QUORUM AND TURNOUT IN REFERENDA

Helios Herrera
SIPA, Columbia University
CIE-ITAM

Andrea Mattozzi
California Institute of Technology

Abstract
We analyze the effect of turnout requirements in referenda in the context of a group turnout model. We show that a participation quorum requirement may reduce the turnout so severely that it generates a "quorum paradox": In equilibrium, the expected turnout exceeds the participation quorum only if this requirement is not imposed. Furthermore, a participation quorum does not necessarily imply a bias for the status quo. We also show that in order to induce a given expected turnout and avoid the quorum paradox, the quorum should be set at a level that is lower than half the target. Finally, we argue that a super majority requirement to overturn the status quo is never equivalent to a participation quorum. (JEL: D72)

1. Introduction

Direct democracy is firmly established in many democratic countries, and the use and scope of direct democracy institutions are increasing all around the world. In Europe, for instance, the average number of referenda held every year was 0.18 in the 1980s, 0.39 in the 1990s, and is around 0.27 in the current decade.[1]

In many countries and in some U.S. states, referenda have to meet certain turnout requirements in order to be valid. Typically, the status quo can be overturned only if a majority of voters is in favor of it and if the turnout reaches a certain level (i.e., a participation quorum is met). In some cases an approval

The editor in charge of this paper was Patrick Bolton.

Acknowledgments: We benefitted from discussions with Alessandra Casella, Federico Echenique, Jacob Goeree, Daniela Iorio, Alessandro Lizzeri, Massimo Morelli, Nicola Persico, Leeat Yariv, and seminar participants at several universities. We thank the managing editor Patrick Bolton and two anonymous referees for their suggestions. Mattozzi gratefully acknowledges financial support from National Science Foundation grant SES-0617901. All usual disclaimers apply.

E-mail addresses: Herrera: hhh2108@columbia.edu; Mattozzi: andrea@hss.caltech.edu

1. For evidence on the increasing use of referenda as tools for policymaking see, for example, Casella and Gelman (forthcoming) and references therein, Qvortrup (2002), and Matsusaka (2005a, 2005b).

Journal of the European Economic Association June 2010 8(4):838–871

quorum is required: The turnout of the majority voting against the status quo has to reach a certain level.[2]

The common rationale for a turnout requirement is that "a low turnout in referendums is seen as a threat to their legitimacy" (Qvortrup 2002, p. 164). In other words, to change the status quo policy a large proportion of citizens should take part in this decision and a high turnout reflects the fact that enough citizens care about the issue at stake. However, the extent to which citizens care about an issue depends on the mobilization effort of political parties. In fact, political parties and allied interest groups spend a great deal of effort and huge amounts of campaign money in order to encourage citizens to vote for one of the alternatives.[3]

The existence of a turnout requirement introduces a crucial asymmetry in the campaign strategy of organized groups, by allowing those in favor of the status quo policy to use a "quorum-busting" strategy. Instead of devoting resources to increase the turnout of voters opposing the reform, the status quo party can exploit the group of apathetic citizens. In fact, if a significant fraction of voters abstains, the referendum will fail due to lack of quorum, and the status quo will remain in place. Examples of referenda on salient policy issues that failed for lack of quorum abound. For instance, a recent controversial case was the 1998 referendum on abortion legalization in Portugal.[4] Furthermore, with regards to U.S. states, an exemplary case is the one of Oregon. Indeed, since 1997, the state of Oregon has a participation quorum requirement of 50% (which is referred to as double majority) for all property tax ballot measures at the local level. Foley (2006) documents that, out of 1,117 measures, 141 did not pass because of the 50% quorum requirement: 80% of the failed measures had a turnout above 40%. A striking 50% of the failed measures would have passed if only the needed votes to reach the quorum had been cast as no votes.[5]

In this paper we analyze the effect of turnout requirements in referenda in the context of a simple game theoretic model of group turnout. The main results of the paper are the following: First, we show that the introduction of a participation quorum requirement, which validates the referendum only if participation is high

2. A list of countries that have participation or approval quorum requirements is provided in Table A.1 in Appendix A.

3. The amount of resources political parties spend in order to mobilize voters has increased significantly in the last 20 years. For example, the American National Election Studies provide evidence of a sharp increase in the percentage of respondents contacted by political parties since 1990.

4. The same referendum was repeated in February 2007. After the referendum failed to pass due to lack of quorum for the second time, in July 2007 the Portuguese parliament approved a law to legalize abortion.

5. The historical evidence goes back to the Weimar Republic. Two referenda on the confiscation of royal property (1926) and on the repudiation of the war guild (1929) obtained a yes vote of 93.3% and 94.5%, respectively. Both referenda were declared void because the Weimar constitution required a majority not only of the votes but also of the eligible voters (see Qvortrup 2002).

enough, may generate in equilibrium a quorum paradox, namely, the equilibrium expected turnout may be smaller than the quorum itself. Interestingly enough, this could occur even if the expected turnout that would result in equilibrium in the absence of a participation quorum was greater than the required quorum. In other words, we show that there are levels of the quorum requirement such that in equilibrium the expected turnout exceeds the participation quorum only if the requirement is not imposed.

Second, a participation quorum requirement does not necessarily imply a bias for the status quo policy. In fact, the expected probability that the status quo is overturned may be higher in the presence of a participation quorum requirement than in its absence. Indeed, there are levels of the quorum requirement such that in equilibrium, either the equilibrium expected turnout is smaller than the quorum, or the probability that the status quo is overturned is strictly higher than when the quorum requirement is absent.

Third, we show that in order to induce in equilibrium a given expected turnout and avoid the quorum paradox, the participation requirement should be set at a level that is lower than half the target.

Because our goal is to analyze how a participation requirement affects the distribution of voting outcomes in large elections, our approach is to consider a framework common in the literature on large elections, and extend it to the case where a turnout requirement is introduced. In particular, our model is based on the group-based model of turnout first developed by Snyder (1989) and Shachar and Nalebuff (1999).[6] In these models two opposed parties spend effort to mobilize their supporters to the polls, while facing aggregate uncertainty on the voters' preferences.

To the best of our knowledge, the only paper that analyzes turnout requirements in referenda is Corte-Real and Pereira (2004). In that paper they study the effects of a participation quorum using a decision-theoretic axiomatic approach. Contrary to our paper, they do not explore how the incentives of parties and interest groups to mobilize voters depend on the turnout requirements.[7]

The remainder of the paper is organized as follows. Section 2 presents the basic model; Section 3 introduces the main results through a simple example. Section 4 contains the full equilibrium characterization. In Section 5, we present the comparative statics of the model, and in Section 6 we show how to induce a given expected turnout while avoiding the quorum paradox. In Section 7 we discuss some generalizations and extensions of the basic model. Section 8 concludes. All proofs are in the appendices.

6. See also Morton (1987, 1991) for other group-based models.

7. For a review of the advantages and limits of decision-theoretic models of turnout as compared to mobilization models of turnout, see Feddersen (2004).

2. The Model

Consider a simple model of direct democracy where individuals have to choose between two alternatives: r (reform) and s (status quo). The voting rule is simple majority and ties are broken randomly. Let $q \in [0, 1]$ denote a participation quorum requirement, that is, the status quo can be replaced if and only if: (i) at least a fraction q of the population shows up at the polls and (ii) a majority of voters vote in favor of r.

There are two exogenously given parties supporting policies r and s, and a continuum of voters of measure 1, of which a proportion $\tilde{r} \in [0, 1]$ supports policy r, and the remaining support policy s . Slightly abusing notation, we will use the same symbol (e.g., s) to denote a policy and the party supporting that policy. We assume that, from the parties' point of view, $\tilde{r}$ is a random variable with uniform distribution. Each voter has a personal cost of voting $c \in [0, 1]$ that is also drawn from a uniform distribution.[8]

Parties decide simultaneously the amount of campaign funds to spend (equivalently, the amount of effort to exert) to mobilize voters in order to win the referendum. The parties' objective functions are

$$\pi_r(S, R) = BP - R,$$
$$\pi_s(S, R) = B(1 - P) - S,$$

where P is the (endogenous) probability that alternative r is selected, R and S are the spending of the parties r and s, respectively, and $B > 0$ is the payoff to parties if their preferred alternative is chosen.[9]

Because our focus is on the strategic interactions between parties, we depart from the pivotal voter approach in modeling voters' behavior.[10] We assume that voters receive a benefit from voting their preferred policy that is strictly concave in parties' mobilization efforts. In particular, if a party spends x, the benefit to a voter who supports that party's policy is $\rho(x)$, where the function $\rho: \mathbb{R}_+ \to [0, 1]$ is continuous, twice differentiable for $x > 0$, strictly increasing, and strictly concave, and satisfies the properties

$$\lim_{x \to 0} x\rho'(x) = 0, \quad \lim_{x \to 0} x\rho''(x) = 0, \quad \lim_{x \to \infty} \rho'(x) = 0.$$

8. See footnote 14 and Section 7 for a discussion of the robustness of our results to alternative distributional assumptions.

9. See Herrera and Mattozzi (2006) for the case in which parties have different payoffs.

10. See, for example, Börgers (2004) for a pivotal voter model of costly voting.

This specification is equivalent to having parties' expenditures affect individual cost of voting.[11] Finally, for the sake of simplicity, for most of the paper we will assume that $\rho(0) = 0$.[12]

For a given level of spending R, a voter who supports policy r and has a voting cost equal to c votes for alternative r if and only if $\rho(R) \geq c$. Because this holds for fraction $\rho(R)$ of the voters supporting policy r, the vote share for that policy (as a fraction of the total population) is $v_R = \tilde{r}\rho(R)$. Likewise, the vote share for policy s is $v_S = (1 - \tilde{r})\rho(S)$.[13]

Alternative r is selected if and only if it has a greater vote share than policy s and the quorum q is achieved. This occurs with probability

$$\begin{aligned} P &= \Pr(v_R \geq v_S \text{ and } v_R + v_S \geq q) \\ &= \Pr\left(\tilde{r} \geq \frac{\rho(S)}{\rho(R) + \rho(S)} \text{ and } (\rho(R) - \rho(S))\tilde{r} \geq q - \rho(S)\right). \end{aligned}$$

By defining

$$Q = \frac{q - \rho(S)}{\rho(R) - \rho(S)}, \quad K = \frac{\rho(S)}{\rho(R) + \rho(S)},$$

we can represent P as a function of $\rho(R)$ and $\rho(S)$ for any given q. In particular, P takes the values shown in Figure 1 (see Appendix B for the construction of the figure).

Note that P is continuous in its arguments on the whole space $(\rho(S), \rho(R)) \in [0, 1]^2$. If $q = 0$, that is, there is no participation quorum requirement, the curved line collapses on the axes, and $P = 1 - K$ on the whole space. In this region the probability that the reform policy is selected is only a function of parties' mobilization efforts. However, as q increases, the curved line moves northeast continuously, and below the curved line the probability that the reform policy is selected also depends on the quorum requirement. Clearly, whenever $\rho(R) < q$ and $\rho(S)$ is sufficiently small, the reform policy cannot prevail in the referendum. When $q = 1$ the curved line collapses to the point $(1, 1)$ and P converges to zero.

Before characterizing the equilibria of this game, it might be useful to consider a simple numerical example that illustrates our results.

11. As Shachar and Nalebuff (1999) among others point out, party spending is effective in driving voters to the polls in several ways: Campaign spending decreases the voters' cost of acquiring information, it decreases the direct cost of voting, it increases the cost of abstaining, and it signals the closeness and importance of the alternatives at stake.

12. Our analysis is qualitatively unchanged if we relax the assumption of $\rho(0) = 0$, as long as $\rho(0)$ is small.

13. The assumption that the personal cost of voting $c \in [0, 1]$ is drawn from a uniform distribution is not crucial for our purposes. If we consider the more general case in which c is drawn from a continuous distribution G with density g, and we assume that g has bounded derivative and $\rho(\cdot)$ is concave enough, then the analysis would be qualitatively unaffected by defining $\tilde{\rho}(\cdot) \equiv G(\rho(\cdot))$.

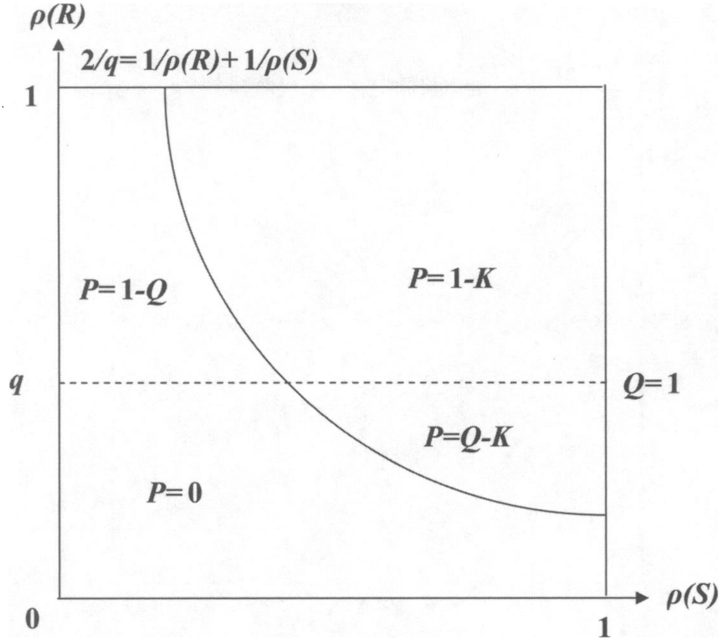

FIGURE 1. Probability of approval.

3. An Example

Consider the case in which $B = 4$, and $\rho(x) = 1 - e^{-x}$. As we will prove in the next section, for each set of parameter values the pure-strategies Nash equilibrium of this game, if it exists, is unique. Furthermore, depending on the level of q, there are only three possible candidates for a Nash equilibrium in pure-strategies, which are represented in Figure 2: Two symmetric profiles denoted by O and C, and one asymmetric profile denoted by A.

Table 1 summarizes the equilibrium level of parties' spending, the expected turnout $E(T)$, and the expected probability $P(q)$ that the reform policy wins a majority of votes for different levels of participation quorum q.

If the quorum requirement is sufficiently low, the probability of winning will not depend on q. Given that there are no asymmetries, it is not surprising that the equilibrium will be symmetric and, in the unique Nash equilibrium outcome, parties spend $R = S = \ln 2 \simeq 0.69$. Moreover, the expected turnout equals 0.5, the expected probability of policy r being selected equals 0.5, and the expected profit of each party equals $2 - \ln 2 \simeq 1.3$. This equilibrium exists if and only if $q < 0.1634$, and is represented by point C in Figure 2.

Suppose now that $q = 0.25$. In this case the status quo party will exploit the asymmetry introduced by the participation quorum requirement. In fact, by

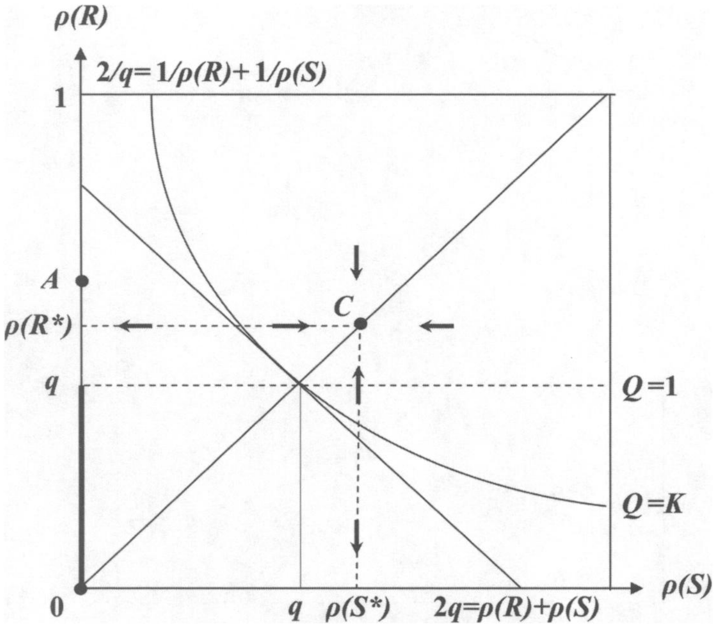

FIGURE 2. Pure-strategy Nash equilibrium candidates.

choosing not to mobilize its supporters, that is, by choosing $S = 0$, party s might be successful in "busting" the quorum at zero cost. In this case, party r's probability of winning is either $1 - (Q)$ or 0, depending on $1 - e^{-R}$ being greater than or smaller than q (see Figure 1). In the unique pure-strategy Nash equilibrium outcome party r spends $R \simeq 0.96$, and party s spends $S = 0$. In this quorum-busting equilibrium the expected turnout drops to 0.31, the expected probability of policy r being selected is strictly bigger than 0.5, and expected profits are such that $E(\pi_r) > 1.3 > E(\pi_s)$.

If the participation quorum requirement is higher, say $q = 0.4$, in equilibrium party r increases its spending to $R \simeq 1.19$, while party s, a fortiori, will spend $S = 0$. The resulting expected turnout is 0.35, the expected probability of policy r being selected is now strictly smaller than 0.5, and expected profits are such that $E(\pi_s) > 1.3 > E(\pi_r)$. The asymmetric pure-strategy Nash equilibrium exists if and only if $q \in (0.1655, 0.491)$, and it is represented by point A in Figure 2.

TABLE 1. Equilibrium in the example.

q	Equilibrium	S	R	$E(T)$	$P(q)$
< 0.1634	C	0.69	0.69	0.50	0.50
$= 0.25$	A	0.00	0.96	0.31	0.59
$= 0.40$	A	0.00	1.19	0.35	0.43
> 0.4910	O	0.00	0.00	0.00	0.00

Finally when q is high enough, because party r expected profits in the asymmetric equilibrium are clearly decreasing in q, and they become eventually negative, the unique equilibrium will be trivially $R = S = 0$. This equilibrium is represented by point O in Figure 2, and it is the unique equilibrium when $q > 0.491$.

Several interesting observations can be derived from this numerical example; we list them in order of importance.

The introduction of a participation quorum requirement is usually motivated by the idea of validating the referendum results only if participation is high enough, that is, if voters care enough about the issue at stake in the referendum. However, a participation quorum limit may generate less participation if voters that turn out to the polls respond to parties' mobilization efforts. The presence of the quorum may reduce participation so much that expected turnout may well be below the quorum itself. In particular, there are values of B such that the symmetric spending profile cannot be supported in equilibrium even if the expected turnout that generates it is greater than q.

This quorum paradox is not a trivial consequence of the existence of a no-spending equilibrium (point O) for high values of q. In fact, it may also occur for parameter values such that the unique equilibrium is the asymmetric one (point A in Figure 2, and $q \in (0.25, 0.40)$ in the numerical example). In this case, while the status quo party is not mobilizing voters as its goal is to win by a lack of participation, the reform party is mobilizing voters to push turnout above the quorum threshold. The tension between parties in this equilibrium is no longer about obtaining the majority of votes as in the symmetric equilibrium, but about having turnout reaching the quorum or not.[14]

The quorum requirement does not necessarily imply a bias for the status quo policy. In fact, the expected probability that the status quo is replaced may be higher in the presence of a participation quorum requirement than in the case of its absence. Moreover, the increase in the expected probability that the status quo is overturned may be associated with a smaller amount of total spending (e.g., see the case of $q = 0.25$ in the numerical example).

For any level of the payoff B there always exists a range of q where there is no equilibrium in pure-strategy. The intuition is simple. For given B, there is a level of q such that party s is indifferent between playing $S = R$ when r is playing R (i.e., the symmetric equilibrium strategy profile), and trying to bust

14. In Table A.2 in Appendix A we list several examples of recent referendum outcomes, which are suggestive of our asymmetric equilibrium-strategies profile. What the different cases have in common is the very low turnout for the status quo party (the NO column in Table A.2 averaging well below 10%) and a much higher turnout for the reform party (the YES column) who wins the popular vote by an extremely large margin. This suggests that the tension between reaching the quorum or not seems to dominate the tension between obtaining the majority of votes or not. The resolution of this tension is uncertain and can go either way (as the last TURNOUT column shows): In some cases the quorum is reached, in others it is not, and sometimes by very small margins.

the quorum by playing $S = 0$. Clearly, at exactly that level of q, the symmetric strategy profile is still the unique equilibrium. However, at a slightly higher level of q, whereas $S = 0$ is now a best response to r playing R, the converse is not true; that is, R cannot be a best response to $S = 0$. In fact, when q is positive and $S = 0$, party r has a higher marginal return from spending. A (discrete) increase in party r's spending above R leads party s to switch back to a strictly positive spending. Therefore, for a subset of (q, B), the game parties are playing can be seen as a "matching pennies" game. We will show that there is at least one natural mixed strategy equilibrium in that region that smooths the transition from the equilibrium in C in Figure 2 to the equilibrium in A.

In the next section we characterize the Nash equilibria of this game for all values of the exogenous parameters (q, B).

4. Equilibrium Characterization

We start by focusing on pure-strategy Nash equilibria. There are only three possible candidates for a Nash equilibrium in pure-strategies: two symmetric profiles, and an asymmetric one. For each set of parameter values the pure-strategies Nash equilibrium of this game, if it exists, is unique. However, a pure-strategy equilibrium may not exist. The next proposition summarizes these results.

PROPOSITION 1. *A pure-strategies Nash equilibrium, if it exists, is unique. Moreover, for all B, there exist unique thresholds $\underline{q}(B)$, $\hat{q}(B)$, $\bar{q}(B)$, with $\underline{q} < \min\{\hat{q}, \bar{q}\}$, such that:*

(i) *the symmetric positive spending profile $C \equiv (S^*, R^*)$, where $R^* = S^* > 0$, is an equilibrium if and only if $q \leq \underline{q}$;*
(ii) *the quorum-busting profile $A \equiv (0, \hat{R})$, where $\hat{R} > 0$, is an equilibrium if and only if $q \in [\hat{q}, \bar{q}]$;*
(iii) *the zero spending profile $O \equiv (0, 0)$ is an equilibrium if and only if $q \in [\bar{q}, 1]$;*
(iv) *if $q \in (\underline{q}, \hat{q})$, there exists a mixed strategy equilibrium where r plays the pure-strategy $\tilde{R}$ while s plays the mixed strategy*

$$S = \begin{cases} 0 & \text{with probability } \alpha, \\ \tilde{S} & \text{with probability } 1 - \alpha, \end{cases}$$

where $\alpha(\underline{q}) = 0$ and $\alpha(\hat{q}) = 1$.

The proof is in Appendix C. Clearly, the spending profile (S^*, R^*) is an equilibrium as long as q is such that party s does not want to deviate to $S = 0$.

In the appendix we show that there is a unique level of q such that this is true, which equals[15]

$$\underline{q} = \left(\frac{1}{2} - \frac{R^*}{B}\right)\rho(R^*). \tag{1}$$

If instead $q > \underline{q}$, then the only pure-strategy equilibrium candidate has party s spending 0. In particular, in the quorum-busting asymmetric spending profile $(0, \hat{R})$, party s spends zero because its optimal strategy is trying to keep the total turnout below quorum, whereas party r spends a positive amount $\hat{R}$ in an effort to mobilize enough supporters to push the turnout above quorum with some probability. A quorum-busting spending profile $(0, \hat{R})$ is an equilibrium if and only if the following two conditions hold:

$$\pi_r(0, \hat{R}) \geq \pi_r(0, 0) = 0,$$

$$\pi_s(0, \hat{R}) \geq \pi_s(\hat{S}, \hat{R}),$$

where $\hat{R}$ and $\hat{S}$ are functions of (q, B) implicitly defined by

$$\hat{R} = \arg\max\left(B\left(1 - \frac{q}{\rho(\hat{R})}\right) - \hat{R}\right), \tag{2}$$

$$\hat{S} = \arg\max\left(B\frac{\rho(\hat{S})}{\rho(\hat{R}) + \rho(\hat{S})} - \hat{S}\right). \tag{3}$$

Note that $\hat{S}$ is the best response of party s to party r spending $\hat{R}$ inside the $P = 1 - K$ region.[16] Intuitively, a quorum-busting equilibrium can exist if and only if q is not so high to make party r's profits negative, and q is not too small to make party s worse off by spending zero than spending $\hat{S}$. Note that $\hat{R}$ and $\hat{S}$ both depend on B and q. By defining the two thresholds on q as

$$\bar{q}(B) : \pi_r(0, \hat{R}) = \pi_r(0, 0), \tag{4}$$

$$\hat{q}(B) : \pi_s(0, \hat{R}) = \pi_s(\hat{S}, \hat{R}), \tag{5}$$

15. Note that, because $\underline{q}$ is always smaller than $1/2$, Proposition 1 implies that if the voting quorum is set at $q = 1/2$, the symmetric spending profile cannot be an equilibrium. This is due to the simplifying assumption that there are no strong partisan voters, namely, $\rho(0) = 0$. If instead $\rho(0) > 0$, and some voters vote even if parties are not mobilizing, it is straightforward to show that

$$\underline{q} = \left(\frac{1}{2} - \frac{R^*}{B}\right)\rho(R^*) + \rho(0)\left(\frac{1}{2} + \frac{R^*}{B}\right),$$

and the symmetric equilibrium can survive even if $q > 1/2$.

16. Note that the assumptions on $\rho(\cdot)$ guarantee that $\hat{R}$ and $\hat{S}$ are well defined.

proving that part (ii) of Proposition 1 shows that for any B the thresholds $\bar{q}$ and $\hat{q}$ are uniquely defined. The quorum-busting equilibrium may not always exist, since for low values of B we may have that $\hat{q} > \bar{q}$ and the interval $[\hat{q}, \bar{q}]$ disappears. However, this never occurs if B is large enough.

Finally, the zero spending profile $O \equiv (0, 0)$ is an equilibrium if and only it is optimal for r to spend zero when s spends zero, that is $0 = \pi_r(0, 0) \geq \pi_r(0, \hat{R})$. For this to be true q has to be high enough. In fact, it is immediately clear that for $q = 0$ the zero spending profile cannot be an equilibrium because, for any B, party r can spend an arbitrarily small amount and increase its probability of winning discretely from 1/2 to 1. Moreover, for all $\rho(R) < q$, $S = 0$ is a dominant strategy for s (the boldfaced line in Figure 2) as s can guarantee itself that $P = 0$. In other words, s can win the referendum with probability one at no cost attaining the maximum possible payoff $\pi_s = B$. Hence, no strictly positive equilibrium spending profile can be in the interior of the $\rho(R) < q$ region. If $\bar{q}$ is uniquely defined, part (iii) of Proposition 1 follows immediately.

The fact that $\underline{q} < \hat{q}$, and $\underline{q} < \bar{q}$, implies that pure-strategy equilibria never coexist, and that they may fail to exist. In fact, because $\underline{q} < \hat{q}$, there is always a region of non-existence in pure-strategies. However, there is a natural mixed strategy equilibrium in that region that smooths the transition from the pure-strategy symmetric equilibrium to the non-zero pure-strategy asymmetric equilibrium. Let $(\tilde{S}, \tilde{R})$ be defined as

$$\tilde{S} = \arg\max \left(B \frac{\rho(S)}{\rho(S) + \rho(\tilde{R})} - S \right),$$

and $\pi_s(0, \tilde{R}) = \pi_s(\tilde{S}, \tilde{R})$. In words, $\tilde{S}$ is the best response to $\tilde{R}$ in the $P = 1 - K$ region, and $\tilde{R}$ is the level of spending by party r such that party s is indifferent between spending $\tilde{S}$ and 0. The mixing probability α is chosen so that $\tilde{R}$ is indeed a best response for party r.

There might be other mixed strategy equilibria. However, an appealing feature of the equilibrium described in Proposition 1 is that as q increases from $\underline{q}$ to $\hat{q}$ we move gradually and continuously from the pure-strategy symmetric equilibrium (S^*, R^*) to the pure-strategy asymmetric equilibrium $(0, \hat{R})$. We summarize the set of pure-strategy Nash equilibria and the mixed strategy equilibrium described above as a function of q and B in Figure 3 (see Appendix D for the construction of the figure).

In the next section we study how the expected turnout, the expected probability of winning, and the expected profits change in the different equilibria as a function of q and B.

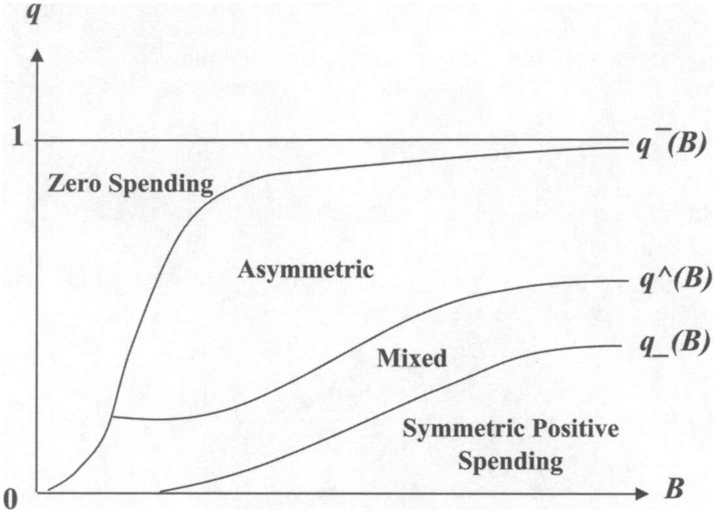

FIGURE 3. Equilibria as a function of q and B.

5. Expected Turnout and Probability of Referendum Approval

In this section we show that the conclusions drawn from the example in Section 2 are general. The introduction of a quorum requirement (motivated by the idea of validating the result of the referendum only if participation is high enough) may generate in equilibrium less participation. Furthermore, a quorum requirement does not necessarily imply a bias for the status quo policy. Indeed, when $q \in (\hat{q}, \bar{q})$, either the equilibrium expected turnout is smaller than the quorum, or the equilibrium probability that the reform policy is adopted is strictly bigger than the case where the quorum requirement is absent. We start analyzing how the expected turnout $E(T)$ varies depending on which region of the parameter space we are in.[17]

In the positive spending symmetric equilibrium region, expected turnout is constant in q, increasing in B, and always above q. Namely, when $q \in (0, \underline{q})$, we have that $E(T) = \rho(R^*) > q$. Clearly, the symmetric spending profile cannot be supported in equilibrium if the expected turnout that generates it is not high enough to meet the quorum, namely, when $q > \rho(R^*)$. However, if the quorum requirement is in the interval $q \in (\underline{q}, \rho(R^*))$, the symmetric spending profile cannot be supported in equilibrium even if the expected turnout that generates is bigger than q.

17. In this section we will assume that B is such that $\hat{q} < \bar{q}$. This is always true when B is large enough, we show in Appendix D.

If $q \in (\underline{q}, \hat{q})$, and parties are playing the mixed strategy profile described in Proposition 1, we have that the expected turnout is equal to

$$E(T) = \frac{\rho(\tilde{R})}{2} + (1 - \alpha(q))\frac{\rho(\tilde{S})}{2},$$

and satisfies the properties that are summarized in the next claim.

Claim 1. *If $q \in (\underline{q}, \hat{q})$ and parties are playing the mixed strategy equilibrium of Proposition 1, then $E(T) > q$,*

$$\lim_{q \to \underline{q}} E(T) = \rho(R^*) > \frac{\rho(\hat{R}(\hat{q}))}{2} = \lim_{q \to \hat{q}} E(T),$$

and $\lim_{q \to \underline{q}^+} \partial E(T)/\partial q < 0$.

The proof is in Appendix E. Claim 1 shows that the expected turnout in the mixed equilibrium is smaller than the expected turnout in the symmetric positive spending equilibrium and it is decreasing in q, for some $q \in (\underline{q}, \hat{q})$.[18]

If $q \in (\hat{q}, \bar{q})$, that is, in the region where parties are playing the asymmetric profile, we have that $E(T) = \rho(\hat{R})/2$, the expected turnout is increasing in q and B, and satisfies the properties that are summarized in the next claim.

Claim 2. *If $q \in (\hat{q}, \bar{q})$, then $E(T)|_{q=\hat{q}} > \hat{q}$. Furthermore, there exist $\bar{B}$ and q' such that for $B > \bar{B}$, $E(T) > q$ if and only if $q < q'$.*

In other words, when the benefit is high enough, there always exists an interval to which q belongs such that the equilibrium expected turnout is strictly smaller than the quorum itself.

Finally for $q \in (\bar{q}, 1)$, we have that $E(T) = 0$. Figure 4 summarizes the results.

As Figure 4 shows, when $q > \underline{q}$ (i.e., outside of the symmetric positive spending region), the introduction of a quorum requirement decreases the expected turnout in equilibrium. More importantly, in the region represented by the bold-faced dotted segment, the equilibrium expected turnout is smaller than the quorum itself (even if the expected turnout that results in equilibrium holding B constant and removing the quorum requirement is bigger than q). Claim 2 guarantees that for B high enough such a region always exists. This is precisely what we call the quorum paradox: In equilibrium the expected turnout exceeds the participation quorum only if the requirement is not imposed. In the next proposition we

18. In the case of $\rho(x) = 1 - e^{-x}$ the expected turnout is decreasing in q for any $q \in (\underline{q}, \hat{q})$.

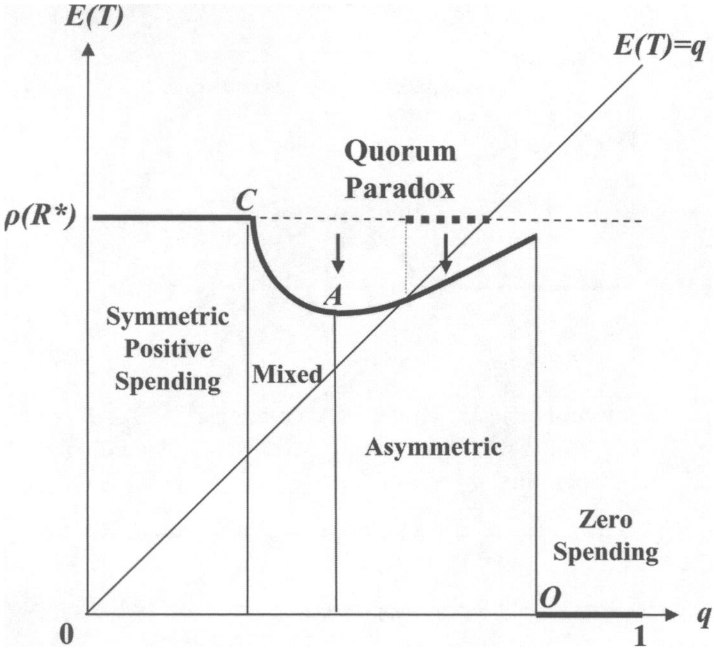

FIGURE 4. Expected turnout as a function of q (B held constant).

summarize what we have learned about the perverse effect that a participation quorum may have on expected turnout.

PROPOSITION 2. *The introduction of a quorum requirement $q > \underline{q}$ decreases the expected turnout in equilibrium. Furthermore, when the benefit B is high enough, there always exists an interval of values of q where the expected turnout exceeds the participation quorum only if the requirement is not imposed.*

To see the intuition behind this result note that, when $q > \underline{q}$, whereas the status quo party has little or no incentive to mobilize voters as its goal is to win by a lack of participation, the reform party is mobilizing voters only to push expected turnout above the quorum threshold. In other words, the tension between parties is no longer about obtaining the majority of votes as in the symmetric equilibrium, but about having turnout reaching the quorum or not. However, in the asymmetric profile the reform party cannot free-ride on the mobilization effort of the status quo party, and returns to mobilization spending are decreasing.

Similarly to the expected turnout, the expected probability $P(q)$ that the reform policy wins a majority of votes varies depending on which region of the parameter space we are in. In particular, $P(q)$ is continuous for $q \neq \bar{q}$, and it is equal to

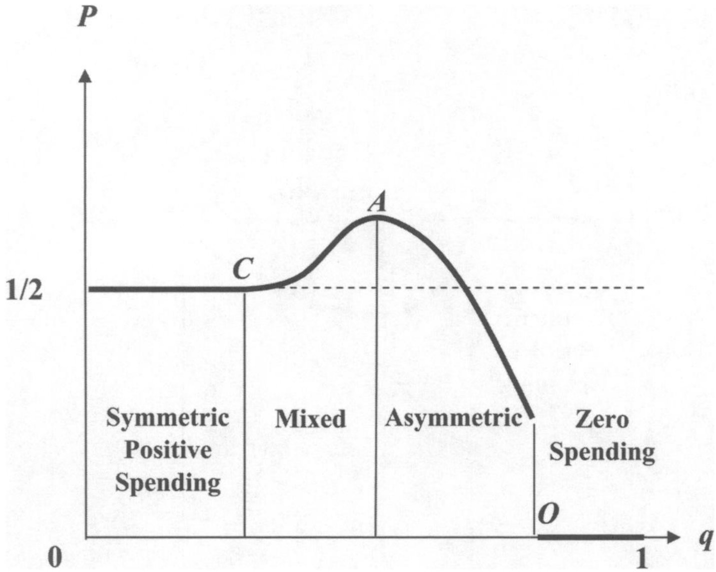

FIGURE 5. Expected probability of approval as a function of q (B held constant).

$$P(q) = \begin{cases} 1/2 & q \in (0, \underline{q}), \\ \alpha(1 - \frac{q}{\rho(\tilde{R})}) + (1-\alpha)(\frac{\rho(\tilde{R})}{\rho(\tilde{R})+\rho(\tilde{S})}) & q \in (\underline{q}, \hat{q}), \\ 1 - \frac{q}{\rho(\hat{R})} & q \in (\hat{q}, \bar{q}), \\ 0 & q \in (\bar{q}, 1). \end{cases}$$

For $q \in (\hat{q}, \bar{q})$, the expected probability $P(q)$ is decreasing in q, as proved in Lemma C.1 in the appendix. Also, it must be that $P(\hat{q}) > 1/2$. In fact, by the definition of $\hat{q}$, the status quo party is indifferent between spending $S = 0$ and $\hat{S}$ at $q = \hat{q}$. Hence, because its profits are equal, the chance of winning must be higher in the case s is spending a positive amount $\hat{S}$. Namely, $1 - P(\hat{q}) < 1 - P(\hat{S}, \hat{R}) < 1/2$, where the last inequality comes from the fact that, when $q \geq \hat{q}$, it follows that $\hat{S} < R^* < \hat{R}$. Finally, if $B > \bar{B}$, because $\rho(\hat{R}(\bar{q}))/2 < \bar{q}$, it follows that

$$P(\bar{q}) = 1 - \frac{\bar{q}}{\rho(\hat{R}(\bar{q}))} < \frac{1}{2} = P(q'),$$

where $\bar{B}$ and q' are defined in Claim 2. Figure 5 summarizes the results.

In particular, we have the following result.

PROPOSITION 3. *There always exists an interval of values of q where a participation quorum requirement increases the expected probability that the reform policy wins a majority of votes.*

Putting together the results of Proposition 2 and 3 we have that when $q \in (\hat{q}, \bar{q})$, either the equilibrium expected turnout is smaller than the quorum, or the equilibrium probability that the reform policy is adopted is strictly bigger than the case where the quorum requirement is absent.

We conclude this section by analyzing parties' expected profits $E(\pi)$ as a function of q. If $q \leq \underline{q}$, parties' expected profits are equal and do not depend on q. Namely, $E(\pi)|_{q \leq \underline{q}} = B/2 - R^*$. If instead $q \in (\underline{q}, \hat{q})$, it is immediately clear that $E(\pi_s) < E(\pi)|_{q \leq \underline{q}}$. Moreover, if $\tilde{R} < 2R^*$ then $E(\pi)|_{q \leq \underline{q}} < E(\pi_r)$.[19] For $q \in (\hat{q}, \bar{q})$, when parties are playing the asymmetric pure-strategies equilibrium, we have that

$$E(\pi_s) = B\frac{q}{\rho(\hat{R})} \quad \text{and} \quad E(\pi_r) = B\left(1 - \frac{q}{\rho(\hat{R})}\right) - \hat{R}.$$

Not surprisingly, the expected profits of the status quo party are strictly smaller than those of the reform party when $q = \hat{q}$, and they are increasing in the quorum requirement. The expected profits of the reform party are instead decreasing in q. Finally, for $q \in (\bar{q}, 1)$, the reform policy cannot win, expected profits equal actual profits, and $\pi_s = B > 0 = \pi_r$.

6. Mending the Participation Quorum

A common rationale for the use of a participation quorum requirement is to make sure that, for a referendum to be valid, there is enough popular interest in the issue at stake. Because this interest is typically associated with the turnout, the quorum requirement should take into account that, if voters respond to parties' mobilization efforts, turnout is endogenous. In this section we address two points. First, we show that in order to induce an expected equilibrium turnout of q and avoid the quorum paradox, the participation quorum requirement should be set at a level that is less than half of q. Second, we argue that a super majority requirement to overturn the status quo is never equivalent to a participation quorum, in the sense of yielding the same Nash equilibrium outcomes.

Suppose that q is the expected equilibrium turnout that we want to induce in a given referendum. In order to avoid the possibility of triggering a quorum

19. This follows from

$$E(\pi_r) + E(\pi_s) = B - \tilde{R} > 2E(\pi)|_{q \leq \underline{q}}.$$

paradox, a spending profile that is an equilibrium without the quorum requirement and yields an expected turnout above q should remain an equilibrium when the quorum requirement is imposed. This occurs if and only if spending zero is not a profitable deviation for the party supporting the status quo. In other words, to avoid the paradox, the quorum-busting strategy (which is always available to party s) should be used only when the interest in the issue at stake (B) is low enough, so that the expected turnout without the quorum requirement is already below q.

Recall from the previous section that in the symmetric positive spending equilibrium, the level of the exogenous benefit B determines the symmetric equilibrium spending $R^*(B)$ and the expected turnout $E(T)$. Hence, for any q, there exists a threshold value B_q below which, in the symmetric positive spending equilibrium, the expected turnout is below q. Namely, if $B < B_q$ then $E(T) < q$ in the positive spending equilibrium. This threshold is implicitly defined by

$$\rho(R^*(B_q)) = q.$$

To avoid the quorum paradox, the status quo party should choose to spend $S = 0$ only when $B < B_q$. Because for given q the zero spending strategy is the best response of the status quo party for values of B such that

$$\underline{q}(B) = \left(\frac{1}{2} - \frac{R^*}{B}\right)\rho(R^*) \leq q,$$

we can map any participation quorum q into what we call an *effective* participation quorum q^e, where

$$q^e = \left(\frac{1}{2} - \frac{R^*(B_q)}{B_q}\right)q.$$

In order to induce an expected equilibrium turnout of q while avoiding a quorum paradox, the participation quorum requirement should be set at q^e instead. Note that this policy achieves two goals. First, the status quo party plays $S = 0$ only whenever $B < B_q$ (which would imply $E(T) < q$ in the positive spending equilibrium). Second, the positive spending equilibrium survives if $B > B_q$ (which implies $E(T) > q$). The effective participation quorum q^e corrects for the endogeneity of parties' mobilization efforts and is less than half of the original participation quorum q.

For example, in the case of $\rho(R) = 1 - e^{-R}$ it is easy to obtain that

$$q^e = \frac{2q + (1-q)\ln(1-q)}{4} < \frac{q}{2}.$$

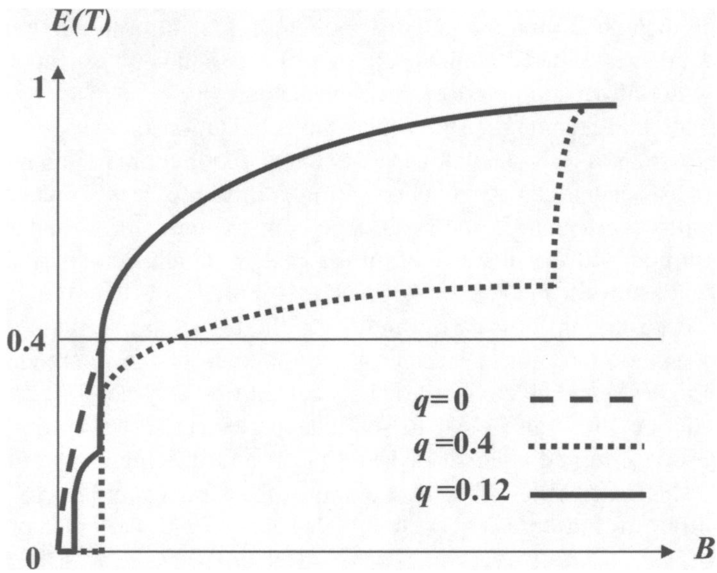

FIGURE 6. Expected turnout as a function of B for different values of q.

In the case of $q = 0.4$, Figure 6 shows how an effective quorum of $q^e(0.4) = 0.12$ can avoid the quorum paradox by inducing an expected turnout smaller than $q = 0.4$ only when expected turnout would have been below quorum anyway.

At this point, a natural question is whether there is a super-majority requirement q_s that is equivalent (i.e., yields the same Nash equilibrium outcomes) to a participation quorum q. The answer is no. To see why this is the case, fix q and define $r_q \in (0, 1)$ as the threshold such that if a proportion of voters $r > r_q$ prefers the reform policy then this policy is selected. Clearly the value of the threshold r_q depends on the equilibrium played. Hence, whereas for fixed q and B, a quota-rule $q_s = r_q$ is indeed equivalent to a participation quorum q, this is not true for any value of B. For example, in the asymmetric equilibrium the threshold r_q decreases with B, which implies that, for fixed participation quorum q, the lower the value of B, the higher the q_s that is needed to make the quota-rule equilibrium outcomes match the participation quorum equilibrium outcomes.

7. Discussion

In this section we discuss the robustness of our results with respect to two simplifying assumptions we adopted in the basic model. First, we explore the consequences of relaxing the assumption that the distribution of r is uniform, allowing for an asymmetric distribution. Second, we consider the case in which spending of one party may also affect supporters of the opposite party.[20]

Consider the general case in which the distribution of r is not uniform, and in particular it is not symmetric. Let the distribution function of r be $F(r)$, with associated density function $f(r)$. In this case, it is a matter of simple algebra to check that, in the symmetric equilibrium, parties' spending and expected turnout will be higher than in the case in which r is distributed uniformly if and only if $f(1/2) > 1$. Clearly, if $f(1/2) = 1$, nothing changes with respect to the uniform case. Intuitively, the higher is the mass of nearly indifferent voters, the more uncertain is the outcome of the referendum. This leads to a higher spending competition between parties, and therefore to a higher expected turnout. It is also clear to see that the expected probability that the status quo is overturned is equal to $1 - F(1/2)$ and it is higher the more left-skewed is the distribution of r. Furthermore, the strategy profile (S^*, R^*) is an equilibrium if and only if $q \in [0, \underline{q}_r]$, where $\underline{q}_r$ may be larger or smaller than $\underline{q}$.[21]

In the special case of $f(r) = 2(1 - r)\alpha + 2r(1 - \alpha)$, where $\alpha \in (0, 1)$, $\underline{q}_r$ is larger than $\underline{q}$ if and only if $\alpha > 1/2$.[22] Hence, in this particular example, $\underline{q}_r$ is larger than $\underline{q}$ if and only if $f(r)$ is right-skewed. In other words, when on average there is a majority of voters in favor of the status quo policy, the status quo party will switch later (i.e., for higher values of q) to the quorum-busting strategy. Intuitively, given our mobilization technology, spending is more effective in mobilizing voters the higher the proportion of supporters a party expects to have. Therefore, if the status quo party is indifferent between $S > 0$

20. We refer the interested reader to our working paper Herrera and Mattozzi (2006) for other generalizations of the basic model, such as the analysis of the case in which parties' payoffs are heterogeneous. In Herrera and Mattozzi (2006) we also provide a comparison between an approval quorum requirement and the participation quorum requirement we have considered so far, and show that all the analysis for the participation quorum case carries over to the approval quorum case.

21. In particular,

$$\underline{q}_r = F^{-1}\left(F\left(\frac{1}{2}\right) - \frac{R^*}{B}\right)\rho(R^*),$$

and $\underline{q}_r$ is larger than $\underline{q}$ if and only if

$$F\left(\frac{1}{2}\right) - \frac{R^*}{B} > F\left(\frac{1}{2} - \frac{R^*}{B}\right).$$

22. Note that when $\alpha = 0$, $f(r) = 2r$, when $\alpha = 1$, $f(r) = 2(1 - r)$, and when $\alpha = 1/2$ we have the uniform distribution.

and $S = 0$ at $\underline{q}$ in the case of a society split evenly, it is strictly better off mobilizing when it expects to have a majority.

In the asymmetric equilibrium, it is a matter of simple algebra to show that spending and expected turnout are higher than in the case in which r is distributed uniformly if and only if $f(q/\rho(\hat{R})) > 1$. In the special case considered herein, it follows that if $\alpha > (<)1/2$, then $f(q/\rho(\hat{R})) > 1$ if and only if $q < (>)\rho(\hat{R})/2$. This means that when the distribution of r is left-skewed (i.e., $\alpha < 1/2$), expected turnout is higher for low values of q such that the asymmetric equilibrium exists (since $\hat{q} > \rho(\hat{R}(\hat{q}))/2$), and it is smaller for high values of q. The analysis shows that in general our equilibrium characterization is not qualitatively affected and therefore, while the uniformity assumption greatly simplifies the model, it is not crucial for our main results.

In the basic model we also assume that the actions of one party only mobilize its own voters/supporters but have no impact on the voters/supporters of the other party. In general we could assume that some spending of one party may affect supporters of the opposite party (as in the case of fear of opponent's mobilization, for example, or negative campaigning).[23] If the effect of spending on the opposite party's turnout is positive, then our results would still hold provided that this effect is not unrealistically large. For otherwise, if for example the reform party could unilaterally prevent the status quo party from reducing the turnout of its own supporters, then trivially a quorum-busting strategy would be unsuccessful. This is not the case as long as the "backfiring" effect is limited. If, on the other hand, the effect of spending on the opposite party's turnout is negative, then ceteris paribus turnout is further reduced and the quorum-busting strategy would be used even more often. [24]

8. Conclusion

We provide an analysis of the consequences of imposing participation requirements in the context of binary elections. Turnout requirements affect the equilibrium turnout, and the chance that one alternative prevails in the referendum. We show that a participation requirement drastically distorts the incentives of parties to mobilize voters in the context of a group-based model of turnout. The result we obtain on equilibrium turnout is unambiguous: A quorum requirement can only depress turnout, sometimes even generating a quorum paradox. Regarding the common argument that a turnout requirement introduces a bias for the status quo, we show that, in the context of group-based models of turnout, this is

23. See, for example, Herrera, Levine, and Martinelli (2008).

24. Clearly, we are focusing on the realistic case in which the direct effect (i.e., mobilizing the party's supporters) is larger than the positive or negative backfiring effect.

not always the case. In fact, the probability that the status quo is overturned may decrease or increase in the presence of a quorum provision.

A natural question, which we do not address in this version of the paper, is to assess the welfare gains/losses of introducing a participation quorum requirement relative to the case in which the quorum is absent. In our working paper (Herrera and Mattozzi 2006), to which we refer the interested reader, we show that the effects on voters' welfare are ambiguous, as in the presence of a quorum limit there is always a welfare loss on the revenue side yet on the cost side there may be a welfare gain. Indeed, a participation quorum requirement never leads to an ex ante revenue gain and, whenever quorum-busting takes place, it causes generically an ex ante revenue loss because the policy supported by the majority does not always prevail in the referendum. However, under some assumptions, it might also reduce the total cost of voting.

The quorum provision could perhaps be an effective safeguard against so-called false majorities, that is, the exploitation of voter apathy by a minority or a special interest group of committed citizens. However, the distortions that a quorum introduces suggest that more stringent requirements to call a referendum might be a better policy if the goal is to introduce a bias for status quo.

Appendix A: Tables

TABLE A.1. Quorum requirements.

States	Participation (%)	Approval (%)
Azerbaijan, Colombia, Venezuela	25	
Hungary		25
Netherlands	30	
Albania, Armenia		33.3
Uruguay[a]	35	
Denmark[a], Scotland		40
Bulgaria, Italy, Latvia, Lithuania, Macedonia, Malta, Poland, Portugal, Romania[a] Slovenia[a], Slovakia, Taiwan	50	
Croatia, Russia	50	50
Belarus, Latvia[a], Serbia, Sweden[b]		50
U.S. States		
Massachusetts[b]	30	
Mississippi[b]	40	
Nebraska[b]	35	
Wyoming[b]		50

[a]Constitutional referendum.
[b]The percentage is with respect to voters in the general election.

TABLE A.2. Examples of recent referenda outcomes.

Year	ISSUE	YES (%)	NO (%)	TURNOUT (%)
Colombia				
1990	Constitutional board	97.6	2.4	$26.1 > q$
2003	Eligibility of candidates	93.3	6.7	$25.1 > q$
2003	All other 14 issues	90	10	$23.4 < q$
Italy				
1990	Hunting	92.2	7.8	$43.4 < q$
1991	Unique preference	95.6	4.4	$62.5 > q$
1999	Proportional quota	91.5	8.5	$49.6 < q$
2005	Stem cell research	88	12	$25.6 < q$
Poland				
1996	Property rights	96.1	3.9	$32.4 < q$
Taiwan				
1996	Relations with China	92	8	$45.1 < q$

Appendix B: Construction of Figure 1

Define

$$M = \frac{1}{\rho(R)} + \frac{1}{\rho(S)}.$$

The equality $M = 2/q$ yields the curved line in Figure 1. $M < 2/q$ is the region above this line; $M > 2/q$ is the region below. Figure 1 shows four regions, which depend on whether $M \gtrless 2/q$ and $\rho(R) \gtrless q$. According to the figure, the value of P is determined as follows in these regions:

	Region		
	$M \gtrless 2/q$	$\rho(R) \gtrless q$	Value of P
(a)	$>$	$<$	0
(b)	$>$	$>$	$1 - Q$
(c)	$<$	$>$	$1 - K$
(d)	$<$	$<$	$Q - K$

We now show that this is true.

Recall that

$$P = \Pr\left(\tilde{r} \geq \frac{\rho(S)}{\rho(R) + \rho(S)} \text{ and } (\rho(R) - \rho(S))\tilde{r} \geq q - \rho(S)\right),$$

and that we defined

$$Q = \frac{q - \rho(S)\cdot}{\rho(R) - \rho(S)}, \quad K = \frac{\rho(S)}{\rho(R) + \rho(S)}.$$

Note that $K \in [0, 1]$, whereas Q may be any real number.

The condition $M = 2/q$ is equivalent to $Q = K$. However, the relationship between $M \gtrless 2/q$ and $Q \gtrless K$ depends on $\rho(R) \gtrless \rho(S)$. Specifically:

1. if $\rho(R) > \rho(S)$ then $M < 2/q \iff Q < K$;
2. if $\rho(R) < \rho(S)$ then $M < 2/q \iff Q > K$.

We therefore treat these two cases separately, showing that, for each case, the given table holds.

Case 1: $\rho(R) > \rho(S)$. Then

$$P = \Pr(\tilde{r} \geq K \text{ and } \tilde{r} \geq Q) = \Pr(\tilde{r} \geq \max\{K, Q\}).$$

Suppose $M < 2/q$. Then $Q < K$ and hence

$$P = \Pr(\tilde{r} \geq K) = 1 - K.$$

Because $M < 2/q$ and $\rho(R) > \rho(S)$ imply $\rho(R) > q$, we have thus established both (c) and (d), where (d) holds vacuously.

Suppose instead $M > 2/q$. Then $Q > K$ and hence $P = \Pr(\tilde{r} \geq Q)$. The conditions $M > 2/q$ and $\rho(R) > \rho(S)$ imply $q > \rho(S)$ and hence $Q > 0$. If $\rho(R) > q$ then also $Q < 1$ and thus $P = 1 - Q$; this establishes (b). If $\rho(R) < q$ then $Q > 1$ and thus $P = 0$; this establishes (a).

Case 2: $\rho(R) < \rho(S)$. Then

$$P = \Pr(\tilde{r} \geq K \text{ and } \tilde{r} \leq Q) = \Pr(K \leq \tilde{r} \leq Q).$$

Suppose $M < 2/q$. Then $Q > K$. If $\rho(R) > q$ then $Q > 1$ and hence $P = 1 - K$; this establishes (c). If $\rho(R) < q$ then $Q < 1$ and hence $P = Q - K$; this establishes (d).

Suppose instead $M > 2/q$. Then $Q < K$ and hence $P = 0$. Because $M > 2/q$ and $\rho(R) < \rho(S)$ imply $\rho(R) < q$, we have thus established both (a) and (b), where (b) holds vacuously.

Appendix C: Proof of Proposition 1

C.1. Part (i)

Consider first the benchmark case of $q = 0$. For all given values of S, the profit function $\pi_r(S, R)$ is continuous for all $R \geq 0$, twice differentiable for all $R > 0$, and single peaked in R, and likewise for $\pi_s(S, R)$. For any pair of values (S^*, R^*) which jointly solve the two first order conditions it must be the case that $S^* = R^*$. In fact, by taking the necessary and sufficient FOCs, we have that

$$\frac{\rho'(R^*)\rho(S^*)}{(\rho(R^*)+\rho(S^*))^2} = \frac{1}{B} = \frac{\rho'(S^*)\rho(R^*)}{(\rho(R^*)+\rho(S^*))^2},$$

which yields

$$\frac{\rho'(R^*)}{\rho(R^*)} = \frac{\rho'(S^*)}{\rho(S^*)}.$$

Therefore, it must be that $S^* = R^*$, where R^* solves

$$\frac{\rho'(R^*)}{4\rho(R^*)} = \frac{1}{B}.$$

Because $\rho'(R)/\rho(R)$ is decreasing in R, and its codomain are the positive real numbers, an equilibrium exists and it is unique for any B. Consider now the case in which $q > 0$. Note that $\pi_r(S^*, R)$ is single peaked in the $P = 1 - K$ region, it is increasing in the $P = Q - K$ region, and non-positive in the $P = 0$ region. Hence, $\pi_r(S^*, R)$ has a global maximum at $R = R^*$. The symmetric profile $S^* = R^*$ for $q = 0$ is an equilibrium for $q > 0$ if and only if both $S^* = R^*$ lies in the $P = 1 - K$ region and s does not have an incentive to deviate to zero, that is, $\pi_s(S^*, R^*) \geq \pi_s(0, R^*)$. This is true if and only if $q \in [0, \underline{q}(B)]$, where

$$\underline{q}(B) = \left(\frac{1}{2} - \frac{R^*}{B}\right)\rho(R^*).$$

C.2. *Parts (ii) and (iii)*

Proving parts (ii) and (iii) of Proposition 1 amounts to show that for all B, there exist unique thresholds $\underline{q}(B)$, $\hat{q}(B)$, and $\bar{q}(B)$ such that $\underline{q} < \min\{\hat{q}, \bar{q}\}$. To do so we first prove two preliminary lemmas.

LEMMA C.1. *Let $\hat{R}$ and $\hat{S}$ be defined by (2) and (3), respectively. Then*

$$\frac{d\hat{R}}{dq} > 0, \quad \frac{d\pi_s(0,\hat{R})}{dq} > 0, \quad \frac{d\pi_r(0,\hat{R})}{dq} < 0, \quad \frac{d\pi_s(\hat{S},\hat{R})}{dq} < 0.$$

Proof. From (2) and the assumptions on $\rho(\cdot)$ it follows that $\hat{R}$ is the unique solution to

$$\frac{q\rho'(\hat{R})}{\rho^2(\hat{R})} = \frac{1}{B}.$$

Since the RHS is constant in q while the LHS is increasing in q and decreasing in $\hat{R}$, then $\hat{R}$ is increasing in q, i.e., $d\hat{R}/dq > 0$. Regarding the profits, we have that

$$\pi_s(0, \hat{R}) = B\frac{q}{\rho(\hat{R})},$$

and

$$\pi_r(0, \hat{R}) = B\left(1 - \frac{q}{\rho(\hat{R})}\right) - \hat{R},$$

and the result follows from noticing that $q/\rho(\hat{R})$ is increasing in q. Finally,

$$\frac{d\pi_s(\hat{S}, \hat{R})}{dq} = -B\frac{\rho(\hat{S})\rho'(\hat{R})}{(\rho(\hat{R}) + \rho(\hat{S}))^2}\frac{d\hat{R}}{dq} < 0.$$

□

LEMMA C.2. *There exists a unique* $\tilde{q} = \rho(R^*)/4 < \underline{q}$ *such that* $\hat{R}(\tilde{q}) = \hat{S}(\tilde{q}) = R^* = S^*$. *Furthermore,* $q \neq \tilde{q}$ *implies* $\hat{S} < S^*$.

Proof. $\hat{R}$ and R^* uniquely solve

$$q\frac{\rho'(\hat{R})}{\rho^2(\hat{R})} = \frac{1}{B} \qquad \text{and} \qquad \frac{\rho'(R^*)}{4\rho(R^*)} = \frac{1}{B},$$

respectively. It is easy to check that when $q = \rho(R^*)/4$,

$$q\frac{\rho'(\hat{R})}{\rho^2(\hat{R})} = \frac{\rho'(R^*)}{4\rho(R^*)}.$$

Next, from the definition of $\underline{q}$, we have that $\tilde{q} < \underline{q}$ if and only if $R^* < B/4$, or

$$\frac{4}{B} > \frac{\rho'\left(\frac{B}{4}\right)}{\rho\left(\frac{B}{4}\right)}.$$

Therefore, $\tilde{q} < \bar{q}$ if and only if

$$\Gamma(x) \equiv \frac{x\rho'(x)}{\rho(x)} < 1.$$

To prove that $\Gamma(x) < 1$, first note that $\Gamma(x)$ is differentiable hence continuous for $x > 0$. Second, $\Gamma(x) \geq 1$ implies that

$$\Gamma'(x) = \left(\frac{\rho'(x)}{\rho(x)}(1 - \Gamma(x)) + \frac{x\rho''(x)}{\rho(x)}\right) < 0.$$

Hence, $\lim_{x\to 0}\Gamma(x) \le 1$ implies $\Gamma(x) < 1$. Because $\lim_{x\to 0}(x\rho'(x)) = 0$, and $\lim_{x\to 0}(x\rho''(x)) = 0$, we have that

$$\lim_{x\to 0}\Gamma(x) = \begin{cases} \lim_{x\to 0}\frac{\rho'(x)+x\rho''(x)}{\rho'(x)} = 1 & \text{if } \rho(0) = 0, \\ 0 & \text{if } \rho(0) > 0. \end{cases}$$

If $q < \tilde{q}$, it follows that $\hat{R} < R^* = S^*$. Because

$$\frac{\rho'(S)\rho(R)}{(\rho(R)+\rho(S))^2}$$

is always decreasing in S, and increasing in R if and only if $S > R$, it follows that

$$\frac{\rho'(S^*)\rho(\hat{R})}{(\rho(\hat{R})+\rho(S^*))^2} < \frac{\rho'(S^*)\rho(R^*)}{(\rho(R^*)+\rho(S^*))^2} = \frac{1}{B},$$

and therefore $\hat{S} < S^*$. If $q > \tilde{q}$, it follows that $\hat{R} > R^* = S^*$, and

$$\frac{\rho'(S^*)\rho(\hat{R})}{(\rho(\hat{R})+\rho(S^*))^2} < \frac{\rho'(S^*)\rho(R^*)}{(\rho(R^*)+\rho(S^*))^2} = \frac{1}{B}.$$

Hence $q \neq \tilde{q}$ implies $\hat{S} < S^*$. □

We are now ready to prove parts (ii) and (iii). We need to show that, for all B the thresholds $\underline{q}(B)$, $\hat{q}(B)$, and $\bar{q}(B)$ are uniquely defined. Furthermore, $\underline{q} < \hat{q}$, and $\underline{q} < \bar{q}$. First, we show that

$$\underline{q} < \hat{q} < \frac{1}{2},$$

and that the thresholds $\underline{q}$ and $\hat{q}$ are well defined. Define

$$C(q) = \pi_s(\hat{S}, \hat{R}) - \pi_s(0, \hat{R}),$$

and

$$D(q) = \pi_s(S^*, R^*) - \pi_s(0, R^*).$$

Hence $\underline{q}$ and $\hat{q}$ are implicitly defined by

$$C(\hat{q}) = 0, \quad D(\underline{q}) = 0.$$

Clearly $D'(q) < 0$, and from Lemma C.1, $C'(q) < 0$. So the thresholds $\underline{q}$ and $\hat{q}$ are uniquely defined. From Lemma C.2, $\tilde{q} < \underline{q}$. Hence, $D(\tilde{q}) = C(\tilde{q}) > 0$, and $\tilde{q} < \hat{q}$. To show that $\underline{q} < \hat{q}$, it suffices to show that for $q \geq \tilde{q}$ it is true that $D'(q) < C'(q)$, that is

$$\frac{1}{\rho(R^*)} > \frac{1}{\rho(\hat{R})} + \frac{d\hat{R}}{dq}\left(\frac{\rho(\hat{S})\rho'(\hat{R})}{(\rho(\hat{R}) + \rho(\hat{S}))^2} - \frac{1}{B}\right).$$

Because for $q \geq \tilde{q}$ we have $\hat{R} \geq R^* \geq \hat{S}$, and because $d\hat{R}/dq > 0$, it follows that the term in brackets in the above inequality is non positive and therefore $D'(q) < C'(q)$. Next, we show that $\underline{q} < \bar{q}$ and that $\bar{q}$ is well defined. Recall that when $q = \bar{q}$, $\pi_r(0, \hat{R}) = 0$ and, by the envelope theorem, we have that

$$\frac{d\pi_r(0, \hat{R})}{dq} = \frac{\partial \pi_r(0, \hat{R})}{\partial q} = -\frac{B}{\rho(\hat{R})} < 0.$$

Hence $\bar{q}$ is uniquely defined. To show that $\underline{q} < \bar{q}$, note that when $q \geq \tilde{q}$, we have that

$$0 < D(\tilde{q}) = B\left(\frac{1}{2} - \frac{\tilde{q}}{\rho(R^*)}\right) - R^* < B\left(1 - \frac{\tilde{q}}{\rho(R^*)}\right) - R^* = \pi_r(0, \hat{R}(\tilde{q})),$$

and

$$\frac{dD(q)}{dq} = -\frac{B}{\rho(R^*)} < -\frac{B}{\rho(\hat{R})} = \frac{d\pi_r(0, \hat{R})}{dq} < 0.$$

Hence, because $D(q)$ is smaller and decreases faster than $\pi_r(0, \hat{R})$, the desired inequality follows.

C.3. *Part (iv)*

In order to prove part (iv), we need the following lemma.

LEMMA C.3. *For all $q \in (\underline{q}, \hat{q})$ there exists a unique $\tilde{R}(q) \in (R^*, \hat{R})$ such that the best response of party s is $S \in \{0, \tilde{S} > 0\}$. Furthermore, $\tilde{R}(\underline{q}) = R^*$, $\tilde{R}(\hat{q}) = \hat{R}$, $\partial\tilde{R}/\partial q > 0$, and $\partial\tilde{S}/\partial q < 0$.*

Proof. Denote $S = S(R)$ as the best response of party s to R and let

$$C(R, q) = \pi_s(S, R) - \pi_s(0, R) = B\left(\frac{\rho(S)}{\rho(R) + \rho(S)} - \frac{q}{\rho(R)}\right) - S.$$

The indifference condition that defines $\tilde{R}(q)$ is $C(R, q) = 0$. Because $S^* = S(R^*)$ and $\hat{S} = S(\hat{R})$, we have that $\tilde{R}(\underline{q}) = R^*$, and $\tilde{R}(\hat{q}) = \hat{R}$. Because

$\partial C/\partial q < 0$, for $q \in (\underline{q}, \hat{q})$ we have that $C(R^*, q) < C(R^*, \underline{q}) = 0$, and $C(\hat{R}, q) > C(\hat{R}, \hat{q}) = 0$. If $\partial C/\partial R > 0$ for all $R \in [R^*, \hat{R}]$ and $q \in (\underline{q}, \hat{q})$, then for any $q \in (\underline{q}, \hat{q})$ there exists a unique $\tilde{R} \in (R^*, \hat{R})$ such that $C(\tilde{R}, q) = 0$. What is left to show is that $\partial C/\partial R > 0$ when $R \in [R^*, \hat{R}]$. Using the fact that

$$B\frac{\rho'(S(R))\rho(R)}{(\rho(R) + \rho(S(R)))^2} = 1,$$

we have that

$$\begin{aligned}\frac{\partial C}{\partial R} &= B\frac{q\rho'(R)}{(\rho(R))^2} - B\frac{\rho'(R)\rho(S(R))}{(\rho(R) + \rho(S(R)))^2} \\ &\quad + \left(B\frac{\rho'(S(R))\rho(R)}{(\rho(R) + \rho(S(R)))^2} - 1\right)\frac{\partial S(R)}{\partial R} \\ &= B\left(\frac{q\rho'(R)}{(\rho(R))^2} - \frac{\rho'(R)\rho(S(R))}{(\rho(R) + \rho(S(R)))^2}\right),\end{aligned}$$

and by using the definition of $\hat{R}$ and R^* we have that for $R \in [R^*, \hat{R}]$,

$$\frac{q\rho'(R)}{(\rho(R))^2} > \frac{1}{B} > \frac{\rho'(R)\rho(S(R))}{(\rho(R) + \rho(S(R)))^2}.$$

Finally, because $C(R, q)$ is differentiable in both arguments, the implicit function theorem implies that $\tilde{R}(q)$ is differentiable and

$$\frac{\partial \tilde{R}}{\partial q} = -\frac{\partial C/\partial \tilde{R}}{\partial C/\partial q} > 0.$$

Because $\tilde{S} > 0$ is the best response to $\tilde{R} > R^*$, then by the proof of Lemma C.2, $\partial \tilde{S}/\partial \tilde{R} < 0$ and therefore

$$\frac{\partial \tilde{S}}{\partial q} = \frac{\partial \tilde{S}}{\partial \tilde{R}}\frac{\partial \tilde{R}}{\partial q} < 0.$$

□

We are now ready to prove part (iv). By construction, $\tilde{R}$ makes party s indifferent between playing 0 and $S(\tilde{R})$. We have an equilibrium if s chooses the mix $(\alpha, 1 - \alpha)$ (with α on $S = 0$) such that the best response of party r is $\tilde{R}$. Let

$$R(\alpha) \equiv \arg\max_R(\alpha\pi_r(0, R) + (1 - \alpha)\pi_r(S(\tilde{R}), R)),$$

be the best response of party r to party s mixing between 0 and $S(\tilde{R})$. We want to find an α such that $R(\alpha) = \tilde{R}$. Note that it must be the case that $R(\alpha) \in (R(0), R(1))$, where

$$R(1) \equiv \arg\max_R \left(B\left(1 - \frac{q}{\rho(R)}\right) - R \right) = \hat{R},$$
$$R(0) \equiv \arg\max_R \left(B\left(\frac{\rho(R)}{\rho(R) + \rho(S(\tilde{R}))}\right) - R \right) = R'',$$

and $R'' < R^* < \tilde{R} < \hat{R}$. Because the objective

$$\alpha\pi_r(0, R) + (1-\alpha)\pi_r(S(\tilde{R}), R)$$

is concave in R for all α, the FOC delivers uniquely our target, namely,

$$\alpha = \left(\frac{1}{B} - \frac{\rho'(\tilde{R})\rho(S(\tilde{R}))}{\left(\rho(\tilde{R}) + \rho(S(\tilde{R}))\right)^2}\right) \Big/ \left(\frac{q\rho'(\tilde{R})}{\rho(\tilde{R})^2} - \frac{\rho'(\tilde{R})\rho(S(\tilde{R}))}{\left(\rho(\tilde{R}) + \rho(S(\tilde{R}))\right)^2}\right).$$

Finally, note that $\alpha(\underline{q}) = \alpha(\tilde{R} = R^*) = 0$, and $\alpha(\hat{q}) = \alpha(\tilde{R} = \hat{R}(\hat{q})) = 1$.

Appendix D: Construction of Figure 3

Regarding $\underline{q}$, note that

$$\frac{d\underline{q}}{dB} = \rho'(R^*)\left(\frac{\partial R^*}{\partial B}\left(\frac{1}{4} - \frac{R^*}{B}\right) + \frac{1}{4}\frac{R^*}{B}\right).$$

Because

$$\frac{\partial R^*}{\partial B} = \left(4 - B\frac{\rho''(R^*)}{\rho'(R^*)}\right)^{-1} > 0,$$

if $R^*/B < 1/4$, it follows that $d\underline{q}/dB > 0$. Finally, $R^*/B < 1/4$ if and only if $\Gamma(x) < 1$ for $x > 0$, which is implied by the proof of Lemma C.2. Because

$$\lim_{B\to 0} R^* = 0, \quad \lim_{B\to\infty} R^* = \infty, \quad \lim_{B\to\infty} \frac{R^*}{B} = \lim_{B\to\infty} \frac{\partial R^*}{\partial B} \leq \frac{1}{4},$$

it follows that

$$\lim_{B\to 0} \underline{q} = 0, \quad \lim_{B\to\infty} \frac{d\underline{q}}{dB} = 0, \quad \lim_{B\to\infty} \underline{q} \in \left[\frac{1}{4}, \frac{1}{2}\right].$$

In particular, a sufficient condition for $\lim_{B\to\infty} \underline{q} = 1/2$ is $\lim_{x\to\infty} \rho''(x)/\rho'(x) = c < 0$ (this is true for example in the case of $\rho(x) = 1 - e^{-\alpha x}$, and $\alpha > 0$). Recall that if $\rho(0) > 0$ we can have that $\lim_{B\to\infty} \underline{q} > 1/2$.

Regarding $\hat{q}(B)$, recall that $\hat{R}$ is a function of q and B, and we have that

$$\frac{\partial \hat{R}}{\partial q} = \frac{1}{q}\left(\frac{2\rho(\hat{R})}{qB} - \frac{\rho''(\hat{R})}{\rho'(\hat{R})}\right)^{-1} \in \left(0, \frac{B}{2\rho(\hat{R})}\right),$$
$$\frac{\partial \hat{R}}{\partial B} = \frac{1}{B}\left(\frac{2\rho(\hat{R})}{qB} - \frac{\rho''(\hat{R})}{\rho'(\hat{R})}\right)^{-1} = \frac{q}{B}\frac{\partial \hat{R}}{\partial q} \in \left(0, \frac{q}{2\rho(\hat{R})}\right).$$

Therefore,

$$\frac{d\hat{q}}{dB} = \frac{\rho(\hat{R}(\hat{q}))\left(\frac{\hat{S}(\hat{q})}{B^2} + \left(B^{-1} - \frac{\rho(\hat{S}(\hat{q}))\rho'(\hat{R}(\hat{q}))}{(\rho(\hat{R}(\hat{q}))+\rho(\hat{S}(\hat{q})))^2}\right)\frac{\partial \hat{R}(\hat{q})}{\partial B}\right)}{1 - \rho(\hat{R}(\hat{q}))\left(B^{-1} - \frac{\rho(\hat{S}(\hat{q}))\rho'(\hat{R}(\hat{q}))}{(\rho(\hat{R}(\hat{q}))+\rho(\hat{S}(\hat{q})))^2}\right)\frac{\partial \hat{R}(\hat{q})}{\partial \hat{q}}} > 0,$$

$\lim_{B\to 0} \hat{q} = 0$, and $\lim_{B\to\infty} \hat{q} \in [\lim_{B\to\infty} \underline{q}, 1/2]$, where we used

$$\frac{d\hat{R}(\hat{q})}{dB} = \frac{\partial \hat{R}(\hat{q})}{\partial \hat{q}}\frac{d\hat{q}}{dB} + \frac{\partial \hat{R}(\hat{q})}{\partial B} > 0,$$
$$\frac{\hat{S}(\hat{q})}{B} \in \left(0, \frac{1}{2}\right),$$
$$\frac{\rho(\hat{S}(\hat{q}))}{\rho(\hat{R}(\hat{q})) + \rho(\hat{S}(\hat{q}))} \in \left(0, \frac{1}{2}\right),$$
$$\frac{1}{2} \geq \lim_{B\to\infty} \hat{q} \geq \lim_{B\to\infty} \underline{q} \in \left[\frac{1}{4}, \frac{1}{2}\right].$$

Regarding $\bar{q}(B)$,

$$\frac{d\bar{q}}{dB} = \frac{1}{B^2}\left[-\left(\left(\frac{\partial \hat{R}(\bar{q})}{\partial \bar{q}}\frac{d\bar{q}}{dB} + \frac{\partial \hat{R}(\bar{q})}{\partial B}\right)B - \hat{R}(\bar{q})\right)\rho(\hat{R}(\bar{q})).\right.$$
$$\left. + B^2\left(1 - \frac{\hat{R}(\bar{q})}{B}\right)\rho'(\hat{R}(\bar{q}))\left(\frac{\partial \hat{R}(\bar{q})}{\partial \bar{q}}\frac{d\bar{q}}{dB} + \frac{\partial \hat{R}(\bar{q})}{\partial B}\right)\right]$$

$$= \left[-\frac{\partial \hat{R}(\bar{q})/\partial B}{B}\rho(\hat{R}(\bar{q})) + \frac{\hat{R}(\bar{q})}{B^2}\rho(\hat{R}(\bar{q})) + \left(1 - \frac{\hat{R}(\bar{q})}{B}\right)\rho'(\hat{R}(\bar{q}))\frac{\partial \hat{R}(\bar{q})}{\partial B}\right]$$

$$\div \left[1 + \frac{\partial \hat{R}(\bar{q})/\partial \bar{q}}{B}\rho(\hat{R}(\bar{q})) - \left(1 - \frac{\hat{R}(\bar{q})}{B}\right)\rho'(\hat{R}(\bar{q}))\frac{\partial \hat{R}(\bar{q})}{\partial \bar{q}}\right]$$

$$= \frac{\hat{R}(\bar{q})}{B^2}\rho(\hat{R}(\bar{q})) > 0,$$

where the last equality is obtained by substituting back the equation for $\bar{q}(B)$. Moreover,

$$\lim_{B\to 0} \bar{q} = 0, \quad \lim_{B\to\infty} \frac{d\bar{q}}{dB} = 0, \quad \lim_{B\to\infty} \bar{q} \geq \frac{1}{2},$$

where we used $\hat{R}(\bar{q})/B \in (0, 1)$,

$$\frac{d\hat{R}(\bar{q})}{dB} = \frac{\partial \hat{R}(\bar{q})}{\partial \bar{q}}\frac{d\bar{q}}{dB} + \frac{\partial \hat{R}(\bar{q})}{\partial B} = \frac{\partial \hat{R}(\bar{q})}{\partial \bar{q}}\left(\frac{d\bar{q}}{dB} + \frac{\bar{q}}{B}\right)$$

$$= \left(2 - \rho(\hat{R}(\bar{q}))\frac{\rho''(\hat{R}(\bar{q}))}{(\rho'(\hat{R}(\bar{q})))^2}\right)^{-1} > 0,$$

and

$$\lim_{B\to 0} \hat{R}(\bar{q}) = 0, \quad \lim_{B\to\infty} \hat{R}(\bar{q}) = \infty, \quad \lim_{B\to\infty} \frac{\hat{R}(\bar{q})}{B} = \lim_{B\to\infty} \frac{d\hat{R}(\bar{q})}{dB} \leq \frac{1}{2}.$$

In particular, if $\lim_{x\to\infty} \rho''(x)/(\rho'(x))^2 = -\infty$, then $\lim_{B\to\infty} \bar{q} = 1$ (this is true for example in the case of $\rho(x) = 1 - e^{-\alpha x}$, and $\alpha > 0$). Summarizing, we have that

$$\frac{d\underline{q}}{dB} > 0, \quad \lim_{B\to 0} \underline{q} = 0, \quad \lim_{B\to\infty} \underline{q} \in \left[\frac{1}{4}, \frac{1}{2}\right]$$

$$\frac{d\hat{q}}{dB} > 0, \quad \lim_{B\to 0} \hat{q} = 0, \quad \lim_{B\to\infty} \hat{q} \in \left[\lim_{B\to\infty} \underline{q}, \frac{1}{2}\right]$$

$$\frac{d\bar{q}}{dB} > 0, \quad \lim_{B\to 0} \bar{q} = 0, \quad \lim_{B\to\infty} \bar{q} \in \left[\frac{1}{2}, 1\right].$$

Appendix E: Proof of Claims 1 and 2

Proof of Claim 1. In order to show that $E(T) > q$ when $q \in (\underline{q}, \hat{q})$, note that in this region

$$B\frac{q}{\rho(\tilde{R}(q))} = B\frac{\rho(S(\tilde{R}(q)))}{\rho(\tilde{R}(q)) + \rho(S(\tilde{R}(q)))} - S(\tilde{R}(q)),$$

and $\hat{R}(\hat{q}) > \tilde{R}(q) > R^* > S(\tilde{R}(q))$. Hence, it must be the case that

$$\frac{1}{2} > \frac{\rho(S(\tilde{R}(q)))}{\rho(\tilde{R}(q)) + \rho(S(\tilde{R}(q)))} > \frac{q}{\rho(\tilde{R}(q))},$$

which implies $\rho(\tilde{R})/2 > q$, and therefore

$$E(T) = \frac{\rho(\tilde{R})}{2} + (1 - \alpha(q))\frac{\rho(\tilde{S})}{2} > q.$$

Continuity of the expected turnout for all $q \neq \bar{q}$ implies that

$$\lim_{q \to \underline{q}} E(T) = \rho(R^*) \quad \text{and} \quad \lim_{q \to \hat{q}} E(T) = \frac{\rho(\hat{R}(\hat{q}))}{2}.$$

Moreover,

$$\frac{\rho(\hat{R}(\hat{q}))}{2} = \frac{\rho'(\hat{R}(\hat{q}))\hat{q}B}{2\rho(\hat{R}(\hat{q}))} < \frac{\rho'(R^*)B}{4} = \rho(R^*),$$

because

$$\rho'(\hat{R}(\hat{q}))\hat{q} < \rho'(R^*)\frac{\rho(\hat{R}(\hat{q}))}{2}.$$

Finally, $\lim_{q \to \underline{q}} \partial E(T)/\partial q < 0$ follows from

$$\begin{aligned}
\lim_{q \to \underline{q}} \frac{\partial E(T)}{\partial q} &= \frac{1}{2}\left(\rho'(R^*) - \lim_{q \to \underline{q}} \frac{\partial \alpha}{\partial \tilde{R}} \rho(R^*)\right) \lim_{q \to \underline{q}} \frac{\partial \tilde{R}}{\partial q} \\
&= \frac{1}{2}\left(\rho'(R^*) - \frac{4\rho'(R^*) - B\rho''(R^*)}{16\underline{q} - B\rho'(R^*)} \rho(R^*)\right) \lim_{q \to \underline{q}} \frac{\partial \tilde{R}}{\partial q} \\
&< \frac{1}{2}\left(\rho'(R^*) - \frac{4\rho'(R^*)}{16\underline{q} - B\rho'(R^*)} \rho(R^*)\right) \lim_{q \to \underline{q}} \frac{\partial \tilde{R}}{\partial q} \\
&= \frac{\rho'(R^*)}{2}\left(\frac{16\rho(R^*)\left(\frac{1}{2} - \frac{R^*}{B}\right) - B\rho'(R^*) - 4\rho(R^*)}{16\rho(R^*)\left(\frac{1}{2} - \frac{R^*}{B}\right) - B\rho'(R^*)}\right) \lim_{q \to \underline{q}} \frac{\partial \tilde{R}}{\partial q} < 0,
\end{aligned}$$

where we used Lemma C.3,

$$\lim_{q \to \underline{q}} \frac{\partial \alpha}{\partial \tilde{R}} = \frac{4\rho'(R^*) - B\rho''(R^*)}{16\underline{q} - B\rho'(R^*)} > 0,$$

and

$$16\rho(R^*)\left(\frac{1}{2}-\frac{R^*}{B}\right)-B\rho'(R^*)-4\rho(R^*)<4\rho(R^*)-B\rho'(R^*)=0.$$

□

Proof of Claim 2. The fact that $E(T)|_{q=\hat{q}} > \hat{q}$ follows directly from Claim 1, by continuity of $E(T)$. Next, because $\bar{q}$ is increasing in B, and $\lim_{B\to\infty}\bar{q} \geq 1/2 > \rho(\hat{R}(\bar{q}))/2$, there exists a $\bar{B}$ such that for $B > \bar{B}$ we have that $\rho(\hat{R}(\bar{q}))/2 < \bar{q}$. Because

$$\frac{\partial \hat{R}(q)}{\partial q} < \frac{B}{2\rho(\hat{R}(q))},$$

it follows that

$$\frac{\partial E(T)}{\partial q} = \frac{\rho'(\hat{R}(q))}{2}\frac{\partial \hat{R}(q)}{\partial q} < 1.$$

Hence, when $B > \bar{B}$, there exists a unique q' such that $E(T) > q$ if and only if $q < q'$. □

References

Börgers, Tilman (2004). "Costly Voting." *American Economic Review*, 94, 57–66.

Casella, Alessandra, and Andrew Gelman (forthcoming). "A Simple Scheme to Improve the Efficiency of Referenda." *Journal of Public Economics*.

Corte-Real, P. Paulo, and Paulo T. Pereira (2004). "The Voter Who Wasn't There: Referenda, Representation, and Abstention." *Social Choice and Welfare*, 22, 349–369.

Feddersen, Timothy J. (2004). "Rational Choice Theory and the Paradox of Not Voting." *Journal of Economic Perspectives*, 18, 99–112.

Foley, Stephanie (2006). "The Local Property Tax Election Study of Oregon. Impact of Double Majority of Ballot Measures 1997–2005." League of Oregon Cities.

Herrera, Helios, David K. Levine, and Cesar Martinelli (2008). "Policy Platforms, Campaign Spending and Voter Participation." *Journal of Public Economics*, 92, 501–513.

Herrera, Helios, and Andrea Mattozzi (2006). "Quorum and Turnout in Referenda." Available at SSRN: <http://ssrn.com/abstract=1003803>.

Matsusaka, John G. (2005a). "Direct Democracy Works." *Journal of Economic Perspectives*, 19, 185–206.

Matsusaka, John G. (2005b). "The Eclipse of Legislatures: Direct Democracy in the 21st Century." *Public Choice*, 124, 157–178.

Morton, Rebecca (1987). "A Group Majority Model of Voting." *Social Choice and Welfare*, 4, 17–31.

Morton, Rebecca (1991). "Groups in Rational Turnout Models." *American Journal of Political Science*, 35, 758–776.

American National Election Studies. "The ANES Guide to Public Opinion and Electoral Behavior." Available at <http://www.electionstudies.org/nesguide/nesguide.htm>.

Qvortrup, Mads (2002). *A Comparative Study of Referendums*. Manchester University Press.

Shachar, Ron, and Barry Nalebuff (1999). "Follow the Leader: Theory and Evidence on Political Participation." *American Economic Review*, 57, 637–660.

Snyder, James M. (1989). "Election Goals and Allocation of Campaign Resources." *Econometrica*, 89, 525–547.