as well as an information shock. She then votes sincerely. The objective of the sender-self is to maximize the likelihood that the receiver-self votes for party $R$. For simplicity of illustration, we formulate the individual’s problem such that she realizes belief-based utility.}

The identity-$R$ agent’s objective can be expressed as:

$$E[U_R] = \mathbb{P}[P[\omega = R | S, s] \geq 0.5]$$

where $S$ is the signal generated by the chosen media (Inside signal structure), while $s$ is the information shock (a signal generated by the Outside signal structure). The agent’s problem is to choose an Inside signal structure that maximizes equation 1.
Conceptualization of media. The conceptualization of an individual’s information environment is a novel aspect of our paper and is a key contribution. In a rich media landscape, an individual can choose from outlets with a wide range of political slants. The Inside signal structure is a finite set of signals ($S = \{S_1, S_2, \ldots, S_n\}$) that are correlated with the state of the world. No restrictions are imposed on the agent’s choice of the Inside signal structure except that it must abide by the Martingale property, namely the expected posterior must equal the prior. We interpret this choice of signal structure as the combination of news outlets from which the agent chooses to receive information.\footnote{Since we allow for a rich structure, the possibility that the agent consumes news from several different outlets is subsumed within our model. Any distribution of posteriors generated by multiple Inside signals can be generated by an appropriately designed single signal structure.}

The diverse set of signal structures represents the rich media landscape in which voters currently gather information. Furthermore, an agent can choose a news media (a signal structure), but not the programming (the realization of a signal).

We also allow for the possibility that individuals might not be able to completely isolate themselves from sources they haven’t chosen. The identity-$R$ agent is involuntarily exposed to an information shock (Outside signal) with the following structure:

$$\mathbb{P}[s = l|\omega = L] = \mathbb{P}[s = r|\omega = R] = t_R$$

where $t_R \in [0.5, 1]$ is the precision of the Outside signal. We interpret the structure of the information shock as reflecting the nature of the media environment as a whole. This structure is one we would expect in a liberal democracy in which each individual outlet may have a political slant but the media, as a whole, is vibrant and free. In other words, the Outside signal structure in a liberal democracy is unbiased. The model is nonetheless amenable to more general structures of the information shock.\footnote{While we restrict our analysis to a binary Outside signal, the model provides the flexibility to consider environments in which the media, as a whole, is biased or believed to be biased. We interpret that bias by means of an Outside signal structure in which $\mathbb{P}[s_R = l|\omega = L] \neq \mathbb{P}[s_R = r|\omega = R]$. An incorrect belief regarding the bias of the Outside signal is interpreted as a form of distrust in the media (see Subsection 3.2.1). We believe that a biased Outside signal can occur when one political side can influence the mainstream media as a whole. We study that scenario in Subsection 3.3.}

The individual holds beliefs over the structure from which the information shock is generated and selects her chosen media accordingly. In the next subsection, we study the individual’s strategic choice of the Inside signal structure in order to possibly counteract the information shock generated by the Outside signal structure.
2.1 Solution of the individual’s decision problem

The solution of the individual’s problem is simply a distillation of the techniques developed in Kamenica and Gentzkow [2011] and Kolotilin [2018]. We first calculate the agent’s expected utility as a function of her interim priors, namely after she has received the Inside signal but before she has received the information shock (the Outside signal). We denote this interim posterior as $P[\omega = R | S_i]$. We will obtain the agent’s optimal signal structure choice as a function of the precision of her information shock.

Figure 3: Expected utility as a function of the interim posteriors

In figure 3, we plot an identity-$R$ agent’s expected utility (which is the likelihood of preserving her political identity) as a function of her interim posterior for an Outside signal structure, such that the precision of the Outside signal ($t_R$) is 0.75. For values of the interim posterior between 0 and 0.25, regardless of the realization of the Outside signal, the agent is unable to preserve her political identity. If the agent has an interim posterior equal to 0.25, and if she receives a favorable Outside signal, then her posterior expectation that the state of the world is $\omega = R$ is equal to 0.5, and she is just able to preserve her political identity. For values of interim posteriors between 0.25 and 0.75, she can preserve her political identity if she receives a favorable Outside signal, $s = r$. The likelihood that she receives a favorable Outside signal increases with her expectation that the state of the world is $\omega = R$. Finally, if the agent’s interim posterior is at least 0.75, then she is able to preserve her political identity regardless of the Outside signal.

**Partisanship and political identity preservation: an aside.** An agent in our model would exhibit partisan behavior if she were born with priors that allowed her to preserve her political identity regardless of any information shock. In that case, she would optimally choose an uninformative Inside signal structure and if there were an election,
she would always vote for her preferred party. However, for intermediate priors and in the presence of information shocks, agents need to persuade themselves in order to preserve their political identity. While they choose media optimally in order to maximize their likelihood of preservation of political identity, they cannot always achieve that objective. The concept of political identity preservation that we develop is, therefore, both weaker than partisanship and broader (nuancing the divide between partisans and non-partisans), thus constitutes a novel contribution to the literature.

**A sufficient set of signals.** For parsimony, we assume that the agent is born with a prior that the two states of the world are equally likely ($P[\omega = R] := w = 0.5$). As we show in Appendix A, three interim posteriors are key to solving the agent’s problem. The first is an interim posterior such that the agent is just able to preserve her political identity in the event of an unfavorable Outside signal ($P[\omega = R | S_i] = 0.75$ in figure 3). We refer to an Inside signal that generates such an interim posterior as a Good ($G$) signal. The second is an interim posterior that allows the agent to just preserve her political identity only if she receives a favorable Outside signal ($P[\omega = R | S_i] = 0.25$ in figure 3). We refer to an Inside signal that generates such an interim posterior as a Bad ($B$) signal. The third is an interim posterior such that the agent is certain that the state does not match her political identity ($P[\omega = R | S_i] = 0$). We refer to an Inside signal that generates such an interim posterior as a Terrible ($T$) signal. Any signals other than the three described above — Good, Bad, and Terrible — are suboptimal. Furthermore, we show that two signal structures, $GT$ and $GB$, are sufficient to solve the agent’s problem.

In figures 4 (a) and 4 (b), we plot the agent’s expected utility as a function of her interim posterior after observing the Inside signal and before observing the Outside signal. For lower values of precision, as in figure 4(a), the concave closure of the expected utility function is such that it would be optimal for the agent to choose a signal structure that mixes between $G$ and $T$, namely $GT$. On the other hand, if the Outside signal is more precise, as in figure 4(b), then the concave closure of the expected utility is such that the optimal signal structure is $GB$. This can also be seen in figure 5, which shows that an agent who faces a less (more) precise Outside signal expects greater utility from choosing a $GT$ ($GB$) signal structure.\(^{19}\)

\(^{19}\)Note that the $GT$ and $GB$ signal structures here are defined for an identity-$R$ agent. For an identity-$L$ agent, a $GT$ signal structure would be one which outputs Good news, from identity-$L$’s perspective always in the state of the world $L$ and sometimes in the state of the world $R$. The Terrible ($T$) signal would only be generated in the state of the world $R$. Similarly, for an identity-$L$ agent, a $GB$ signal structure would be modified analogously.
Interpreting the signal structures. Note that the GT signal structure is one-sided, in the sense that only signal $G$ is realized in the favorable state of the world. Therefore, on observing a signal $T$, the agent is certain that the state of the world is unfavorable. We interpret the agent’s choice of GT as the choice of media with a strong political bias.\footnote{For instance, media sources like 	extit{Breitbart News} for Republicans or the 	extit{The Daily Kos} for Democrats. See Budak et al. [2016] for one approach to organizing media outlets by slant.} Favorable news is routinely reported by these outlets and is not very informative for Bayesian agents. When these outlets report unfavorable news, it is highly informative for Bayesian agents and will convince them that the state of the world is not favorable to their side.
On the other hand, GB is two-sided in that either G or B can be realized in either state of the world, according to a predetermined probability distribution. This is akin to the agent consuming more balanced media because it provides her with a mix of positive and negative news in either state of the world.\(^{21}\) The probability of realization of each signal (G or B) depends on the state of the world.\(^{22}\)

The fact that a negative signal is sent by the GB Inside media in either state of the world makes it less informative, allowing a critique to not be irredeemably bad. Specifically, a favorable Outside signal can counteract an unfavorable Inside signal making the Outside news potentially crucial in the preservation of the individual’s political identity. In contrast, with the more slanted GT Inside signal structure, the preservation of political identity does not depend on the realization of Outside news.\(^{23}\)

It is important to note that the nature of the media outlets endogenously chosen by the agent (her Inside signal structure) depends on her beliefs about the media environment as a whole. If she believes that the media environment is imprecise or biased against her, then she will choose to consume news from a more politically biased media outlet.\(^{24}\)

**Robustness checks.** Adding mean-zero noise to the belief thresholds that agents require to preserve their political faith results in a smooth expected utility (as a function of interim posteriors) graph. Extension A in Kamenica and Gentzkow [2011] shows that adding stochasticity in the receiver’s action makes the sender’s expected payoff function smooth and that an optimal signal structure can still be found using the usual procedure of creating the concave closure of the expected utility function. The agent’s expected utility at any given interim posterior will remain almost unchanged for a sufficiently small noise. As such, the concave closure of the expected utility function and the optimal signal structures remain almost the same. Given that the key results of information aggregation failure noted in propositions 1, 2, 3, and 4, are not knife-edge, they are robust to the

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\(^{21}\)We interpret the GB signal structure to be media sources like the Wall Street Journal for Republicans or the Los Angeles Times for Democrats.

\(^{22}\)The signal structures GT and GB are not Blackwell ranked because neither signal structure is a garbling of the other. An alternative intuition for the fact that the two signal structures are not Blackwell ranked is that a Bayesian agent who receives a large outsized disutility from voting for the incorrect party in the favorable (resp. unfavorable) state of the world would prefer the GT (resp. GB) signal structure.

\(^{23}\)While we do not attempt to rank the informativeness of GT versus GB signal structures in a Blackwell sense, it is clear that conditioning on either a GT or a GB signal structure being chosen, the more informative the Outside signal, the more informative the GT or GB signal structure will need to be. This is in contrast with the crowding out effect observed in other models of information acquisition. The more informative the Outside signal, the more informative the Inside signal will need to be (conditioning on it being either GT or GB).

\(^{24}\)In Appendix A, we consider a general model and calculate the optimal signal structure choice for all binary Outside signal structures, which includes biased signal structures. In Subsections 3.2.2 and 3.3, agents who believe that the Outside signal is biased against them choose a one-sided signal structure.
addition of a small amount of mean-zero noise to the belief thresholds for agents. In Appendix A.3, we illustrate how the optimal signal structure remains in an example.

In Appendix A.4, we show that all the results are robust to an additive function according to which the agent has a relatively small gain from voting for the correct party. This is because, for low values of gain from being correct, the set of signal structures that solve the agent’s problem remains the same as in the benchmark. If there is a region of information aggregation failure present in the benchmark, then it does not disappear when we include a small gain from voting for the correct party, although the region might shrink in size.\(^{25}\)

We further show in Appendix A.5 that the results remain unchanged if the agent gains linearly from holding posteriors favorable to her party, in addition to the gain from political identity preservation. This is because the choice of an Inside signal structure is invariant to the gain from favorable posteriors, and all electoral outcomes are determined by that choice. This robustness holds for all levels of that gain in the utility function.

Therefore, the results are robust to small changes to the shape of the utility function such as noise, a preference for being correct, or holding more favorable beliefs. The key driver of the results is a sharp change in the agent’s utility when her beliefs cross a predetermined threshold which is in the vicinity of one-half.

3 The model: Electoral outcomes

In the previous section, we considered an identity-\(R\) agent’s decision-theoretic problem (the identity-\(L\) agent’s problem is specified and solved analogously). In order to study information aggregation in elections, which is our main focus, we assume that there are countably infinite agents who vote for the party they believe to be superior after they form posterior beliefs.\(^{26}\)

Since we are aggregating individual decisions, we can consider any distribution of voter characteristics. These include identity (\(R\) or \(L\)), priors, exposure to the information shock, access to inside media, and possibly incorrect beliefs about the information shock, among other things. We can also consider asymmetries in the news environment such as

\(^{25}\)As we note in the Appendix A.4, as long as the gain from voting for the correct party is less than the gain from political faith preservation, the agent will manipulate her learning to increase the likelihood of preserving her political faith and we can find a region of information aggregation failure in the parameter space.

\(^{26}\)Ties are broken in favor of the party matching the agent’s identity. Since there are infinite voters, pivotality is not a concern. We could, therefore, specify a linearly additive utility function that puts some weight on the superior party winning the election and some on holding preferred ex-post beliefs, and obtain identical results. Further, in elections with large finite populations, pivotality concerns are minimal and would tend to be dwarfed by a small weight on belief preservation.
a bias in the process that generates the information shock.\textsuperscript{27}

In each case, we can compare the electoral outcomes in our model with those in a benchmark case in which agents are not motivated by identity preservation, or in which an Inside signal is not available. Focusing on electoral outcomes allows us to not only illustrate whether information is aggregated correctly but also to highlight the margin of victory. In each application, we study the role of a particular asymmetry between the two political sides. To highlight the impact of that asymmetry in each environment, we suppose that the countably infinite agents are either identity-\textsuperscript{R} or identity-\textsuperscript{L} in equal proportion. We also assume that all agents share a common and symmetric prior regarding the state of the world (\(\mathbb{P}[\omega = \text{R}] := w = 0.5\)).\textsuperscript{28}

3.1 Asymmetric exposure to information shocks

What happens if voters of one political side are systematically less exposed to information shocks than the other side?\textsuperscript{29} In what follows, we suppose that identity-\textsuperscript{R} agents contend with a less precise information shock than identity-\textsuperscript{L} agents. We considered this question in the example that was discussed in the introduction; a more detailed illustration of the solution and the mechanisms is presented here.

Each agent receives an Outside signal (\(s \in \{r, l\}\)). The Outside signal for identity-\textsuperscript{L} agents has the following structure:

\[
\mathbb{P}[s = l | \omega = L] = \mathbb{P}[s = r | \omega = R] = t
\]

where \(t \in [0.5, 1]\) is the precision of the Outside signal.

For identity-\textsuperscript{R} agents, the Outside signal has the following structure:

\[
\mathbb{P}[s = l | \omega = L] = \mathbb{P}[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \leq t
\]

where \(t \in [0.5, 1]\) is the precision of the Outside signal for identity-\textsuperscript{L} agents and \(\tau \in [0, 1]\) is the relative exposure of identity-\textsuperscript{R} agents to the Outside signal.\textsuperscript{30}

\textsuperscript{27}While we will assume conditionally independent realizations of the information shock for agents, we can also allow for correlated realizations.

\textsuperscript{28}In each application, we illustrate the key results using graphs and numerical examples and prove that the result holds for values around the combinations of parameters considered.

\textsuperscript{29}This asymmetry could be a response by agents to asymmetric distrust (see figure 2), rural-urban sorting (see Scala and Johnson [2017]), or educational attainment differences (see Marshall [2019]).

\textsuperscript{30}One can interpret \((1 - \tau)\) as the extent of noise mixed in with the outside information shock for identity-\textsuperscript{R} agents. Equal exposure to outside information shocks can be considered by setting \(\tau = 1\). When \(\tau = 0\), identity-\textsuperscript{R} agents receive an outside information shock that is pure noise. An alternative isomorphic setup would simply specify different levels of precision for the Outside signal received by agents of both identities, such that the Outside signal’s precision for an identity-\textsuperscript{R} agent is lower than that for an identity-\textsuperscript{L} agent.
One way to interpret precision and exposure in the context of the Outside signal is as attention. Attention, or lack thereof, might be a feature of the media landscape or one’s social circle, which will determine the intensity, frequency, or clarity with which agents receive the signal from outside their chosen media diet. For instance, in the U.S., an asymmetry in information insularity between the two political parties may be due to the rural-urban sorting between Republicans and Democrats.\textsuperscript{31} An alternative interpretation of attention would be that signal precision and exposure capture the openness of an agent to receiving a signal from outside her chosen Inside media. This openness, or lack thereof, may be due to the agent’s preference or beliefs regarding the trustworthiness of the Outside information, which may lead her to actively avoid exposure to mainstream media. The asymmetry in exposure can therefore be thought of as reflecting either the media landscape, or the agent’s preferences or beliefs, or some combination thereof.

Individuals in this environment have access to a wide array of news and opinion outlets that allow them to consume a specific diet of chosen media. We are interested in their choice of Inside media as a function of their exposure to Outside media and its effect on electoral outcomes.\textsuperscript{32}

\textbf{Example.} In the example appearing in the introduction, we set $t_R = 0.51$ and $t_L = 0.75$, which is equivalent to setting $t = 0.75$ and $\tau = 0.04$. For these parameter values, we find that introducing the ability to choose one’s media transforms a fully symmetric election into one with a failure of information aggregation, where party $R$ wins the election in both states of the world. This result is obtained despite agents being rational in their information processing.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
 & \multicolumn{2}{c|}{Without Inside media} & \multicolumn{2}{c|}{With Inside media} \\
\hline
 & Ex-Ante $\omega = R$ & $\omega = L$ & Ex-Ante $\omega = R$ & $\omega = L$ \\
\hline
R Win Margin & 0\% & +26\% & -26\% & +29\% & +56\% & +2\% \\
\hline
R Win Prob & 50\% & 100\% & 0\% & 100\% & 100\% & 100\% \\
\hline
\end{tabular}
\caption{Results with and without Inside media}
\end{table}

Table 1 shows that without Inside media, asymmetric exposure to mainstream media does not provide the $R$-side with any advantage in winning margin or winning proba-

\textsuperscript{31}While of course there are urban US Republicans too, this average urban-rural sorting is very stark in the US and in several countries too, and has become starker in the last decades. Other drivers may also be different average education level among the two camps.

\textsuperscript{32}In this subsection, both agents receive unbiased Outside signals that are not fully informative, and we suppose that identity-$R$ agents receive noisier Outside signals than identity-$L$ agents. The reverse case yields analogous results.
bility. The correct side wins in both states of the world. When the state of the world is \( R \), then 51% of identity-\( R \) agents and 75% of identity-\( L \) agents receive an Outside signal indicating that the state of the world is more likely to be \( R \), and they vote for party \( R \), which implies that the \( R \)-side wins with a margin of 26%. Because the Outside signal is unbiased, party \( L \) wins with the same margin in state \( L \).

The mechanism. The introduction of the Inside signal in the second column of table 1 results in an information aggregation failure that allows party \( R \) to win regardless of the state of the world. Below, we provide the intuition for this result.

It is important to recall two facts from Section 2, where we considered the problem for a single agent. First, the \( GT \) (resp. \( GB \)) signal structure is optimal for low (resp. high) values of precision of the Outside signal. We see this illustrated in figure 5 and the intuition is provided in figures 4 (a) and 4 (b). \(^{33}\) Second, recall that the \( GT \) signal structure is one-sided while the \( GB \) signal structure is more balanced. That is, in the favorable state of the world, a \( GT \) signal structure outputs only the signal \( G \), while a \( GB \) signal structure generates either \( G \) or \( B \) as the signal. In the unfavorable state of the world, both signal structures (\( GT \) and \( GB \)) generate either one of their respective signals according to their probability distributions. Under the parameter values described above (\( t = 0.75 \) and \( \tau = 0.04 \)), an identity-\( R \) agent chooses a \( GT \) signal structure, whereas an identity-\( L \) agent chooses a \( GB \) signal structure. \(^{34}\)

When the state of the world is \( R \), identity-\( R \) agents receive the \( G \) signal from their chosen \( GT \) signal structure with probability 1. Therefore, all identity-\( R \) agents vote for party \( R \) in state \( R \). Identity-\( L \) agents who optimally choose the \( GB \) signal structure receive a \( G \) signal with probability 0.25 and a \( B \) signal with probability 0.75. If they receive a \( G \) signal, they preserve their political faith and vote for party \( L \). If they receive a \( B \) signal, they preserve their political faith and vote for party \( L \) only if they receive the favorable Outside signal (\( l \)), which happens with probability \( t = 0.25 \). Identity-\( L \) agents vote for party \( L \) with probability 44% (0.25 + 0.75 \times 0.25 \approx 0.44 \). Therefore, party \( R \) wins in the state of the world \( R \).

When the state of the world is \( L \), identity-\( R \) agents receive the \( G \) signal with probability 0.96 and the \( T \) signal with probability 0.04. They preserve their political faith and

\(^{33}\)Note that the precision of the Outside signal for an identity-\( R \) agent is \( t_R = \tau t + \left(\frac{1-\tau}{2}\right) \).

\(^{34}\)In Appendix B.1, we show that the \( GT \) signal structure is optimal for identity-\( R \) agents if and only if \( \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \leq \frac{1}{\sqrt{2}} \). Because identity-\( L \) agents receive an Outside signal with a perceived precision \( t \), they choose a \( GT \) signal structure for all values of \( t \leq \frac{1}{\sqrt{2}} \) and a \( GB \) signal otherwise. Note also, that if the two sides faced the same precision of Outside signal structure, their Inside media choice would still be different from each other. A \( GT \) (similarly, \( GB \)) signal structure chosen by an identity-\( R \) agent and one chosen by an identity-\( L \) agent are analogous to each other and have opposite biases.
vote for party $R$ if only if they receive the $G$ signal. Recall that a *Good* Inside signal is one that *just* allows an agent to preserve her political faith upon receiving an unfavorable Outside signal. The low precision of the Outside signal for party $R$ implies that the $G$ signal can be generated with a high probability in the unfavorable state of the world and still provide just enough informativeness to counteract an unfavorable Outside signal. Identity-$L$ agents, who choose a $GB$ signal structure, receive a $G$ signal with probability 0.75 and a $B$ signal with probability 0.25. As before, an identity-$L$ agent preserves her political faith for sure upon receiving the $G$ signal and only if a favorable Outside signal (which happens with probability $t = 0.75$) is realized upon receiving the $B$ signal. This means that identity-$L$ agents preserve their political faith and vote for party $L$ with probability 0.94 ($0.75 \times 0.75 + 0.25 \times 0.75$). Because 96% of identity-$R$ agents preserve their political faith and vote for party $R$ while 94% of identity-$L$ agents preserve their political faith and vote for party $L$, party $R$ wins in the state of the world $L$.

Figure 6: Political identity preservation conditional on the state of the world

![Figure 6: Political identity preservation conditional on the state of the world](image)

Next, we consider the same result through the illustrations in figure 6. Here, the red (blue) lines indicate a $GT$ ($GB$) signal structure which is chosen for low (high) levels of precision of the Outside signal. The solid lines indicate ex-ante expected utility, while the dotted line indicates expected utility in the favorable state of the world and the dashed line indicates expected utility unfavorable state of the world. The expected utility of an agent is the chance that she preserves her political faith and votes for her preferred party.

When the state of the world is $R$, all identity-$R$ agents preserve their political faith. The dotted red line indicates the fact that in the favorable state of the world, a $GT$ signal structure only generates the $G$ signal and the identity-$R$ agent preserves her political faith for sure. The dashed blue line is the expected utility an agent choosing the $GB$
signal structure receives in the unfavorable state of the world. The expected utility of the identity-$L$ agent in the state of the world $R$ can be measured using this line. Since expected utility maps directly into votes for an agent’s preferred party, this graph illustrates how party $R$ wins in the state of the world $R$.

When the state of the world is $L$, the likelihood of political identity preservation by identity-$R$ agents is denoted by the red arrow on the dashed red line, while the corresponding likelihood for identity-$L$ agents is denoted by the blue arrow on the dotted blue line. Given the parameter values in table 1, 96% of identity-$R$ agents vote for party $R$ while 94% of identity-$L$ agents vote for party $L$. Therefore, party $R$ wins even in state $L$.

This kind of information misaggregation occurs when identity-$R$ agents receive a very imprecise Outside signal, while identity-$L$ agents must contend with a moderately precise Outside signal. This configuration is particularly salient because it corresponds to identity-$R$ agents being relatively closed to Outside information while identity-$L$ agents are relatively open, which might be the result of a rural-urban divide between the parties, differences in educational attainment between voters of the two parties, or an asymmetry between the parties in terms of trust placed in the mainstream media.

Discussion. Every agent individually maximizes her likelihood of political identity preservation, but, for some parameters, this implies that party $L$ loses regardless of the state of the world.\textsuperscript{35} If instead, identity-$L$ agents choose $GT$ as their Inside signal structure, then party $L$ can win in the correct state — implying that an ex-ante suboptimal individual choice can allow for correct information aggregation. In other words, party $L$ would benefit if it could convince its electorate to consume more politically biased news. In the U.S. context, this would occur if the Democratic party notes that Republicans consume more biased news, realizes that they could benefit electorally if Democrats also do so, and successfully influences their news consumption patterns. We believe that, in line with the structure of the model, it is more likely that individual news consumption is driven by individual preferences, rather than by mandates from political parties, or by strategic considerations as to which media is being consumed by members of the opposing political camp.

It is important to note that information aggregation failure occurs for low values of exposure ($\tau$) and intermediate values of Outside signal precision ($t$). If the Outside signal were less precise, for instance, if $t = 0.7$, then agents of both identities would choose a $GT$ signal structure, and the correct party would always win. On the other hand, if the

\textsuperscript{35}In aggregate, if each agent maximizes her likelihood of political identity preservation, then that is equivalent to each party ($R$ and $L$) maximizing their expected vote share.
Outside signal were very precise, then again, the correct party would always win. If the state of the world is $L$, then identity-$L$ agents are more likely to receive a Good signal and vote for party $L$ in state $L$ relative to the outcomes in table 1. Similarly, identity-$R$ agents are more likely to receive a Terrible signal and vote for party $L$. Party $L$, therefore, receives more votes than party $R$ in state $L$.

Figure 7: Signal choices and results with asymmetric exposure

In figure 7(a), we consider all values of signal precision ($t$) and exposure ($\tau$). In the red-shaded area, agents of both identities choose a $GT$ signal structure, while in the blue-shaded area, they choose a $GB$ signal structure. In the purple-shaded region, identity-$R$ agents choose a $GT$ signal structure, while identity-$L$ agents choose a $GB$ signal structure. There is no region where identity-$R$ agents choose a $GB$ signal structure and identity-$L$ agents choose a $GT$ signal structure. As claimed in proposition 1 below, there is a region within the purple-shaded region where information aggregation fails, such that identity-$R$ individuals choose a $GT$ signal structure while identity-$L$ agents choose a $GB$ signal structure.

Proposition 1 In the environment specified in Subsection 3.1, the correct candidate wins except in a region with intermediate Outside signal precision and low exposure among identity-$R$ agents. Party $R$ wins regardless of the state of the world if the following conditions are satisfied:

36We focus on the region of information aggregation failure because it represents the region of parameters where the wrong party wins and has clear implications as an outcome. One alternative measure could be the region where the ex-ante winning margin advantage deviates from 50% for either party while another measure could be the percentage of the population that votes for the incorrect party. Agents are free to choose any signal structure, including full revelation. In the absence of a belief-preservation motive, they can choose to learn with complete accuracy, and the correct party would always receive all the votes.
• Identity-R agents choose a GT signal structure, which implies $\tau t + \frac{1-t}{2} \leq \frac{1}{\sqrt{2}}$

• Identity-L agents choose a GB signal structure, which implies $t \geq \frac{1}{\sqrt{2}}$

• Supposing identity-R agents choose a GT signal structure while identity-L agents choose a GB signal structure, a greater share of identity-R agents preserve their political faith, which implies $\tau < \frac{(1-t)^2}{(3-t)(2t-1)}$

If any of the above conditions are not satisfied, then the party matching the state of the world wins the election. If all the conditions above are satisfied, then information aggregation failure occurs with strictly positive winning margins. The maximum possible winning margin for party R if $\omega = L$ is $\frac{3-2\sqrt{2}}{2} \approx 8.58\%$. This occurs when $\tau = 0$ and $t = \frac{1}{\sqrt{2}}$.

While the detailed proof is provided in Appendix B.1, we provide a sketch of the argument here. When agents of both identities choose a GT signal structure for their respective Inside media and if the state of the world is $R$, then all identity-R agents and some identity-L agents vote for party $R$. If the state of the world is $L$, then all identity-L agents and some identity-R agents vote for party $L$. In this case, information aggregation failure is not possible. We show in Appendix B.1 that the party that matches the state of the world wins for all values of $\tau$ and $t$, such that a GB signal structure is optimal for both identities. Furthermore, the parameter space is such that there is no situation in which identity-R agents choose a GB signal structure and identity-L agents choose a GT signal structure.

Party R can win regardless of the state of the world only if identity-R agents choose a GT signal structure while identity-L agents choose a GB signal structure. In this case, party R always receives a greater vote share when the state of the world is $R$. Party R receives a greater vote share when the state of the world is $L$ if and only if the likelihood of the preservation of political faith is greater for identity-R agents than identity-L agents. It is also important to note that the region of information aggregation failure is non-empty, as can be seen in figure 7(a). Furthermore, the winning margins are not knife-edge. When $\omega = L$, party R’s winning margins are maximized if: (i) agents of identity-R receive a completely uninformative Outside signal, allowing them to preserve their political faith for sure, and (ii) agents of identity-L face an Outside signal that makes the choice of GB as their Inside signal structure just optimal. If they face a more precise Outside signal, identity-L agents would be able to preserve their political faith in the state of the world $L$ with a greater likelihood.

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37 Here, the two lower conditions — that identity-L agents choose a GB signal structure and that party R wins if $\omega = L$ — imply the first condition which requires identity-R agents to choose a GT signal structure. This is because if identity-R agents choose a GB signal structure, then party L would win in state of the world L.
Party R achieves an ex-ante winning margin advantage as a result of the lower exposure of identity-R citizens. This is a straightforward implication of the fact that it is easier for identity-R citizens to preserve their political identity because they are contending with a less precise Outside signal than identity-L citizens. It is, however, striking that the political advantage enjoyed by party R can be so substantial for some parameters that it can win regardless of the state of the world. We can exit the region of information aggregation failure by increasing the relative exposure of identity-R agents or by increasing or decreasing the precision of the Outside signal. Furthermore, while the region of information aggregation failure in figure 7(a) appears to be small, those parameter values may be particularly relevant in the case of the U.S. The low values of \( \tau \) correspond to a lower exposure to the Outside signal for identity-R agents. The intermediate values of Outside media precision at which information misaggregation takes place are high enough that identity-L agents choose a two-sided GB Inside signal structure, but not high enough that party L wins in state L.

**Correlated signals.** In this model, we have assumed that the Inside and Outside signals realize independently of each other and for each individual. We saw in the example above that party R can win with certainty in either state of the world. Because its winning margin is positive (specifically, bounded away from zero), we know that the result of information aggregation failure is robust to some — possibly small — degrees of correlation between the two signals.

In Appendix B.2, we describe the implications of three cases of correlated signals. The parameter values are the same as those in the example above, i.e., there is an equal proportion of infinite identity-R and identity-L agents. Identity-R agents receive a less precise Outside signal than identity-L agents (\( t_R = 0.51 \) and \( t_L = 0.75 \)). The common prior belief is that each state of the world is equally likely.

First, we consider the case in which the Outside signal realization is fully correlated across agents of the same identity. We then consider the case in which the Inside signal realization is fully correlated in the same way. Finally, we study the situation in which both signals are fully correlated in the way they realize for all agents of the same identity. In each scenario, since the Outside signals realize independently of the Inside signal, the optimal signal structure choice for individuals remains the same as in the baseline model. This implies that the expected winning margin advantage for party R remains the same, both ex-ante (+29%), and conditional on the state being R (+56%) or L (+2%). The aggregate implications of this expected winning margin advantage vary according to

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38The policy implication of this fact is that information aggregation failure is not inevitable and changes in the media environment can improve outcomes.
the kind of correlation we consider. In each case, party $R$ wins with a higher probability than party $L$. Therefore, while correlation introduces some aggregate uncertainty, the flavor of the key results remains largely unchanged.

**Non-common priors.** In addition to having a political affiliation, it may be that agents hold more favorable priors towards their party, and we can very simply extend the baseline model to consider the implication of such non-common priors. Suppose that an identity-$R$ agent holds a prior $P_R[\omega = R] := w_R$, which is higher than that of an identity-$L$ agent ($P_L[\omega = R] := w_L$). Here, we illustrate robustness to non-common priors by considering a particular example. We provide calculations and the conditions for information aggregation failure in Appendix B.3.

Figure 7(b) considers the case in which $w_R = 0.6$ and $w_L = 0.4$ and shows that the region of information aggregation failure, where party $R$ wins regardless of the state of the world, is much larger with non-common priors than with common and symmetric ones ($P[\omega = R] := w = 0.5$).³⁹

In this case, if the precision of the Outside signal ($t_R = \tau t + \frac{(1-\tau)}{2}$) for an identity-$R$ agent is less than 0.6, then she can preserve her political identity regardless of the realization of the Outside signal by simply choosing a non-informative Inside signal structure. This is akin to a citizen consuming news commentary from an outlet with a political bias that does not claim to be providing journalistic facts. For higher levels of perceived precision of the Outside signal, the identity-$R$ agent would choose a $GT$ or $GB$ signal structure as before. Holding favorable priors means that identity-$R$ individuals preserve their political identity more often.

Similarly, identity-$L$ citizens are able to preserve their political identity more often when they hold priors favorable to their party. However, because they face a more informative Outside signal, the advantage gained from their priors is limited. If the precision of the Outside signal is low enough for identity-$L$ agents to always preserve their political identity, then identity-$R$ agents would also be able to do the same. Furthermore, in the presence of non-common priors, for some parameter values, identity-$R$ agents always preserve their political identity by choosing a non-informative Inside signal structure, while identity-$L$ agents choose a $GB$ signal structure that allows them to preserve their political identity with a probability not equal to one. Naturally, non-common priors would expand the parameter space within which we observe information misaggregation.

The assumption of agents sharing common priors reduces the region of information misaggregation, which is a key result of the model. In that sense, we have tied our own

³⁹The calculations are presented in Appendix B.3.
hands by assuming that agents share common priors.

### 3.2 Asymmetric media distrust

We now add behavioral cognitive biases to the benchmark model which until now featured fully rational information processing. Figure 2 illustrates the markedly different levels of trust in the mainstream media by voters on either end of the political spectrum. In the polling data for years following 2016, around 15% of Republicans have expressed a great deal or fair amount of trust in mass media.\(^4^0\) The corresponding proportion for Democrats has hovered around 70%. Here, we consider two models of asymmetric media distrust. The mechanism underlying the results is similar to the one presented earlier in Subsection 3.1, albeit now in the contest of a misspecified model.

#### 3.2.1 Distrust in mainstream media precision

We interpret here distrust as reflecting an agent’s perception of the quality of mainstream media, namely the source of the information shock. As noted earlier, agents’ beliefs regarding the Outside signal structure are based on their beliefs regarding the mainstream media landscape as a whole. If some agents incorrectly believe that mainstream media reporting is imprecise, then they will believe that the precision of the mainstream media is lower than it really is.

We suppose that identity-\(L\) agents hold correct beliefs regarding the Outside signal and perceive the following structure:

\[
P[s = l | \omega = L] = P[s = r | \omega = R] = t
\]

where \(t \in [0.5, 1]\) is the true precision of the Outside signal. On the other hand, identity-\(R\) agents incorrectly believe that the Outside signal has the following structure:

\[
P[s = l | \omega = L] = P[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2}
\]

where \(\tau \in [0, 1]\) is the extent of an identity-\(R\) agent’s trust in the Outside signal.

The choice of signal structures by agents of either identity is the same as in Subsection 3.1. Again, we find there to be misaggregation of information within the region where

\(^{40}\)It could be that Republicans are right to distrust the mainstream media because a far greater proportion of journalists tend to identify as Democrats than Republicans (see Willnat et al. [2022]). We study the implications of bias in the mainstream media in Section 3.3, which studies Propaganda. However, this situation is more commonly observed in illiberal democracies where the ruling party can exert pressure on the mainstream media.
identity-R agents choose a $GT$ signal structure and identity-L agents choose a $GB$ signal structure. Because identity-L agents are correct about the process generating the Outside signal and because the Outside signal does not impact the likelihood of political identity preservation for identity-R agents, the region of misaggregation of information (where party $R$ wins regardless of the state of the world) is identical to the region presented in figure 7(a). In sum, the results are identical whether we consider the asymmetry to be in exposure to Outside media or distrust in the quality of Outside media.

3.2.2 Distrust in mainstream media unbiasedness

What happens if there is no systematic bias present yet some individuals believe there is? This obviously reflects distrust in the unbiasedness of the mainstream media. Therefore, we now suppose that identity-R agents believe the mainstream media to be biased when actually it is not.

Suppose the Outside signal structure is unbiased and has a precision of $t \in [0.5, 1]$. Then, identity-L agents correctly believe that the Outside signal structure follows:

$$P[s = l | \omega = L] = P[s = r | \omega = R] = t$$

Identity-R agents hold an incorrect belief and perceive the media to be biased when it is not. Thus, they believe that the Outside signal structure follows:

$$P[s = l | \omega = L] = \tau \cdot t + (1 - \tau) \cdot 1, \quad P[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot 0$$

where $\tau \in [0, 1]$ is an identity-R agent’s belief in the extent of the Outside signal’s unbiasedness.

Figure 8: Signal choices and the perception of mainstream media unbiasedness
Figure 8 shows the Inside signal structure choices for agents of each identity as well as the region of information aggregation failure. In this region, identity-R agents perceive the media to be highly biased, and the Outside signal is at least moderately precise. The perception of bias in the Outside signal structure makes a favorable Outside signal more informative for identity-R agents and an unfavorable Outside signal less so. This allows identity-R voters to preserve their political identity with greater likelihood. Therefore, asymmetry in the perception of bias by the mainstream gives party R an advantage that allows it to win regardless of the state of the world for a subset of parameters (the calculations are provided in Appendix B.5).

Proposition 2 In the environment specified in Subsection 3.2.2, the correct candidate wins except in a region where the Outside signal has intermediate precision and low perceived unbiasedness. Party R wins regardless of the state of the world if the following conditions are satisfied:

- Identity-R agents choose a GT signal structure, which implies \(2 - \tau - 2 \tau t - 2 \tau^2 t^2 + 2 \tau^2 t \geq 0\)
- Identity-L agents choose a GB signal structure, which implies \(t \geq \frac{1}{\sqrt{2}}\)
- Supposing identity-R agents choose a GT signal structure while identity-L agents choose a GB signal structure, a greater share of identity-R agents preserve their political faith, which implies \(\tau < \frac{(1-t)^2}{(1-(1-t))^2}\)

If any of the above conditions are not satisfied, then the party matching the state of the world wins the election. If all the conditions above are satisfied, then information aggregation failure occurs with strictly positive winning margins. The maximum possible winning margin for party R if \(\omega = L\) is \(\frac{3 - 2\sqrt{2}}{2} \simeq 8.58\%\). This occurs when \(\tau = 0\) and \(t = \frac{1}{\sqrt{2}}\).

Thus, creating the perception that the mainstream media produces propaganda is a strong political tool that can allow the incorrect party to win. One implication is that party R benefits if it can convince identity-R voters that the mainstream media is biased in favor of party L. As noted in the introduction, President Trump railed against the mainstream media, accusing it of being biased. Our model suggests that if identity-R voters believe that the mainstream media is biased in favor of party L, then that will influence the Inside media choices of identity-R voters. This gives party R a major electoral advantage and, for a substantial subset of parameters, party R will be able to win regardless of the state of the world.

In this subsection, we considered an interpretation of media distrust in which some agents incorrectly believe that the media is biased and found that there exists a substantial
region of information aggregation. In Subsection 3.2.1, we interpreted distrust in the mainstream media as an incorrect belief held by identity-R citizens regarding the Outside signal’s precision and found a comparable result. In view of the similarities in the regions of information misaggregation in figure 8 and 7(a), it is apparent that the phenomenon of an electoral benefit accruing to the side with greater distrust of mainstream media is robust to our interpretation of mistrust.

3.3 Propaganda

A telltale sign of a decaying democracy is the state’s use of the mainstream media for propaganda purposes. We define propaganda as a bias in the information shock process that stems from asymmetry in the realization of news that is favorable or unfavorable to the political parties, depending on the state of the world. We are specifically interested in the impact of propaganda on the agents’ choice of Inside-media as well as its effect on electoral outcomes. In particular, the model allows us to explore whether an agent’s ability to choose Inside media, can counteract propaganda, and furthermore, whether that ability can lead to information aggregation failure if the agent incorrectly believes that the mainstream media is biased.

Suppose that the Outside signal is biased in favor of party L such that the signal l is realized more often. Specifically, the Outside signal structure follows:

\[ P[s = l|\omega = L] = \tau \cdot t + (1 - \tau) \cdot 1, \quad P[s = r|\omega = R] = \tau \cdot t + (1 - \tau) \cdot 0 \]

where \( t \in [0.5, 1] \) is the precision of the Outside signal, and \( \tau \in [0, 1] \) is the extent of unbiasedness in state-influenced media. The higher \( \tau \) is, the less biased is the signal. This bias is commonly known by all agents and is the true process that generates the Outside signal.

Such a signal structure for the Outside media can exist when the state exerts control over mainstream media outlets. Suppose, for instance, that a strongman leader can force the mainstream media to frequently run positive stories, but cannot prevent the occasional negative story. In such an environment, he may also be able to censor media outlets in order to prevent agents from receiving an Inside signal. In the next two subsections, we therefore consider the implications of propaganda with and without censor-

\footnote{The extent of unbiasedness, denoted by \( \tau \), is analogous to the probability with which a signal is credible, denoted by \( \chi \), in Lipnowski et al. [2019]. A similar notion of partial commitment is analyzed theoretically and experimentally in Fréchette et al. [2019].}

\footnote{We also assume that a strongman leader is unable to force the media to be biased against his side. In practice, biased media reporting may occur without explicit orders and may simply be the media owners’ attempts to curry favor with the government of the day.}
ship.

3.3.1 With censorship

Suppose that no agents, regardless of their identity, have access to any information other than their prior and the information shock. This scenario may be interpreted either as a situation in which the ruling party shuts down all media other than the propagandized state-controlled media or as a sparse media environment such as that which existed prior to the internet. We define this as the benchmark with censorship, which will serve to highlight the role played by the ability to choose Inside media.

It is straightforward to show that because the agents share common and symmetric priors, and because the Outside signal is informative, the realization of the Outside signal determines whether the agent is able to preserve her political identity. The condition for party $L$ to win in state $L$ is simply that the Outside signal $l$ be realized more often than the signal $r$, which, in fact, always holds (the details of this claim and the ones that follow are presented in Appendix B.6).

If the state of the world is $R$, then the Outside signal $l$ is realized more often if $\tau t < 0.5$. If this condition holds, then there is information misaggregation, and party $L$ wins regardless of the realization of the state of the world. This region is illustrated in figure 9(a).

Figure 9: Propaganda with and without censorship

(a) Results with censorship

(b) Signal choices and results without censorship

If the ruler of an illiberal democracy can influence the mainstream media and also prevent citizens from independently accessing information, then she can ensure electoral victory regardless of the state of the world for a large subset of the parameter space.
Censorship along with propaganda is, therefore, a powerful combination of tools in an illiberal democracy.

### 3.3.2 Without censorship

Now suppose that agents can select an Inside signal structure. Being rational, they perceive an information shock that is favorable to the propagandizing side (party $L$) to be less informative than a shock unfavorable to it. Recall that a Good signal from the Inside signal structure is designed to just counteract an unfavorable information shock (Outside signal). For identity-$R$ agents, an unfavorable Outside signal is relatively easy to counteract because it is less effective. For identity-$L$ agents, it is more difficult to counteract. As can be seen in figure 9(b), there is a large subset of parameters within which identity-$R$ agents optimally choose a $GT$ signal structure while identity-$L$ agents choose a $GB$ signal structure. The region of information misaggregation lies within this subset of parameter values.

In the region of information misaggregation, party $R$ wins regardless of the state of the world. The intuition behind this result is that if the state of the world is $R$, then party $R$ must win because all identity-$R$ agents and some identity-$L$ agents vote for party $R$. The fact that an Outside signal unfavorable to party $L$ is very informative implies that if the state of the world is $L$, then an insufficient proportion of identity-$L$ agents preserve their political identity and vote for party $L$. For propaganda to backfire, it must be that the Outside media is sufficiently biased and the precision of the Outside signal is strong enough to push identity-$L$ agents to choose a $GB$ signal structure as their Inside media. However, if the precision of the Outside signal is high, then party $L$ will win in state $L$, and no misaggregation of information is present.

<table>
<thead>
<tr>
<th>Table 2: Results with and without censorship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters: $t = 0.6$, $\tau = 0.3$</td>
</tr>
<tr>
<td>With censorship</td>
</tr>
<tr>
<td>Ex-Ante $\omega = R$</td>
</tr>
<tr>
<td>L Win Margin</td>
</tr>
<tr>
<td>L Win Prob</td>
</tr>
</tbody>
</table>

Table 2 presents the case of $t = 0.6$ and $\tau = 0.3$ (see Appendix B.6 and B.7 for the calculations). These parameter values correspond to a situation where the Outside signal is fairly precise, and the party-$L$–influenced media is known to be particularly biased. It can be seen that under censorship, party $L$ always wins, and with high margins (reminiscent of the electoral results seen in some Eastern European “democracies” that were
controlled by strongmen). On the other hand, in the absence censorship, party \( L \) loses in both states of the world. Therefore, for a substantial subset of parameters, there is a reversal of electoral outcomes when censorship is disallowed. Suppose that party \( L \) cannot perfectly target propaganda and that there is a possibility of mistakes such that there is a positive probability for all levels of bias and precision. Then, party \( L \) must also institute censorship in order to benefit from propaganda. Otherwise, with a positive probability, propaganda backfires.\(^{43}\)

**Proposition 3** In the environment specified in Subsection 3.3.2, the correct candidate wins except in a region with low-to-intermediate precision and a highly biased Outside signal. Party \( R \) wins regardless of the state of the world if the following conditions are satisfied:

- **Identity-R agents choose a GT signal structure, which implies** \( 2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \geq 0 \)
- **Identity-L agents choose a GB signal structure, which implies** \( \tau(1 - 2t - 2\tau t^2 + 2\tau t) \leq 0 \)
- **Supposing identity-R agents choose a GT signal structure while identity-L agents choose a GB signal structure, a greater share of identity-R agents preserve their political faith, which implies** \( \frac{1 - \tau t}{1 + \tau t} > 1 - \tau(1 - t)(1 - \tau t) \)

If any of the above conditions are not satisfied, then the party matching the state of the world wins the election. If all the conditions are satisfied, then information aggregation failure occurs with strictly positive winning margins. The maximum possible winning margin for party \( R \) if \( \omega = L \) is approximately 5.99\%. This occurs when \( \tau \simeq 0.236 \) and \( t \simeq 0.558 \).

The calculations and the proof can be found in Appendix B.7.

Proposition 3 states that without censorship, propaganda is not simply weak, it actually backfires. Propaganda, by its very nature, implies that news favorable to the propagandizing party is discounted by Bayesian agents and does not affect their posteriors to a substantial degree. News unfavorable to the propagandizing party is particularly informative because it is so rare. If individuals can independently access information, then identity-R agents need very little of it to counteract the propaganda in favor of party \( L \). On the other hand, identity-L agents need a much stronger signal to counteract unfavorable news from the biased media. For all parameters, party \( R \) wins in the state of the world it is meant to (state \( R \)), and for a substantial parameter space, also in state \( L \).\(^{44}\)

\(^{43}\)Li et al. [2022] find that a truthful alternative media can counteract propaganda, a result that is similar in spirit to ours.

\(^{44}\)In the above proposition, we see that biasing the Outside signal structure in favor of party \( L \) backfires. A simple corollary of this proposition is that if party \( L \) were to optimally design the Outside signal structure, it
3.3.3 Propaganda that individuals are oblivious to

We now consider an even less desirable situation in which the mainstream media is biased in favor of party $L$ but individuals are convinced that it is unbiased. We first consider the case in which individuals of both identities are oblivious to the bias in the information shock process. Later on, we will also consider cases in which only identity-$R$ or only identity-$L$ individuals are oblivious to the bias. In each of these cases, at least some citizens choose their Inside signal structure under incorrect beliefs. The mainstream media, which is biased toward party $L$, generates a signal favorable to party $L$ more often than one favorable to party $R$.

The true process that generates the Outside signal is biased in favor of party $L$ and takes the following form:

\[
P[s = l | \omega = L] = \tau \cdot t + (1 - \tau) \cdot 1, \quad P[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot 0
\]

where $t \in [0.5, 1]$ is the precision of the Outside signal and $\tau \in [0, 1]$ is the Outside signal’s true level of unbiasedness. In the first case we consider, agents of both identities incorrectly believe that $\tau = 1$.

Figure 10: Outcomes when individuals are oblivious to propaganda

As can be seen in figure 10(a), agents of both identities choose a $GB$ signal structure for their Inside media in the region of information aggregation failure (see Appendix B.8 would (for some parameters) choose to bias it in favor of party $R$. This corollary seems less plausible because it implies that party $L$ would influence the mainstream media to produce programming favorable to party $R$ more often.
for the detailed calculations).

Although the agents are Bayesian, they update based on incorrect beliefs and choose their Inside signal structure sub-optimally. Identity-\(R\) agents update excessively upon receiving an unfavorable Outside signal, and cushion themselves against the Outside signal by choosing an overly informative Inside signal structure. They thus preserve their political identity less often than they could have. Similarly, identity-\(L\) citizens update too little upon receiving unfavorable outside information, which works to their benefit. As we show in proposition 4, for low values of Outside media unbiasedness (\(\tau\)) and moderate-to-high values of Outside media precision (\(t\)), party \(L\) enjoys such a large advantage that it can win regardless of the realized state of the world.

**Proposition 4** In the environment specified in Subsection 3.3.3, the correct candidate wins, except in a region with an intermediate-precision and highly biased Outside signal. For party \(L\) to win regardless of the state of the world, it is necessary that agents of both identities choose a GB signal structure, which occurs when \(t \in \left[\frac{1}{\sqrt{2}}, 1\right]\). Further, party \(L\) wins in the state of the world \(R\) with a strictly positive margin if \(t < \frac{\sqrt{1+\tau}}{1+\tau}\). If \(\omega = R\), party \(L\)’s winning margin is maximized when \(\tau = 0\) and \(t = \frac{1}{\sqrt{2}}\).

The proof and detailed calculations are to be found in Appendix B.8. The existence of such a region of information aggregation failure can explain why propagandizing news outlets go to great lengths to portray themselves as accurate and balanced.

**Censorship:** We consider the implications of censorship in an environment where the Outside media is biased but citizens wrongly believe it is not. Party \(L\) wins regardless of the state of the world if the Outside signal favorable to party \(L\) is generated more often in both. As in Subsection 3.3.1, this condition holds if \(\tau t < 0.5\). The electoral results are therefore also identical and are illustrated in figure 9(a). The only difference with Subsection 3.3.1 lies in the intensity of an individual’s belief in the party they vote for.

**\(L\) agents are oblivious:** So far in this subsection, we have considered the case in which voters of both identities are oblivious to the bias in the Outside signal. Now, suppose that only identity-\(L\) agents believe that the Outside signal is unbiased, while identity-\(R\) citizens know that it is biased. Identity-\(L\) agents update less than they should when faced with an unfavorable Outside signal, and their Inside signal structure choice is the same as in figure 10(a). Identity-\(R\) agents choose their Inside signal structure as shown in figure 9(b). The correct side always wins under this specification, which for party \(L\) is an improvement over the backfiring of propaganda we saw in Subsection 3.3.2.
R agents are oblivious: Now suppose that only Identity-R agents are unaware of the bias in the Outside signal. Identity-L agents know that the Outside signal structure is biased in favor of Party L and choose their Inside signal structure correctly as shown in figure 9(b). Identity-R agents hold incorrect beliefs and choose their Inside signal structure as in figure 10(a).

As shown in figure 10(b), the region where party L wins regardless of the state of the world expands if only identity-R agents — rather than both identities — are oblivious to the bias in the Outside signal.

Summarizing Subsection 3.3, we have demonstrated that propaganda backfires in the presence of a rich media landscape. Bayesian citizens largely discard information shocks favorable to the propagandizing side and update their beliefs to a much greater extent upon receiving unfavorable information shocks. A rich media landscape allows citizens on the non-propagandizing side to preserve their political identity more effectively than those on the propagandizing side. Censorship allows the propagandizing side to benefit from the propaganda. Alternatively, the propagandizing side benefits if citizens, particularly those on the opposing side, are unaware of the mainstream media’s bias. In many illiberal democracies, the ruling party usually invests heavily in propaganda. According to our model, that effort should be complemented by censorship or attempting to an attempt to legitimize the propagandized news.

3.4 Discussion: Belief-based partisanship

We now analyze the influence of belief-based partisanship by varying the proportion of the population that identifies with party R under two sets of conditions.

Figure 11: Information aggregation failure and belief-based partisanship

(a) Asymmetric exposure

(b) Precision counters partisanship
In figure 11 (a), we consider asymmetric exposure to the information shock by building on the exposition in Subsection 3.1. Specifically, the precision of the information shock is \( t = 0.75 \), which is the same as in the example considered earlier. The y-axis measures the exposure of identity-R citizens to the information shock \( (\tau) \), while the x-axis measures the proportion of the population that identifies with party \( R \). It can be seen that when there are sufficiently few identity-R voters, party \( L \) always wins. Conversely, when there are enough identity-R voters, party \( R \) always wins. The region of information aggregation failure — in which party \( R \) always wins — tends to shrink as the exposure of \( R \) citizens to the Outside signal increases. There is, however, a threshold value of exposure at which identity-R citizens switch from a GT signal structure to a GB signal structure (at approximately \( \tau = 0.83 \)). At that point, there is a discontinuous expansion of the region of information aggregation failure.

Figure 11 (b), illustrates the subtlety of this formalization of partisanship even more strikingly. Unlike behavioral partisans, belief-based partisans must contend with Outside information, and they must convince themselves to vote for their preferred party. In the state of the world that favors the other party, this becomes more difficult as the precision of the information shock increases. In the graph, we suppose that citizens of both identities contend with an information shock of equal precision. Precision appears on the y-axis and as before, the x-axis is the proportion of the population that identifies with party \( R \). When the precision of the information shock is high, the correct party wins in both states of the world even when the population is divided very unequally between the political identities.

4 Takeaways and further research

Supporting a political party has become nowadays similar to supporting a sports team: emotional attachment and the desire to believe your team is the best come first and the urge to learn the truth often comes second. People seem to follow extreme media outlets, such as Breitbart News (see Budak et al. [2016]), because it makes them feel good about themselves: it makes them believe, most of the time, they are right to support the side they support. When an election comes, this selective media exposure has relevant information aggregation consequences that can subvert more standard electoral model results. The framework we propose is very simple and thus may accommodate several alternative assumptions/extension including non-strategic information acquisition by a proportion of voters, non-Bayesian updating (such as correlation neglect, affine distortions in updating, motivated updating, etc.), or strategic voting with a finite electorate. While we do rely on asymmetries in the media environment, it seems evident that some
asymmetry does exist, and our model is robust enough to consider several different interpretations of that asymmetry. In this paper, we had to limit our analysis to a few specific contexts and obtained several takeaway points, which we summarize here below, but we believe our simple framework provides fertile ground for further studies.

- With symmetric common priors, failure of information aggregation in liberal democracies can only happen when one side is fairly closed to outside news and the other is fairly open (but not too open either). In this case and with an otherwise fully symmetric setup, the more closed side wins the election regardless of the state of the world.

- If we replace exposure to, with trust in, mainstream news (a misspecified model, in which one side believes the news is less reliable/precise than it actually is), the above results hold: in an otherwise symmetric framework, high distrust on one side and average trust on the other side generate systematic information aggregation failures.

- If we allow for non-common priors (a nonstandard assumption, but perhaps plausible nowadays) the failure of aggregation above is exacerbated. The same happens if the two identity camps are asymmetric in size: the larger this initial asymmetry the more exposure to outside news is needed to maintain correct information aggregation.

- If distrust in media is modeled instead as misperceived mainstream media bias (namely, mainstream media is unbiased but is perceived as biased by one side), the aggregation failure results are qualitatively unchanged.

- Government propaganda, i.e. common knowledge of biased official media, results in an electoral advantage only in conjunction with the crackdown of independent media or when voters (on one side or on both sides) are unaware of the propaganda (i.e. of the official media bias, a misperceived model). Otherwise known propaganda is easy to defeat, and the opposition side can even have an advantage and win the election regardless of the state. Thus, when propaganda is common knowledge cracking down on non-government media is key.

Thus, in general, our results speak to several salient recent events such as the election of (and then the very close defeats of) Trump and Bolsonaro, and whether the current emotional partisanship and rich media environment played a key role in such elections, or in non-liberal democracies (such as Russia, Turkey, Syria, and Hungary) where the
regime fully controls the official media message, why the government also engages in a politically costly and often extensive crackdown on independent media.

Media choice (instrumentally driven by political identity) is demand-driven and therefore media outlets are passive signal providers in our model. More work needs to be done on the supply side of news, in particular on the supply of misinformation and how effective this might be in aggregate. As noted in Jerit and Zhao [2020] misinformation has become ingrained in our political landscape. Such claims are more likely to be consumed by voters with a certain desire to preserve their political identity.

Strategic information acquisition in order to hold preferred beliefs can have several other applications. We believe that analyzing selective fact-checking of potentially fake news or the relation between such motivated learning and political accountability are particularly promising avenues for future research.

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A Solution for the general model

Suppose that an agent believes (whether correctly or incorrectly) that

\[ P[s = l | \omega = L] = k, \quad P[s = r | \omega = R] = m \]

The agent’s expected utility still follows equation 1 and the signal structure must satisfy the Martingale constraint.
The agent’s interim posteriors on observing the Outside signal and before observing
the signal from the chosen signal structure are such that:

\[
P[\omega = L|s = l] = \frac{k}{1+k-m}, \quad P[\omega = L|s = r] = \frac{1-k}{1+m-k}
\]

The \(G\), \(B\), and \(T\) signals are described Section 2. An identity-\(R\) agent is able to preserve
her political identity as long as her posterior upon observing both signals is such that
\(P[\omega = R|s, S] \geq 0.5\). Therefore, the \(G\), \(B\), and \(T\) signals must be such that:

\[
\begin{align*}
P[\omega = R|S = G] &= \frac{k}{1+k-m} \\
P[\omega = R|S = B] &= \frac{1-k}{1+m-k} \\
P[\omega = R|S = T] &= 0
\end{align*}
\]

Claim 1 It is sufficient to consider three signals used in two possible signal structures when
solving the agent’s problem described above.

Proof. The proof follows directly from Kolotilin [2018] and Kamenica and Gentzkow
[2011]. We use the linear optimization technique of comparing marginal utility to price
ratios (MU-Price ratios) for the different signals. In figure 3, the MU-Price ratio of a signal
is represented by the slope of the line from the origin to the point on the expected utility
curve that corresponds to the interim posterior generated by that signal. In that sense,
this technique is equivalent to finding the optimal signal structure using the concave
closure of the expected utility function.

Lemma 2 Any signal \(M_i\) that generates a posterior \(P[\omega = R|M_i] \in (\frac{k}{1+k-m}, 1]\) is sub-optimal
when compared to a signal \(M_G\) where \(M_G\) is such that \(P[\omega = R|M_G] = \frac{k}{1+k-m}\).

Proof. Regardless of whether the agent observes \(M_i\) or \(M_G\), her expected utility will be
the same. This is because the agent is able to preserve her political identity regardless of
the realization of the Outside signal.

However, it is more costly (according to the Martingale constraint) to generate the
signal \(M_i\). Therefore, the MU-Price ratio of generating \(M_i\) is less than that for generating
\(M_G\). This implies that any signal structure where \(P[M_i] > 0\) will have a lower ex-ante
expected utility than a signal structure that assigns \(P[M_i] = 0\) and adds \(\frac{P[\omega = R|M_G]}{P[\omega = R|M_G]} \cdot P[M_i]\)
to the probability that \(M_G\) is generated. Therefore, no \(M_i\) such that \(P[\omega = R|M_i] \in (\frac{k}{1+k-m}, 1]\) will be chosen by the agent. Equivalently, \(M_i\) is sub-optimal when compared
to \(M_G\). ■
Lemma 3  Any signal $M_i$ that generates a posterior $P[\omega = R|M_i] \in (\frac{1-k}{1+m}, \frac{k}{1+m-k})$ is sub-optimal when compared to a signal $M_B$ where $M_B$ is such that $P[\omega = R|M_B] = \frac{1-k}{1+m-k}$.

Proof. For $M_i$, the ex-ante expected utility of the agent is given by:

$$P \left[ P[\omega = R|M_i, s] \geq 0.5 \right] = mP[\omega = R|M_i] + (1-k)(1 - P[\omega = R|M_i])$$

$$= 1 - k + P[\omega = R|M_i](m + k - 1)$$

This implies that the MU-Price ratio is:

$$(m + k - 1) + \frac{(1-k)}{P[\omega = R|M_i]}$$

For $M_B$, the ex-ante expected utility is:

$$P \left[ P[\omega = R|M_B, s] \geq 0.5 \right] = m\left(\frac{1-k}{1+m-k}\right) + (1-k)(1 - \left(\frac{1-k}{1+m-k}\right)) = 2m\left(\frac{1-k}{1+m-k}\right)$$

This means that the MU-Price ratio is:

$$2m = (m + k - 1) + \frac{(1-k)}{\left(\frac{1-k}{1+m-k}\right)}$$

Since $P[\omega = R|M_i] > \frac{1-k}{1+m-k}$, the MU-Price ratio for generating a signal structure posterior $M_i$ is lower than for $M_B$. Therefore, no $M_i \in (\frac{1-k}{1+m}, \frac{k}{1+m-k})$ will be chosen by the agent. Equivalently, $M_i$ would be sub-optimal when compared to $M_B$.

Lemma 4  Any signal $M_i$ that generates a posterior $P[\omega = R|M_i] \in (0, \frac{1-k}{1+m-k})$ is sub-optimal when compared to a signal $M_T$ where $M_T$ is such that $P[\omega = R|M_T] = 0$.

Proof. For $M_i$, the ex-ante expected utility is zero. This is because regardless of the realization of the Outside signal, the agent is never able to preserve her political identity, and the same is true for $M_T$. However, $M_i > 0$, which implies that the cost for generating $M_i$ is higher than that for generating $M_T$. Therefore, no $M_i \in (0, \frac{1-k}{1+m-k})$ will be chosen by the agent. Equivalently, $M_i$ is sub-optimal when compared to $M_T$.

The agent requires only three signals to solve her problem. In fact, any signal that generates a posterior different from them would be sub-optimal. $M_G$, which generates a posterior $P[\omega = R|M_G] = \frac{k}{1+k-m}$, is abbreviated to $G$. $M_B$, which generates a posterior $P[\omega = R|M_B] = \frac{1-k}{1+m-k}$, is abbreviated to $B$. Finally, $M_T$, which generates a posterior $P[\omega = R|M_T] = 0$, is abbreviated to $T$. 

46
A signal structure is a combination of signal realizations, and the three possible signals are $G$, $B$, and $T$. Therefore, the possible signal structures are $GT$, $GB$, and $GBT$.\footnote{There are a number of signal structures that are ruled out because they violate the Martingale constraint, specifically, $G$, $B$, $T$, and $BT$. While we assume that the agents share a common symmetric prior belief that $\Pr[\omega = R] = 0.5$, this result is robust to values of $\Pr[\omega = R]$ such that $\frac{1-k}{1+m-k} < \Pr[\omega = R] < \frac{k}{1+k+m}$.}

We argue that while $GBT$ is feasible according to the budget constraint, and might even be an optimal choice for some parameters, it can be ignored since whenever it is optimal, a simpler signal structure ($GB$ or $GT$) is as well. In other words, this signal structure never offers strictly greater expected utility (than the max of $GB$, and $GT$), and is therefore not required to solve the agent’s problem. Either the MU-Price ratio of $G$ is higher than that of $B$ in which case $GT$ should be implemented, rather than $GBT$; or the MU-Price ratio of $B$ is higher than that of $G$ in which case $GB$ should be implemented, rather than $GBT$; or, the MU-Price ratios of $G$ and $B$ are equal, in which case either $GB$ or $GT$ provides the agent with the same expected utility as $GBT$, and therefore, $GBT$ can be ignored.

Therefore, $GT$ and $GB$ alone are sufficient to solve the agent’s problem.

A.1 The identity-$R$ agent’s problem

Recall that:

$$
\Pr[s = l|\omega = L] = k, \quad \Pr[s = r|\omega = R] = m
$$

This is a linear optimization problem, and therefore, the agent chooses to employ the signals with the highest MU-Price ratio.

For signal $G$, the MU is 1. This is because regardless of the Outside signal, the agent is able to preserve her political identity. For signal $B$, the MU is equal to the likelihood that the Outside signal is favorable ($r$, for an identity-$R$ agent) given that $B$ is realized, i.e. $2m(\frac{1-k}{1+m-k})$. Finally, for $T$, the agent is never able to preserve her political identity, and therefore the MU is 0.

The price of each of these signals is determined according to the coefficient corresponding to it in the Martingale constraint, i.e. $(\frac{k}{1+k+m}) \cdot P_G + (\frac{1-k}{1+m-k}) \cdot P_B + 0 \cdot P_T = 0.5$. This price is simply the intermediate posterior generated by the signal.

The MU-Price ratio is $\frac{1}{1+k+m}$ for signal $G$, $\frac{2m(\frac{1-k}{1+m-k})}{(\frac{1-k}{1+m-k})} = 2m$ for signal $B$, and undefined for signal $T$.

The signal structure $GT$ is optimal when MU-Price ratio for signal $G$ is at least as large as that for signal $B$, which simplifies to $1 + k - m - 2km \geq 0$. If $1 + k - m - 2km \leq 0$, then signal structure $GB$ is optimal. This is equivalent to saying that the concave closure of
the expected utility curve shown in figure 4 has a kink if and only if \(1 + k - m - 2km < 0\). If it does, then a GB signal provides the agent with a higher expected utility than a GT signal.

We can now calculate the probability of realization of different signals, the utility achieved, and the likelihood of voting for the preferred party under the signal structures GT and GB.

The probabilities of realizing the different signals will help us calculate expected utilities as well as the outcome of the election.

**Signal structure GT:** Unconditional on the state, the likelihood that the signal G is realized is \(P_G = \frac{1+k-m}{2k}\), which is also the agent’s ex-ante expected utility.

Conditional on the state being \(\omega = R\), the signal G is always realized, and therefore the agent’s expected utility is \(E[U_R|GT \cap \omega = R] = P[G|\omega = R] = 1\).

Conditional on the state being \(\omega = L\), the likelihood that signal G is realized is \(\frac{1-m}{k}\). Whenever signal G is realized, the agent is able to preserve her political identity. Therefore, this also equals the agent’s expected utility \(E[U_R|GT \cap \omega = L]\).

**Signal structure GB:** This signal structure is somewhat more complicated, and therefore, we use the following three equations.

\[
\begin{align*}
P[G|GB] + P[B|GB] &= 1 \quad (2) \\
P[G|\omega = L] &= \frac{1-m}{k} \quad (3) \\
P[B|\omega = L] &= \frac{m}{1-k} \quad (4)
\end{align*}
\]

Given that the signal structure is GB,

\[
1 = P[G] + P[B] = \left( P[G|\omega = R] \cdot \frac{1}{2} + P[G|\omega = L] \cdot \left( \frac{1}{2} \right) \right) + \left( P[B|\omega = R] \cdot \frac{1}{2} + P[B|\omega = L] \cdot \left( \frac{1}{2} \right) \right)
\]

\[
= \frac{1}{2} \left( P[G|\omega = R] \right) \left( 1 + \frac{P[G|\omega = L]}{P[G|\omega = R]} \right) + \left( P[B|\omega = R] \right) \left( 1 + \frac{P[B|\omega = L]}{P[B|\omega = R]} \right)
\]

which simplifies to:

\[
1 = P[G|\omega = R] \frac{(1 + k - m)}{2k} + (1 - P[G|\omega = R]) \frac{(1 + m - k)}{2(1 - k)}
\]
Therefore, conditional on the state,
\[ P[G|GB \cap \omega = R] = k, \quad P[B|GB \cap \omega = R] = 1 - k \]
\[ P[G|GB \cap \omega = L] = 1 - m, \quad P[B|GB \cap \omega = L] = m \]

and unconditional on the state:
\[ P[G|GB] = \frac{1 + k - m}{2}, \quad P[B|GB] = \frac{1 + m - k}{2} \]

To calculate the likelihood of political identity preservation, and therefore, expected utility, it is helpful to recall that
\[ P[s = l|\omega = L] = k, \quad P[s = r|\omega = R] = m \]

The agent’s expected utility conditional on the state being \( \omega = R \) is:
\[ E[U_R|GB \cap \omega = R] = P[G|GB \cap \omega = R] + P[r|B \cap \omega = R] \cdot P[B|GB \cap \omega = R] \]
\[ = k + m (1 - k) \]

while the agent’s expected utility conditional on the state being \( \omega = L \) is:
\[ E[U_R|GB \cap \omega = L] = P[G|GB \cap \omega = L] + P[r|B \cap \omega = L] \cdot P[B|GB \cap \omega = L] \]
\[ = (1 - m) + (1 - k)m \]

The unconditional expected utility is simply a weighted average of the conditional expected utilities. If the agent’s beliefs about the signal structure of the Outside signal are correct, then:
\[ E[U_R|GB] = \frac{1}{2} \cdot E[U_R|GB \cap \omega = R] + \frac{1}{2} \cdot E[U_R|GB \cap \omega = L] \]
\[ = \frac{1 + k + m - 2km}{2} \]

A.2 The identity-L agent’s problem

Recall that:
\[ P[s = l|\omega = L] = k, \quad P[s = r|\omega = R] = m \]
The intuition behind the calculations is similar to the identity-R agent’s problem. Furthermore, all one needs to do to arrive at these results is to use the calculations from the previous subsection, and replace \( w \) with \( 1 - w \) and \( k \) with \( m \).

The MU for signal \( G \) is 1. For signal \( B \), the MU is \( 2k(\frac{1}{1+k-m}) \). Finally, for signal \( T \), the MU is 0. The price of each of these signals is determined according to the coefficient corresponding to it in the Martingale constraint \( \left( \frac{m}{1+m-k} \right) \cdot P_G + \left( \frac{1-m}{1+k-m} \right) \cdot P_B + 0 \cdot P_T = 0.5 \). This price is simply the intermediate posterior generated by the signal. The MU-Price ratio is \( \frac{1}{m+1} \) for signal \( G \), \( \frac{2k}{(1+1/m)} \) = 2k for signal \( B \), and undefined for signal \( T \).

The signal structure \( GT \) is optimal when \( 1 + m - k - 2km \geq 0 \). If \( 1 + m - k - 2km \leq 0 \), then signal structure \( GB \) is optimal.

We can now calculate the probability of realization of different signals, the utility achieved, and the likelihood of voting for the preferred party under the signal structures \( GT \) and \( GB \).

**Signal structure \( GT \):** Unconditional on the state, the likelihood that the signal \( G \) is realized is \( P[G] = \frac{1+m-k}{2m} \), which is also the agent’s ex-ante expected utility.

Conditional on the state being \( \omega = L \), the signal \( G \) is always realized and so the agent’s expected utility is \( E[U_L | GT \cap \omega = L] = P[G | \omega = L] = 1 \).

Conditional on the state being \( \omega = R \), the likelihood that signal \( G \) is realized is \( \frac{1-k}{m} \). Whenever it is realized, the agent is able to preserve her political identity. Therefore, this also equals the agent’s expected utility \( E[U_L | G \cap | \omega = R] \)

**Signal structure \( GB \):** This signal structure is somewhat more complicated, and therefore, we use the following three equations.

\[
P[G|GB] + P[B|GB] = 1 \tag{5}
\]

\[
P[G|\omega = R] = \frac{1-k}{m} \tag{6}
\]

\[
P[B|\omega = R] = k
\]

\[
P[G|\omega = L] = \frac{1-m}{1-m}
\]

Conditional on the state,

\[
P[G|GB \cap \omega = L] = m, \quad P[B|GB \cap \omega = L] = 1 - m
\]

\[
P[G|GB \cap \omega = R] = 1 - k, \quad P[B|GB \cap \omega = R] = k
\]
and unconditional on the state:

\[ P[G|GB] = \frac{1 + m - k}{2}, \quad P[B|GB] = \frac{1 + k - m}{2} \]

To calculate the likelihood of political identity preservation, and therefore expected utility, it is helpful to recall that:

\[ P[s = l|\omega = L] = k, \quad P[s = r|\omega = R] = m \]

The agent’s expected utility conditional on the state being \( \omega = L \) is:

\[
E[U_L|GB \cap \omega = L] = P[G|GB \cap \omega = L] + P[l|B \cap \omega = L] \cdot P[B|GB \cap \omega = L] \\
= m + k(1 - m)
\]

The agent’s expected utility conditional on the state being \( \omega = R \) is:

\[
E[U_L|GB \cap \omega = R] = P[G|GB \cap \omega = R] + P[l|B \cap \omega = R] \cdot P[B|GB \cap \omega = R] \\
= (1 - k) + (1 - m)k
\]

The unconditional expected utility is simply a weighted average of the conditional expected utilities. If the agent’s beliefs about the signal structure of the Outside signal are correct, then:

\[
E[U_L|GB] = \frac{1}{2} \cdot E[U_L|GB \cap \omega = R] + \frac{1}{2} \cdot E[U_L|GB \cap \omega = L] \\
= \frac{1 + k + m - 2km}{2}
\]

### A.3 Robustness to noise

Suppose we add a small, mean-zero noise term to an identity-\( R \) agent’s objective function. That is, the agent now maximizes \( E[U_R] = P[\omega = R|S, s] \geq 0.5 + \epsilon] \), where \( \epsilon \) is generated from \( \mathcal{N}(0, \sigma^2) \).

The noise in the threshold implies that the agent’s preservation of her political identity is now stochastic, and varies continuously with the agent’s interim posterior. The smoothness of the agent’s expected utility function in figure 12 is due to this stochasticity. In this example, we suppose that \( \sigma = 0.02 \). In figure 12 (a), we see that a \( G^+T \) signal is optimal when the Outside signal is unbiased and has a precision equal to 0.6. When contending with a more precise Outside signal (precision = 0.8), the agent opti-
Figure 12: Expected utility as a function of interim posteriors once noise is added

(a) Lower precision Outside signal structure

(b) Higher precision Outside signal structure

mally chooses a $G^+ B^-$ signal structure. The $G^+$ signal generates an interim posterior that is more favorable to party $R$ than a $G$ signal and in the case of normally distributed noise, it is optimal because it allows the agent to preserve her political identity for a large proportion of the possible realizations of the noise. Analogously, the $B^+$ signal generates an interim posterior that is a bit more favorable than the $B$ signal generates for an identity-$R$ agent.

The key results in propositions 1, 2, 3, and 4 are not knife-edge. For noise with sufficiently low variance, the optimal signal structures change very little and information aggregation failure would still occur.

A.4 Robustness to gain from being correct

We now show for the general model that the key results are robust to including a small gain from being correct in the utility function. In other words, in addition to a gain from political identity preservation, agents also gain utility from being correct about the state of the world. The utility function of an identity-$R$ agent is, therefore, modified to become:

$$
U_R = \begin{cases} 
(1 - \gamma) + \gamma & \text{if } \mathbb{P}[\omega = R | S, s] \geq 0.5 \text{ and } \omega = R \\
(1 - \gamma) & \text{if } \mathbb{P}[\omega = R | S, s] \geq 0.5 \text{ and } \omega = L \\
\gamma & \text{if } \mathbb{P}[\omega = R | S, s] < 0.5 \text{ and } \omega = L \\
0 & \text{if } \mathbb{P}[\omega = R | S, s] < 0.5 \text{ and } \omega = R
\end{cases}
$$

(8)
In this case, $\gamma \in (0,1)$ captures the extent to which the agent gains utility from being correct as opposed to a signal that allows her to preserve her political identity.\(^{46}\)

We now show two key results for an identity-R agent. First, for a low value of $\gamma$, $GT$ and $GB$ signal structures are sufficient to solve the agent’s problem. Second, as $\gamma$ increases, the threshold at which the agent switches from a $GT$ signal structure to a $GB$ signal structure changes continuously. These two results together imply that the key results of the model are robust to small values of $\gamma$. For an identity-L agent, the same results hold analogously.

Based on the above utility function, there are four interim posteriors (generated after the agent observes her Inside signal, and before she observes the Outside signal) that are key to solving the agent’s problem. Three are derived from the $G$, $B$, and $T$ signals, while the fourth is an $Excellent$ or $E$ signal. The signals must be such that:

\[
\begin{align*}
P[\omega = R | S = G] &= \frac{k}{1 + k - m} \\
P[\omega = R | S = B] &= \frac{1 - k}{1 + m - k} \\
P[\omega = R | S = T] &= 0 \\
P[\omega = R | S = E] &= 1
\end{align*}
\]

The set of signal structures that satisfy the Martingale property is

\{\(ET, GT, EB, GB, EBT, EGT, GBT, EGBT\)\}

We disregard the signal structures $EBT$, $EGT$, $GBT$, and $EGBT$ because whenever one of them is optimal, a simpler signal structure will also be.

In the next step, we compare the ex-ante expected utilities of each of these signal structures to show that for low values of $\gamma$, $GT$ and $GB$ are sufficient to solve the agent’s problem.

**Signal structure $ET$:**

\[
\begin{align*}
E[U_R | ET \cap \omega = R] &= 1, \\
E[U_R | ET \cap \omega = L] &= \gamma \\
E[U_R | ET] &= \frac{1 + \gamma}{2}
\end{align*}
\]

\(^{46}\) $\gamma = 0$ is the benchmark model. $\gamma = 1$ corresponds to a case in which the agent only wants to know the correct state. In that case, the agent will choose a fully revealing echo chamber signal structure.
Signal structure $EB$:

$$E[U_R|EB \cap \omega = R] = 1 - \frac{(1-k)(1-m)}{m}, \quad E[U_R|EB \cap \omega = L] = 1 - \gamma - m + 2\gamma m$$

$$E[U_R|EB] = 1 - \frac{1}{2} \left[ \frac{(1-k)(1-m)}{m} + \gamma + m - 2\gamma m \right]$$

Signal structure $GT$:

$$E[U_R|GT \cap \omega = R] = 1, \quad E[U_R|GT \cap \omega = L] = \gamma + \frac{1-m}{k} - 2\gamma \frac{1-m}{k}$$

$$E[U_R|GT] = \frac{1+\gamma}{2} + \left( \frac{1-m}{k} \right) \left( \frac{1}{2} - \gamma \right)$$

Signal structure $GB$:

$$E[U_R|GB \cap \omega = R] = k + m - km, \quad E[U_R|GB \cap \omega = L] = \gamma + (1-2\gamma)(1-km)$$

$$E[U_R|GB] = \frac{1-\gamma + k + m}{2} - (1-\gamma)km$$

Claim 5 Suppose that $\gamma \leq 0.5$. The signal structures $GT$ and $GB$ are then sufficient to solve the agent’s problem.

Proof. We now show that the signal structure $GT$ provides the agent with at least as much expected utility as $ET$ or $EB$ as long as $\gamma \leq 0.5$.

$$E[U_R|GT] - E[U_R|ET] \geq 0$$

simplifies to $\left( \frac{1-m}{k} \right) \left( \frac{1}{2} - \gamma \right) \geq 0$, and holds if $\gamma \leq 0.5$.

Similarly, $E[U_R|GT] - E[U_R|EB] \geq 0$ simplifies to $(1-m)(1-k) \left( \frac{1-2\gamma}{k} + \frac{1}{m} \right) \geq 0$, and holds if $\gamma \leq 0.5$. ■

Because $E[U_R|GT]$ and $E[U_R|GB]$ are continuous functions of $\gamma$, the agent’s choice of Inside signal structure depends on a threshold that varies continuously with $\gamma$. Therefore, for low values of $\gamma$, the region of information aggregation failure doesn’t disappear completely, although it may shrink.

In general, as long as $\gamma \leq 0.5$, there exists a region of information aggregation failure. If $\gamma > 0.5$, then all agents optimally choose an $ET$ signal structure and there is no information aggregation failure.

Consider the parameters used in the example in the introduction. Specifically, suppose the precision of the Outside signal is 0.51 for identity-$R$ agents and 0.75 for identity-$L$ agents. Then, as long as $\gamma < 0.2$, identity-$R$ agents would optimally choose a $GT$ signal
structure and identity-$L$ agents would prefer a $GB$ signal structure, and information aggregation failure would occur.

A.5 Robustness to gain from holding more favorable posteriors

We now show that if the agents also gain utility from holding posteriors that are more favorable to their preferred party, then for all levels of that gain, the results are identical.

The utility function of an identity-$R$ agent is now modified to become:

$$U_R = \begin{cases} 
(1 - \lambda) + \lambda P[\omega = R | S, s] & \text{if } P[\omega = R | S, s] \geq 0.5 \\
\lambda P[\omega = R | S, s] & \text{if } P[\omega = R | S, s] < 0.5 
\end{cases}$$

(9)

As in the earlier setup, the agent gains utility from preserving her political identity. She also gains some utility from holding favorable posteriors. Here, $\lambda \in [0, 1)$ captures the weight that the agent places on holding more favorable posteriors, while $1 - \lambda$ is the agent’s utility from preserving her political identity.\footnote{\(\lambda = 0\) is the benchmark model. \(\lambda = 1\) corresponds to a case in which the agent’s utility is linear in how favorable her posterior belief is towards her party.}

Based on the above utility function, there are three interim posteriors (generated after the agent observes her Inside signal, and before she observes the Outside signal) which are key to solving the agent’s problem, and which are the same as in the benchmark model:

$$P[\omega = R | S = G] = \frac{k}{1 + k - m}$$
$$P[\omega = R | S = B] = \frac{1 - k}{1 + m - k}$$
$$P[\omega = R | S = T] = 0$$

As in the benchmark model, only two signal structures are required to solve the agent’s problem, i.e. GT and GB. Furthermore, the trade-off between the two signals remains unchanged. Specifically, the agent chooses GT if the MU-Price ratio of the G signal is at
least as large as that of the $B$ signal. This simplifies to:

$$\frac{\text{MU}_G}{P_G} \geq \frac{\text{MU}_B}{P_B}$$

$$\lambda \frac{k}{1+k-m} + (1 - \lambda) \geq \lambda \frac{(1-k)}{1+m-k} + (1 - \lambda) \frac{2m(1-k)}{1+m-k}$$

$$\lambda + \frac{(1 - \lambda)(1+k-m)}{k} \geq \lambda + (1 - \lambda)(2m)$$

$$1 + k - m - 2km \geq 0$$

This condition is identical to the one in the benchmark model. Therefore, the agent’s choice of Inside signal structure remains unchanged in this modified model. This also implies that the results of the modified model are identical to those of the benchmark for all values of $\lambda \in [0, 1)$.

B Applications

B.1 Asymmetric exposure to an unbiased Outside signal

An identity-$L$ agent receives an Outside signal such that:

$$\mathbb{P}[s = l | \omega = L] = t \in [0.5, 1], \quad \mathbb{P}[s = r | \omega = R] = t \in [0.5, 1]$$

An identity-$R$ agent receives a less precise Outside signal:

$$\mathbb{P}[s = l | \omega = L] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \in [0.5, 1], \quad \mathbb{P}[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \in [0.5, 1]$$

where $\tau \in [0, 1]$.

B.1.1 The identity-$R$ agent’s problem

We use the results developed in Appendix A.1 and simply plug in $k = m = \frac{1 - \tau}{2} + \tau t$.

**Signal structure GT:** This signal structure is chosen if $\tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \leq \frac{1}{\sqrt{2}} \simeq 0.71$.

The agent’s expected utilities are the same as her likelihood of political identity preservation and are equal to:

$$E[U_R | GT \cap \omega = R] = 1, \quad E[U_R | GT \cap \omega = L] = \frac{1 + \tau - 2\tau t}{1 - \tau + 2\tau t}$$
Unconditioned on the realization of the state, the ex-ante expected utility is

\[ E[U_R|GT] = \frac{1}{1 - \tau + 2\tau t} \]

**Signal structure GB:** This signal structure is chosen if \( \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \geq \frac{1}{\sqrt{2}} \approx 0.71 \).

The agent’s expected utilities are the same as the likelihood of political identity preservation for the agent, and they equal:

\[ E[U_R|GB \cap \omega = R] = \frac{3}{4} + \tau t - \frac{t}{2} - \tau^2 \left( \frac{1}{4} + t^2 - t \right), \quad E[U_R|GB \cap \omega = L] = 1 - \left( \frac{1 - \tau + 2\tau t}{2} \right)^2 \]

Unconditioned on the realization of the state, the ex-ante expected utility is

\[ E[U_R|GB] = \frac{3}{4} - \tau^2 \left( t^2 + \frac{1}{4} - t \right) \]

**B.1.2 The identity-L agent’s problem**

**Signal structure GT:** This signal structure is chosen if \( t \leq \frac{1}{\sqrt{2}} \approx 0.71 \).

\[ E[U_L|GT \cap \omega = R] = \frac{1 - t}{t}, \quad E[U_L|GT \cap \omega = L] = 1 \]

**Signal structure GB:** This signal structure is chosen if \( t \geq \frac{1}{\sqrt{2}} \approx 0.71 \).

\[ E[U_L|GB \cap \omega = R] = 1 - t^2, \quad E[U_L|GB \cap \omega = L] = 2t - t^2 \]

**B.1.3 Proof of proposition 1**

**Claim 6** For party R to win regardless of the state of the world, it is necessary that identity-R agents choose a GT signal structure, while identity-L agents choose a GB signal structure.

**Proof.**

If agents of both identities choose a GT signal structure, and if the state of the world is R, then all identity-R agents vote for party R, and some identity-L agents also vote for party R. If the state of the world is L, then all identity-L agents vote for party L and some identity-R agents also vote for party L. Clearly, the correct party wins in either state.

Furthermore, there is no parameter space in which identity-R agents choose a GB signal structure while identity-L agents choose a GT signal structure.
If agents of both identities choose a GB signal structure and if the state of the world is \( R \), then party \( R \) wins if:

\[
\frac{3}{4} + \tau t - \frac{\tau^2}{2} - \tau^2 \left( \frac{1}{4} + t^2 - t \right) > 1 - t^2
\]

which simplifies to:

\[
\tau \left( t - \frac{1}{2} \right) \left( 1 - \tau \left( t - \frac{1}{2} \right) \right) + \left( t^2 - \frac{1}{4} \right) > 0
\]

which always holds.

Similarly, if the state of the world is \( L \), then party \( L \) wins if:

\[
1 - \left( \frac{1 - \tau + 2\tau t}{2} \right)^2 < 2t - t^2
\]

which simplifies to:

\[
\left( \frac{1 - \tau + 2\tau t}{2} \right)^2 - (1 - t)^2 > 0
\]

This always holds because \( 1 - t \in [0, 0.5] \) while \( \frac{1 - \tau + 2\tau t}{2} \in [0.5, 1] \).

\[\blacksquare\]

**Claim 7** If identity-\( R \) agents choose a GT signal structure while identity-\( L \) agents choose a GB signal structure, then party \( R \) wins regardless of the state of the world if \( \tau < \frac{(1-t)^2}{(3-t)(2t-1)} \)

**Proof.** Since we have assumed independent realizations of signals, the likelihood of political faith preservation (or expected utility) of an agent of identity-\( R \) (respectively \( L \)) is the same as the proportion of identity-\( R \) (respectively \( L \)) that are able to preserve their political faith and vote for their preferred party.

Suppose identity-\( R \) agents choose a GT signal structure while identity-\( L \) agents choose a GB signal structure. Then in the state of the world \( R \), all identity-\( R \) agents preserve their political faith, while only some identity-\( L \) agents do so. Party \( R \) wins in the state of the world \( R \). If the state of the world is \( L \), party \( R \) wins only if \( E[U_R|GT \cap \omega = L] > E[U_L|GB \cap \omega = L] \), which implies \( \frac{1 + \tau - 2\tau t}{1 - \tau + 2\tau t} > 2t - t^2 \), which simplifies to \( \tau < \frac{(1-t)^2}{(3-t)(2t-1)} \).

\[\blacksquare\]

The conditions for information aggregation failure are, therefore as follows:

- Identity-\( R \) agents choose a GT signal structure, which implies \( \tau t + \frac{1-t}{2} \leq \frac{1}{\sqrt{2}} \)
- Identity-\( L \) agents choose a GB signal structure, which implies \( t \geq \frac{1}{\sqrt{2}} \)

58
• Supposing identity-R agents choose a GT signal structure while identity-L agents choose a GB signal structure, a greater share of identity-R agents preserve their political faith, which implies $\tau < \frac{(1-t)^2}{(3-t)(2t-1)}$.

Claim 8 If all the conditions above are satisfied, then information aggregation failure occurs with strictly positive winning margins. The maximum possible winning margin for party $R$ if $\omega = L$ is $\frac{3-2\sqrt{2}}{2} \simeq 8.58\%$. This occurs when $\tau = 0$ and $t = \frac{1}{\sqrt{2}}$.

Proof.
If $\tau < \frac{(1-t)^2}{(3-t)(2t-1)}$ holds, then $E[U_R|GT \cap \omega = L] > E[U_L|GB \cap \omega = L]$, which means that a strictly greater proportion of identity-R agents preserve their political faith if $\omega = L$ than identity-L agents. This implies party $R$ wins in the state of the world $L$ with strictly positive margins. If the state of the world is $R$, then all identity-R agents receive the $G$ signal which allows them to preserve their political faith for sure. Some identity-L agents receive a $B$ signal, which means that if $\omega = R$, then party $R$ wins with a strictly positive margin.48

The winning margin for party $R$ in case $\omega = L$ is maximized when $\tau = 0$ and $t = \frac{1}{\sqrt{2}}$. These parameters are such that the Outside signal is completely uninformative for an identity-R agent, allowing such agents to preserve their political faith with probability 1 by simply choosing an uninformative Inside signal. Now consider identity-L agents when $\omega = L$ — the likelihood of political faith preservation by such agents is minimized when $t = \frac{1}{\sqrt{2}}$ and these agents optimally choose a GB signal structure.49 Given these parameter values, if the state of the world is $L$, then all identity-R agents preserve their political faith while approximately 91.42% of identity-L agents do so. As such, party $R$ wins 54.29% of the votes while party $L$ wins 45.71% (winning margin = 8.58%). □

B.2 Correlated signals

In our model, we suppose that the signals (Inside and Outside) realize independently. That is, the two signals are independent of each other and they realize independently for each agent. In the key example in the Introduction, we saw that party $R$ wins for sure in either state of the world. Because the winning margin of party $R$ is positive (specifically, bounded away from zero), we know that the result of information aggregation failure is robust to some, possibly small, degree of correlation of the two signals.

48When $\tau = 0$ and $t = \frac{1}{\sqrt{2}}$, then if $\omega = R$, 50% of identity-L agents preserve their political faith and party $R$ wins with a margin of 50%.

49It is also optimal for an identity-L agent to choose a GT signal. We are breaking the tie in favor of the GB signal here. We could, alternatively, have chosen $t = \frac{1}{\sqrt{2}} + \epsilon$. 
Below, we consider three cases of correlated signals. In each case, we suppose that the two signals (Inside and Outside) realize independently. For ease of illustrating the effects, we also suppose that there is full correlation in the realization of the signal across all agents of a particular identity. Finally, we fix parameter values to be the same as studied in the key example of the paper. That is, there is an equal proportion of infinite identity-R and identity-L agents. Identity-R agents receive a less precise Outside signal than identity-L agents ($t_R = 0.51$ and $t_L = 0.75$). The common prior belief is that each state of the world is equally likely.

First, we consider the case that the Outside signal realization is fully correlated across agents of the same identity. Second, we suppose that Inside signal realization is fully correlated in the same way. Finally, we study the situation where both signals are fully correlated in the way they realize for all agents of the same identity. In each scenario, since the Outside signals realize independently of the Inside signal, the optimal signal structure choice for agents remains the same as in the baseline model. This implies that the expected winning margin advantage for party R remains the same ex-ante (+29%) and conditional on the state being R (+56%) or L (+2%). The aggregate implications of this expected winning margin advantage vary according to the kind of correlation we consider. In each case, party R wins with a higher probability than party L. Table 3 illustrates the outcomes and the probabilities with which those occur, conditional on the state of the world and for each case of correlated signals we consider.

**Outside signals:** Suppose the Outside signals are fully correlated such that either all identity-R agents receive favorable news from the Outside, or all of them receive unfavorable news. The correlation functions similarly for all identity-L agents. Here, we find that the likelihood that party R wins is 50%, while the same for party L is 37.5%. The chance of a tie is 12.5%.  

**Inside signals:** Suppose the Inside signals are fully correlated such that if GT is the chosen signal structure for identity-R agents, then either they all receive a G signal, or all of them receive a T signal. The correlation functions similarly for all identity-L agents. Here, we find that the likelihood that party R wins is 49.51%, while the same for party L is 1.96%. The chance of a tie is 48.53%.

---

Note that a tie occurs with a probability of 25% in state R. This is when all identity-R individuals, having observed a G Inside signal, preserve their political identity, and all identity-L citizens also preserve their political identity because all of them receive a favorable Outside signal. If the correlation of the realization of the Outside signal was anything less than full, then instead of a tie, party R would win.
Table 3: Outcomes and probabilities with correlated signals

<table>
<thead>
<tr>
<th></th>
<th>Correlated Outside signal</th>
<th></th>
<th>Correlated Inside signal</th>
<th></th>
<th>Correlated Inside and Outside signals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \omega = R )</td>
<td></td>
<td>( \omega = L )</td>
<td></td>
<td>( \omega = R )</td>
</tr>
<tr>
<td>Probability</td>
<td>Win Margin R</td>
<td>Probability</td>
<td>Win Margin R</td>
<td>Probability</td>
<td>Win Margin R</td>
</tr>
<tr>
<td>0.75</td>
<td>+75%</td>
<td>0.25</td>
<td>+21.08%</td>
<td>0.5625</td>
<td>+100%</td>
</tr>
<tr>
<td>0.25</td>
<td>0%</td>
<td>0.75</td>
<td>−3.92%</td>
<td>0.4375</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.03675</td>
</tr>
</tbody>
</table>

Inside and Outside signals: Suppose that both, the Inside and Outside signals are fully correlated in how they realize for agents of the same identity. Crucially, they realize independently of each other. Here, we find that the likelihood that party \( R \) wins is 31.13%, while the same for party \( L \) is 1.84%. The chance of a tie is 67.03%.

B.3 Non-common priors

Suppose that identity-\( R \) and identity-\( L \) agents have different priors. Specifically,

\[
P_R[\omega = R] := w_R \quad \quad \quad P_L[\omega = R] := w_L
\]

where \( w_L < 0.5 < w_R \).

As in Subsection 3.1, an identity-\( L \) agent receives an Outside signal, such that:

\[
P[s = l|\omega = L] = t \in [0.5, 1], \quad P[s = r|\omega = R] = t \in [0.5, 1]
\]
An identity-\(R\) agent receives a less precise Outside signal, such that:
\[
\mathbb{P} [s = l | \omega = L] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \in [0.5, 1], \quad \mathbb{P} [s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \in [0.5, 1]
\]
where \(\tau \in [0, 1]\)

### B.3.1 The identity-\(R\) agent’s problem

We use the results developed in Appendix A.1 and allow for \(w_R > 0.5\) while plugging in \(k = m = \frac{1 - \tau}{2} + \tau t\).

**Signal structure \(N\):** This signal structure is chosen if \(\tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \leq w_R\)

The agent is able to preserve her political identity regardless of the realization of the Outside signal.

\[
E[U_R | N \cap \omega = R] = 1, \quad E[U_R | N \cap \omega = L] = 1
\]

**Signal structure \(GT\):** This signal structure is chosen if \(w_R < \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \leq \frac{1}{\sqrt{2}} \simeq 0.71\).

The agent’s expected utilities are the same as her likelihood of political identity preservation, and are equal to:

\[
E[U_R | GT \cap \omega = R] = 1, \quad E[U_R | GT \cap \omega = L] = \frac{w_R(1 + \tau - 2\tau t)}{(1 - w_R)(1 - \tau + 2\tau t)}
\]

**Signal structure \(GB\):** This signal structure is chosen if \(\tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \geq max\{\frac{1}{\sqrt{2}}, w_R\}\).

The agent’s expected utilities are the same as her likelihood of political identity preservation, and are equal to:

\[
E[U_R | GB \cap \omega = R] = \frac{(2\tau t + 1 - \tau)(2\tau t - 1 - \tau)}{4w_R \tau(2t - 1)} + \frac{(2\tau t + 1 - \tau)(2\tau t + 1 - \tau - 2w_R)(1 + \tau - 2\tau t)}{8w_R \tau(2t - 1)}
\]

and

\[
E[U_R | GB \cap \omega = L] = \frac{(1 + \tau - 2\tau t)(2\tau t - 1 - \tau)}{4(1 - w_R) \tau(2t - 1)} + \frac{(2\tau t + 1 - \tau)(2\tau t + 1 - \tau - 2w_R)(1 + \tau - 2\tau t)}{8(1 - w_R) \tau(2t - 1)}
\]

### B.3.2 The identity-\(L\) agent’s problem

We use the results developed in Appendix A.2 and allow for \(w_L < 0.5\) while plugging in \(k = m = t\).
Signal structure $N$: This signal structure is chosen if $t \leq 1 - w_L$
The agent is able to preserve her political identity regardless of the realization of the Outside signal.

\[ E[U_L | N \cap \omega = R] = 1, \quad E[U_L | N \cap \omega = L] = 1 \]

Signal structure $GT$: This signal structure is chosen if $1 - w_L < t \leq \frac{1}{\sqrt{2}} \simeq 0.71$.

\[ E[U_L | GT \cap \omega = R] = \frac{(1 - w_L)(1 - t)}{w_L t}, \quad E[U_L | GT \cap \omega = L] = 1 \]

Signal structure $GB$: This signal structure is chosen if $t \geq \max\{\frac{1}{\sqrt{2}}, 1 - w_L\}$.

\[ E[U_L | GB \cap \omega = R] = \frac{(1 - t)(t - w_L)}{w_L(2t - 1)} + \frac{t(1 - t)(t + w_L - 1)}{w_L(2t - 1)} \]

and

\[ E[U_L | GB \cap \omega = L] = \frac{t(2t - 1 - t^2 + (1 - w_L) t)}{(1 - w_L)(2t - 1)} \]

Conditions for information aggregation failure: The conditions under which party $R$ wins in both states of the world are:

- Identity-$R$ agents choose a $GT$ or $N$ signal structure, which implies $\tau t + \frac{1 - \tau}{2} \leq \max\{\frac{1}{\sqrt{2}}, w_R\}$
- Identity-$L$ agents choose a $GB$ signal structure, which implies $t \geq \max\{\frac{1}{\sqrt{2}}, 1 - w_L\}$
- Identity-$R$ agents preserve their political faith with a greater likelihood if $\omega = L$
  - If $\tau t + \frac{1 - \tau}{2} \leq w_R$ and identity-$R$ agents choose an $N$ signal structure, we require
    \[ \frac{t(2t - 1 - t^2 + (1 - w_L) t)}{(1 - w_L)(2t - 1)} < 1 \]
  - If $w_R < \tau t + \frac{1 - \tau}{2} \leq \frac{1}{\sqrt{2}}$ and identity-$R$ agents choose an $GT$ signal structure,
    we require
    \[ \frac{t(2t - 1 - t^2 + (1 - w_L) t)}{(1 - w_L)(2t - 1)} < \frac{w_R(1 + \tau - 2\tau t)}{(1 - w_R)(1 - \tau + 2\tau t)} \]

B.4 Distrust in the mainstream media’s precision

Suppose that identity-$R$ agents believe (incorrectly) that the media is less precise than it actually is. Specifically, identity-$L$ agents correctly believe that the process generating the Outside signal is such that:

\[ \mathbb{P}[s = l | \omega = L] = t \in [0.5, 1], \quad \mathbb{P}[s = r | \omega = R] = t \in [0.5, 1] \]
In contrast, identity-\( R \) agents incorrectly believe that the process that generates the Outside signal is more noisy, such that:

\[
\mathbb{P}[s = l|\omega = L] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \in [0.5, 1], \quad \mathbb{P}[s = r|\omega = R] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \in [0.5, 1]
\]

where \( \tau \in [0, 1] \).

This is similar to an asymmetry in exposure to mainstream media. As such, the signal choices and expected utilities are identical to those calculated in Subsection B.1.\footnote{For identity-\( R \) agents, the expectations are based on incorrect beliefs. We will, therefore, separately calculate the probability of political identity preservation.}

For identity-\( L \) agents, the probability of political identity preservation is identical to the expected utilities calculated in Subsection B.1.

For identity-\( R \) agents, if the chosen signal is of type \( GT \), then the probability of political identity preservation is identical to the expected utility calculated in Subsection B.1. If the chosen signal is of type \( GB \), then an identity-\( R \) agent’s probability of political identity preservation is calculated conditioned on the state.

If the state is \( \omega = R \), then:

\[
\mathbb{P}[\text{PFP}_R|GB \cap \omega = R] = \mathbb{P}[G|GB \cap \omega = R] + \mathbb{P}[r|B \cap \omega = R] \cdot \mathbb{P}[B|GB \cap \omega = R]
\]

\[
= \left( \frac{1 - \tau}{2} + \tau t \right) + t \cdot \left( \frac{1 + \tau}{2} - \tau t \right)
\]

\[
= \frac{1}{2} (1 - \tau + t + 3t - 2\tau t^2)
\]

If the state is \( \omega = L \), then:

\[
\mathbb{P}[\text{PFP}_R|GB \cap \omega = L] = \mathbb{P}[G|GB \cap \omega = L] + \mathbb{P}[r|B \cap \omega = L] \cdot \mathbb{P}[B|GB \cap \omega = L]
\]

\[
= \left( \frac{1 + \tau}{2} - \tau t \right) + (1 - t) \cdot \left( \frac{1 - \tau}{2} + \tau t \right)
\]

\[
= \frac{1}{2} (2 - t + \tau t - 2\tau t^2)
\]

B.5 Distrust in unbiasedness of mainstream media

Suppose that the true process of the Outside signal is:

\[
\mathbb{P}[s = l|\omega = L] = t \in [0.5, 1], \quad \mathbb{P}[s = r|\omega = R] = t \in [0.5, 1]
\]
Identity-\(L\) agents know the true process, while identity-\(R\) agents believe that the process of the Outside signal is biased in the following way:

\[
P[s = l|\omega = L] = \tau \cdot t + (1 - \tau) \cdot 1, \quad P[s = r|\omega = R] = \tau \cdot t + (1 - \tau) \cdot 0
\]

where \(\tau \in [0, 1]\).

**B.5.1 The identity-\(R\) agent’s problem**

**GT signal structure:** This signal structure is chosen if \(2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \geq 0\).

The agent’s expected utilities are:

\[
E[U_R|GT \cap \omega = R] = 1, \quad E[U_R|GT \cap \omega = L] = \frac{1 - \tau t}{1 + \tau t - \tau}
\]

If the agent chooses a GT signal structure, then the likelihood of political identity preservation does not depend on the realization of the Outside signal. Therefore, the likelihood of political identity preservation is identical to the expected utility.

**GB signal structure:** This signal structure is chosen if \(2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \leq 0\).

The agent’s expected utilities (under incorrect beliefs) are:

\[
E[U_R|GB \cap \omega = R] = 1 - \tau + \tau t + \tau^2 t - \tau^2 t^2, \quad E[U_R|GB \cap \omega = L] = 1 - \tau t + \tau^2 t - \tau^2 t^2
\]

In this case, the likelihood of political identity preservation differs from the agent’s expected utility, such that:

\[
P[PFP_R|GB \cap \omega = R] = 1 - \tau + 2\tau t - \tau t^2, \quad P[PFP_R|GB \cap \omega = L] = 1 - \tau t^2
\]

**B.5.2 The identity-\(L\) agent’s problem**

**GT signal structure:** This signal structure is chosen if \(t \leq \frac{1}{\sqrt{2}}\).

The agent’s expected utilities are:

\[
E[U_L|GT \cap \omega = R] = \frac{1 - t}{t}, \quad E[U_L|GT \cap \omega = L] = 1
\]

**GB signal structure:** This signal structure is chosen if \(t \geq \frac{1}{\sqrt{2}}\).

The agent’s expected utilities are:

\[
E[U_L|GB \cap \omega = R] = 1 - t^2, \quad E[U_L|GB \cap \omega = L] = 2t - t^2
\]
B.5.3 Proof of proposition 2

Claim 9 For party R to win regardless of the state of the world, it is necessary that identity-R agents choose a GT signal structure, while identity-L agents choose a GB signal structure.

Proof. If agents of both identities choose a GT signal structure, and if the state of the world is R, then all identity-R agents vote for party R, and some identity-L agents also vote for party R. If the state of the world is L, then all identity-L agents vote for party L and some identity-R agents also vote for party L. Clearly, the correct party wins in either state.

There is no parameter space where identity-R agents choose a GB signal structure while identity-L agents choose a GT signal structure.

If agents of both identities choose a GB signal structure, and if the state of the world is R, then party R wins if:

\[ 1 - \tau + 2\tau t - \tau t^2 > 1 - t^2 \]

This simplifies to:

\[ \tau (2t - 1) + t^2 (1 - \tau) > 0 \]

which always holds.

Similarly, if the state of the world is L, then party L wins if:

\[ 2t - t^2 > 1 - \tau t^2 \]

which simplifies to:

\[ (2t - 1) - t^2 (1 - \tau) > 0 \]

which holds for values of \( t \in [\frac{1}{\sqrt{2}}, 1] \) and \( \tau \in [\frac{2}{3}, 1] \). The region where agents of both identities choose a GB signal structure is a subset of the region where \( t \in [\frac{1}{\sqrt{2}}, 1] \) and \( \tau \in [\frac{2}{3}, 1] \). Therefore, if agents of both identities choose a GB signal structure, then the correct party wins. ■

Claim 10 If identity-R agents choose a GT signal structure while identity-L agents choose a GB signal structure, then party R wins regardless of the state of the world if \( \tau < \frac{(1-t)^2}{t(1-(1-t)^2)} \).

Proof. Since we have assumed independent realizations of signals, the likelihood of political faith preservation (or expected utility) of an agent of identity-R (respectively L) is the same as the proportion of identity-R (respectively L) that are able to preserve their political faith and vote for their preferred party.

Suppose identity-R agents choose a GT signal structure while identity-L agents choose a GB signal structure. Then in the state of the world R, all identity-R agents preserve
their political faith, while only some identity-L agents do so. Party R wins in the state of the world R. If the state of the world is L, party R wins only if 
\[ E[U_R|GT \cap \omega = L] > E[U_L|GB \cap \omega = L], \] which implies 
\[ \frac{1 - \tau t}{1 + \tau t - \tau} > 2t - t^2, \] which simplifies to 
\[ \tau < \frac{(1 - t)^2}{t(1 - (1 - t)^2)}. \] 

The conditions for information aggregation failure are, therefore as follows:

- Identity-R agents choose a GT signal structure, which implies 
\[ 2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \geq 0. \]

- Identity-L agents choose a GB signal structure, which implies 
\[ t \geq \frac{1}{\sqrt{2}}. \]

- Supposing identity-R agents choose a GT signal structure while identity-L agents choose a GB signal structure, a greater share of identity-R agents preserve their political faith, which implies 
\[ \tau < \frac{(1 - t)^2}{t(1 - (1 - t)^2)}. \]

Claim 11 If all the conditions above are satisfied, then information aggregation failure occurs with strictly positive winning margins. The maximum possible winning margin for party R if \( \omega = L \) is 
\[ \frac{3 - 2\sqrt{2}}{2} \simeq 8.58\%. \] This occurs when \( \tau = 0 \) and \( t = \frac{1}{\sqrt{2}}. \)

Proof. This claim is identical to Claim 8. Even though the Outside signal structure for identity-R agents is different here, the proof of this claim remains unchanged. ■

B.6 Propagandized Outside signal with censorship

Here that the Outside signal is biased towards party L, which is common knowledge. The structure of the Outside signal is:

\[ \mathbb{P}[s = l | \omega = L] = \tau \cdot t + (1 - \tau) \cdot 1, \quad \mathbb{P}[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot 0 \]

where \( t > 0.5. \)

We can use the results from Appendix A.1 and A.2 by simply plugging in \( k = 1 + \tau t - \tau \) and \( m = \tau t. \)

Because we are considering censorship, no agent of either identity has access to any information other than her prior and the realized Outside signal. The agents share common and symmetric priors, and their posteriors on observing the Outside signals are:

\[ Pr[\omega = L|s = l] = \frac{1 + \tau t - \tau}{2 - \tau} \]

Since \( t > 0.5, Pr[\omega = L|s = l] > 0.5 \)
and

\[ Pr[\omega = L|s = r] = 1 - t \]

Since \( t > 0.5 \), \( Pr[\omega = L|s = r] < 0.5 \)

For party \( L \) to win in state \( R \), we require that \( Pr(s = l|\omega = R) > Pr(s = r|\omega = R) \) or \( 1 - \tau t > \tau t \). That is, \( \tau t < 0.5 \).

For party \( L \) to win in state \( L \), we require that \( Pr(s = l|\omega = L) > Pr(s = r|\omega = L) \) or \( 1 + \tau t - \tau > \tau - \tau t \). That is, \( \tau(1 - t) < 0.5 \). If \( \tau \in [0, 1] \), and \( t > 0.5 \), or if \( \tau \in (0, 1] \) and \( t \geq 0.5 \) then this always holds.

Therefore, party \( L \) can win in both states of the world if \( \tau t < 0.5 \).

### B.7 Propagandized Outside signal without censorship

Here, the Outside signal is biased towards party \( L \), and this bias is common knowledge. The structure of the Outside signal is:

\[
\begin{align*}
P[s = l|\omega = L] &= \tau \cdot t + (1 - \tau) \cdot 1, \\
P[s = r|\omega = R] &= \tau \cdot t + (1 - \tau) \cdot 0
\end{align*}
\]

where \( t > 0.5 \).

We can use the results from Appendix A.1 and A.2 by simply plugging in \( k = 1 + \tau t - \tau \) and \( m = \tau t \).

#### B.7.1 The identity-\( R \) agent’s problem

**GT signal structure:** Chosen if \( 2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \geq 0 \).

The agent’s expected utilities are:

\[
\begin{align*}
E[U_R|GT \cap \omega = R] &= 1, \\
E[U_R|GT \cap \omega = L] &= \frac{1 - \tau t}{1 + \tau t - \tau}
\end{align*}
\]

**GB signal structure:** Chosen if \( 2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \leq 0 \).

The agent’s expected utilities are:

\[
\begin{align*}
E[U_R|GB \cap \omega = R] &= 1 - \tau + \tau t + \tau^2 t - \tau^2 t^2, \\
E[U_R|GB \cap \omega = L] &= 1 - \tau t + \tau^2 t - \tau^2 t^2
\end{align*}
\]
B.7.2 The identity-\textit{L} agent’s problem

\textit{GT signal structure:} Chosen if \( \tau(1 - 2t - 2\tau t^2 + 2\tau t) \geq 0 \) The agent’s expected utilities are:

\[
E[U_L|GT \cap \omega = R] = \frac{1-t}{t}, \quad E[U_L|GT \cap \omega = L] = 1
\]

\textit{GB signal structure:} Chosen if \( \tau(1 - 2t - 2\tau t^2 + 2\tau t) \leq 0 \) The agent’s expected utilities are:

\[
E[U_L|GB \cap \omega = R] = 1 - \tau t + \tau^2 t - \tau^2 t^2, \quad E[U_L|GB \cap \omega = L] = 1 - \tau + \tau t + \tau^2 t - \tau^2 t^2
\]

B.7.3 Proof of proposition 3

Claim 12 For party R to win regardless of the state of the world, it is necessary that identity-\textit{R} agents choose a GT signal structure, while identity-\textit{L} agents choose a GB signal structure.

Proof.

If agents of both identities choose a \textit{GT} signal structure, and if the state of the world is \textit{R}, then all identity-\textit{R} agents vote for party \textit{R}, and some identity-\textit{L} agents also vote for party \textit{R}. If the state of the world is \textit{L}, then all identity-\textit{L} agents vote for party \textit{L} and some identity-\textit{R} agents also vote for party \textit{L}. Clearly, the correct party wins in either state.

There is no parameter space where identity-\textit{R} agents choose a \textit{GB} signal structure while identity-\textit{L} agents choose a \textit{GT} signal structure.

If agents of both identities choose a \textit{GB} signal structure, and if the state of the world is \textit{R}, then party \textit{R} wins if:

\[
1 - \tau + \tau t + \tau^2 t - \tau^2 t^2 > 1 - \tau t + \tau^2 t - \tau^2 t^2
\]

which simplifies to:

\[
\tau(2t - 1) > 0
\]

which always holds.

Similarly, if the state of the world is \textit{L}, then party \textit{L} wins if:

\[
1 - \tau + \tau t + \tau^2 t - \tau^2 t^2 > 1 - \tau t + \tau^2 t - \tau^2 t^2
\]

which simplifies to:

\[
\tau(2t - 1) > 0
\]
which always holds. ■

**Claim 13** If identity-R agents choose a GT signal structure while identity-L agents choose a GB signal structure, then party R wins regardless of the state of the world if \( \frac{1 - \tau}{1 + \tau(1 - t)} > 1 - \tau(1 - t)(1 - \tau t) \)

**Proof.** Since we have assumed independent realizations of signals, the likelihood of political faith preservation (or expected utility) of an agent of identity-R (respectively L) is the same as the proportion of identity-R (respectively L) that are able to preserve their political faith and vote for their preferred party.

Suppose identity-R agents choose a GT signal structure while identity-L agents choose a GB signal structure. Then in the state of the world \( R \), all identity-R agents preserve their political faith, while only some identity-L agents do so. Party R wins in the state of the world \( R \). If the state of the world is \( L \), party R wins only if \( E[U_R|GT \cap \omega = L] > E[U_L|GB \cap \omega = L] \), which implies \( \frac{1 - \tau}{1 + \tau(1 - t)} < 1 - \tau + \tau t + \tau^2 t - \tau^2 t^2 \), which simplifies to \( \frac{1 - \tau}{1 + \tau(1 - t)} > 1 - \tau(1 - t)(1 - \tau t) \).

The conditions for information aggregation failure are, therefore as follows:

- Identity-R agents choose a GT signal structure, which implies \( 2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \geq 0 \)
- Identity-L agents choose a GB signal structure, which implies \( \tau(1 - 2t - 2\tau^2 t + 2\tau t) \leq 0 \)
- Supposing identity-R agents choose a GT signal structure while identity-L agents choose a GB signal structure, a greater share of identity-R agents preserve their political faith, which implies \( \frac{1 - \tau}{1 + \tau(1 - t)} > 1 - \tau(1 - t)(1 - \tau t) \)

**Claim 14** If all the conditions above are satisfied, then information aggregation failure occurs with strictly positive winning margins. The maximum possible winning margin for party R if \( \omega = L \) is approximately 5.99%. This occurs when \( \tau \approx 0.236 \) and \( t \approx 0.558 \).

**Proof.** If \( \frac{1 - \tau}{1 + \tau(1 - t)} > 1 - \tau(1 - t)(1 - \tau t) \) holds, then \( E[U_R|GT \cap \omega = L] > E[U_L|GB \cap \omega = L] \), which means that a strictly greater proportion of identity-R agents preserve their political faith if \( \omega = L \) than identity-L agents. This implies party R wins in the state of the world \( L \) with strictly positive margins. If the state of the world is \( R \), then all identity-R agents receive the G signal which allows them to preserve their political faith for sure.
Some identity-L agents receive a B signal, which means that if \( \omega = R \), then party R wins with a strictly positive margin.\(^{52}\)

Suppose \( \tau = 0.236 \) and \( t = 0.558 \). The percent of identity-R agents that preserve their political faith and vote for party R in state of the world L equals \( \frac{1-t\tau}{1+t\tau} \simeq 96.94\% \). The percent of identity-L agents that preserve their political faith and vote for party L in state of the world L equals \( 1 - \tau(1 - t)(1 - t\tau) \simeq 90.94\% \). The agents who do not preserve their political faith vote for the other party. Therefore, party R wins in state of the world L with a margin of approximately 5.99\%. We used numerical simulation to establish that this is the maximum possible value.\(^{53}\)

B.8 Propaganda with oblivious citizens

In this case, the true process of the Outside signal is:

\[
P[s = l|\omega = L] = \tau \cdot t + (1 - \tau) \cdot 1, \quad P[s = r|\omega = R] = \tau \cdot t + (1 - \tau) \cdot 0
\]

where \( \tau \in [0, 1] \) and \( t \in [0.5, 1] \). Agents don’t know the true process, and believe that the process generating the Outside signal is unbiased, such that:

\[
P[s = l|\omega = L] = t, \quad P[s = r|\omega = R] = t
\]

B.8.1 The identity-R agent’s problem

**GT signal structure:** This signal structure is chosen if \( t \leq \frac{1}{\sqrt{2}} \)

The agent’s expected utilities are:

\[
E[U_R|GT \cap \omega = R] = 1, \quad E[U_R|GT \cap \omega = L] = \frac{1 - t}{t}
\]

If the agent chooses a GT signal structure, then the likelihood of political identity preservation does not depend on the realization of the Outside signal. Therefore, the likelihood of political identity preservation is identical to the expected utility.

\(^{52}\)When \( \tau = 0.236 \) and \( t = 0.558 \), then if \( \omega = R \), 88.20\% of identity-L agents preserve their political faith and party R wins with a margin of 11.80\%.

\(^{53}\)MATLAB/Mathematica code available on request. Alternatively, see this link for the solution of this optimization problem on Wolfram Alpha.
**GB signal structure:** This signal structure is chosen if \( t \geq \frac{1}{\sqrt{2}} \). The agent’s expected utilities (under incorrect beliefs) are:

\[
E[U_R|GB \cap \omega = R] = 2t - t^2, \quad E[U_R|GB \cap \omega = L] = 1 - t^2
\]

Here, the likelihood of political identity preservation differs from the agent’s expected utility.

\[
P[PFP_R|GB \cap \omega = R] = t(1 + \tau - \tau t), \quad P[PFP_R|GB \cap \omega = L] = (1 + \tau t)(1 - t)
\]

**B.8.2 The identity-L agent’s problem**

**GT signal structure:** This signal structure is chosen if \( t \leq \frac{1}{\sqrt{2}} \). The agent’s expected utilities are:

\[
E[U_L|GT \cap \omega = R] = \frac{1 - t}{t}, \quad E[U_L|GT \cap \omega = L] = 1
\]

If the agent chooses a GT type signal structure, then the likelihood of political identity preservation does not depend on the realization of the Outside signal. Therefore, the likelihood of political identity preservation is identical to the expected utility.

**GB signal structure:** This signal structure is chosen if \( t \geq \frac{1}{\sqrt{2}} \). The agent’s expected utilities (under incorrect beliefs) are:

\[
E[U_L|GB \cap \omega = R] = 1 - t^2, \quad E[U_L|GB \cap \omega = L] = 2t - t^2
\]

In this case, the likelihood of political identity preservation differs from the agent’s expected utility, such that:

\[
P[PFP_L|GB \cap \omega = R] = 1 - \tau t^2, \quad P[PFP_L|GB \cap \omega = L] = 1 - \tau(1 - t)^2
\]

**B.8.3 Proof of proposition 4**

**Claim 15** For party L to win regardless of the state of the world, it is necessary that agents of both identities choose a GB signal structure, which occurs when \( t \in \left[ \frac{1}{\sqrt{2}}, 1 \right] \). Further, party L wins in the state of the world R with a strictly positive margin if \( t < \frac{1}{1 + \tau} \).

**Proof.**

If agents of both identities choose a GT signal structure, and if the state of the world is R, then all identity-R agents vote for party R, and some identity-L agents also vote for
party \( R \). If the state of the world is \( L \), then all identity-\( L \) agents vote for party \( L \) and some identity-\( R \) agents also vote for party \( L \). Clearly, the correct party wins in either state.

There is no parameter space in which agents of the two identities choose different signal structures for their respective Inside media consumption.

If agents of both identities choose a \( GB \) signal structure \(( t \geq \frac{1}{\sqrt{2}}) \), and if the state of the world is \( L \), then party \( L \) wins if:

\[
1 - \tau (1 - t)^2 > (1 + \tau t)(1 - t)
\]

This simplifies to:

\[
t + \tau t - \tau > 0
\]

which always holds because \( \tau \in [0, 1] \) and \( t \in [0.5, 1] \).

Furthermore, if the state of the world is \( R \), then party \( L \) wins if:

\[
1 - \tau t^2 > t(1 + \tau - \tau t)
\]

which simplifies to

\[
t < \frac{1}{1+\tau}
\]

Therefore, party \( L \) wins regardless of the state if \( t \in \left[ \frac{1}{\sqrt{2}}, \frac{1}{1+\tau} \right] \). There are no parameter values for which party \( R \) can win in state \( L \).

\[\nabla\]

**Claim 16** If \( \omega = R \), party \( L \)'s winning margin is maximized when \( \tau = 0 \) and \( t = \frac{1}{\sqrt{2}} \).

**Proof.** The winning margin for party \( L \) in case \( \omega = R \) is maximized when \( \tau = 0 \) and \( t = \frac{1}{\sqrt{2}} \). These parameters are such that the Outside signal is wholly biased in favor of party \( L \) and generates the signal \( l \) regardless of the state of the world and regardless of the value of \( t \). Since agents are oblivious to this and \( t = \frac{1}{\sqrt{2}} \), all agents choose a \( GB \) signal structure, this allows identity-\( L \) agents to preserve their political faith with probability 1 in either state of the world. When \( \omega = L \), \( t(1 + \tau - \tau t) \) proportion of identity-\( R \) agents preserve their political faith. If \( \tau = 0 \), this proportion is minimized by choosing \( t = \frac{1}{\sqrt{2}} \). When \( \tau = 0 \) and \( t = \frac{1}{\sqrt{2}} \), party \( L \) wins in the state of the world \( R \) with a margin of 29.29% and in the state of the world \( L \) with a margin of 70.71%. \(\nabla\)