Sorting in Risk-Aversion and Asset Price Volatility

Helios Herrera (ITAM) *

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Abstract

I analyze how an exogenous cost of entry in a risky asset market affects two endogenous variables: the degree of market participation and price volatility. I show that different entry costs generate different participation equilibria and that a multiplicity of equilibria may arise, but that the new market entrants are always more risk-averse than the rest of the participants. Every participation equilibrium is associated with a volatility of the asset price. Increased market participation leads to increased asset price volatility and higher welfare.

(JEL: G12, D40, C70)

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"If the arbitrageur is risk-averse, his interest in such arbitrage will be limited. With a finite risk-bearing capacity of arbitrageurs as a group, their aggregate ability to bring prices of broad groups of securities into line is limited as well." Schleifer (2000).

1 Introduction

During the past years, an increase in the number and in the diversity of traders has generated changes in financial asset markets. For instance, US stock market participation has increased consistently since the fifties and this increase has been most dramatic in the eighties and nineties. The number of shareholders in the United States increased by more than 60% from 1989 to 1998, when it reached approximately 84 million individuals.[1] Since 1995, a great number of new investors began purchasing assets in financial markets worldwide especially through the Internet.[2]

*ITAM-Centro de Investigacion Economica, Av. Camino a Santa Teresa No.930, 10700 Mexico D.F. Tel: +52 55 5628 4000 ext.2961, Fax: +52 55 5628 4058, Email: helios@itam.mx

[1] According to the Survey of Consumer Finances (1998).

[2] From 1995 through mid-2000, investors opened 12.5 million on-line brokerage accounts, as Barber and Odean document (JEP 2001).

It is interesting to understand the effects of this change in participation on prices of financial assets and in particular on the volatility of those prices. One explanation for this increase in participation in the stock market is a decrease in the cost of entry in the market. Whereas in the past higher transaction and information costs kept certain types of investors out of risky asset markets, nowadays the easier access has attracted new types of investors into financial markets. As a result of information technology and telecommunications improvements, there is no doubt that the cost of acquiring information on assets declined dramatically in the past years. However, while there seems to be a consensus that the lowering of the pecuniary (brokerage) and non-pecuniary (information and setup) costs of participation has encouraged the entry of new investor-types into asset markets, whether these new market participants have *increased or decreased* the asset price volatility is a question that still needs theoretical and empirical exploration. The goal of this paper is to address theoretically the following question. Under what conditions does increased market participation increase or decrease the asset price volatility ? In the quest to explore the "origins" of stock price volatility, I use an endogenous market participation model to show that one source of volatility emerges from the self-selection of potential investors. As I show, this source of volatility is an immediate consequence of the change in the overall composition of the attitudes towards risk of investors. The change in the market level of risk-aversion follows from the easier access to the stock market allowing the entry of new investors.

A standard argument in the related literature on market participation states that higher market participation should *decrease* volatility, because the idiosyncratic demand shocks of participants tend to cancel one another out as the number of participants rises. As Pagano (ReStud. 1989) shows in his seminal work, this dampening of the price volatility is caused by the cancellation of the independent demand shocks of the market participants due to a *law of large numbers* effect. A lower entry cost induces more investors to enter the market thus dampening volatility. Pagano obtains the latter effect assuming that all potential market participants are identical when deciding whether or not to enter the market. Hence, his explanation does not consider that as the cost of entry decreases not only may the number of participants change but the new participants may also have characteristics that differ from the old participants. An interesting avenue of research is to explore the effects of sorting of investors on market participation and price volatility. In this paper I explore the effects of heterogeneity in risk-aversion.

I show that if the potential market participants have different levels of risk-aversion, more participation *increases* volatility. I analyze the different participation equilibria that arise from the market entry game. Different participation equilibria imply different price volatilities, depending on what types of investors enter the asset market. I find that the less risk-averse types enjoy a *higher* benefit than the more risk-averse types from participating in a risky asset market. The main reason is that, once they have entered the market, less risk averse individuals require a *lower* compensation than the relatively more risk averse individuals for bearing the risk of holding the risky asset. As a consequence, for any given cost of market participation, if a given type participates then all the types less risk-averse than him participate too. As market participation increases (due to a lower entry cost or multiplicity of equilibria) and more risk-averse investors join the less risk-averse in the market, the *less aggressive* reaction to the price fluctuations of the new participants leads to an *increase* of the volatility of the price of the asset. When multiple participation equilibria arise, these are Pareto ranked, the equilibrium with larger participation dominating the others and so forth. This implies that higher welfare is associated with higher volatility.

Orosel (1998) analyzes a model in which participation is determined endogenously and fluctuates over time. He shows that there is a positive link between movements in prices and fluctuations in participation. In his model the reason why prices fluctuate are changes in endogenous participation, triggered by innovation in dividends that follow a Markov process. The endogenous fluctuations of market participation lead to increased volatility of the share price. In my model instead, volatility is not related to endogenous fluctuations in market participation. Although different across equilibria, market participation is fixed within each equilibrium. The reason being that I want to explore what forces affect volatility as a consequence of a change in the composition of market participants. Another important work on endogenous market participation is Allen & Gale (1994). They show that limited market participation can have the effect of amplifying price volatility relative to full market participation. The main difference with respect to this paper is that in Allen & Gale their "excess" volatility is generated by a shortage of cash in the market. That is, as participant-types with more cash in their portfolios enter the market, the liquidity provided by these new investor-participants can absorb more trades hence dampening volatility. Along with this heterogeneity in liquidity preferences (cash) of participants, Allen & Gale also introduce heterogeneity in risk-aversion. However, since they model a market characterized by liquidity shortage, the heterogeneity in risk-aversion, which is needed to generate the different participation equilibria, is not crucial for the volatility result. The vanishing of the excess volatility with full participation they obtain, has little to do with the fact that the new participants are more risk-averse than the rest, but with the fact that the new entrants introduce more cash, i.e., liquidity into the market. In my paper instead, the "risk-aversion effect" is a major force driving the volatility. The fact that in equilibrium new participants are more risk-averse than the rest is crucial in my model because the fluctuations of the equilibrium price are positively affected by the *average* risk-tolerance of the market. Most of the other work in the limited participation literature takes a different approach, see for instance Merton (1987), Basak and Cuoco (1998) and Shapiro (2002). These papers do not treat participation endogenously, but assume that certain exogenously chosen agents are prevented from investing in some given financial assets.

The rest of the paper is organized as follows. Section 2 sets up the formal model, finds the link between price volatility and the composition of the market participants and finds the gain from participation. Section 3 looks at the participation equilibria, volatility and welfare. Section 4 concludes.

2 The Model

This is a model of endogenous market participation. There is a continuum of agents which maximize their expected utility over their final wealth W. All agents have CARA utility and are heterogenous in terms of their absolute risk-aversions a^{-1} (I refer to its inverse a as the risk-tolerance). Agent types are indexed by $i \in [0, +\infty)$ and are ordered according to their absolute risk-aversion a_i^{-1}, from the less risk-averse (more risk-tolerant) to the more risk-averse (less risk-tolerant). The cumulative distribution of agent types can allow for mass points and it is denoted by $F(i)$. The total mass of agents (all potential participants) is $N = \int_0^{+\infty} dF(i) \in \mathcal{R}^+$. The average risk-tolerance $\overline{a}_{[0,+\infty)}$ of all agents, is defined as:

$$\overline{a}_{[0,+\infty)} = \frac{\int_0^{+\infty} (a_i)\, dF(i)}{N}$$

Call $\widetilde{F}(i)$is the distribution of the types that do participate in the market and $n = \int_0^{+\infty} d\widetilde{F}(i) \in [0, N]$ their mass, that is the size of the market. A fundamental variable for this analysis is $\overline{a}$ defined as the average risk-tolerance of the market participants only:

$$\overline{a} = \frac{\int_0^{+\infty} (a_i)\, d\widetilde{F}(i)}{n}$$

There are two assets: a risky asset with normally distributed return

$$x \sim \mathcal{N}\left(\mu_x, \sigma_x^2\right)$$

only traded in the market under consideration and a risk-free asset with return $R = 1$ normalized to one (i.e. zero interest rate without loss of generality).

There are three dates and two periods where the agents make decisions. In the first period all agents simultaneously decide whether to enter the market or not. Entry involves a cost $c > 0$ and the possibility of trading the risky asset which is precluded without entry. In the second period only the agents that entered the market decide how much of the risky asset to trade.

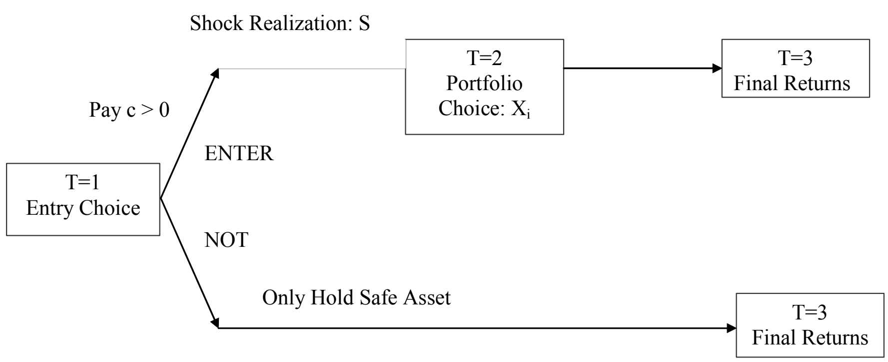
Agents have no demand shocks their only heterogeneity being their risk-aversion. To generate price variability I assume that the supply S of the risky asset is random. In particular I assume that the supply has zero mean, is normally distributed and is independent of other random variables:

$$S \sim \mathcal{N}\left(0, (n\sigma)^2\right) \tag{1}$$

To guarantee that this exogenous source of randomness does not fade as the market size increases, I assume that the noise in the market supply (its standard deviation) is proportional to the size of the market n. This assumption is there to guarantee scale invariance: with identical agents (all agents of one type only) the properties of the market (such as its volatility and the gain from entry) would be constant, that is independent of the mass of agents n that enters the market. It is important to have this invariance with homogeneous investors to capture the effect of heterogeneity of investors on market variables, which is the goal of this paper.

In the third date returns u and x are realized. In the second date, the shock S is realized. In the first date, that is, before the agents decide whether to enter or not, agents have an initial endowment of risk-free asset (cash) w. This endowment could in general be different for every agent. That would not change the results due to the absence of wealth effects in this CARA-Normal framework.

Timing of Decisions



As customary in the endogenous participation literature (see for instance, Pagano (1989) or Allen and Gale (1994)), the entry decision is separated and precedes the portfolio decision. This entry cost represents the cost of gathering information about how the stock market works in general (setup cost and information cost). Only after an investor has gathered the necessary information (entered the market), is he able to decide how much to invest in the asset. The resolution of this uncertainty can be represented by the realization of the random variable S.

I look for the equilibria of this two stage problem by solving by backward induction. Starting with the second stage decision, I find every investor's demand for the risky asset and aggregate these demands, taking as given the composition of the market participants $\widetilde{F}(i)$, which is derived later when I solve the first stage problem. Equating aggregate demand to the supply, I obtain the equilibrium price of the asset and the indirect utility from entry of every market participant as a function of the composition of the market. Comparing the expected utility from entry net of the entry cost to the expected utility of not entering, I obtain the entry condition which determines the first stage entry decision. Finally, I find the equilibria of the entry game and determine the patterns of participation. Different equilibrium levels of participation imply different volatilities of the asset price, which was previously derived in the second stage. This link between participation and volatility is the main objective.

2.1 Second Stage

Here I find the demand of market participants assuming a given pattern of participation $\widetilde{F}(i)$ and a given supply realization S.

If any investor i with risk aversion a_i^{-1} decides to purchase a quantity X_i of the risky asset at price p, his final wealth is:

$$W_i = (w - c - pX_i) + xX_i$$

To obtain the demand, I maximize the expected utility objective, which, given the CARA-Normal framework, becomes:

$$U_i \equiv \mathrm{E}_2\left(-\exp\left(-a_i^{-1}W_i\right)\right) = -\exp\left(-a_i^{-1}\left(\mathrm{E}_2(W_i) - \frac{a_i^{-1}}{2}\mathrm{Var}_2(W_i)\right)\right) \tag{2}$$

where the expectations are taken at date 2 and:

$$\begin{aligned} \mathrm{E}_2(W_i) &= (w-c) + (\mu_x - p)X_i \\ \mathrm{Var}_2(W_i) &= X_i^2\sigma_x^2 \end{aligned}$$

By differentiating the objective with respect to X_i and setting the result to zero, I obtain the demand:

$$X_i(p) = a_i\frac{\mu_x - p}{\sigma_x^2} \tag{3}$$

The market clearing condition is obtained by integrating the demand over the distribution of types that participate in the market and equating that to the random supply S:

$$\left(\int_0^{+\infty}(a_i)\,d\widetilde{F}(i)\right)\frac{\mu_x - p}{\sigma_x^2} = S$$

Defining X as:

$$X = \frac{S}{\int_0^{+\infty} d\widetilde{F}(i)} = \frac{S}{n}$$

we obtain:

$$\overline{a}\frac{\mu_x - p}{\sigma_x^2} = X$$

where, given the distribution (1) of S, X is distributed as:

$$X \sim \mathcal{N}\left(0, \sigma^2\right)$$

The equilibrium price becomes:

$$p = \mu_x - \frac{X}{\overline{a}}\sigma_x^2$$

A quantity useful for later is the quantity demanded in equilibrium by any type i that can be obtained by substituting the equilibrium price in the demand of that type:

$$X_i = \frac{a_i}{\overline{a}} X \tag{4}$$

Finally, the price volatility is the variability of the price as the supply varies given a certain market composition:

$$\mathrm{Var}_1(p) = \frac{1}{\overline{a}^2} \sigma_x^4 \sigma^2 \tag{5}$$

Since volatility depends negatively on average risk-tolerance, we have the following result.

Proposition 1 *A more risk-averse market has higher price volatility.*

The intuition for this result is that, agents with higher tolerance to risk tend to react more aggressively by buying or selling larger amounts of the asset (than agents with lower tolerance to risk) when its price is different from its expected return. This effect becomes clear by observing that the demand function (3) of each type is proportional to a_i. Hence, higher risk-tolerance results in an equilibrium price (which is a random variable) that tends to be closer to the expected return. Namely, high risk-tolerance reduces this size of price fluctuations. An insightful example is the extreme case of a risk-neutral market, where $a_i^{-1} = 0$ for all participants. Such a market would nail the price to the expected return in this model since $\overline{a} \to +\infty$, thereby eliminating all price variability. Conversely, a market which is on average more risk-averse would allow larger fluctuations of the price around the mean return. This is the meaning of the quote by Schleifer at the beginning of the paper.

2.2 First Stage

Having established the result that volatility increases with risk-aversion, what remains to be done is to explore the link between volatility and participation. This is obtained in the next two subsections by establishing the link between participation and risk-aversion.

In this subsection using the results of the first stage I obtain the net expected benefit from participation in the market. The final wealth for any investor type i that does not enter the market is:

$$W_i^0 = w$$

The utility from not-entering the market for any agent i is:

$$U_i^0 = -\exp\left(-a_i^{-1} w\right)$$

The utility from entering the market for any agent i is $\mathrm{E}_1(U_i)$, i.e. the expectation taken at date 1 of U_i defined in (2).

Proposition 2

$$U_i = -\exp\left(-a_i^{-1}(w-c) - \frac{1}{2}\frac{\sigma_x^2}{\overline{a}^2}X^2\right)$$

Proof. See Appendix. ■

The next step is to evaluate the entry condition for any type i, which is:

$$\mathrm{E}_1\left(U_i - U_i^0\right) > 0$$

Proposition 3 *The expected gain from entering the market is:*

$$E_1\left(U_i - U_i^0\right) = -\exp\left(-a_i^{-1}w\right)\left(\exp\left(a_i^{-1}c\right)\left(\frac{1}{\sqrt{1+\frac{\sigma_x^2}{\overline{a}^2}\sigma^2}}\right) - 1\right)$$

Proof. See Appendix. ■

The entry condition becomes:

$$\exp\left(a_i^{-1}c\right) < \sqrt{1+\frac{\sigma_x^2}{\overline{a}^2}\sigma^2}$$

After taking logs we can define the value function V_i that determines the entry condition:

$$\begin{aligned} V_i\left(a_i, \overline{a}\right) &= \frac{a_i}{2}\ln\left(1+\frac{\sigma_x^2}{\overline{a}^2}\sigma^2\right) > c \\ &= \frac{a_i}{2}\ln\left(1+\sigma_x^2 \mathrm{Var}\left(p\right)\right) > c \end{aligned}$$

In this market all types like volatility because it makes beneficial trading possible: nobody would participate in a market that has no price variations (that is no noise $\sigma^2 = 0$), because no type with positive risk-aversion would want to participate in a market where the only purpose is to trade a risky asset whose price always equals its expected return. In fact, with there would be no trading in such a market. Positive volatility translates into an equilibrium price generically different from the expected returns. This implies that there are opportunities for trading (buying and selling significant amounts if volatility is high), which is the only reason for entering the risky asset market in first place. The fact that higher price variability makes entry more beneficial for all types should not be that surprising. Indirect utility functions are under general conditions quasi-convex in prices, namely convex in this case where there is only one relative price. As a consequence, price variability is beneficial in this case.

As for the relative benefit of entry of different investor-types, note that the trading/arbitrage that occurs after entry is risky because the returns are random (I use risky arbitrage in the sense of Schleifer (2000)). Hence, the more risk-tolerant types benefit from trading any given amount of the risky asset more than the more risk-averse types do. The revealed preference of the more risk-tolerant types is indeed to trade more than the more risk-averse types. Hence, the more risk-tolerant types have a higher indirect utility from participating. The higher utility of the more risk-tolerant types is a result that holds in such a stark way because of the absence of hedging purposes. The presence of hedging motives would give additional incentives to trade and raise the gains from entry of the more risk-averse types relative to the more risk-tolerant types that have less need for trading for hedging reasons. With the addition of hedging motives it would not be clear what types have higher incentives to enter the market.[3]

Finally, there are externalities from entering this market. The more risk-averse the market is, the more price variability it allows and the more beneficial is the entry in this market, because of enhanced trading possibilities in expected terms for all types. If a mass of agents that are more risk-averse than the average are in the market, they trade less than the average and leave more scope for such trading for all other participants, increasing their gains from entry in that market.

3 Participation Equilibria and Volatility

I can now characterize the participation equilibria and derive the relationship between participation and volatility. Here is the following *separation property* of any equilibrium.

Proposition 4 *In any equilibrium, if a type i participates then all types $j < i$ participate.*

Proof. The value function $V_i(a_i, \overline{a})$ is increasing in the first argument. Hence, if an agent with a given risk-aversion finds it worthwhile to participate in the market then all the agents that are less risk-averse than him must find it even more worthwhile. ■

As a consequence of this separation property, any equilibrium can be described by identifying a marginal type, every type before which is a participant and no type after which participates. Moreover, the expected gain from entry of any agent i depends also on the composition of the market.

Proposition 5 *Ceteris paribus, a more-risk averse market gives higher expected benefit from participation for any type.*

Proof. The value function of all types decreases in the average risk-tolerance $\overline{a}$. Hence a market with a higher proportion of risk-averse agents gives higher expected gains from entry to all participants. ■

As a consequence, this entry game has the flavor of a coordination game with possible multiplicity of equilibria depending on the distribution of risk aversion across agents. Equilibria with different degrees of participation may arise depending on whether the more risk-averse agents coordinate to enter or not.

[3] An earlier version of this paper assumes a model with a positive non-random supply of the risky asset and idiosyncratic hedging motives. In that model the more risk-tolerant types have higher gains from entry provided that the supply is large.

Proposition 6 *For any given entry cost, multiple equilibria may exist. When they do, they are Pareto ranked: the ones with larger participation exhibit higher welfare and higher price volatility.*

Proof. Comparing any two equilibria, given proposition (4), the extra agents that participate in the market only in the equilibrium with larger participation are more risk-averse than the agents that participate in both equilibria. Hence, average risk-tolerance is smaller in the equilibria with larger participation. Recalling that volatility depends negatively on average risk-tolerance ($\mathrm{Var}(p) = \frac{1}{\overline{a}^2}\sigma_{xu}^2\sigma^2$), equilibria with more participation are welfare improving and at the same time exhibit higher price volatility. ■

3.1 Two-Type Example

The two-type case is a simple illustrative example of how multiple equilibria can arise. In this case I can characterize all the equilibria that arise for all values of the entry cost.

Suppose that the distribution of types has a two point support (two mass points), i.e. there are only two types of agents with risk-tolerances $a_1 > a_2$. The less risk averse types are a measure $N_1 > 0$ and the more risk-averse a measure $N_2 > 0$. The vector (n_1, n_2) with $n_1 \in [0, N_1]$ and $n_2 \in [0, N_2]$, denotes any possible composition of the market, that is, the measure of market participants of every type.

The value function of each type is:

$$\begin{aligned} V_i(n_1, n_2) &= \frac{a_i}{2}\ln\left(1 + \frac{\sigma_x^2}{\overline{a}^2}\sigma^2\right) \qquad i = 1, 2 \\ \text{with} &: \quad \overline{a} = \frac{a_1 n_1 + a_2 n_2}{n_1 + n_2} \in [a_2, a_1] \end{aligned}$$

We have the following equilibria.

Proposition 7 *There are multiple equilibria in which partial participation and full participation equilibria coexist in the range:*

$$V_2(N_1, 0) < c < V_2(N_1, N_2)$$

There is a unique equilibrium in the following cost ranges:

High Cost: $c > V_1(N_1, 0)$ *No participation.*

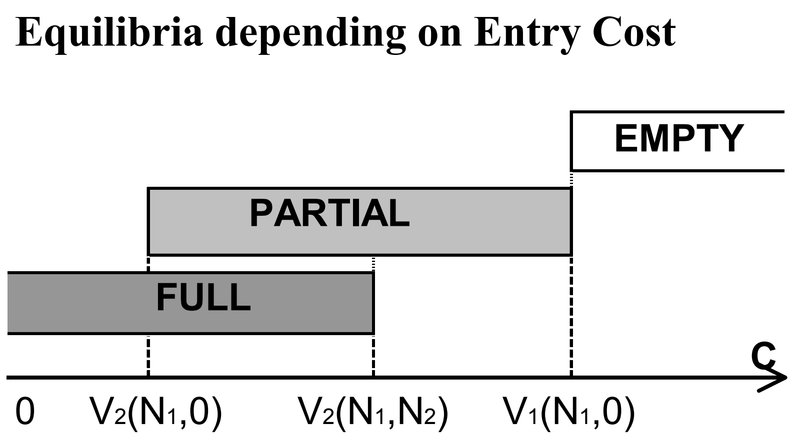
Intermediate Cost: $V_2(N_1, N_2) < c < V_1(N_1, 0)$ *Partial Participation.*

Low Cost: $c < V_2(N_1, 0)$ *Full participation.*

Proof. Note that $V_2(n_1, n_2)$ is increasing in n_2, implying that if any type 2 participates in the market then ally types 2 will, since: $V_2(n_1, N_2) > V_2(n_1, n_2) > c$. Hence, in any equilibrium we can only have $n_2^* \in \{0, N_2\}$.

If $n_2^* = N_2$ then by proposition (4) we have a full participation equilibrium $(n_1^* \in N_1, n_2^* = N_2)$. If $n_2^* = 0$ then the value function of the type 1 agents $V_1(n_1, 0)$ does not depend on n_1. Hence, depending on the entry cost, generically we have that in any equilibrium n_1 can take one of two possible values: $n_1^* \in \{0, N_1\}$, depending on whether $V_1(N_1, 0) = V_1(n_1, 0) \gtrless c$. ■

The following picture summarizes the above proposition:



In the multiple equilibria region the more risk averse types face a coordination problem: it is profitable for them to enter only if a critical mass of them does. The full participation equilibrium exhibits higher volatility and higher welfare.

3.2 Finite Types Case

This two-type case is more than just an illustrative example, similar patterns of equilibria (with possibly more than two equilibria coexisting for certain values of the entry cost) arise for any distribution of agents over a discrete set of types. Consider the case of m types with $a_1 > a_2 > ... > a_m$. The composition of the market participants market composition is described by the vector:

$$\begin{aligned} & (n_1, n_2, .., n_m) \\ \text{with} \quad : \quad & n_i \in N_i, \quad i = 1, 2, .., m \end{aligned}$$

where N_i is the measure of potential participants of type i.

Proposition 8 *In the m type case, all participation equilibrium outcomes are of the form:*

$$\begin{aligned} (n_1, n_2, .., n_m) &= (N_1, N_2, .., N_j, 0, .., 0) \\ \textit{with} \quad &: \quad j = 0, 1, 2, .., m \end{aligned}$$

Proof. The proof depends on two inequalities. First:

$$V_j(N_1,..,N_{j-1},N_j,0,..,0) > V_j(N_1,..,N_{j-1},n_j,0,..,0) > c$$

which tells you that agents of a given type have strategic complementarities so they enter in blocks (either all or nobody). Second:

$$\begin{aligned} V_{j-k}(N_1,..,N_{j-1},N_j,0,..,0) &> V_j(N_1,..,N_{j-1},N_j,0,..,0) > c \\ \text{with} &: k = 1,2,..,j-1 \end{aligned}$$

which tells you that if agents of a given risk-aversion type participate then less risk-averse types will too. ■

For a given entry cost, two or more equilibria may arise depending on how many types coordinate to enter: the coordination of agents of a given type may create gains from coordination for the next higher types, and so forth; leading in this case to three or more equilibria. All equilibria with larger participation have higher volatility and welfare.

4 Summary

There are many factors that affect volatility in one direction or the other. This paper identifies one of them. Using an endogenous market participation model I showed that one source of volatility emerges from a change in the overall composition of the attitudes toward risk of investors. If the potential market participants differ in their attitudes toward risk, then more participation can increase volatility. When looking for the possible sources of volatility, we cannot disregard the diversity of investor characteristics that arises from self-selection of potential investor-participants.

While I have explored the effect of heterogeneity in risk-aversion, it would be interesting to explore the effect of other kinds of heterogeneity, the most important of these being, perhaps, information and wealth. This task is left to further research.[4]

5 Appendix

Proof. 2

$$U_i = -\exp\left(-a_i^{-1}\left(\mathrm{E}(W_i) - \frac{a_i^{-1}}{2}\mathrm{Var}(W_i)\right)\right)$$

[4] For the effects of heterogeneity in information on volatility see Dubra and Herrera (2002).

Note that the certainty equivalent term $\left(\mathrm{E}(W_i) - \frac{a_i^{-1}}{2}\mathrm{Var}(W_i)\right)$ is:

$$\begin{aligned}
\left(\mathrm{E}(W_i) - \frac{a_i^{-1}}{2}\mathrm{Var}(W_i)\right) &= w - c + (\mu_x - p) X_i - \frac{a_i^{-1}}{2} X_i^2 \sigma_x^2 \\
&= w - c + \left(a_i^{-1} X_i \sigma_x^2\right) X_i - \frac{a_i^{-1}}{2} X_i^2 \sigma_x^2 \\
&= w - c + \frac{a_i}{2} \frac{\sigma_x^2}{\overline{a}^2} X^2
\end{aligned}$$

where in the second step we substituted the demand function of agent i (3) and in the third step the quantity demanded by type i in equilibrium (4). Hence:

$$U_i = -\exp\left(-a_i^{-1}(w-c) - \frac{1}{2}\frac{\sigma_x^2}{\overline{a}^2} X^2\right)$$

■

Proof. 3 Given that:

$$\left(U_i - U_i^0\right) = -\exp\left(-a_i^{-1} w\right)\left(\exp\left(a_i^{-1} c\right)\exp\left(-\frac{1}{2}\frac{\sigma_x^2}{\overline{a}^2} X^2\right) - 1\right)$$

taking expectations we have:

$$\begin{aligned}
\mathrm{E}\left(U_i - U_i^0\right) &= -\exp\left(-a_i^{-1} w\right)\left(\exp\left(a_i^{-1} c\right)\mathrm{E}\left[\exp\left(-\alpha X^2\right)\right] - 1\right) \\
\text{with} &: \alpha = \frac{1}{2}\frac{\sigma_x^2}{\overline{a}^2}
\end{aligned}$$

The expectation in the square brackets is over the normal distribution of $X \sim \mathcal{N}\left(0, \sigma^2\right)$, namely:

$$\begin{aligned}
\mathrm{E}\left[\exp\left(-\alpha X^2\right)\right] &= \int\limits_{-\infty}^{+\infty} \exp\left(-\alpha X^2\right)\left(\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{X^2}{2\sigma^2}\right)\right) dX \\
&= \frac{1}{\sigma\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} \exp\left(-\left(\alpha + \frac{1}{2\sigma^2}\right) X^2\right) dX
\end{aligned}$$

Knowing the result from the following standard integral:

$$\int_{-\infty}^{\infty} \exp\left(-X^2\right) dX = \sqrt{\pi}$$

we have:

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(\left(\alpha + \frac{1}{2\sigma^2}\right) X^2\right) dX = \frac{1}{\sigma\sqrt{2\pi}} \frac{\sqrt{\pi}}{\sqrt{\alpha + \frac{1}{2\sigma^2}}} = \frac{1}{\sqrt{1 + \frac{\sigma_x^2}{\overline{a}^2}\sigma^2}}$$

Hence:

$$\mathrm{E}\left(U_i - U_i^0\right) = -\exp\left(-a_i^{-1}w\right)\left(\exp\left(a_i^{-1}c\right)\left(\frac{1}{\sqrt{1+\frac{\sigma_x^2}{\overline{a}^2}\sigma^2}}\right) - 1\right)$$

■

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