Key Concepts — Consumption

Definition 3.1. The Consumption Set is defined as the set of commodities (or goods) consumers can consume. Let there be J goods then we define the consumption set by $X = \mathbb{R}^J_{\geq 0}$. A typical member of this set is $\mathbf{x} = (x_1, x_2, \ldots, x_J)$ where x_j is the consumer's consumption of good j, for $j \in \{1, 2, \ldots, J\}$
Definition 3.2. The budget constraint equation is: $p_1 x_1 + \ldots + p_J x_J \leq M$.
This leads us onto the next definition:
Definition 3.3. The Walrasian budget set is the set of bundles our consumer can choose between

$$\left\{(x_1, \ldots x_J) \in \mathbb{R}^J_{\geq 0} \mid p_1 x_1 + \ldots + p_J x_J \leq M\right\}$$

Key Concepts — Preferences

Definition 3.5. Given $\succeq$ we define:

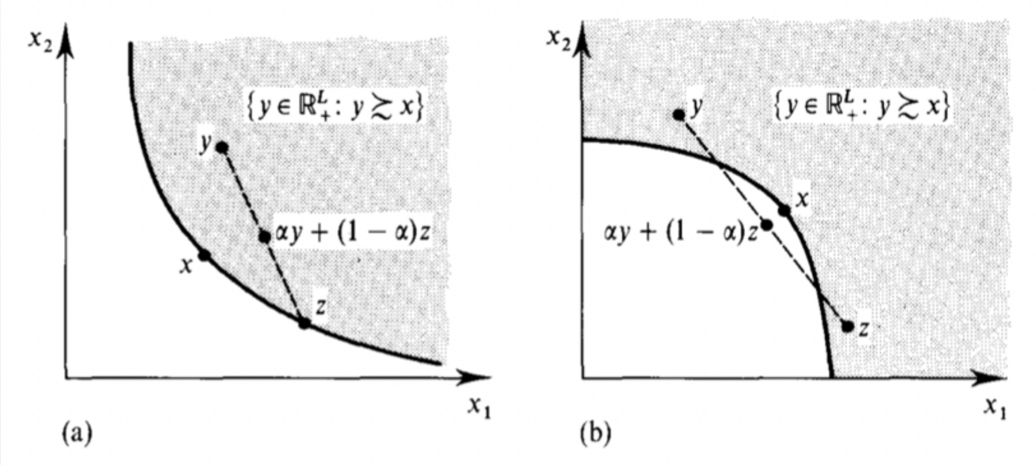
The strict preference relation: $\hat{\mathbf{x}} \succ \overline{\mathbf{x}} \iff \hat{\mathbf{x}} \succeq \overline{\mathbf{x}}$ and not $\hat{\mathbf{x}} \preceq \overline{\mathbf{x}}$.

The indifference relation: $\hat{\mathbf{x}} \sim \overline{\mathbf{x}} \iff \hat{\mathbf{x}} \succeq \overline{\mathbf{x}}$ and $\hat{\mathbf{x}} \preceq \overline{\mathbf{x}}$.

Two important axioms on preferences are completeness and transitivity:

Definition 3.6. Completeness: For any two bundles of goods in $X, \hat{\mathbf{x}} = (\hat{x}_1, \ldots \hat{x}_J)$ and $\overline{\mathbf{x}} = (\bar{x}_1, \ldots, \bar{x}_J)$, at least one of $\hat{\mathbf{x}} \succeq \overline{\mathbf{x}}$ or $\overline{\mathbf{x}} \succeq \hat{\mathbf{x}}$ must hold. (If both hold then $\overline{\mathbf{x}} \sim \hat{\mathbf{x}}$)

Definition 3.7. Transitivity: If we prefer bundle 1 to bundle 2 and bundle 2 to bundle 3 , then we should also prefer bundle 1 to bundle 3 . If $\mathbf{x}^{(1)} \succeq \mathbf{x}^{(2)}$ and $\mathbf{x}^{(2)} \succeq \mathbf{x}^{(3)}$ then $\mathbf{x}^{(1)} \succeq \mathbf{x}^{(3)}$



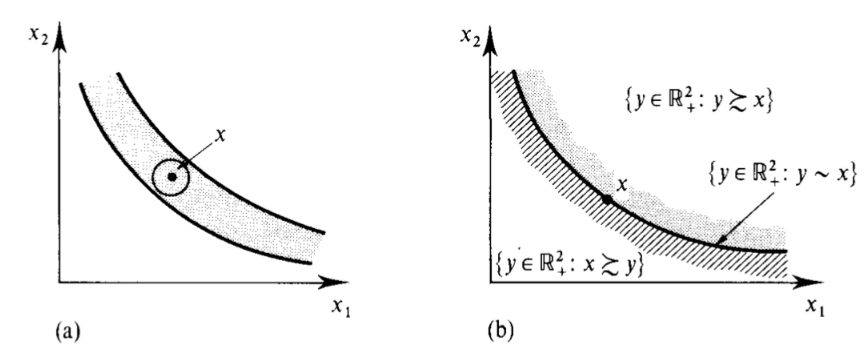


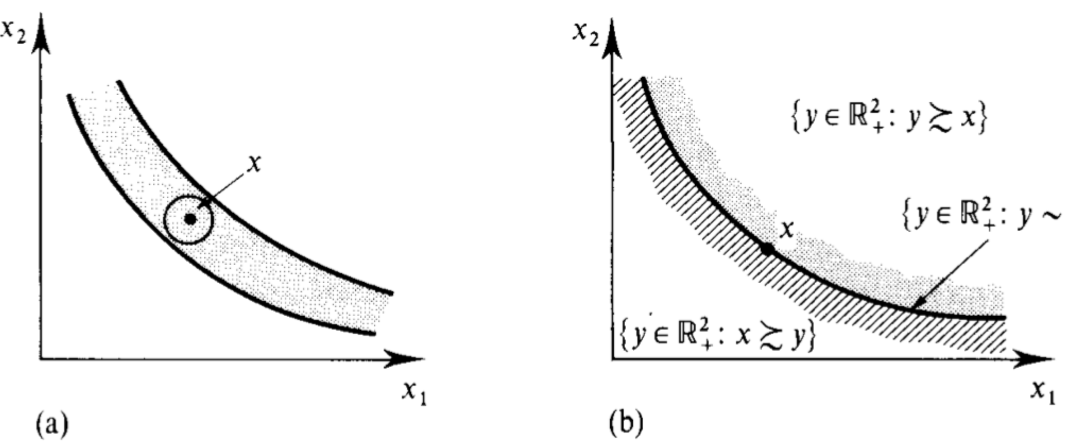
Figure 3.B.2
(a) A thick indifference set violates local nonsatiation.
(b) Preferences compatible with local nonsatiation.

Definition 3.8. Convex preferences: For every $\mathbf{x} \in X$, the upper contour set $\{\hat{\mathbf{x}} \in X \mid \hat{\mathbf{x}} \succeq \mathbf{x}\}$ is a convex set. In other words, if $\hat{\mathbf{x}} \succeq \mathbf{x}$ and $\overline{\mathbf{x}} \succeq \mathbf{x}$ then $\alpha\hat{\mathbf{x}} + (1-\alpha)\overline{\mathbf{x}} \succeq \mathbf{x}$ for any $\alpha \in [0,1]$

Definition 3.11. Preferences are Monotone if for every $\mathbf{x}, \hat{\mathbf{x}} \in X$, if $x_j < \hat{x}_j \forall j \in J$ then $\hat{\mathbf{x}} \succ \mathbf{x}$.

Definition 3.12. Preferences are locally non-satiated if for every $\mathbf{x} \in X, \forall \varepsilon > 0, \exists \hat{\mathbf{x}} \in X$ such that $\|\mathbf{x} - \hat{\mathbf{x}}\| < \varepsilon^7$ and $\hat{\mathbf{x}} \succ \mathbf{x}$

Key Concepts — Preferences



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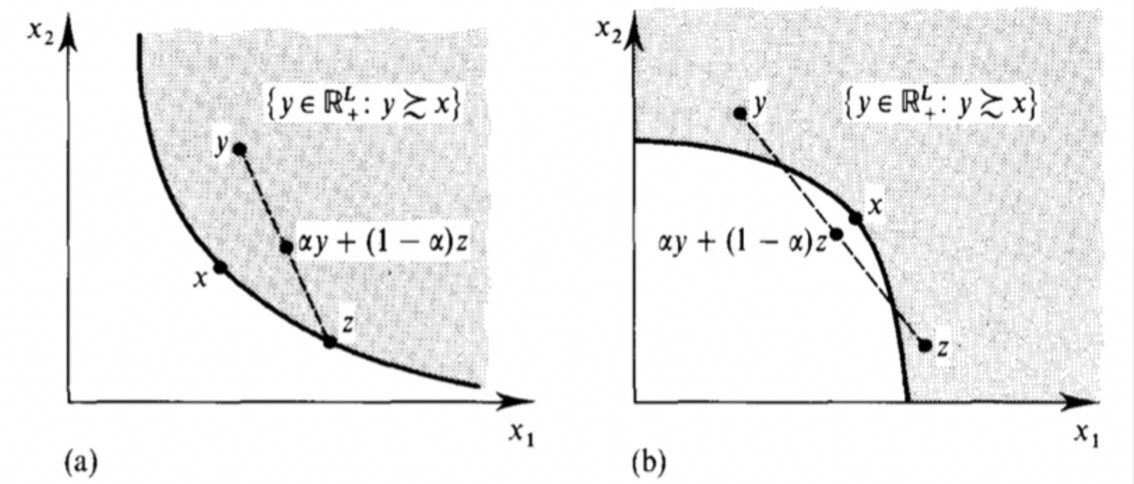
(a) A thick indifference set violates local nonsatiation.
(b) Preferences compatible with local nonsatiation.



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Definition 3.9. Strictly convex preferences: For every $\mathbf{x} \in X$, if $\hat{\mathbf{x}} \succeq \mathbf{x}$ and $\overline{\mathbf{x}} \succeq \mathbf{x}$, with $\hat{\mathbf{x}} \neq \overline{\mathbf{x}}$ then for any $\alpha \in (0, 1)$, $\alpha\hat{\mathbf{x}} + (1 - \alpha)\overline{\mathbf{x}} \succ \mathbf{x}$

Definition 3.10. Preferences are Strongly Monotone if for every $\mathbf{x}, \hat{\mathbf{x}} \in X$, if $x_j \leq \hat{x}_j \forall j \in J$ and $\exists j \in J$ with $x_j < \hat{x}_j$ then $\hat{\mathbf{x}} \succ \mathbf{x}$.
Definition 3.11. Preferences are Monotone if for every $\mathbf{x}, \hat{\mathbf{x}} \in X$, if $x_j < \hat{x}_j \forall j \in J$ then $\hat{\mathbf{x}} \succ \mathbf{x}$.

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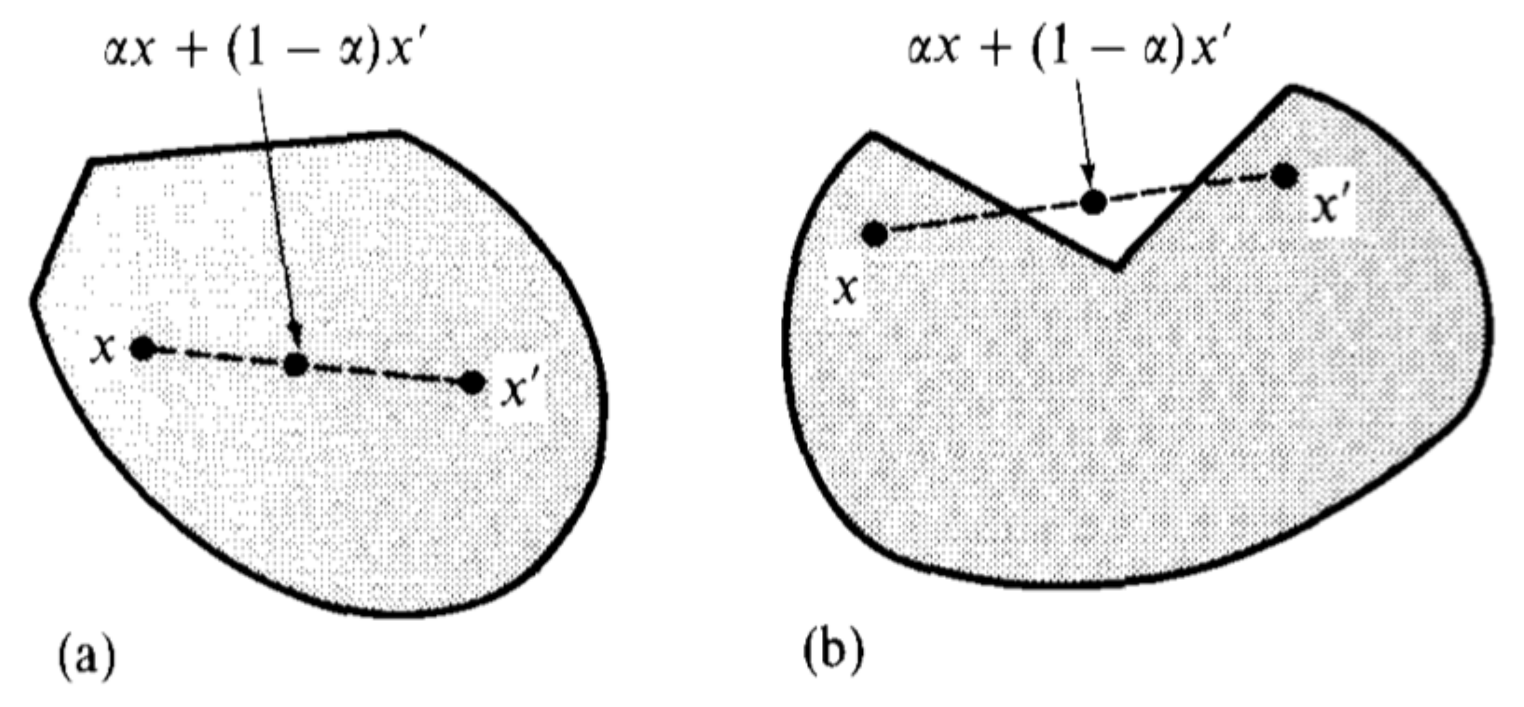
Key Concepts — Utility Functions and Sets

Definition 3.13. The preference relation $\succeq$ can be represented by a utility function $u : X \to \mathbb{R}$ if for every pair of bundles $\overline{\mathbf{x}}, \hat{\mathbf{x}} \in X, \hat{\mathbf{x}} \succeq \overline{\mathbf{x}} \iff u(\hat{\mathbf{x}}) \geq u(\overline{\mathbf{x}})$

Definition 3.14. A function $u : X \to \mathbb{R}$ is quasi-concave if its upper level sets, $\{x \in X : u(x) \geq c\}$, are convex for every $c \in \mathbb{R}$.

Definition M.G.1: The set $\boldsymbol{A} \subset \mathbb{R}^N$ is convex if $\alpha x + (1 - \alpha)x' \in A$ whenever $x, x' \in A$ and $\alpha \in [0, 1]$.



Def 3.0.0 Convexity and Concavity

A set is convex

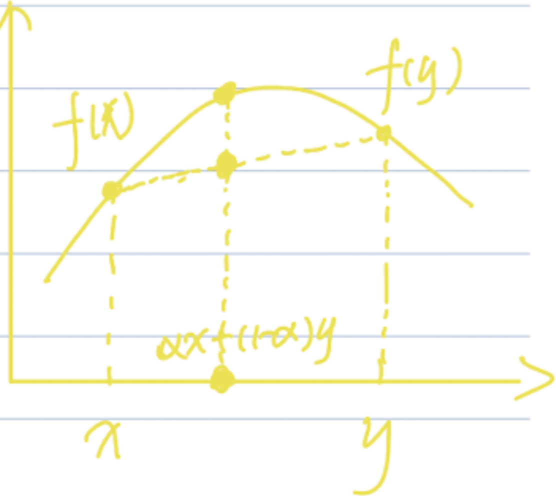
if, for all x and y in X, we have $[\alpha x + (1-\alpha)y] \in X$, for any $\alpha \in [0,1]$. A set that's not convex is called non-convex set.

A function is concave

if, $f[\alpha x + (1-\alpha)y] \geq \alpha f(x) + (1-\alpha)f(y)$
for all x, y, $\alpha \in [0,1]$

A function is strictly concave

if, $f[\alpha x + (1-\alpha)y] > \alpha f(x) + (1-\alpha)f(y)$
for all x, y, $x \neq y$, $\alpha \in (0,1)$



A function is quasiconcave

if, $f[\alpha x + (1-\alpha)y] \geq \min\{f(x), f(y)\}$
for all x, y, $\alpha \in [0,1]$

$\leq \max\{f(x), f(y)\}$
quasiconvex

or, the upper contour sets $\{x \in A : f(x) \geq t\}$ are convex sets,

$\{x : f(x) \leq t\}$
convex
quasiconvex

which means, $f(x) \geq t$ and $f(y) \geq t$ implies that $f[\alpha x + (1-\alpha)y] \geq t$.
for any $t \in R$, all x, y.
and $\alpha \in [0,1]$