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SHARING PRODUCTIVE KNOWLEDGE IN INTERNALLY FINANCED R & D CONTESTS*

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We examine the optimal design of two-stage research and development (R & D) joint ventures. At the second stage, researchers choose R & D effort levels independently in an attempt to achieve an innovation. In the first stage, researchers have an opportunity to share endowments of productive knowledge. Initial pecuniary resources are limited, so rewards for disclosing knowledge and succeeding at the second stage must be financed from successful innovation.

We derive conditions under which full sharing of knowledge and the socially desired levels of R & D effort can be motivated, and examine the optimal incentive structure when this ideal outcome cannot be implemented: full sharing will always be motivated at the first stage, but inefficient R & D effort will be induced to foster information sharing.

I. INTRODUCTION

THE DESIGN of incentive schemes to motivate research and development (R & D) effort has received considerable attention in the literature. (Early works that focused on the incentives created by markets include those of Dasgupta and Stiglitz [1980] and Loury [1979]. More recent work that examines the value of coordinating research through joint ventures includes that of Gandal and Scotchmer [1989] and Katz [1986].) A related issue that has received comparatively little analysis is the possibility that researchers might be motivated to share productive knowledge with each other before they undertake R & D effort. (Exceptions are Bhattacharya and Ritter [1983] and Grossman and Shapiro [1987].) Given the prevalence of joint research ventures in the world economy today, the possibility of sharing relevant knowledge seems to be an important one to explore. In this research, we determine how best to motivate sharing of productive knowledge among competing researchers to maximize *ex ante* welfare.

Sharing productive knowledge with subsequent competitors in an R & D race makes them more formidable opponents. Thus, incentives to share knowledge may be limited unless a researcher is compensated directly for the knowledge he shares with others. In fact, it can be shown that if researchers

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have sizable initial endowments of wealth, rules can be designed (involving payments from the recipients of knowledge to the donors) which ensure that: (1) all productive knowledge is fully shared among researchers initially; and (2) all researchers subsequently put forth the level of effort that is most preferred from the social point of view.[1]

In practice, though, the initial wealth of reseachers will be limited, and their access to capital markets will be restricted (in part because of the unavoidable losses that occur when technological information is disclosed to potential financiers.) Therefore, incentives for the sharing of knowledge may have to be provided by rules which define how the total gain from a successful R & D venture is divided among the researchers. Intuitively, researchers who provide more information must be promised a greater share of the profits from "success" if sharing of knowledge is to be induced. A potential problem is introduced by such sharing rules, however. Those researchers who stand to gain substantially from a successful innovation under the proposed rules may exert too much R & D effort (relative to the social optimum), while those with little to gain may exert too little effort. This potential conflict is the focus of our analysis.

We find that the conflict is not always debilitating. We derive necessary and sufficient conditions for the First-Best outcome to be attainable, even when researchers have *no* initial endowments of wealth. Under the First-Best outcome, all information is shared among researchers, and each subsequently exerts the socially optimal level of R & D effort. We also derive the properties of the Second-Best optimal R & D contest when the First-Best outcome is not feasible. One interesting feature of this contest is that full sharing of knowledge is always induced. Hence, any distortions that arise will occur at the second stage, where R & D effort decisions are made. It is also the case that the entire surplus generated by a successful innovation will not always be distributed to the researchers under the optimal contest. Limited distribution can be optimal to avoid excessive R & D effort.

The "optimal" R & D contest we characterize is ideal in the following sense. The rules we derive are selected to maximize the expected social returns from R & D net of the costs of R & D effort expended by the researchers. Expectations are with respect to the unknown knowledge endowments of the researchers. Thus, our welfare criterion is an *ex ante* one, and our concern is with rules that provide the best incentives from a social perspective.[2] One might think of

[1] This conclusion is proved in Bhattacharya, Glazer and Sappington [1990]. The focus of that paper is precisely on when this ideal (First-Best) outcome can be ensured in a fairly general environment by two exogenously specified incentive schemes. In contrast, the present paper examines the nature of the optimal incentive scheme, and focuses on its properties when the First-Best outcome cannot be ensured.

[2] An alternative perspective would be the private one, where incentive schemes are designed to maximize the expected profits of the researchers. Some thoughts on this perspective are offered in section VI.

our approach as characterizing the optimal policy for a government agency charged with directing the activities of a joint research venture. The agency is permitted to determine how realized gains are distributed among the members of the joint venture, but cannot tax other sectors of the economy to subsidize the joint venture participants. Regulators seldom have such power to tax in the United States.

Our analysis proceeds as follows. In section II, the model under consideration is described in detail, and a formal statement of the social problem is presented. Two benchmark solutions to the problem are recorded in section III. The first is the solution in which both knowledge levels and R & D effort levels are observed publicly. The second is the solution when there are no restrictions on the initial wealth of the research "team". The First-Best outcome is feasible in both these settings. A third benchmark solution is considered in section IV. There it is assumed that knowledge endowments are publicly observed, but R & D effort levels are not. We derive the maximum level of expected profit that can be guaranteed to the researcher who reveals the most knowledge without inducing him to undertake too much R & D effort, and without inducing others to undertake too little R & D.

These benchmark solutions help motivate the solution to the actual problem of interest, in which knowledge endowments and R & D effort levels are privately observed and initial wealth is limited. The circumstances under which the First-Best outcome can be achieved in this setting are reported in section V. The Second-Best outcome is also characterized there.

To demonstrate more clearly when full sharing of knowledge is optimally motivated, we consider a setting in section VI where the ability to structure rewards in the second stage of the R & D contest is constrained by *ex post* (product market) competition. In the setting considered, some sharing of information can be motivated, but full sharing is not feasible. One implication of the analysis in section VI, which has relevance for the optimal structuring of R & D joint ventures, is that even though "winner-take-all" R & D competition may provide the proper incentives for R & D effort (as noted by Sah and Stiglitz [1987]), it will not provide the proper incentives for sharing of valuable knowledge in all circumstances. Concluding remarks are offered in section VII, where we emphasize the links between our findings and related investigations of R & D joint ventures and technology licensing.

II. THE MODEL

For simplicity, we focus on the case where there are only two researchers (A and B). Each researcher is endowed with a level of knowledge (y^A and y^B, respectively) at the start of the game. This level of knowledge is private information for each researcher. To avoid non-concavity problems that arise in more general settings, we assume there are only two possible levels of knowledge, y_1 and y_2, with $y_1 < y_2$.[3]

Greater knowledge is valuable in that it reduces the private cost to the researcher of achieving any given probability of successful innovation. Intuitively, there is a particular project on which the researchers are engaged to work. The social value of succeeding on the project (e.g. of achieving the sought-after innovation) is V. The social value is the same whether one or both researchers succeed. Potentially, this value can be appropriated fully by the research team to reward its members. These properties will hold, for example, when the innovation is a cost reduction and the aggregate demand curve for the product in question is inelastic.

The process of research and development is stochastic, and involves a one-time choice of R&D effort by each researcher.[4] We let P represent the probability that a researcher will realize the innovation (i.e. "succeed"). This probability is an increasing function of the researcher's effort. $C(P, y)$ denotes the personal cost to the researcher of achieving a success probability of P when his operating knowledge is y. This personal cost is an increasing, convex function of P (i.e. $C_P(P, y) > 0$ and $C_{PP}(P, y) > 0 \;\forall P > 0$). It is assumed costless to achieve a zero probability of success (i.e. $C(0, y) = 0$). Furthermore, to ensure interior solutions to the problems we explore, we presume $C_P(0, y) = 0$ and $C_P(1, y) > V$.

With the higher level of knowledge (y_2), both the total and marginal cost of achieving any probability of success is lower, i.e. $C(P, y_2) < C(P, y_1)$ and $C_P(P, y_2) < C_P(P, y_1) \;\forall P > 0$. This set-up implies that a higher level of knowledge can be thought of as greater ability to distinguish between good and bad research techniques, where it is equally costly for the researcher to experiment with any technique in an attempt to succeed. To illustrate, consider the current R&D race to develop superconductors. In this race, the knowledge that a researcher can share might be a list of the materials that are best suited to be superconductors. A researcher can reveal the entire list of materials he has tested and his findings. Alternatively, he can reveal only part of his knowledge by disclosing only some of the materials he has tested, or by reporting a large set of candidate materials that includes the set he knows to be good superconductors.

The problem we are concerned with is best thought of as a two-stage game. In the first stage, each researcher makes a private representation of his knowledge endowment to a "benevolent" and "intelligent" third party, who

[3] The essential problem is that the marginal cost to a researcher of revealing his knowledge may increase with the level of his knowledge. Consequently, the necessary conditions for an optimum may not be sufficient when a continuum of knowledge levels is admitted. (See Bhattacharya, Glazer and Sappington [1990] for details.) This problem is avoided in our binary setting.

[4] Thus, we do not consider R&D races where researchers can adjust their effort over time according to their position in the race. Fudenberg *et al.* [1983], Grossman and Shapiro [1986, 1987], Harris and Vickers [1985, 1987], and Judd [1985] examine races of this type.

we will call the "coordinator". (In the context of joint ventures on superconductors and supercomputers, this coordinator might be a representative of the US government.) The coordinator is intelligent in the sense that she can (costlessly) verify whether a claim of high knowledge, y_2, is a truthful claim. If instructed to do so, this coordinator can also share any fraction of the high knowledge with the researcher who is initially endowed (or who claims to be endowed) with the lower level of knowledge. The coordinator is benevolent in that she need not be motivated to act as instructed. This assumed benevolence allows us to focus on the problem of motivating researchers, although the problem of motivating "expert consultants" of the type we introduce here remains an important area for future research. What the coordinator cannot do is verify whether a claim of "low knowledge" is true or false. Thus, although the coordinator can determine whether a proposed method for discriminating among research techniques is effective, she cannot discern whether a researcher is understating his ability to discriminate among techniques. Given the presence of this coordinator and a fixed number of researchers, it is apparent that our analysis is more relevant in the context of a prenegotiated research joint venture than a market setting with free entry and exit by firms.

The second stage of the game under consideration is the stage at which research and development actually takes place. This stage is assumed to last for a fixed period of time, during which each researcher either fails (i.e. achieves no social gain) or succeeds (i.e. realizes the innovation of value V).[5] At the start of the second stage, the researchers choose, simultaneously and independently, an immutable level of R & D effort, or equivalently, a probability (P) of success. Each researcher chooses his success probability to maximize the difference between expected payment and personal cost. Expected payments are determined by the announced incentive structure, which specifies rewards contingent on initial disclosures and ultimate successes.

Ideally, the researchers should share all their knowledge at the first stage, and then choose the R & D efforts at the second stage that maximize the expected social return net of costs. Of central concern in this paper is when this ideal (i.e. *First-Best*) outcome can be achieved even though: (i) each researcher's knowledge endowment is private information; (ii) the R & D effort undertaken by the researchers is unobservable; and (iii) initial wealth endowments are zero, so all incentives must be created through sharing the gain (V) of a successful innovation. Another potential concern is that of ensuring voluntary participation in the R & D venture by the researchers. For simplicity, we represent this concern by assuming that their net payoffs must exceed their reservation expected profit level, which is normalized to zero.

[5] This simplification, relative to a continuous-time R & D race, allows us to analyze in greater detail the externalities that arise when information is shared.

Rewards to each researcher can be based upon whether his counterpart succeeds, and upon the levels of knowledge initially reported by the two researchers. We let B, S, and F denote payments to a researcher when both researchers succeed, when he is the only one to succeed, and when he is the only one to fail at the second stage, respectively. Superscripts 1, 2, H, and L refer to first-stage announcements of knowledge. 1 (respectively, 2) implies that both researchers claimed to have low (respectively, high) knowledge, y_1 (respectively, y_2). H (respectively, L) denotes the researcher who revealed a higher (respectively, lower) level of knowledge than his counterpart. Thus, for example, S^L is the payment to the researcher who successfully develops the innovation alone after reporting less knowledge than his counterpart.[6]

In addition to these payment variables, there is another policy instrument that may influence the behavior of the researchers. This instrument is the fraction of the higher knowledge (y_2) revealed at the first stage that is transferred to the researcher who reported low knowledge (y_1). Full sharing of knowledge can lower the social cost of achieving any aggregate probability of success; but a natural incentive is for researchers to conceal some of their knowledge, and thereby secure a competitive advantage at the second stage. We let n represent the amount of knowledge transferred to the researcher who originally reports y_1 at the first stage when his counterpart reveals the higher knowledge level, y_2. n can take on any value in the interval $[y_1, y_2]$.

Before proceeding to a formal statement of the social problem, we briefly review the information structure and timing in the model. First, each researcher privately learns his knowledge endowment. Second, an incentive scheme is designed to maximize expected net social welfare. The scheme describes how the value (V) of a successful innovation will be distributed as a function of: (1) reported knowledge levels; and (2) which of the researchers succeed. The *ex ante* probability that a researcher is endowed with knowledge y_i is $\phi_i \in (0,1)$ with $\phi_1 + \phi_2 = 1$.[7] Third, each researcher makes a private report to the coordinator about his knowledge endowment. The coordinator verifies any report of high knowledge, y_2, and transfers knowledge $n \in [y_1, y_2]$ if the reported knowledge levels differ. Then, the researchers choose simultaneously and independently a level of R & D effort. Success or failure by each researcher is then observed, and payments are made as promised.

Formally, the social problem $[SP]$ is the following;

$$\underset{P,B,F,S,n}{\text{Maximize}} \sum_{i=1}^{2} (\phi_i)^2\{[2P^i - (P^i)^2]V - 2C(P^i, y_i)\}$$
$$+ 2\phi_1\phi_2\{[P^L + P^H - P^L P^H]V - C(P^L, n) - C(P^H, y_2)\}$$

[6] Implicit in this formulation are two restrictions. First, randomization in payoffs is not permitted. Second, attention is focused on symmetric schemes. Thus, if both agents report the same level of knowledge at the first stage and both subsequently succeed, they receive the same payment.

[7] Thus, y^A and y^B are independent and identically distributed random variables.

subject to:

(1) $$\phi_1\{P^H[P^L B^H + (1-P^L)S^H] + [1-P^H]P^L F^H - C(P^H, y_2)\} + \phi_2\{P^2[P^2B^2 + (1-P^2)S^2)] + [1-P^2]P^2F^2 - C(P^2, y_2)\} \geqslant \phi_1\{\hat{P}^1[P^1B^1 + (1-P^1)S^1] + [1-\hat{P}^1]P^1F^1 - C(\hat{P}^1, y_2)\} + \phi_2\{\hat{P}^L[P^H B^L + (1-P^H)S^L] + [1-\hat{P}^L]P^H F^L - C(\hat{P}^L, y_2)\};$$

(2) $$P^i[B^i - F^i] + [1-P^i]S^i - C_p(P^i, y_i) = 0 \qquad i = 1, 2;$$

(3) $$P^1[B^1 - F^1] + [1-P^1]S^1 - C_P(\hat{P}^1, y_2) = 0;$$

(4) $$P^H[B^L - F^L] + [1-P^H]S^L - C_P(P^L, n) = 0;$$

(5) $$P^H[B^L - F^L] + [1-P^H]S^L - C_P(\hat{P}^L, y_2) = 0;$$

(6) $$P^L[B^H - F^H] + [1-P^L]S^H - C_P(P^H, y_2) = 0;$$

(7) $$F^i + S^i \leqslant V \qquad i = 1, 2;$$

(8) $$F^L + S^H \leqslant V;$$

(9) $$F^H + S^L \leqslant V;$$

(10) $$B^i \leqslant \frac{1}{2}V \qquad i = 1, 2;$$

(11) $$B^L + B^H \leqslant V;$$

(12) $n \leqslant y_2$; and

(13) $n \geqslant y_1$.

The objective in [SP] is to maximize expected social returns (consumer plus producer surplus) less research and development costs. The truthful disclosure constraint (1) ensures that the researcher with high knowledge will truthfully report his knowledge to the coordinator, rather than claim to have low knowledge. Our focus is on the separating Bayesian–Nash equilibrium in which truthful revelation of knowledge levels is a best response.[8] The effort selection constraints (2)–(6) identify the profit-maximizing choices of success probabilities by the researchers. $\hat{P}^1$ and $\hat{P}^L$ represent the off-equilibrium success probabilities that would be chosen by a researcher were he to

[8] By a "separating" equilibrium, we mean that at the start of the second stage, each researcher knows exactly the level of knowledge his opponent possesses. The equilibrium will be separating if, for example, the coordinator decided to ensure that all knowledge is shared. The same will be true even if no transfer of knowledge occurs, but the coordinator truthfully reports the knowledge levels of the researchers. In contrast, in a "pooling" equilibrium, each researcher enters the second stage with his prior beliefs about the knowledge level of his competitor intact; the coordinator neither shares any knowledge nor does she reveal anything about the true capabilities of the researchers. In footnote 16, we explain our focus on the separating equilibrium by showing that it is always preferred to the pooling equilibrium.

misrepresent his knowledge level as y_1, and if his counterpart were to truthfully report his own knowledge level as y_1 and y_2, respectively. Constraints (7)–(11) are the internal financing constraints. They guarantee that the sum of the payments to the two researchers do not exceed the realized value of their efforts. Of course, all payments must also be non-negative. Constraints (12) and (13) simply identify the feasible range of knowledge levels that might be shared with the researcher who is initially disadvantaged.

III. TWO BENCHMARK SOLUTIONS

We begin to analyze the solution to $[SP]$ by examining two benchmark solutions. The first benchmark is labeled the First-Best solution. It represents the outcome that would be implemented if the researchers' knowledge endowments as well as their second-stage R & D effort levels could be observed publicly. In this ideal solution, full sharing of knowledge will be implemented, and each researcher will be directed to put forth the level of effort that equates his marginal cost of effort with the marginal expected social benefit. Formally, we have:

Definition. Under the First-Best solution,

(i) $n = y_2$; and
(ii) $[1-P^{*i}]V = C_P(P^{*i}, y_i)$

where P^{*i} is the level of R & D effort put forth by both researchers, and

$$y_i = \max\{y^A, y^B\}.$$

Throughout the ensuing analysis, we use an asterisk to denote First-Best levels of R & D effort. Notice that the marginal social benefit of effort by any researcher includes only the value of a successful innovation when the other agent has failed to innovate (which occurs with probability $(1-P)$). We assume throughout that it is always optimal to have both researchers participate in research and development activities at the second stage.

The second benchmark we consider is that in which no internal financing constraints are imposed. In this hypothetical setting, the research team has access to personal wealth and/or perfect capital markets. This access enables agents to make transfer payments to each other whether or not V is realized; it also allows payments to the two researchers that exceed the value of a successful innovation.

Absent any internal financing constraints, the First-Best solution can always be implemented. This observation is recorded formally as Proposition 1.

Proposition 1. If constraints (7)–(11) are not imposed, then the First-Best solution is a feasible solution to $[SP]$.

The intuition behind the finding is the following.[9] There are two incentive problems to deal with: full sharing of knowledge must be motivated, and efficient levels of R & D intensity must be induced. The latter problem is readily resolved by compensating an agent for a successful innovation only when he succeeds alone.[10] This payment structure internalizes for each agent the fact that the social payoff is no greater when both researchers succeed than when only one researcher succeeds. The former problem can also be resolved even when one's knowledge endowment is private information if the researcher who reveals the higher level of knowledge is awarded the entire social surplus. More precisely, the agent (A) who claims to have less knowledge than his counterpart (B) must pay agent B an amount equal to A's expected profit from sharing B's knowledge and participating in the second-stage R & D competition. In addition, B receives an up-front payment equal to $P^{*2}V$, which is the total expected second-stage surplus less expected payments to the researchers.

Thus, when researchers are endowed with funds or can borrow them without disclosing their private knowledge, the two stages of information disclosure and R & D competition can effectively be decoupled, allowing the incentive problems at both stages to be resolved. Such resolution will not always be possible, however, when initial wealth is limited and external financing necessarily involves disclosure of public knowledge that can be used by competitors.

IV. PUBLICLY-OBSERVED KNOWLEDGE LEVELS

In this section, we consider a third benchmark solution. In the setting considered here, no external financing is available and the R & D effort levels of the researchers are not publicly observed. However, in contrast to the general problem [SP] under consideration, we assume that the knowledge levels with which the researchers are endowed are common knowledge. Our concern is with how generously the researcher with greater knowledge can be rewarded for sharing his knowledge with his second-stage competitor, while still ensuring the First-Best levels of R & D effort.

It is apparent that when the two researchers initially share the same level of knowledge (y_i), the First-Best solution can be implemented. In this setting, the problem of motivating information sharing does not arise, and efficient effort levels can be readily induced via $S^i = V$, $B^i = F^i = 0$, for example. By

[9] Proposition 1 is proved in an expanded setting in our [1990] companion paper. As noted above, that paper proposes two exogenously specified incentive schemes and examines when the schemes are able to ensure the First-Best outcome. In contrast, the present paper examines a simpler setting, but undertakes a more ambitious task. Here, we determine precisely when the First-Best solution is attainable, and most importantly, characterize fully the properties of the Second-Best solution when internal financing constraints preclude attainment of the First-Best.

[10] This observation is recorded in Sah and Stiglitz [1987].

compensating researchers with the full value of their efforts only when they succeed alone, the socially desirable levels of R & D effort will be induced. Observe that with this particular incentive scheme, not all of the surplus is distributed to the researchers; they receive nothing in the event of "ties". Other schemes can be designed, however, which induce the desired effort levels from the researchers and distribute *all* the surplus to them. One such scheme has $F^i = \frac{1}{2}P^{*i}V$, $B^i = \frac{1}{2}V$, and $S^i = [1-\frac{1}{2}P^{*i}]V$. In fact, this incentive scheme can be shown to provide the greatest expected joint profits for the two researchers among all schemes that induce First-Best success probabilities.

The more interesting case in this setting with publicly-observed knowledge endowments is where the two researchers initially possess different levels of knowledge. Here a potential conflict arises between motivating efficient second-stage effort levels and rewarding the researcher who supplies his competitor with productive information. Too generous a reward may induce him to undertake too much R & D effort and his competitor to undertake too little; but too scant a reward for sharing knowledge may induce too little sharing.

Lemma 1. Suppose it is common knowledge that $y^A = y_1$ and $y^B = y_2$. Then the incentive scheme that ensures First-Best success probabilities (P^{*2}) and full sharing of knowledge, and also provides the greatest expected profit to agent B has the following properties:

$$F^H = B^H = P^{*2}V; \quad S^H = V; \quad F^L = 0; \quad B^L = S^L = [1-P^{*2}]V$$

Notice that all realized surplus is distributed to the researchers under the scheme described in Lemma 1. The researcher (B) initially endowed with superior knowledge is rewarded the entire social gain if he is the only one to innovate successfully. He also receives $P^{*2}V$ whenever A succeeds, even if B fails. Thus, as a "bonus" for sharing his superior knowledge with his counterpart, agent B is insured to some extent against failure. Researcher A receives positive compensation $[1-P^{*2}]V$ only when he succeeds, whether it be alone or together with B. Under the incentive scheme reported in Lemma 1, the expected profit of researcher B is equal to his total (as opposed to incremental) contribution to expected social surplus.

Lemma 2. Under the conditions of Lemma 1, the maximum expected profit that can be afforded researcher B while still ensuring the First-Best outcome is $P^{*2}V - C(P^{*2}, y_2)$.

We now proceed to section V, where the researchers' knowledge endowments, as well as their R & D effort levels, are privately known.

V. SOLUTION TO THE SOCIAL PROBLEM

In this section, we derive the solution to the social problem [SP] recorded in section II. Recall that in this setting, each agent's endowment of knowledge is known only to himself. Thus, the researcher with superior knowledge must be motivated to reveal his greater capabilities if sharing of knowledge is to occur. Despite this additional complication and the internal financing requirements, it may be possible to implement the First-Best solution in this setting. And even when the First-Best is not feasible, it will still be optimal to motivate full sharing of knowledge.

We develop these findings with an analysis that parallels the analysis in section IV. First, in Lemma 3, we identify the incentive scheme that provides the greatest incentive for revealing one's high knowledge among all schemes that share all information that is revealed and that induce First-Best success probabilities. Then, in Proposition 2, we present a necessary and sufficient condition for feasibility of the First-Best solution. Finally, in Proposition 3, we characterize the incentive scheme that is optimal when the First-Best solution cannot be implemented.

Lemma 3. Among all feasible solutions to [SP] that implement First-Best success probabilities and sharing of all knowledge, the one that provides the greatest increment in profit to the researcher with high knowledge for truthfully revealing his knowledge has the following properties:

$$F^L = 0, \quad F^H = P^{*2}V,$$

$$B^L = [1-P^{*2}]V, \quad B^H = P^{*2}V,$$

$$S^L = [1-P^{*2}]V, \quad S^H = V;$$

$$F^2 = \frac{1}{2}P^{*2}V, \quad F^1 = 0,$$

$$B^2 = \frac{1}{2}V, \quad B^1 \in \left[0, \min\left\{\frac{1}{2}V, \frac{[1-P^{*1}]}{P^{*1}}V\right\}\right],$$

$$S^2 = \left[1-\frac{1}{2}P^{*2}\right]V; \quad S^1 = V - \frac{P^{*1}}{[1-P^{*1}]}B^1.$$

The strong parallels between Lemmas 1 and 3 warrant mention. Whether knowledge endowments are publicly or privately observed, the same payment structure provides the maximum incentive for the (revelation and) sharing of knowledge with a "less able" competitor. In particular, the researcher who reveals superior knowledge is awarded the entire surplus if he subsequently succeeds alone. He is also insured against his own failure, provided his counterpart succeeds. Of course, when knowledge endowments are privately observed, the researcher with high knowledge cannot be certain whether his knowledge is superior to that of his counterpart. Should the two researchers both report high knowledge, they are both insured in the event they fail alone, although each receives only half of what he would receive if his knowledge were strictly superior to that of his counterpart. Furthermore, the two researchers split the entire surplus if they both succeed, and the reward for succeeding alone is not the entire surplus, but something more than half of it.

To best dissuade the researchers from understanding their knowledge endowments, it is optimal to withhold all payment from the researcher who claims to have the lower level of knowledge and subsequently fails alone (i.e. $F^1 = F^L = 0$). By construction, sufficient payment is made to ensure the implementation of First-Best success probabilities, but the payment is made to the researcher who benefits from the sharing of knowledge only when he eventually succeeds.

Notice that all realized surplus is distributed to the two researchers whenever one or both of them initially reveal the higher level of knowledge. Full distribution of the realized surplus does not occur, however, when both researchers claim to be endowed with low knowledge.

Lemma 3 describes the incentive scheme that provides the greatest incentive for truthful revelation of high knowledge among all schemes that ensure full sharing of knowledge and First-Best R & D effort levels. A question that arises is when this maximum incentive is sufficient. Proposition 2 provides the answer to this question.

Proposition 2. The First-Best solution is a feasible solution to $[SP]$ if and only if:

$$(14) \qquad [P^{*2}V - C(P^{*2}, y_2)] - [\hat{P}^1(1-P^{*1})V - C(\hat{P}^1, y_2)] \geqslant -\frac{1}{2}\frac{\phi_2}{\phi_1}[P^{*2}]^2 V,$$

where $\hat{P}^1 = \underset{P}{\operatorname{argmax}} \{P[1-P^{*1}]V - C(P, y_2)\}$.

Proof. Inequality (14) is derived by substituting the relations identified in Lemma 3 into the truthful disclosure constraint (1) in $[SP]$. It need only be noted that with full disclosure and full sharing of knowledge, $\hat{P}^L = P^{*2}$. ■

Some insight concerning when the First-Best solution is likely to be feasible can be gleaned from inequality (14) by examining a limiting case. Suppose that both P^{*1} and ϕ_2 are close to zero. Thus, the presumption is that high knowledge is very unlikely, and the First-Best level of R & D effort is very small when only low knowledge is available to the two researchers. Then, since the right-hand side of (14) is close to zero while the left-hand side is strictly negative (since with $P^{*1} \approx 0$, $\hat{P}^1 \approx \underset{P}{\operatorname{argmax}} \{PV - C(P, y_2)\}$), the inequality cannot be satisfied and so the First-Best solution cannot be implemented. An interpretation of this finding is that when high knowledge is both rare and valuable, it will be more difficult to provide sufficient incentive for the sharing of this information. On the other hand, when high knowledge is very common (i.e. when ϕ_2 is close to unity, so that ϕ_1 is close to zero), little inducement is needed to motivate disclosure and sharing, so the First-Best solution will be feasible.

It is also the case that when even the low level of knowledge is sufficiently productive from the social viewpoint, the First-Best solution will always be feasible. Intuitively, it will not be very difficult to induce a researcher to truthfully reveal high knowledge when that knowledge adds relatively little from the social viewpoint. This intuition is made precise in Corollary 1.

Corollary 1. Suppose $P^{*1} \geqslant \frac{1}{2}$. Then the First-Best solution is a feasible solution to [SP].[11]

Proof. Inequality (14) will necessarily be satisfied if

$$[P^{*2} - \hat{P}^1(1-P^{*1})]V + [C(\hat{P}^1, y_2) - C(P^{*2}, y_2)] \geqslant 0.$$

The second of the two terms in square brackets is positive since $\hat{P}^1 > P^{*2} > P^{*1}$. The term in the first set of square brackets is also positive because $P^{*2} > P^{*1} \geqslant \frac{1}{2}$, and $\hat{P}^1 < 1$, *so* $\hat{P}^1(1-P^{*1}) < \frac{1}{2}$. ■

It is also straightforward to verify that as $y_1 \rightarrow y_2$ (so that low knowledge becomes nearly as productive as high knowledge), inequality (14) will always be satisfied, and so the First-Best solution can be implemented. Again, then, the smaller the advantage of the researcher with high knowledge, the more likely it is that truthful revelation and full sharing can be ensured despite the internal financing restrictions.

When the First-Best solution cannot be implemented, it is because there is an unresolvable conflict between providing incentives for sharing of knowledge at the first stage and motivating the desired success probabilities at the second stage. The researcher (B) who is endowed with the high level of knowledge may be reluctant to share it with his counterpart (A) because doing so puts the two researchers on more equal footing in the second-stage R & D contest. And even though some "handicapping" is instituted to compensate researcher B for his revelation of superior knowledge, internal financing restrictions may prohibit handicapping that is fully compensatory unless inefficient success probabilities are induced and/or less than full sharing of knowledge is implemented. It turns out, however, that the latter distortion will never be introduced. This fact is recorded in Proposition 3, and explained following Corollary 2.

Proposition 3. Suppose inequality (14) does not hold. Then the solution to [SP] has the following properties:

[11] It is shown in Bhattacharya, Glazer and Sappington [1990] that this conclusion extends to more general environments. In particular, if the number of researchers is $N \geqslant 2$, then the sufficient condition for First-Best attainability is $P^{*1} \geqslant \frac{1}{N}$, even when a continuum of knowledge endowments is admitted.

(i) $n = y_2$;
(ii) $F^2 = \frac{1}{2}P^2V, B^2 = \frac{1}{2}V, S^2 = [1-\frac{1}{2}P^2]V$, and $P^2 = P^{*2}$;
(iii) $F^L = 0, B^L + B^H = V$;
(iv) $F^1 = 0$, while $B^1 < \frac{1}{2}V$ and/or $S^1 < V$; and
(v) $S^H = V$ and/or $F^H + S^L = V$.

The proof of Proposition 3 is rather lengthy, and so is relegated to the Appendix. Among the important features of the "second-best" solution is the fact that full sharing of knowledge is always implemented. Corollary 2 helps provide an explanation of this finding.

Corollary 2. Suppose inequality (14) does not hold. Then at the solution to [*SP*], the value of the objective function would increase if the researcher endowed with the smaller level of knowledge chose a higher success probability.

The Corollary reports that even when all available knowledge is fully shared, the researcher A who was initially endowed with the lower level of knowledge is motivated to put forth "too little" R & D effort at the second stage. Thus, relatively little payoff is optimally promised to the researcher A who reports the lower level of knowledge, y_1. Doing so helps deter the researchers from understating their knowledge endowments. The greater share of realized surplus is awarded to the researcher B who reveals high knowledge, which encourages revelation of y_2. The important point is that incentives for revelation are instituted most effectively through adjustments in monetary rewards for successful innovation, rather than restricting information sharing. Because social value is also derived from surplus that is realized but not distributed to the researchers, no direct social costs are incurred when less than full distribution of the surplus occurs. However, if the flow of information to researcher A were restricted to "handicap" him, direct social costs would be incurred, since his costs of achieving any desired success probability would be higher than they need be.

There are other features of the Second-Best solution to [*SP*] that warrant brief mention. To begin, note that all surplus is distributed to the researchers and they are partially ensured against failure when they both reveal high knowledge, y_2. As noted above, these features help provide the greatest possible incentive for truthful revelation of high knowledge. Furthermore, deterring understatement of one's knowledge endowment is best accomplished by withholding all payment from a researcher who fails alone after reporting the lower level of knowledge, y_1. In addition, not all of the surplus generated will be distributed to the researchers when one or both succeed following reports of y_1 by both.

Whether the entire realized surplus will be distributed to the researchers when their initial reports differ is difficult to predict *a priori*. The difficulty stems from the aforementioned possibility of conflicting incentives at the first and second stages. To motivate full disclosure and sharing of knowledge, it is helpful to promise as large a reward as possible to the researcher who reveals y_2 alone. On the other hand, promising too great a reward in this instance may induce inappropriate second-stage R & D effort levels. In particular, if the payments for succeeding, B^H and S^H, are too large, too much effort may be induced; and if the payment for failing, F^H, is too large, too little effort may result.

Some additional insight as to when this potential conflict will arise is provided by Corollary 3. The statement of the Corollary refers to γ^H, which is the Lagrange multiplier associated with the effort selection constraint (6) in [*SP*]. $\gamma^H > 0$ at the solution to [*SP*] implies that the expected value of the coordinator's objective function would increase if the researcher with the superior knowledge at the first stage would increase his level of R & D effort at the second stage, *ceteris paribus.*

Corollary 3. If inequality (14) does not hold and $F^H + S^L < V$ in the solution to [*SP*], then $S^H = B^H = V$, $\gamma^H > 0$, and $S^L > B^L = F^L = 0$.

The proof of Corollary 3 is in the Appendix. The corollary reports that if the realized surplus is not fully distributed when the recipient of knowledge at the first stage succeeds alone at the second stage, then it must be the case that: (a) the recipient of knowledge receives a second-stage payment only when he succeeds alone; (b) the donor of knowledge receives the entire surplus whenever he succeeds; and yet (c) the knowledge donor is still putting forth "too little" R & D effort. Loosely speaking, the recipient of knowledge is receiving the minimal reward consistent with the desired level of R & D effort while the donor of knowledge is receiving the maximal reward. To increase S^L further would make it more attractive for the donor to conceal his superior information; and to increase F^H further would reduce the donor's level of R & D effort (which is already too small). Hence, the coordinator will not distribute all of the realized surplus.

Similarly, it is straightforward to verify that if all of the surplus is not distributed when the donor of knowledge succeeds alone (i.e. if $S^H < V$), then the donor is putting forth "too much" R & D effort at the solution to [*SP*] (i.e. $\gamma^H < 0$), and all surplus is distributed when the recipient of knowledge succeeds alone (i.e. $F^H + S^L = V$).[12] Here too, then, it is the inappropriate second-stage incentives that would be created if all surplus were distributed to the researchers that rules out full distribution.

VI. LIMITED CONTROL

To this point, we have allowed considerable flexibility in designing incentive structures. Only internal financing constraints were imposed. In this setting,

[12] This conclusion follows immediately from inequality (A12) and the complementary slackness restrictions associated with the internal financing constraints (8) and (9).

we showed that full sharing of productive knowledge is always motivated. In this section, we briefly note how this conclusion may be modified when available policy instruments are more limited. Our analysis in this section also extends the observations of Sah and Stiglitz [1987] to allow for the possibility of information exchange prior to R & D competition.

For simplicity, the discussion focuses on the extreme case where the second-stage competition must be of the "winner-take-all" variety. Thus, no control over the second-stage R & D competition is possible: a researcher who succeeds alone receives V; otherwise, he receives nothing. Such an outcome would arise, for example, under Bertrand competition when the industry demand curve is inelastic. The only control available in this setting is over the transfer of knowledge among researchers: thus, we assume that the services of the benevolent coordinator are still available, but no taxation of surplus is available to distribute gains among researchers. We call this the "winner-take-all environment".

Because the second-stage incentive structure is the same in this environment regardless of how much knowledge is revealed and shared, it is impossible to motivate a researcher to truthfully reveal his high knowledge unless the coordinator is instructed not to share all of the revealed knowledge with the "less able" researcher. This fact is recorded as Lemma 4.

Lemma 4. In the winner-take-all environment, full sharing of knowledge (i.e. $n = y_2$) is not feasible.

Proof. Suppose $n = y_2$. Then, since $\hat{P}^L = P^H = P^L = P^2$, $B^H = B^L = B^1 = B^2 = 0$, $F^H = F^L = F^1 = F^2 = 0$, and $S^H = S^L = S^1 = S^2 = V$, the truthful disclosure constraint (1) becomes:

$$P^H[1-P^L]V - C(P^H, y_2) \geqslant \hat{P}^1[1-P^1]V - C(\hat{P}^1, y_2)$$

But this inequality cannot hold because $P^1 < P^L$, and it is straightforward to verify that

$$\frac{\partial}{\partial \tilde{P}}\left\{\operatorname*{argmax}_{P} P[1-\tilde{P}]V - C(P, y_2)\right\} < 0. \qquad \blacksquare$$

Thus, even though "winner-take-all" R & D contests may provide proper incentives for R & D effort *given* the researchers' levels of operating knowledge (as noted in Bhattacharya, Glazer and Sappington [1990] and Sah and Stiglitz [1987]), they generally will not ensure the proper incentives for *sharing* of knowledge.

What may be less obvious is the fact that some nontrivial amount of knowledge sharing can be motivated in the winner-take-all environment. The researcher endowed with high knowledge is willing to share some (but not all) of his knowledge in return for being "certified" by the coordinator as a researcher with knowledge level y_2.[13] This certification causes one's opponent to reduce his equilibrium level of R & D effort at the second stage, since he knows for certain that his competitor has the highest level of knowledge and consequently will put forth significant R & D effort. This observation is recorded formally as Proposition 4. The proof of the proposition requires that the following stability condition (SC) hold:

$$\text{(SC)} \qquad C_{PP}(P,n)C_{PP}(P,y_2) > [V]^2 \quad \forall P \in (0,1) \quad \text{and} \quad n \in [y_1, y_2]$$

This is a standard condition which ensures the stability of the equilibrium under consideration. More precisely, the condition ensures that the reaction function of the researcher with high knowledge (y_2) in (P^H, P^L)-space is less steeply sloped than the corresponding reaction function for the researcher with lower knowledge (n).[14] Given these relative slopes, the unique equilibrium ($\tilde{P}^H, \tilde{P}^L$) will be restored following a perturbation of the success probabilities around their equilibrium values. The stability condition ensures that the greater the level of knowledge one's opponent is thought to have, the smaller will be one's own effort level in equilibrium, *ceteris paribus*. Consequently, gains to having one's higher level of knowledge "certified" are ensured.

Proposition 4. Suppose (SC) holds. Then in any separating equilibrium in the winner-take-all environment, some nontrivial sharing of knowledge is optimally implemented (i.e. $n > y_1$).

Proposition 4, whose proof is in the Appendix, indicates that some sharing of knowledge can be induced even absent direct control over the second-stage payment structure.[15,16] Taken together with Lemma 4, the Proposition is meant to be indicative of the more general proposition that full sharing of information is not always optimal when powers to structure payoffs from research and development are limited. We conjecture that less than full sharing of knowledge will also often characterize the optimal policy when the

[13] Comparing this conclusion to Katz' [1986] finding that no sharing of information will occur when the product market is characterized by Bertrand competition points out an important role the coordinator can play in our model.

[14] The reaction function for a researcher specifies his profit-maximizing choice of P given the corresponding choice by his opponent.

[15] The coordinator could be instructed to accurately report the knowledge level of each researcher without actually transferring any knowledge. Proposition 4 reports that this policy will never be optimal for the coordinator.

[16] Proposition 4 helps prove the assertion made in footnote 8 that the separating equilibrium identified in section IV leads to a higher level of expected welfare than the pooling equilibrium in which no knowledge is transferred. The proof of the assertion proceeds as follows. Suppose the coordinator is instructed to institute the winner-take-all competition at the second stage and to implement the sharing of knowledge identified in Proposition 4. Then, relative to the pooling equilibrium, more knowledge is transferred and, given the distribution of knowledge that prevails at the second stage, First-Best R & D effort levels are chosen. Hence, the separating equilibrium is preferred.

relevant goal is to maximize only the expected profits of the researchers.[17] But verification of this conjecture is left for future research.

VII. CONCLUSIONS

We have examined a two-stage model of R & D contests in which researchers compete against each other at the second stage after having the opportunity to share productive knowledge at the first stage. Our focus was on the interaction between the incentive problems that arise in the two stages. We derived necessary and sufficient conditions for it to be possible to induce researchers to both share all of their productive knowledge at the first stage, and then put forth the efficient levels of R & D effort at the second stage. These conditions took account of limited wealth endowments. We also characterized the optimal departures from the First-Best outcome when it is not feasible.

Because our simple model has a number of special features, it would be unwise to attempt to draw conclusive policy implications from our analysis. Nevertheless, our findings provide some insight into such concerns as how R & D joint ventures should be structured in the social interest. The design of second-stage (product market) payoff structures is of particular importance. As we showed, sharing of knowledge is facilitated when it is possible to sign agreements, akin to licensing agreements, that specify how the gains from success will be divided among researchers. In fact, these agreements can result in sharing of productive knowledge even when subsequent product market competition is characterized by Bertrand-like price-setting behavior. *Ex ante* agreements to share rewards from innovation can reduce the intensity of competition that characterizes Bertrand competition, and thereby restore incentives for sharing of knowledge.[18]

Our analysis also suggests the importance of *ex ante* rules governing the division of realized surplus between consumers and research firms. In particular, the fraction of the social value of an innovation that is awarded to researchers may optimally depend upon their aggregate prediction about the likelihood of success. More precisely, if an innovation is realized after all firms

[17] The intuition behind this conjecture is the following. When the concern is only with the profits of the researchers, it is more likely that all of the realized surplus will be distributed to them. But full distribution can make truthful disclosure of knowledge more difficult to motivate. Such motivation is facilitated, however, by sharing less than all of the knowledge that is revealed.

[18] Such findings serve to qualify the conclusions of Katz [1986] and Ordover and Willig [1985], for example. These studies demonstrate that no sharing of knowledge will occur in the absence of licensing agreements when the product market is characterized by Bertrand competition.

[19] Our analysis can readily be extended to the setting where the cost to each researcher of acquiring a knowledge endowment is known. But when these costs are not common knowledge, a nontrivial third stage would have to be added to our model. This initial stage would introduce an additional incentive problem that could interact in interesting ways with the two incentive problems we have explored in this paper.

claim to have little chance of succeeding (and therefore share little knowledge), then more of the surplus generated should flow to consumers rather than the firms.

An entirely comprehensive analysis of research and development would have to take account of the fact that knowledge "endowments" are generally endogenous. Thus, incentives to acquire (independently or jointly) as well as to share productive knowledge need to be considered.[19] It also seems important to allow for the possibility of different degrees of success or failure. Furthermore, the problem of sharing knowledge becomes considerably more complex when a coordinator is either unavailable or self-interested; bargaining solutions among researchers need to be developed. In addition, more general representations of knowledge should be explored. In particular, knowledge is generally not a uni-dimensional variable that admits a simple ranking; furthermore important complementarities in knowledge may exist. These extensions remain for future research.

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APPENDIX

Proof of Proposition 3 and Corollary 2

Let λ, γ^i, $\hat{\gamma}^1$, γ^L, $\hat{\gamma}^L$, γ^H, ξ^i, ξ^L, ξ^H, β^i, β, η_2 and η_1 be the Lagrange multipliers associated with constraints (1)–(13), in that order. Also, let π^i_{Pj} and π^i_{PjPk} represent the obvious first- and second-partial derivatives of the researcher's expected profit for $i = 1, 2, H, L$. Let $\hat{\pi}^i_{Pj}$ and $\hat{\pi}^i_{PjPk}$ represent the corresponding partial derivatives for $i, j, k = 1, L$ when the researcher's true knowledge endowment is y_2. Using this notation, the necessary conditions for a solution to [SP] include the following:

(A1) $\quad P^1\{-\lambda\phi_1\hat{P}^1+\gamma^1+\hat{\gamma}^1\}-\beta^1 \leqslant 0$

(A2) $\quad P^2\{\lambda\phi_2 P^2+\gamma^2\}-\beta^2 \leqslant 0$

(A3) $\quad P^H\{-\lambda\phi_2\hat{P}^L+\gamma^L+\hat{\gamma}^L\}-\beta \leqslant 0$

(A4) $\quad P^L\{\lambda\phi_1 P^H+\gamma^H\}-\beta \leqslant 0$

(A5) $\quad [1-P^1]\{-\lambda\phi_1\hat{P}^1+\gamma^1+\hat{\gamma}^1\}-\xi^1 \leqslant 0$

(A6) $\quad [1-P^2]\{\lambda\phi_2 P^2+\gamma^2\}-\xi^2 \leqslant 0$

(A7) $\quad [1-P^H]\{-\lambda\phi_2\hat{P}^L+\gamma^L+\hat{\gamma}^L\}-\xi^H \leqslant 0$

(A8) $\quad [1-P^L]\{\lambda\phi_1 P^H+\gamma^H\}-\xi^L \leqslant 0$

(A9) $\quad P^1\{-\lambda\phi_1[1-\hat{P}^1]-\gamma^1-\hat{\gamma}^1\}-\xi^1 \leqslant 0$

(A10) $\quad P^2\{\lambda\phi_2[1-P^2]-\gamma^2\}-\xi^2 \leqslant 0$

(A11) $\quad P^H\{-\lambda\phi_2[1-\hat{P}^L]-\gamma^L-\hat{\gamma}^L\}-\xi^L \leqslant 0$

(A12) $\quad P^L\{\lambda\phi_1[1-P^H]-\gamma^H\} = \xi^H \leqslant 0$

(A13) $\quad -2\phi_1\phi_2 C_n(P^L,n)-\gamma^L C_{Pn}(P^L,n)+\eta_1-\eta_2 = 0$

(A14) $\quad 2(\phi_1)^2\{[1-P^1]V-C_P(P^1,y_1)\}+\gamma^1\pi^1_{P^1P^1}-\lambda\phi_1\hat{\pi}^1_{P^1}+\hat{\gamma}^1\hat{\pi}^1_{P^1\hat{P}^1} = 0$

(A15) $\quad 2(\phi_2)^2\{[1-P^2]\,V-C_P(P^2,y_2)\}+\gamma^2\pi^2_{P^2P^2}+\lambda\phi_2\pi^2_{P^2} = 0$

(A16) $\quad 2\phi_1\phi_2\{[1-P^H]\,V-C_P(P^L,n)\}+\gamma^H\pi^H_{P^HP^L}+\gamma^L\pi^L_{P^LP^L}+\lambda\phi_1\pi^H_{P^L} = 0$

(A17) $\quad 2\phi_1\phi_2\{[1-P^L]\,V-C_P(P^H,y_2)\}+\gamma^L\pi^L_{P^LP^H}+\gamma^H\pi^H_{P^HP^H}+\lambda\phi_1\pi^H_{P^H}$
$$-\lambda\phi_2\hat{\pi}^L_{P^H}+\hat{\gamma}^L\hat{\pi}^L_{P^LP^H} = 0$$

(A18) $\quad -\lambda\phi_1\hat{\pi}^1_{\hat{P}^1}+\hat{\gamma}^1\hat{\pi}^1_{\hat{P}^1\hat{P}^1} = 0$

(A19) $\quad -\lambda\phi_2\hat{\pi}^L_{\hat{P}^L}+\hat{\gamma}^L\hat{\pi}^L_{\hat{P}^L\hat{P}^L} = 0$

It is immediate from (A18) and (A19) that $\hat{\gamma}^1 = \hat{\gamma}^L = 0$ (since $\hat{\pi}^1_{\hat{P}^1} = \hat{\pi}^L_{\hat{P}^L} = 0$ by definition). Therefore, since $B^1+S^1 > 0$ and $B^L+S^L > 0$ to motivate $P^1 > 0$ and $P^L > 0$, and since $\lambda > 0$ (when inequality (14) does not hold), it follows from (A1) and (A5) that $\gamma^1 > 0$, and from (A3) and (A7) that $\gamma^L > 0$. This constitutes the proof of Corollary 2.

Now, given the maintained assumption that higher levels of knowledge reduce total and marginal costs of effort, i.e. $C_n(\cdot) < 0$ and $C_{Pn}(\cdot) < 0$, it is immediate from (A13) that $\eta_2 > 0$, so $n = y_2$ by complementary slackness.

Since $\xi^1 \geqslant 0$ and $\xi^L \geqslant 0$, it also follows from the complementary slackness conditions associated with (A9) and (A11) that $F^1 = F^L = 0$.

If $\gamma^2 \geqslant 0$, then $\xi^2 > 0$ from (A6). And if $\gamma^2 \leqslant 0$, then $\xi^2 > 0$ from (A10). Hence $\xi^2 > 0$ and $F^2+S^2 = V$ by complementary slackness.

To prove that $B^L+B^H = V$, suppose $\beta = 0$. Then if $\xi^H = 0$, $\lambda\phi_1 P^H+\gamma^H \leqslant 0$ from (A4); hence $\lambda\phi_1[1-P^H]-\gamma^H > 0$ since $\lambda > 0$, which contradicts (A12). Now, if $\xi^H > 0$, then $\xi^H = [1-P^H][\gamma^L-\lambda\phi_2\hat{P}^L] > 0$ from (A7) (since $S^L = V-F^H > 0$ for $P^H > 0$). But then $\beta > 0$ from (A3).

Now suppose $\xi^L = \xi^H = 0$. Then $\lambda\phi_1 P^H+\gamma^H \leqslant 0$ from (A8), so $-\lambda\phi_1 P^H-\gamma^H \geqslant 0$, which implies that $\lambda\phi_1[1-P^H]-\gamma^H > 0$. But this contradicts (A12).

Now suppose $S^1 = V$ and $B^1 = \frac{1}{2}V$. Then it is straightforward to verify that $\pi^1_{P^1P^1} = -\frac{1}{2}V-C_{PP}(P^1,y_1)$, and $\hat{\pi}^1_{P^1} = -\frac{1}{2}V\hat{P}^1$. Also, it follows from equation (1) that $[1-P^1]-C_P(P^1,y_1) = -\frac{1}{2}P^1 V$. Therefore, equation (A14) reduces to

$$-(\phi_1)^2 P^1 V-[\gamma^1-\lambda\phi_1\hat{P}^1]\tfrac{1}{2}V-\gamma^1 C_{PP}(P^1,y_1) = 0$$

But this is impossible since each of the three terms is negative. (Note that with $B^1 > 0$, $\gamma^1 - \lambda\phi_1\hat{P}^1 = \beta^1/P^1 > 0$ from (A1).)

To prove condition (ii) of the proposition, note that if constraint (2) is not imposed for $i = 2$, we have $\beta^2 > 0$ from (A2) and $\xi^2 > 0$ from (A6). Hence $B^2 = \frac{1}{2}V$ and $F^2 = V - S^2$. Also, from (A15), $P^2 = P^{*2}$. Therefore, with $F^2 = \frac{1}{2}P^2V$, all of the necessary conditions for a solution to [SP] are satisfied, as is constraint (2).

Proof of Corollary 3

Using the notation and the relationships from the proof of Proposition 3, note that if $F^H + S^L < V$, then $\xi^H = 0$. Hence, from (A12), $\gamma^H > 0$. Consequently, from (A8), $\xi^L > 0$ which implies $S^H = V$. Also, $\beta > 0$ from (A4), so $B^L + B^H = V$.

Now, suppose $B^L > 0$. Then from (A3) and (A7), $0 \geqslant \gamma^L - \lambda\phi_2\hat{P}^L = \beta/P^H > 0$, which is a contradiction. Therefore, $B^L = 0$ and $B^H = V$. ■

Proof of Proposition 4

The necessary conditions for a solution to the social problem in the winner-take-all environment are readily shown to include the following, where the notation corresponds exactly to that employed in the proof of Proposition 3:

(A20) $\quad 2\phi_1\phi_2|C_n(P^L, n)| + \gamma^L|C_{pn}(P^L, n)| - \eta_2 + \eta_1 = 0;$

(A21) $\quad \lambda\phi_1 P^H V = -\gamma^L C_{PP}(P^L, n) - \gamma^H V;$ and

(A22) $\quad \lambda\phi_2\hat{P}^L V = \gamma^H C_{PP}(P^H, y_2) + \gamma^L V.$

Suppose $n = y_1$. Then the truthful disclosure constraint in this environment can be rewritten as:

(A23) $$\phi_1\{P^H[1-P^L]V - C(P^H, y_2)\} + \phi_2\{P^2[1-P^2]V - C(P^2, y_2)\}$$
$$\geqslant \phi_1\{\hat{P}^1[1-P^1]V - C(\hat{P}^1, y_2)\} + \phi_2\{\hat{P}^L[1-P^H]V - C(\hat{P}^L, y_2)\}$$

But $P^H > P^2$ and $P^1 > P^L$ when (SC) holds. Hence, since

$$\frac{\partial}{\partial\tilde{P}}\left\{\underset{P}{\text{argmax}}\, P[1-\tilde{P}]V - C(P, y_2)\right\} < 0,$$

(A22) holds as a strict inequality, so $\lambda = 0$. Furthermore, (A21), (A22) and (SC) imply $\gamma^H = \gamma^L = 0$ when $\lambda = 0$. Therefore, since $\eta_2 = 0$ when $n = y_1$, we have a contradiction of (A20). ■

REFERENCES

BHATTACHARYA, S., GLAZER, J. and SAPPINGTON, D., 1990, 'Motivating the Sharing of Knowledge in R&D Ventures: First-Best Implementation', Bellcore Technical Memorandum, March.

BHATTACHARYA, S. and RITTER, J., 1983, 'Innovation and Communication: Signaling with Partial Disclosure', *Review of Economic Studies*, 50, pp. 331–346.

DASGUPTA, P. and STIGLITZ, J., 1980, 'Uncertainty, Industrial Structure, and the Speed of R&D', *Bell Journal of Economics*, 11, pp. 1–28.

FUDENBERG, D., GILBERT, R., STIGLITZ, J. and TIROLE, J., 1983, 'Pre-emption, Leapfrogging and Competition in Patent Races', *European Economic Review*, 22, pp. 3–31.

GANDAL, N. and SCOTCHMER, S., 1989, 'Coordinating Research Through Research Joint Ventures', Boston University mimeo, December.

GROSSMAN, G. and SHAPIRO, C., 1986, 'Optimal Dynamic R&D Programs', *Rand Journal of Economics*, 17, pp. 581–593.

GROSSMAN, G. and SHAPIRO, C., 1987, 'Dynamic R&D Competition', *Economic Journal*, 97, pp. 372–387.

HARRIS, C. and VICKERS, J., 1985, 'Perfect Equilibrium in a Model of a Race', *Review of Economic Studies*, 52, pp. 193–209.

HARRIS, C. and VICKERS, J., 1987, 'Racing with Uncertainty', *Review of Economic Studies*, 54, pp. 1–21.

JUDD, K., 1985, 'Closed-Loop Equilibrium in a Multistage Innovation Race', Northwestern University Discussion Paper.

KATZ, M., 1986, 'An Analysis of Cooperative Research and Development', *Rand Journal of Economics*, 17, pp. 527–543.

LOURY, G., 1979, 'Market Structure and Innovation', *Quarterly Journal of Economics*, 93, pp. 395–410.

ORDOVER, J. and WILLIG, R., 1985, 'Antitrust for High-Technology Industries: Assessing Research Joint Ventures and Mergers', *Journal of Law and Economics*, 28, pp. 311–333.

SAH, R. and STIGLITZ, J., 1987, 'The Invariance of Market Innovation to the Number of Firms', *Rand Journal of Economics*, 18, pp. 98–108.