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Pricing and Coordination: Strategically Stable Equilibria

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This paper investigates a class of two-period games in which the first-period interactions have the features of a prisoner's dilemma game and the second-period interactions have the features of a coordination game. This class of games seems relevant to problems of industrial organization where firms not only compete in prices but also have to coordinate in some other dimensions. We find that equilibrium outcomes that we can reject on intuitive grounds are not generated by elements of stable sets à la Kohlberg and Mertens. *Journal of Economic Literature* Classification Numbers: 026, 611. © 1990 Academic Press, Inc.

1. INTRODUCTION

This paper investigates a class of two-period games in which the first-period interactions have the features of a prisoner's dilemma game and the second-period interactions have the features of a coordination game. We find that equilibrium outcomes that we can reject on intuitive grounds are not elements of stable sets.[1,2]

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[1] Stability refers to strategic stability as defined in Kohlberg and Mertens (1986).

[2] Other papers that studied the "forward induction" properties of different games are Ben-Porath and Dekel (1987), Osborne (1987), and van Damme (1989).

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We first show that outcomes in which collusive behavior in the first period is enforced through threats to play equilibria in the second period that are *unfavorable* to the deviator are not "strategically stable." We next show that if players have the option of taking a self-destructive first-period action, strategic stability eliminates some Pareto inferior (low payoff) outcomes from the set of subgame-perfect Nash equilibria. In particular, players are able to ensure themselves a minimum payoff which may exceed what they would get in some subgame-perfect equilibria. The intuition for rejecting both the collusive and low payoff outcomes is based on forward induction.

Finally we show that the set of strategically stable outcomes is sensitive to the quantitative features of the first- and second-period payoff matrices. These quantitative features also affect the persuasiveness of the forward induction argument. In particular, changes in the values of the off-diagonal elements of the coordination game that would not affect any of the qualitative features of the second-period coordination game (were that game considered in isolation), affect whether outcomes with collusion in the first period or low payoffs in the second period are strategically stable. Only if the off-diagonal elements of the coordination game are sufficiently small do considerations of strategic stability eliminate these outcomes.

The class of games studied in this paper seems relevant for problems of industrial organization. Firms often interact through choices of advertising campaigns, directions of research and development, types of investment, manufacturing technologies, and product characteristics, as well as through choices of prices and quantities. The former interactions often have features of a coordination game. For instance, if one peanut butter manufacturer mentions the carcinogenic properties of its rival's product, the profit maximizing response could be to point out the harmful effects of the first firm's product. However, if neither firm mentions any harmful effect of peanut butter, it would be in the interest of both firms to only advertise the positive attributes of peanut butter. Similarly, two risk averse firms may want to use the same technologies or pursue the same line of resarch. Alternatively, profit maximizing firms may wish to choose different technologies or directions of research. Of course these interactions typically occur over many periods. Analytic tractability and space limitations lead us to only analyze two-period interactions in this paper. In Glazer and Weiss (1987) we investigated a particular three-period extension of the model. We conjecture that if we extend the game and allow firms to play the prisoner's dilemma game for more than one period prior to the coordination game, then collusion will not take place in the later periods when there are few periods remaining before the coordination game will be played. The open question is, however, whether collusion will take place in the first periods when the prisoner's dilemma will be played many times before the co-ordination game is played.

		Firm 2	
		H	L
Firm 1	H	x, x	z,w
	L	w,z	y,y

a

		Firm 2	
		C	N
Firm 1	C	a, a	−c,0
	N	0,−c	0.0

b

FIG. 1. (a) First-period interaction. (b) Second-period interaction.

2. THE MODEL

Consider the following situation. Two identical firms interact for two periods. In the first period each firm chooses either a high price, H, or a low price, L. In the second period each firm chooses between two different actions C or N. (The reader can think of C as complementary advertising and N as negative advertising.)

In each period the firms move simultaneously. The payoffs from the first interaction are shown in Fig. 1a, where the first element in each entry is the payoff to firm 1. We assume that $w > x > y > z$. Note that these payoffs describe a prisoner's dilemma. It always pays for a firm to undercut its rival's price.

Figure 1b describes the payoffs from the interaction in the second period. We assume that $a > 0 > -c$. Consequently, the isolated second interaction has the structure of a coordination game. There are two pure strategy Nash equilibria in this game: the "cooperative" one, both firms playing C; and the "noncooperative" one, both firms playing N.

Let s_i denote a pure strategy of firm i in the two-period game. A deterministic outcome is described by $(P_1, A_1; P_2, A_2)$. P_i specifies firm i's move in the first period, and A_i specifies firm i's move in the second period for $i = 1, 2$. P_i can be either L or H and A_i can be either C or N.[3]

The following two strategy combinations generate the "noncollusive-cooperative" outcome $(L, C; L, C)$ as a subgame-perfect Nash equilibrium.

(1a) Both play L in the first period and play C in the second period, regardless of what prices were charged in the first period.

(1b) Both play L in the first period. Each plays C in the second period if and only if its opponent played L in the first period.

[3] In this paper we focus only on deterministic outcomes. Note, though, that deterministic outcomes can also be generated by mixed strategies. This will be important later when we study the stability properties of these outcomes. Clearly, in the two-period game we study there are also (subgame perfect) Nash equilibria that generate random outcomes. It does not appear that there is much we can gain by studying these random outcomes.

The following two strategy combinations can generate the "low payoff" outcome (L, N; L, N) as a subgame-perfect Nash equilibrium.

(2a) Both play L in the first period and play N in the second period, regardless of what prices were charged in the first period.

If $z + a < y$, then the following strategy combination is also a subgame-perfect equilibrium:

(2b) Both play L in the first period. Each plays N in the second period if and only if its opponent played L in the first period.

If $z + a > y$, then (3) describes subgame-perfect equilibria that generate the "two-price" outcomes (H, C; L, C) and (L, C; H, C).

(3) Firm i plays H and firm j plays L in the first period, $i \neq j$. In the second period both firms play C if i played H, otherwise both play N.

If $x + a > w$, then the following strategy combination is a subgame-perfect equilibrium that generates the "collusive" outcome (H, C; H, C).

(4) Both firms play H in the first period and play C in the second period, if and only if both played H in the first period.

The strategy combination described by (4) supports collusion as part of the outcome of a subgame-perfect equilibrium of this game.

Even though the strategy combinations described above are subgame-perfect equilibria (and hence satisfy "backward induction"), the equilibria described by (3), (4), and sometimes (2a) fail to satisfy forward induction. Our formal treatment of this problem applies the Kohlberg–Mertens (1986, referred to throughout as K-M) notion of stability.

3. Stable Sets of the Two-Period Model

K-M introduce the notion of strategically stable sets of equilibria which satisfy a version of forward induction. We shall show that none of the equilibria that generate the "collusive" outcome (H, C; H, C) or the "two-price" outcomes (H, C; L, C) and (L, C; H, C) are elements of a stable set. We also show that if $z + a > y$, then neither equilibrium (2a) nor any other equilibrium that generates the "low payoff" outcome (L, N; L, N) is an element of a stable set. If, however, $z + a < y$, then equilibrium (2a) is an element of a stable set. Finally we show that the "noncollusive-cooperative" equilibria (1a) and (1b), as well as other equilibria that generate the outcome (L, C; L, C), are elements of a stable set.

To analyze the stable outcomes of our game we first write the reduced normal form of the game. This is done in Fig. 2. The strategy of a firm is represented by a triple X, Y, Z. The first term is the price the firm charges in the first period. The second and third terms each take the values C and N and describe how the firm responds to its opponent charging H and L, respectively, in the first period. Thus L, C, N represents the strategy of charging price L in the first period and playing C in the second period if

	L,N,N	L,C,C	L,N,C	L,C,N	H,N,N	H,C,C	H,N,C	H,C,N
L,N,N	y,y	y,y−c	y,y−c	y,y	w,z	w,z−c	w,z−c	w,z
L,C,C	y−c,y	y+a,y+a	y+a,y+a	y−c,y	w−c,z	w+a,z+a	w+a,z+a	w−c,z
L,N,C	y−c,y	y+a,y+a	y+a,y+a	y−c,y	w,z	w,z−c	w,z−c	w,z
L,C,N	y,y	y,y−c	y,y−c	y,y	w−c,z	w+a,z+a	w+a,z+a	w−c,z
H,N,N	z,w	z,w−c	z,w	z,w−c	x,x	x,x−c	x,x	x,x−c
H,C,C	z−c,w	z+a,w+a	z−c,w	z+a,w+a	x−c,x	x+a,x+a	x−c,x	x+a,x+a
H,N,C	z−c,w	z+a,w+a	z−c,w	z+a,w+a	x,x	x,x−c	x,x	x,x−c
H,C,N	z,w	z,w−c	z,w	z,w−c	x−c,x	x+a,x+a	x−c,x	x+a,x+a

FIG. 2. The reduced normal form of the two-period game.

the other firm charged price H, and playing N in the second period if the other firm charged price L. Because we are only writing the reduced normal form of the game, we omit the firm's reaction to itself charging price H or L.

Let $\bar{S}_i$ denote the set of all pure strategies of player i, in the game presented in Fig. 2, and let $\sigma_i: \bar{S}_i \to [0, 1]$ be a mixed strategy of player i, thus $\sum_{s_i \in S_i} \sigma_i(s_i) = 1$. Let $u_i(s_1, s_2)$ be the payoff to player i if player 1 plays the pure strategy s_1 and player 2 plays the pure strategy s_2. Let $u_i(\sigma_1, \sigma_2) = \sum_{s_1 \in S_1} \sum_{s_2 \in S_2} \sigma_1(s_1)\sigma_2(s_2)u_i(s_1, s_2)$ be the expected payoff to player i if player 1 plays the mixed strategy σ_1 and player 2 plays σ_2. Sometimes we use the convention $u_i(s_1, \sigma_2) = \sum_{s_2 \in S_2} \sigma_2(s_2)u_i(s_1, s_2)$.

PROPOSITION 1. *Suppose $x + a > w$, and let*

$$\bar{C}(H, H) \equiv \Big\{(\sigma_1, \sigma_2) \mid \sigma_i(H, C, C) = 1 - \sigma_i(H, C, N) \leq \frac{x + a - w + c}{a + c} \text{ for } i = 1, 2\Big\}.$$

Then,

(a) $(\sigma_1, \sigma_2) \in \bar{C}(H, H)$, if and only if (σ_1, σ_2) is a Nash equilibrium that generates the outcome $(H, C; H, C)$.

(b) Neither the set $\bar{C}(H, H)$ nor any of its subsets is stable.

Proof. See Appendix.

The "forward induction" argument against the "collusive" equilibria can be best understood by focusing on the collusive equilibrium (4) described in the previous section.

In this equilibrium both firms play H in the first period and C in the second. If, however, one of the players deviates to L in the first period, the equilibrium strategies call for both firms to play N in the second period. The equilibrium payoff is $x + a$ for both firms. The reason this equilibrium (and all the other equilibria that generate the "collusive" outcome) does not satisfy forward induction is as follows. Suppose firm 1 deviates to L in the first period. After observing this deviation, the only reasonable inference for firm 2 to draw is that firm 1 will play C in the second period. If firm 1 had deviated in the first period intending to play N in the second period, that deviation would give it a payoff of w (regardless of the response of firm 2 to its deviation) which is less than its "equilibrium" payoff of $x + a$. Only by playing C in the second period is there any possibility of the deviation making firm 1 better off. Furthermore, firm 2 knows that firm 1 knows that if this deviation were to induce it (firm 2) to believe that firm 1 will play C in the second period, then it (firm 2) would also play C. Consequently, firm 1 would play C in the second period, as would firm 2, and the deviation would be profitable for firm 1, since its payoff would be $w + a$, which is greater than the equilibrium payoff $x + a$. Knowing this, neither firm would play the initial equilibrium strategies.

We now discuss the stability properties of the "low payoff" outcome $(L, N; L, N)$. Recall that this outcome can always be generated by the subgame-perfect Nash equilibrium (2a). However, we show that this outcome is stable if and only if $z + a < y$.

PROPOSITION 2. *Suppose that* $z + a > y$ *and let*

$$\overline{N}(L, L) \equiv \Big\{(\sigma_2, \sigma_2) \mid \sigma_i(L, N, N) = 1 - \sigma_i(L, N, N) \le \frac{y - z + c}{a + c} \text{ for } i = 1, 2\Big\}.$$

Then,

(*a*) $(\sigma_1, \sigma_2) \in \overline{N}(L, L)$, *if and only if* (σ_1, σ_2) *constitutes a Nash equilibrium that generates the outcome* $(L, N; L, N)$.

(*b*) *Neither the set* $\overline{N}(L, L)$ *nor any of its subsets is stable.*

Proof. See Appendix.

To gain some intuition as to why the outcome $(L, N; L, N)$ is not stable in the case where $z + a > y$, let us suppose firm 1 deviates to H in the first period. Because in this deviation it loses money in the first period ($z < y$), firm 1 can "use" this first-period loss to convince firm 2 that it (firm 1) will play C in the second period. This is the only second-period action that justifies the deviation. If this argument is persuasive, both firms will play C, and the deviator's second-period profit will be a instead of zero. Since

$a > y - z$, the increase in the second-period profit is enough to compensate the deviator for its "loss" in the first period.

Note that the opportunity to lose a "small" amount of money eliminates $(L, N; L, N)$ as a strategically stable outcome. If, however, $z + a < y$, the argument that led us to reject the "low payoff" equilibrium does not hold. When a firm deviates from L to H in the first period, its "loss" of $y - z$ cannot be compensated even if, as a result of this deviation, the two firms abandon their second-period strategies, play C instead of N, and collect the second-period profit of a instead of zero. We now prove that in such a case the outcome $(L, N; L, N)$ is stable.

PROPOSITION 3. *Suppose that $z + a < y$. Then,*

(a) $(\sigma_1, \sigma_2) \in \overline{N}(L, L)$, if and only if (σ_1, σ_2) is a Nash equilibrium that generates the outcome $(L, N; L, N)$.

(b) The set $\overline{N}(L, L)$ is stable.

Proof. See Appendix.

Finally, we show that the "noncollusive-cooperative" outcome $(L, C; L, C)$ that is generated by (among others) equilibria (1a) and (1b) is always stable.

PROPOSITION 4. *Let*

$$\overline{C}(L, L) \equiv \left\{(\sigma_1, \sigma_2) \mid \sigma_i(L, N, C) = 1 - \sigma_i(L, C, C) \text{ for } i = 1, 2\right\}.$$

Then,

(a) $(\sigma_1, \sigma_2) \in \overline{C}(L, L)$ if and only if (σ_1, σ_2) is a Nash equilibrium that generates the outcome $(L, C; L, C)$.

(b) The set $\overline{C}(L, L)$ is stable.

Proof. Similar to the Proof of Proposition 3.

4. A MODIFICATION

In this section we modify the second-period interaction by assuming that the "off-diagonal" payoffs to the player who plays N are not zero. This analysis demonstrates the sensitivity of "forward induction" arguments to changes in the "off-equilibrium" payoffs. Since the method of proof here is similar to that in the previous section, we present results and discuss them without providing the proofs.

Figure 3 illustrates the modified second-period interaction assuming $0 < b < a$. In Glazer and Weiss (1987) we proved that if b is not too large, none of our results change. In particular, we showed that if and only if $b <$

		Firm 2	
		C	N
Firm 1	C	a,a	−c,b
	N	b,−c	0,0

FIGURE 3

$(a + c)(x + a - w)/(x + a + c - w)$ is the "collusive" outcome not stable.[4] This condition can be rewritten as: $w - x < a - cb/(a + c - b)$. The last term on the right-hand side of the inequality is the payoff from the mixed strategy equilibrium of the second-period subgame. Therefore, if this inequality is not satisfied, a firm observing a price deviation by its rival *could* think that the rival was anticipating playing the mixed strategy equilibrium in the second period. A best response by the nondeviator would be to choose N. However, that response to a deviation would dissuade a firm from cutting prices in the first period, thus supporting the collusive equilibrium.

Thus if b is sufficiently large, there does not appear to be any persuasive forward induction argument that would convince firm 2 that firm 1 was certainly going to play C after having deviated to L in the first period. Consequently, each firm's threat to play N in the second period, if its opponent played L in the first, is credible.

On the other hand for b sufficiently small, so that the above inequality holds, the only equilibrium of the second-period subgame that would make a price cut profitable is one in which both firms choose C. Therefore, after observing a price deviation forward induction would lead the nondeviator to play C.

In Glazer and Weiss (1987) we also showed that if $b > (y - z)(a + c)/(y - z + c)$, the low payoff outcome of both firms playing L in the first period and N in the second is a stable outcome. Thus, for b sufficiently large, both collusive pricing and the low payoff noncooperative outcomes are strategically stable.

On the other hand, if in the first-period interaction there are sufficiently many prices from which firms can choose, so that the interval between the highest and the lowest price is large and the differences between the payoffs from prices near to one another are small, then neither collusive pricing nor the low payoff noncooperative outcomes are strategically sta-

[4] Note that if c is close to zero this condition is always satisfied. Also, from $x < w$, if the difference in payoffs from playing N against C versus playing N against N is less than $x + a - w$, this condition is satisfied.

ble, *for any value of b*. The intuition is that a fine enough price space enables a firm to make a deviation that would only satisfy forward induction if it were followed by both firms playing C in the second period. Once again, the reader is referred to Glazer and Weiss (1987) for a formal proof.

5. Some Concluding Remarks

This paper illustrates the power of the Kohlberg and Mertens (1986) notion of stability for analyzing multidimensional interactions in multiperiod games.

The interactions that we studied were ones in which a prisoner's dilemma game was linked to (and followed by) a coordination game. In that two-period interaction we showed that if the off-diagonal elements of the coordination game were near zero, then collusive behavior in the prisoner's dilemma cannot be sustained as an outcome of a strategically stable equilibrium.

Although this particular result could also be generated by an equilibrium refinement that eliminates Pareto inefficient equilibria in all subgames, in general the two approaches generate very different results. To see this, recall the case in which the off-diagonal elements of the coordination game are sufficiently large. In this case both collusive behavior in the first period and noncooperation in the second period can be sustained as outcomes of different strategically stable equilibria. In contrast, so long as the off-diagonal elements of the coordination game do not affect the ranking of the Nash equlibria, they have no force in equilibrium refinements that rely on elimination of Pareto inefficient equilibria. The contrast between the two approaches was illustrated even more sharply in a previous version of this paper (Glazer and Weiss, 1987) in which we showed that subgame-perfect collusive equlibria can be eliminated by strategic stability (forward induction) arguments even if the second-period coordination game does not have a Pareto inferior equlibrium. In that version we also showed that none of our qualitative results are substantively altered if the model is modified to allow firms to interact over three periods. In that extension firms must incur setup costs in the first period to participate in the coordination game in the last period.

Appendix

In the following proofs we make extensive use of Proposition 6 in K-M. We shall, therefore, repeat that proposition here.

Proposition (K-M). (A) *A stable set contains a stable set of any game obtained by deletion of a dominated strategy.* (B) *A stable set* *contains a stable set of any game obtained by deletion of a strategy which is an inferior response in all the equilibria of the set.*

The proof of part (A) of each of the propositions is straightforward and therefore omitted. We shall prove part (B) of the propositions.

Proof of Proposition 1. Suppose that $(\sigma_1, \sigma_2) \in \overline{C}(H, H)$ and let $s_1' \equiv L, N, C$. Then, $u_1(s_1', \sigma_2) = w < x + a = u_1(\sigma_1, \sigma_2)$. Thus firm 1's pure strategy L, N, C is an inferior response to all equilibria in the set $\overline{C}(H, H)$. In a similar way it can be shown that firm 1's pure strategy L, N, N is an inferior response to all strategies of firm 2 in the set $\overline{C}(H, H)$. After deletion of these two pure strategies of firm 1, firm 2's pure strategy H, C, N becomes inadmissible. It is now dominated by its pure strategy H, C, C. However, the Nash equilibria of the game obtained after the deletion of firm 2's (now dominated) pure strategy H, C, N will not contain any subset of $\overline{C}(H, H)$ since $\sigma_2(H, C, N) > 0$ for all $(\sigma_1, \sigma_2) \in \overline{C}(H, H)$.

Proof of Proposition 2. Suppose that $(\sigma_1, \sigma_2) \in \overline{N}(L, L)$ and let $s_1' \equiv H, C, N$ and $s_1'' \equiv H, N, N$. Then, $u_1(s_1', \sigma_2) = u_1(s_1'', \sigma_2) = z < y = u_1(\sigma_1, \sigma_2)$. Thus firm 1's pure strategies H, C, N and H, N, N are inferior responses to all equilibria in the set $\overline{N}(L, L)$. Note, though, that after deletion of these two pure strategies of firm 1, firm 2's pure strategy L, N, N becomes inadmissible since it is now dominated by its pure strategy L, C, N. However, the game obtained after the deletion of firm 2's pure strategy L, N, N will not contain any subset of $\overline{N}(L, L)$ since $\sigma_2(L, N, N) > 0$ for all $(\sigma_1, \sigma_2) \in \overline{N}(L, L)$.

Proof of Proposition 3. Note first that $\overline{N}(L, L)$ is the set of all (σ_1, σ_2) such that σ_i is a convex combination of firm i's pure strategies L, C, N and L, N, N for $i = 1, 2$. It is also easy to see that if $(\sigma_1, \sigma_2) \in \overline{N}(L, L)$ and $s_1 \in \overline{S}_1 \backslash \overline{N}(L, L)$ then $u_1(\sigma_1, \sigma_2) > u_1(s_1, \sigma_2)$ and that the same holds for firm 2. Thus, all strategies outside the set $\overline{N}(L, L)$ are strictly inferior responses to all equilibria in the set $\overline{N}(L, L)$, for both firms. This immediately implies that for an $\varepsilon > 0$ there exists some $\delta_0 > 0$ such that for any completely mixed strategy tuple (σ_1, σ_2) and for any δ_1, δ_2 such that $0 < \delta_i < \delta_0$ the perturbed game where every strategy s_i of firm i is replaced by $(1 - \delta_i)s_i + \delta_i\sigma_i$ has an equilibrium ε-close to $\overline{N}(L, L)$. Thus, the set $\overline{N}(L, L)$ is stable.

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