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Efficient Allocation of a "Prize"—King Solomon's Dilemma*

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A planner is interested in allocating an indivisible good (a "prize") to one of many agents in the economy. His objective is to *give* the prize to the agent who values it most, without any payments being made by the recipient. The planner, however, does not know the identity of this agent, while the agents themselves do. This paper shows how the planner can construct simple, multistage mechanisms with a unique subgame perfect equilibrium outcome. At this outcome, the agent who values the prize most gets it without any transfer of money being made by any of the agents or the planner. © 1989 Academic Press, Inc.

1. INTRODUCTION

The title of this paper refers to the Old Testament, Kings A, Chapter 3: Two mothers came to King Solomon with a baby and both claimed to be the baby's genuine mother. King Solomon faced the problem of finding out which of two women was the true mother of the baby. The development of modern economics, we hope, will enable us to recast King Solomon's problem in an interesting way. This paper attempts to use implementation theory to solve King Solomon's and similar problems. It is perhaps best to start with a simple example of King Solomon's dilemma.

Suppose there are two mothers, Mother A and Mother B, and they both claim to be the true mother of a baby. Let the "true" mother's valuation of the baby be v_t and the "false" mother's valuation be v_f. v_t and v_f are

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interpreted as willingness to pay, and $v_t > v_f$. This defines what we mean by "true" mother and "false" mother. There is a monetary instrument so that when a mother pays m, her utility is $v - m$ ($v = v_f$ or v_t) if she gets the baby, whereas her utility is $-m$ if she does not get the baby. The identity of the mother whose valuation is v_t is known to both mothers, but not to King Solomon. In other words, the mothers have complete information about the "state of the world," while King Solomon does not. The valuations v_t and v_f and the utility functions are common knowledge.

We can now describe King Solomon's problem precisely. His goal is to "give" the baby to Mother A if her valuation is v_t and to "give" the baby to Mother B otherwise. Now, it is straightforward to find out the identity of the true mother—simply ask the mothers to participate in a second price auction. The point that makes this problem interesting and nontrivial is that King Solomon, as a result of his benevolent nature, is not interested in collecting money; his goal is to *give* the baby to the true mother, not to auction it. In the implementation language, King Solomon wants to design a mechanism, or a game form, so that the *unique* equilibrium outcome of the mechanism is to give the baby to Mother A when her valuation is v_t and to Mother B otherwise, without any monetary transfers *in equilibrium.*

In this paper, we want to study a generalization of the situation described above: imagine that an indivisible "prize" is to be allocated between two (or more) individuals. We would like to find an assignment procedure so that the person with the highest valuation will be the recipient of this prize. In other words, we propose to construct a mechanism to implement the social choice rule that assigns the prize to the individual with the highest valuation. This seems to be a natural and interesting social choice rule to implement; in any case, this choice rule implements the Pareto efficient allocations.[1]

In this paper we assume that the individuals' valuations of the prize are common knowledge among the individuals themselves. This is in line with a number of papers in the implementation theory literature, e.g., Maskin (1977), Moore and Repullo (1988), Abreu and Sen (1987), and Palfrey and Srivastava (1986). These authors have studied mechanism design in gen-

[1] There are many examples of which our model is a realistic description: the allocation of a piece of technology among many countries by a development agency, the assignment of a piece of equipment among many researchers, etc. An especially interesting application is the granting of a license for a research and development project. Suppose there are many potential researchers. The planner does not know their abilities, but he wants to select the ablest one and to motivate him to perform efficiently. In an earlier version of this paper, we have analyzed this problem extensively and have shown that formally the planner's problem is equivalent to King Solomon's. In order to achieve the first-best (i.e., the ablest researcher choosing the efficient effort level), the planner would have to grant the license to the ablest researcher without collecting any payment from him.

eral models and have obtained interesting results. Our paper is closest to those by Moore and Repullo, and Abreu and Sen—the game forms we use are multistage mechanisms and the implementation solution concept is subgame perfect equilibrium.[2] However, there are two elements that make our contribution distinct from theirs. First, the mechanisms constructed by these authors are quite complex and are not easy to interpret. (This is neither surprising nor meant to be a criticism, since their mechanisms apply to very general environments. On the other hand, we consider a relatively simple set of feasible allocations and a simple social choice function.) We think the mechanisms in this paper are simple and intuitive. Second, many general mechanisms in the literature exploit the nonattainability of some supremum (for example, the agent shouting out the biggest integer earns a bonus) in order to pin down the set of equilibria in the game form. The mechanisms we construct do not have this problem. In our mechanisms, at each stage, there is only one player who makes a move and she chooses from a finite or compact set.

The paper proceeds as follows. Section 2 begins with the solution to King Solomon's problem described earlier. We then define and solve the general model. We find it useful to consider separately the discrete and continuous valuation cases. In each, we provide a mechanism to implement the social choice function that awards a prize to the player with the highest valuation. Conclusions are drawn in Section 3.

2. King Solomon's Problem

We first provide a solution to King Solomon's problem as defined in the Introduction. Recall that we have assumed that the valuations of the true and false mothers (v_t and v_f) are common knowledge. The only missing information to King Solomon is which mother values the baby at v_t. Happily, King Solomon can use the following mechanism to find out the true mother.

Stage 1: Mother A may say that the baby is either "mine" or "hers." If she says "hers," then the baby is given to Mother B. If she says "mine," then proceed to Stage 2.

Stage 2: Mother B can "agree" to or "challenge" Mother A's claim. If she agrees, then the baby is given to Mother A. If she challenges, then she gets the baby, but must pay King Solomon a sum v, where $v_f < v < v_t$. In turn, Mother A is penalized and pays King Solomon $\delta > 0$.

Figure 1 gives a sketch of the mechanism.

[2] It is clear that the social choice rule we want to implement is nonmonotonic (see Maskin, 1977). Hence, it is not implementable in Nash equilibrium.

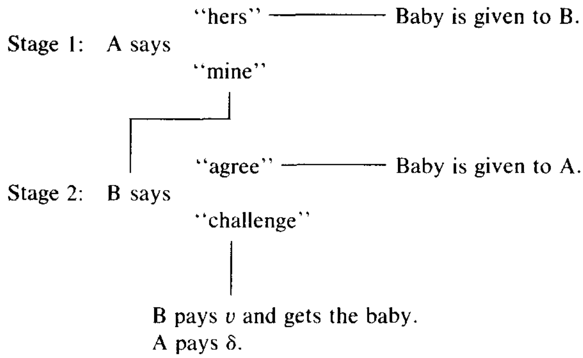

FIGURE 1

The outcome of the unique (subgame) perfect equilibrium of the above game involves the true mother getting the baby without any transfers of money. The strategies of the unique perfect equilibrium are: The true mother says "mine" at Stage 1 and "challenge" at Stage 2. The false mother says "hers" at Stage 1 and "agree" at Stage 2. The point is, the payment v of the challenging mother satisfies $v_f < v < v_t$ so that at Stage 2, Mother B challenges only if she is the true mother; otherwise, allowing Mother A to claim the baby is a better move. On the other hand, Mother A must say "hers" at Stage 1 when her valuation is v_f in order to avoid the penalty δ.

Note that the above game form relies on the assumption that there are only two possible valuations. Our objective in this paper is to construct mechanisms to solve "King Solomon's dilemma" and similar problems when this assumption is relaxed. This will be done in two steps. First, we discuss the case where each mother's valuation comes from a discrete but arbitrary set. Then we consider the case where each mother's valuation comes from an interval. We shall now deal with the first case.

There are two players, A and B. King Solomon wants to award a prize (the "baby") to the player who values the prize higher. We assume that A and B have complete information of each other's valuations. However, King Solomon does not, but he knows that A's valuation is one of

$$u_1 < u_2 < \cdots < u_i < \cdots < u_I,$$

and B's valuation is one of

$$v_1 < v_2 < \cdots < v_j < \cdots < v_J.$$

Therefore, King Solomon is able to deduce that:

$$\text{If } u_i \neq v_j, \text{ then there exists } \varepsilon > 0 \text{ s.t. } |u_i - v_j| > \varepsilon, \text{ all } i, j. \qquad (*)$$

Formally, King Solomon wants to award the "prize" to the player with the higher valuation. In the case of a tie in valuations, King Solomon gives the prize to either A or B. The mechanism we have previously discussed cannot be used directly to solve his problem. However, the following mechanism does achieve the implementation of efficient prize allocation.

Stage 1: Player A says whether the prize is "mine" or "hers." If she says "hers," then the prize is given to Player B. If she says "mine," then proceed to Stage 2.
Stage 2: Player B may say "agree" or "challenge." If she says "agree," then Player A gets the prize. If she says "challenge," both players pay King Solomon $\varepsilon/4$ (ε being defined by (*)) and then they proceed to the following sequential bidding game.
Stage 3: Player A announces a bid $\hat{u}$ from $\{u_1, u_2, \ldots, u_I\}$.
Stage 4: Player B announces a bid $\hat{v}$ from $\{v_1, v_2, \ldots, v_J\}$. The player who has placed the higher bid (the winner) gets the prize and pays $\max[\hat{u}, \hat{v}] - \varepsilon/2$. If there is a tie, Player A gets the prize and pays $\hat{u} - \varepsilon/2$. In any case the loser does not pay her bid.

Suppose that the actual valuations of the players are u_i and v_j. The game form together with the utility functions of the players then defines a game; call this game $\Gamma(u_i, v_j)$. See also Fig. 2.

THEOREM 1. *Consider the game $\Gamma(u_i, v_j)$. (1) Suppose $u_i \geq v_j$. The unique subgame perfect equilibrium outcome awards the prize to Player A. (2) Suppose $u_i < v_j$. The unique subgame perfect equilibrium outcome awards the prize to Player B. At this equilibrium outcome, no player ever pays King Solomon anything.*

Proof. $\Gamma(u_i, v_j)$ is a finite game and must possess a subgame perfect equilibrium. We now characterize the (unique) subgame perfect equilibrium outcome.

(1) Consider the case $u_i \geq v_j$. First, consider the subgame beginning at Stage 3. In any subgame perfect equilibrium, A's payoff is at least $\varepsilon/2$ and B's payoff is 0. To prove this, we consider A's strategy of bidding $\hat{u} = u_i$, i.e., her true valuation. What is B's best response at Stage 4? If B bids $\hat{v} > \hat{u}$, then B wins and gets the prize. Her payoff is $v_j - \hat{v} + \varepsilon/2$. But since $\hat{v} > \hat{u} = u_i \geq v_j$, $\hat{v} - v_j \geq \hat{v} - u_i > \varepsilon$ by (*). Then her payoff is at most $-\varepsilon/2 < 0$. Thus, B is better off bidding $\hat{v} \leq \hat{u}$, given $\hat{u} = u_i$ at Stage 3. Hence, A can guarantee herself a utility of at least $u_i - \hat{u} + \varepsilon/2 = \varepsilon/2$ by simply bidding $\hat{u} = u_i$ at Stage 3. Moreover, we know that A will never lose, since that gives her 0. In other words, B always loses and her equilibrium payoff at Stage 3 is 0.

Now consider Stage 2. Will B say "agree" or "challenge?" If B says "challenge," she pays $\varepsilon/4$ and therefore nets a negative utility overall.

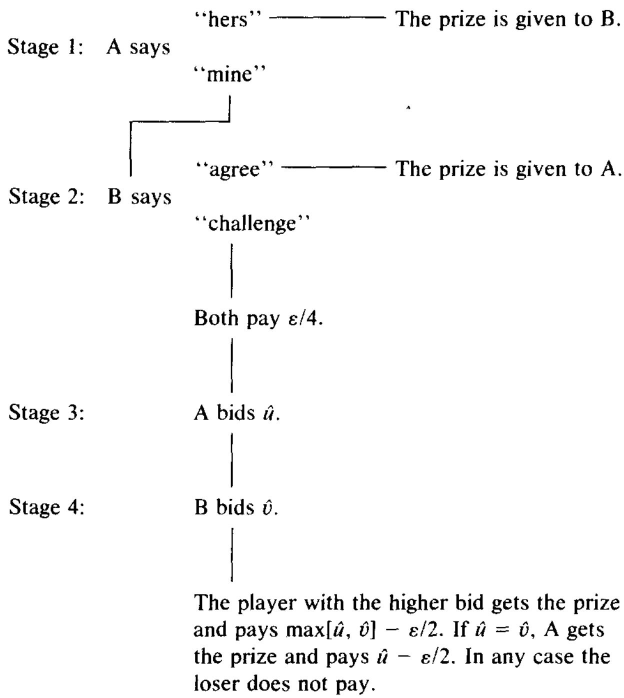

FIGURE 2

Therefore, B is better off saying "agree." Clearly, given that B says "agree" at Stage 2, A will say "mine" at Stage 1, and A gets the prize in equilibrium.

(2) Consider the case $u_i < v_j$. Consider the subgame beginning at Stage 3. In any subgame perfect equilibrium A's payoff is 0 and B's payoff is at least $\varepsilon/2$. First, note that A's equilibrium payoff cannot be negative, since she can bid $\hat{u} = u_i$. Can A ever have a strictly positive utility? Suppose A bids $\hat{u} \geq v_j$; i.e., A bids at least B's true valuation. Given $\hat{u} \geq v_j$, B never bids $\hat{v} > \hat{u}$. [If B did, her payoff was $v_j - \hat{v} + \varepsilon/2 \leq \hat{u} - \hat{v} + \varepsilon/2 < -\varepsilon/2 < 0$, by (*). But B could have 0 by bidding $\hat{v} \leq \hat{u}$.] Hence B bids $\hat{v} \leq \hat{u}$. Then A wins and gets a payoff of $u_i - \hat{u} + \varepsilon/2 \leq \hat{u} - v_j + \varepsilon/2$, which, by (*), is less than $-\varepsilon/2$. Therefore, A never bids $\hat{u} \geq v_j$. But given $\hat{u} < v_j$, B will bid some $\hat{v}$ such that $v_j \geq \hat{v} > \hat{u}$; thus $v_j - \hat{v} + \varepsilon/2 \geq \varepsilon/2$. In sum, A never wins and her payoff is 0, while B's payoff is at least $\varepsilon/2$.

Next, consider Stage 2. If B challenges, she pays $\varepsilon/4$ so that in equilibrium she still obtains a strictly positive payoff from "challenging" (since she gets at least $\varepsilon/2$ from Stage 3). Therefore, B will challenge at Stage 2. However, A has to pay $\varepsilon/4$ whenever B challenges. Moreover, A gets 0 at Stage 3. Thus A's unique best choice at Stage 1 is to say "hers," and the prize is given to B.

To summarize, we have shown that in any subgame perfect equilibrium, Player A gets the prize if $u_i \geq v_j$ and Player B gets the prize if $u_i < v_j$. In equilibrium, no player ever pays anything. Q.E.D.

It is readily verified that the low valuation player can never get the prize in the "bidding" subgame starting at Stage 3, while the high valuation player always gets the prize and her payoff is at least $\varepsilon/2$. The bidding game entry fee, $\varepsilon/4$, at Stage 2 (if B challenges) deters a low valuation player from entering Stage 3 but does not change the high valuation player's incentives in the bidding game. We finally note that Theorem 1 can easily be extended to three or more players. (This will be discussed in the Appendix for the more general case to be presented below.)

It must be noted that we have made use of the discrete nature of players' valuations—see (*) and players' strategy sets above. Clearly, when players' possible valuations are not discrete but are from distributions with continuous supports, there does not exist $\varepsilon > 0$ such that (*) is satisfied. Then the game form we have constructed may fail to implement King Solomon's choice rule. We now show how King Solomon can solve his problem in this circumstance.

Let Player A's valuation be from a (closed) interval U, and Player B's valuation be from a (closed) interval V. For simplicity, we assume $U = V$. Let $c = \min U$ and $d = \max U$. We normalize U and let $c = 0$. Again, King Solomon's objective is to award the prize to the player with the higher valuation. The following mechanism allocates the prize efficiently.

Stage 1: Player B announces a real number ε, where ε is in the interval $[0, d]$. If $\varepsilon = 0$, Player A gets the prize. Otherwise, proceed to Stage 2.

Stage 2: Player A says whether the prize is "mine" or "hers." If she says "hers," then the prize is given to Player B. If she says "mine," then proceed to Stage 3.

Stage 3: Player B may say "agree" or "challenge." If she says "agree," then Player A gets the prize. If she says "challenge," both players pay King Solomon ε, and then they proceed to the next Stage.

Stage 4: Player A announces a bid $\hat{u}$ from the set $[0, d]$ and pays $\hat{u}$.

Stage 5: Player B announces a bid $\hat{v}$ from the set $[0, d]$ and pays $\hat{v}$. The player with the higher bid gets the prize. If there is a tie, Player B gets the prize.

Let the true valuations of the players be u and v. The mechanism together with the utility functions of the players then define a game; call this game $G(u, v)$. Figure 3 illustrates the mechanism.

THEOREM 2. *Consider the game $G(u, v)$. (1) Suppose $u > v$. The unique subgame perfect equilibrium outcome awards the prize to Player*

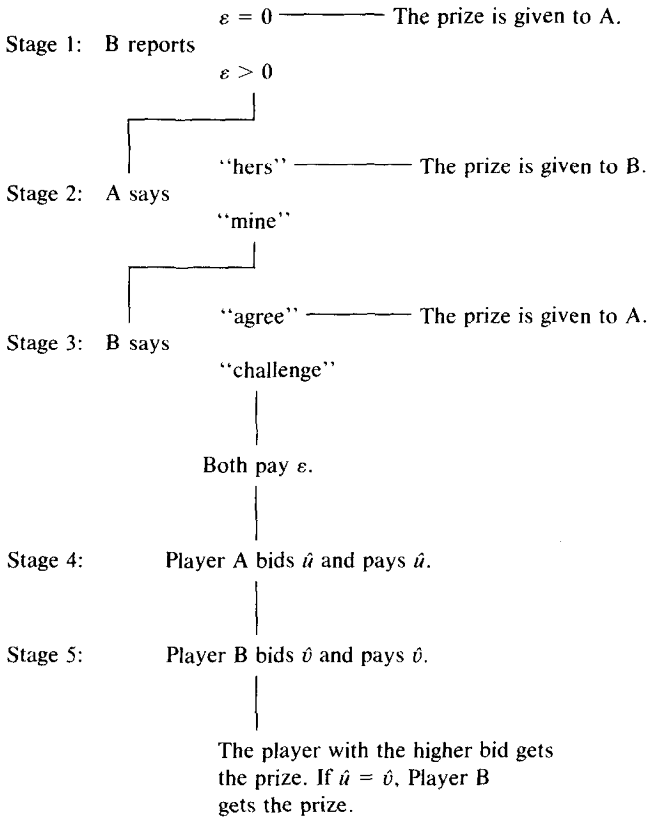

FIGURE 3

A. (2) Suppose $u < v$. The unique subgame perfect equilibrium outcome awards the prize to Player B. (3) Suppose $u = v$, either A or B may get the prize in subgame perfect equilibrium. At this equilibrium outcome, no player ever pays King Solomon anything.

Proof. (1) Consider the case where $u > v$; i.e., Player A should get the prize. The following are Player B's best responses at Stage 5 given that A has bided $\hat{u}$ at Stage 4. If $\hat{u} > v$, then B cannot get a positive utility from Stage 5, and she therefore bids $\hat{v} = 0$. However, if $\hat{u} < v$, B will certainly match A's bid ($\hat{v} = \hat{u}$), obtains the prize, and earns a positive payoff $v - \hat{v}$ from Stage 5. If $\hat{u} = v$, then B is indifferent between $\hat{v} = 0$ and $\hat{v} = \hat{u} = v$ and these are her only best choices.

Consider the subgame beginning at Stage 4. The following are the unique subgame perfect equilibrium strategies: Player A bids $\hat{u} = v$ at Stage 4, Player B bids $\hat{v} = \hat{u}$ if $\hat{u} < v$ and $\hat{v} = 0$ if $\hat{u} \geq v$ at Stage 5. It is obvious that these strategies constitute a subgame perfect equilibrium. We now show that these are unique subgame perfect equilibrium strategies. Consider Player B first. The argument in the last paragraph establishes that B's best responses against $\hat{u} < v$ and $\hat{u} > v$ are unique and are given by his strategy just described. Hence these must also be his moves (conditional on $\hat{u} < v$ or $\hat{u} > v$) in any subgame perfect equilibrium. We now show that in any subgame perfect equilibrium, B always bids $\hat{v} = 0$ against $\hat{u} = v$. Suppose not; i.e., suppose in a subgame perfect equilibrium, in response to A's bid of $\hat{u} = v$, B chooses $\hat{v} = v$ and $\hat{v} = 0$ with probabilities $1 \geq p > 0$ and $1 - p$, respectively. Consider Player A. His payoff for bidding $\hat{u} = v$ is now $p(-\hat{u}) + (1 - p)(u - \hat{u}) = (u - v) - pu$. Consider now the strategy of bidding $\hat{u} = v + \eta$. For $\eta > 0$, B's unique best response is $\hat{v} = 0$ and Player A receives the prize. Hence A's payoff is $u - v - \eta$. For η small enough, $u - v - \eta > u - v - pu$, since $p,u > 0$. However, for any $\eta > 0$ (and small enough), Player A bidding $v + \eta$ cannot be an equilibrium strategy: bidding $v + \eta/2$ will win the prize for certain with a smaller payment. Contradiction. Hence in any subgame perfect equilibrium, B always bids $\hat{v} = 0$ against $\hat{u} = v$. It is then clear that the pair of strategies described forms the unique subgame perfect equilibrium, since A's strategy of $\hat{u} = v$ is the unique equilibrium response.

Next, we consider Stage 3. Clearly, Player B's best choice is to say "agree"; if she says "challenge," she incurs a cost $\varepsilon > 0$ and gets 0 from Stage 4.

Given the strategies in subsequent stages, saying "mine" is Player A's unique best choice at Stage 2. Moreover, any announcement of ε is optimal for B at Stage 1.

In sum, in a subgame perfect equilibrium, A says "mine" at Stage 2 (if reached) and bids $\hat{u} = v$ at Stage 4; B says any ε at Stage 1, "agree" at Stage 3 (if reached), and at Stage 5 bids $\hat{v} = 0$. In equilibrium, Player A gets the prize and no payment is ever made.

(2) Consider the case where $v > u$; i.e., Player B should get the prize. Consider the subgame starting at Stage 4. We argue that in subgame perfect equilibrium, Player A must bid $\hat{u} = 0$. From the optimal strategies of Player B at Stage 5 described in (1), Player A can never get the prize and obtain a nonnegative utility in equilibrium. Thus Player A's optimal move at Stage 4 is to bid $\hat{u} = 0$. Hence, Player B gets the prize and her utility from the subgame at Stage 4 is v.

At Stage 3, B will say "challenge" if and only if $\varepsilon \leq v$—she gets v from Stage 4 and pays ε at Stage 3 when she challenges. Next, consider Stage 2. It is obvious that Player A will say "hers" if and only if B challenges at Stage 3. (Player A gets 0 from Stage 4 and must pay $\varepsilon > 0$ at Stage 3 if Player B challenges.)

Now, B clearly prefers A to say "hers" in order to get her maximum utility v. Note that Player B can always announce $0 < \varepsilon \leq v$ at Stage 1. Therefore, in subgame perfect equilibrium, B will say $0 < \varepsilon \leq v$ at Stage 1 and "challenge" at Stage 3. Given B's strategy, A's best choice is to say "hers" at Stage 2.

To summarize, in a subgame perfect equilibrium, A says "hers" at Stage 2 and, if reached, bids $\hat{u} = 0$ at Stage 4; B says ε, $0 < \varepsilon \leq v$, at Stage 1, "challenge" at Stage 3 (if reached), and bids $\hat{v} = 0$ at Stage 5. In equilibrium, Player B gets the prize and no payment is ever made.

(3) Consider the case where $u = v$; i.e., either Player A or Player B may get the prize. Consider the subgame at Stage 5. Player B's best responses are given by (1). Next, consider the subgame at Stage 4. We first show that in any subgame perfect equilibrium, Player B must bid $\hat{v} = 0$ in response to A's bid of $\hat{u} = u$ ($= v$) at Stage 4. Note that A can always guarantee a utility of 0 at Stage 4 by bidding $\hat{u} = 0$. Now if against $\hat{u} = u$ ($= v$), B chooses $\hat{v} = v$ with positive probability, then A will get a strictly negative payoff, which contradicts the fact that A's equilibrium payoff must be nonnegative. Therefore, in a subgame perfect equilibrium, B must bid $\hat{v} = 0$ against $\hat{u} = u$ ($= v$). It can easily be shown that for each $\theta \in [0, 1]$, the following is a subgame perfect equilibrium at Stage 4: Player A chooses $\hat{u} = 0$ and $\hat{u} = u$ with probabilities θ and $1 - \theta$ respectively, while Player B bids $\hat{v} = \hat{u}$ if $\hat{u} < v$ and bids $\hat{v} = 0$ if $\hat{u} \geq v$. Note that for each θ (i.e., in each equilibrium), A's equilibrium payoff is 0, while B's equilibrium payoff is θv. For later use, call a subgame perfect equilibrium (at Stage 4) in which A chooses $\hat{u}$ with probability $\theta = \hat{\theta}$, a "$\theta = \hat{\theta}$ equilibrium."

It is straightforward to verify that the set of subgame perfect equilibrium of the whole game is characterized by θ, where $\theta \in [0, 1]$. For $\theta = 0$, the equilibrium is: Player B announces any $\varepsilon \geq 0$ at Stage 1. Player A says "mine" at Stage 2 and Player B says "agree" at Stage 3. The continuation equilibrium at Stage 4 is a "$\theta = 0$ equilibrium." For $\theta = \hat{\theta} > 0$, the equilibrium is: At Stage 1, Player B announces any ε, where $0 < \varepsilon \leq \hat{\theta} v$. Player A says "hers" at Stage 2 and Player B says "challenge" at Stage 3. The continuation equilibrium at Stage 4 is a "$\theta = \hat{\theta}$ equilibrium."

To conclude, when $u = v$, either A or B may get the prize, and no player pays anything in equilibrium. Q.E.D.

Theorem 2 can be easily extended to the case of $N \geq 3$ players. In the Appendix, we have written the mechanism that implements the efficient prize allocation rule for $N \geq 3$ players. We wish to note that the mechanism in Theorem 2 is more general than that in Theorem 1, in that (*) is not required. We have chosen to discuss also the discrete case because we are able to construct a simpler mechanism (it has one stage less than the mechanism in Theorem 2).

3. Conclusion

This paper has studied the following situation: A planner is interested in allocating an indivisible good or project, which we call a "prize," to one of many agents in the economy. His objective is to give the prize to the agent who values it most, without any payments being made by the recipient. The planner, however, does not know the identity of this agent, while the agents themselves do. We have shown how the planner can construct simple, multistage mechanisms with a unique subgame perfect equilibrium outcome. At this outcome, the agent who values the prize most gets it without any transfer of money being made by any of the agents or the planner. In each of the mechanisms we construct, the players move sequentially and have perfect information about all previous moves. In these games the solution concept we use, subgame perfect equilibrium, is natural and intuitive.

We have assumed that the agents have complete information. Even though this may be a realistic assumption in some contexts and is in conformity with a number of recent papers in implementation, we feel that the simple mechanisms we have proposed (or some modifications of them) may work well under less restrictive assumptions, for example, when agents know who has the highest valuation but not the exact values, or when each agent knows only whether his valuation is highest or not. These extensions await future research.

Appendix

We now describe a mechanism that implements the efficient prize allocation choice rule when there are $N \geq 3$ players. We retain the assumption used in Theorem 2: each player's possible valuation is from the interval $[0, d]$.

Stage 0: Each player k, $k = 2, \ldots, N$, announces a real number ε_k from the interval $[0, d]$. Let $\varepsilon \equiv \min(\varepsilon_k, k = 2, \ldots, N)$. If $\varepsilon = 0$, the prize is awarded to Player 1. Otherwise, proceed to Stage i, where $i = 1$.

Stage *i:* Player i says whether the prize is "mine" or "not mine." If she says "not mine," then proceed to Stage $i + 1$, $i = 1, \ldots, N - 2$. If she says "mine," then proceed to Stage $i.i + j$, where $j = 1$. If at Stage N-1, Player N-1 says "not mine," then Player N gets the prize.

Stage $i.i + j$: Player $i + j$ says "challenge" or "not challenge." If Player $i + j$ says "not challenge" and $i + j + 1 \leq N$, then proceed to Stage $i.i + j + 1$; if Player $i + j$ says "not challenge" and $i + j = N$, then Player i gets the prize. If Player $i + j$ says "challenge," Player i and Player $i + j$ each pays ε. Then they proceed to game $\gamma(i, j)$.

$\gamma(i, j)$: Player i bids $\hat{u}$ from $[0, d]$ and pays $\hat{u}$. Then Player $i + j$ bids $\hat{v}$ from $[0, d]$ and pays $\hat{v}$. The player with the higher bid gets the prize. If there is a tie, Player $i + j$ gets the prize.

In the above mechanism each player is asked in sequence to claim the "prize" or not. Whenever a player claims, then another player may challenge her. This will lead the two players to play the sequential bidding game as described in Stages 4 and 5 of the mechanism in Theorem 2.

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