EC202 Seminar
Week 4
Covering Materials from Week 2

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Key Concepts

- Exogenous parameters — variables that we take as given
  - Initial allocation — For agent $i \in I$, we let $\mathbf{e}_i = (e_{i1}, e_{i2}, \ldots, e_{iJ})$ be consumer i 's initial allocation or endowment. In this notation, e_{ij} is consumer i 's initial endowment of good j.
  - Preference — For agent $i \in I$, we let $\succeq_i$ be the preferences of agent i. Normally we will assume preferences are representable and so an agent i has some utility function $u_i : X \to \mathbb{R}$
- Endogenous parameters — variables of interest to calculate using a model
  - Post-trade allocation — For agent $i \in I$, we let $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iJ})$ be consumer i 's posttrade allocation. In this notation, x_{ij} is consumer i 's post-trade allocation of good j.
  - Price of each good — For each good $j \in J$, we have a per-unit price p_j, giving us a price vector $\mathbf{p} = (p_1, p_2, \ldots, p_J)$
- Pareto efficiency
  - Market clearing
  - Excess demand/supply
  - Feasible set of allocations
  - Pure exchange economy
  - Pareto optimality

Definition 2.3. The feasible set of allocations is

$$\left\{ \mathbf{x} = (\mathbf{x}_i)_{i \in I} \in \mathbb{R}^{I \times J}_{\geq 0} \mid \sum_{i \in I} \mathbf{x}_i = \sum_{i \in I} \mathbf{e}_i \right\}$$

Fact 2.1. In a 2×2 pure exchange economy an allocation is feasible if:

$$x_{A1} + x_{B1} = e_{A1} + e_{B1} \text{ and } x_{A2} + x_{B2} = e_{A2} + e_{B2}$$

In-class Question

Q3. Consider a 2×2 pure exchange economy where the initial endowment is $\mathbf{e}_A = (3,2)$,$\mathbf{e}_B = (2,1)$ and preferences represented by $u_A, u_B : \mathbb{R}^2_{\geq 0} \to \mathbb{R}$ given by

$$u_A(x_{A1}, x_{A2}) = \frac{x_{A1}}{4} + \ln(1 + x_{A2}) \qquad u_B(x_{B1}, x_{B2}) = x_{B1}x_{B2}$$

a) Draw indifference curves on an Edgeworth box and verify that preferences are convex.

b) Calculate Marginal Rates of Substitution for both agents and evaluate them at the initial endowment.

c) Use b) to draw indifference curves through the initial endowment and on your Edgeworth box depict a lens of allocation that Pareto dominates the initial endowment.

d) Argue that we can find an allocation that Pareto dominates the initial endowment by having Andy give Bob 0.6ε good 2 in exchange for ε more good 1 for some small $\varepsilon > 0$. Test this by setting $\varepsilon = 0.1$ and calculating utilities at $\mathbf{x}_A = (3.1, 1.94)$,$\mathbf{x}_B = (1.9, 1.06)$. You should find this Pareto dominates the initial endowment and give some intuition for this.

e) Find the Pareto Set and depict it on your Edgeworth box.

Q3. Consider a 2×2 pure exchange economy where the initial endowment is $\mathbf{e}_A = (3, 2), \mathbf{e}_B = (2, 1)$ and preferences represented by $u_A, u_B : \mathbb{R}^2_{\geq 0} \to \mathbb{R}$ given by

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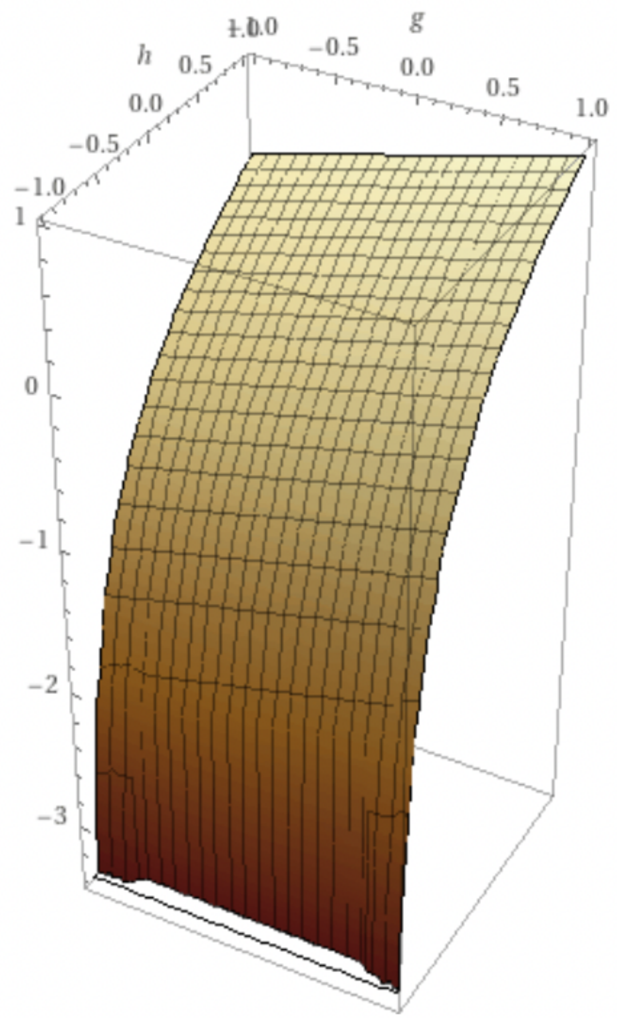
a) Draw indifference curves on an Edgeworth box and verify that preferences are convex.

WolframAlpha.com

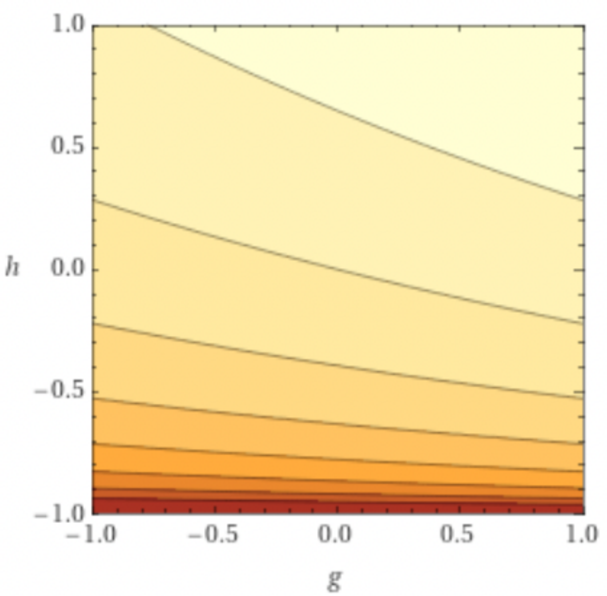
Input

$$u(g, h) = \frac{g}{4} + \log(1 + h)$$

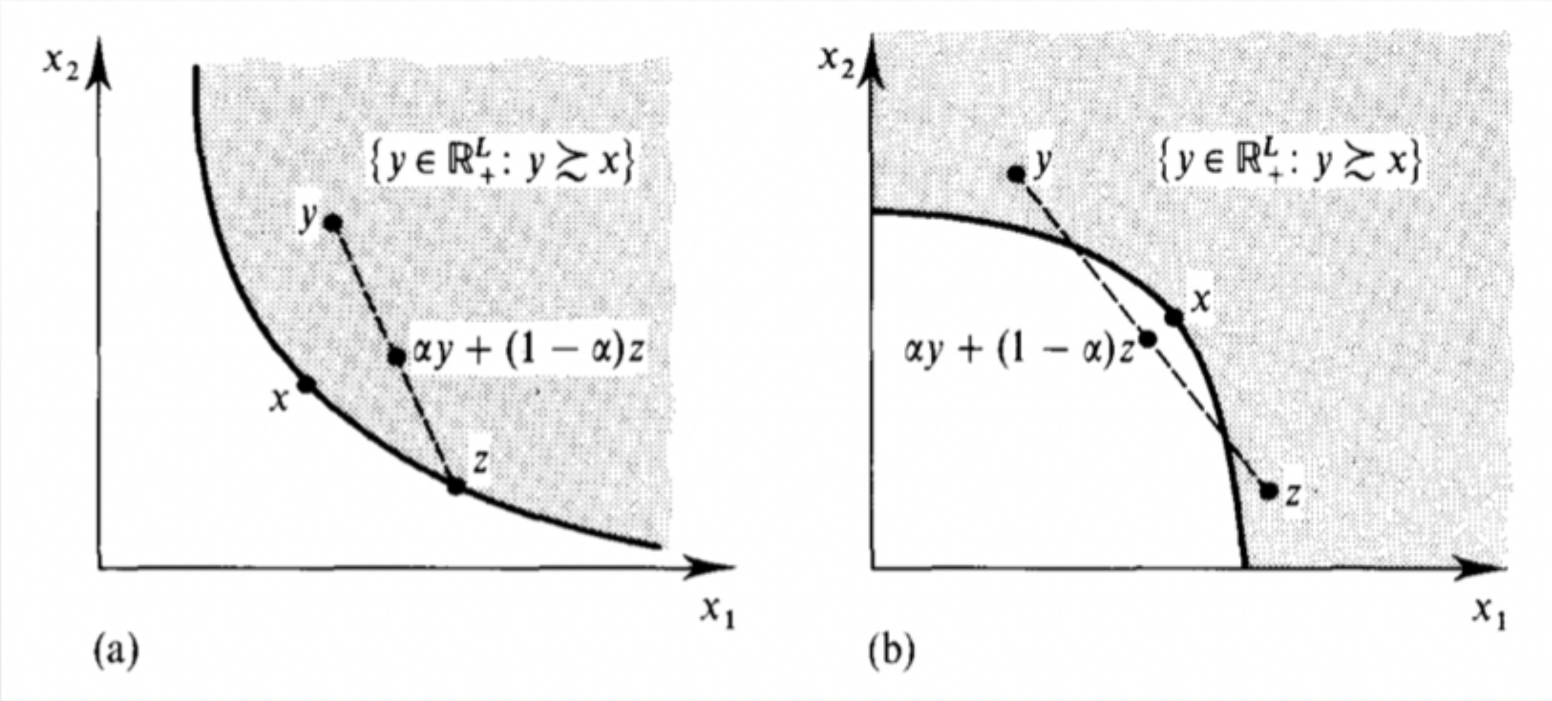
3D plot



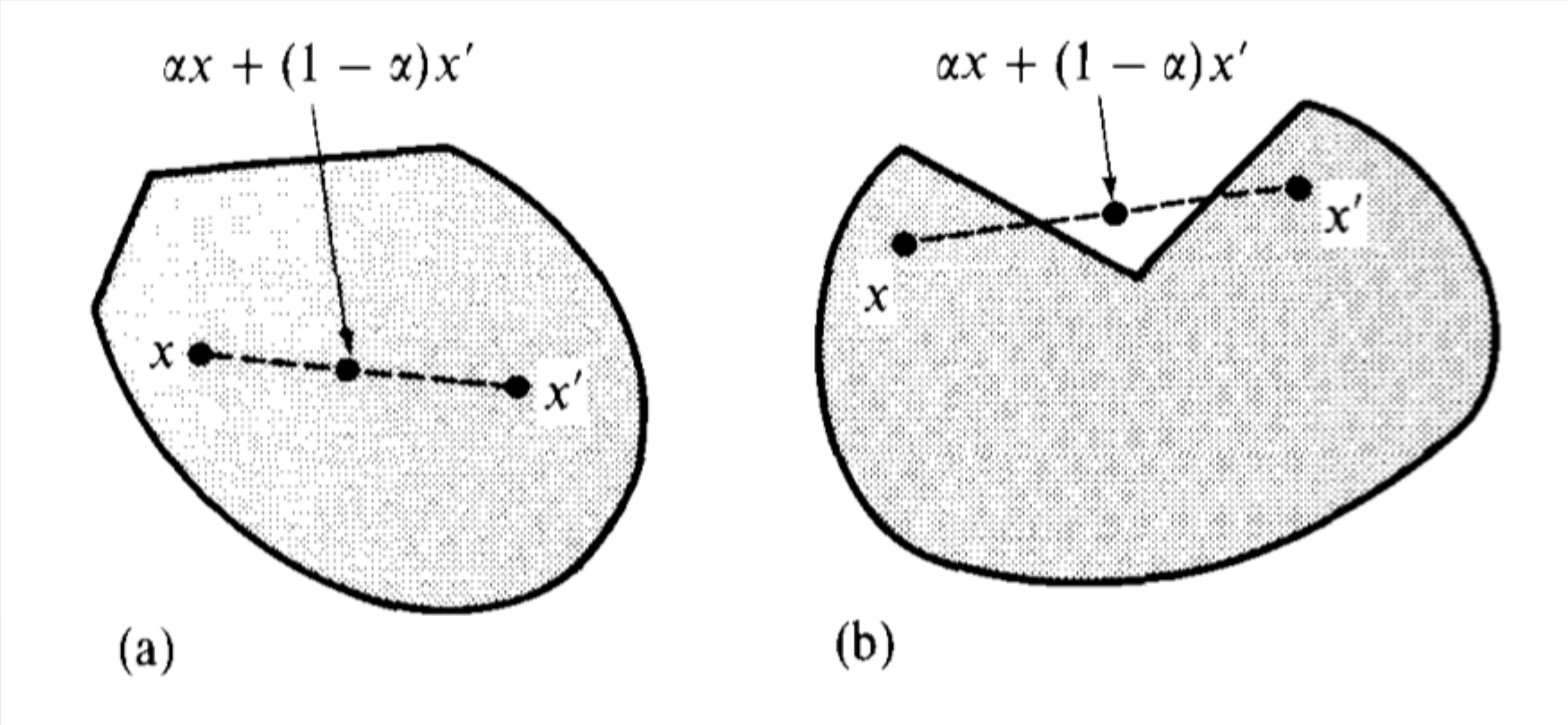
Contour plot



Definition 3.8. Convex preferences: For every $\mathbf{x} \in X$, the upper contour set $\{\hat{\mathbf{x}} \in X \mid \hat{\mathbf{x}} \succeq \mathbf{x}\}$ is a convex set. In other words, if $\hat{\mathbf{x}} \succeq \mathbf{x}$ and $\overline{\mathbf{x}} \succeq \mathbf{x}$ then $\alpha\hat{\mathbf{x}} + (1-\alpha)\overline{\mathbf{x}} \succeq \mathbf{x}$ for any $\alpha \in [0,1]$



Definition M.G.1: The set $\boldsymbol{A} \subset \mathbb{R}^N$ is convex if $\alpha x + (1-\alpha)x' \in A$ whenever $x, x' \in A$ and $\alpha \in [0,1]$.



Q3. Consider a 2×2 pure exchange economy where the initial endowment is $\mathbf{e}_A = (3,2), \mathbf{e}_B = (2,1)$ and preferences represented by $u_A, u_B : \mathbb{R}^2_{\geq 0} \to \mathbb{R}$ given by

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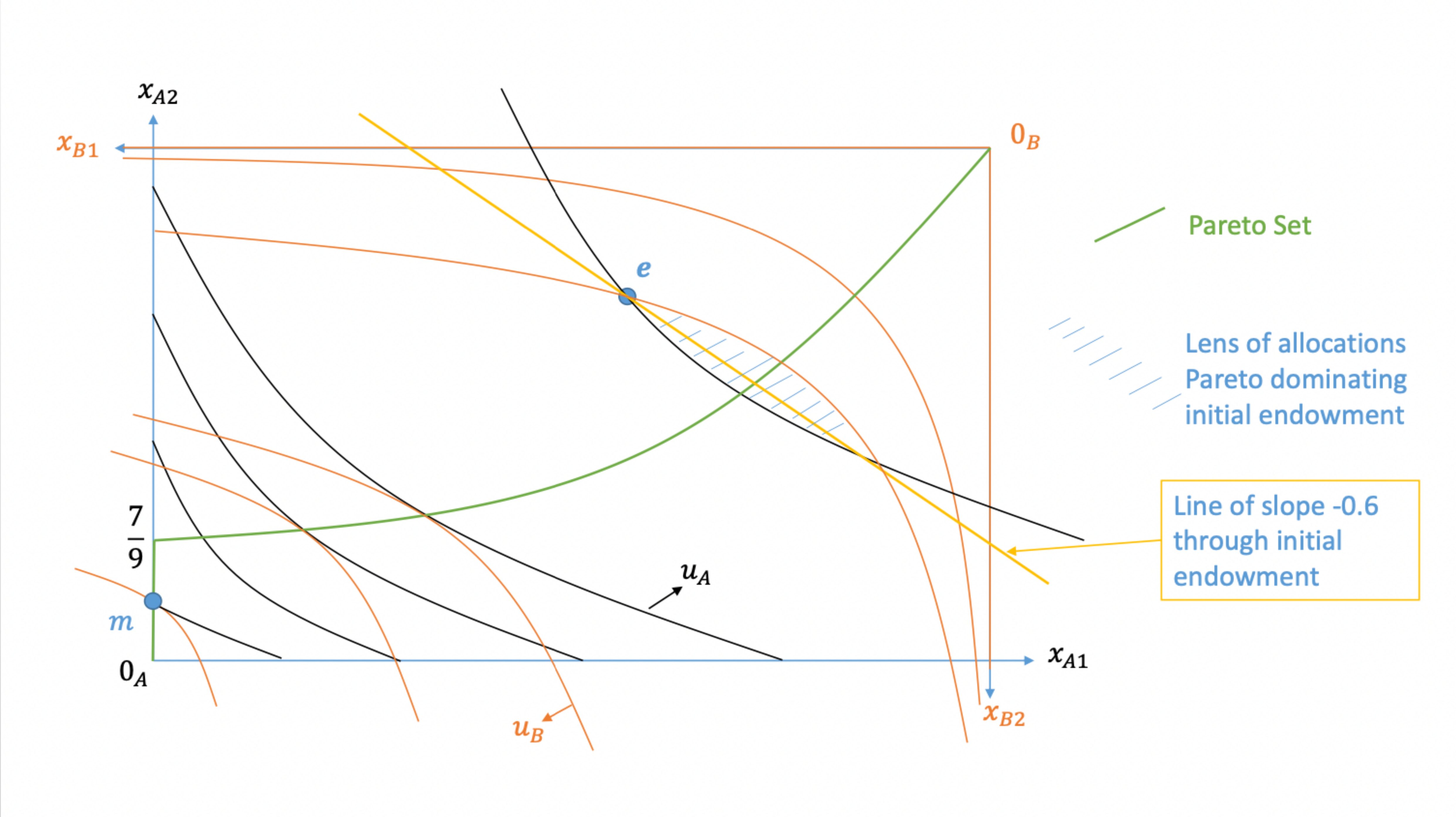
a) Draw indifference curves on an Edgeworth box and verify that preferences are convex.



Q3. Consider a 2×2 pure exchange economy where the initial endowment is $\mathbf{e}_A = (3, 2), \mathbf{e}_B = (2, 1)$ and preferences represented by $u_A, u_B : \mathbb{R}^2_{\geq 0} \to \mathbb{R}$ given by

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b) Calculate Marginal Rates of Substitution for both agents and evaluate them at the initial endowment.

b) Assuming $x_{B1} > 0$, the marginal rates of substitution are

$$MRS^A_{1,2} = -\frac{MU_1}{MU_2} = -\frac{\frac{1}{4}}{\frac{1}{1+x_{A2}}} = -\frac{1 + x_{A2}}{4}$$

$$MRS^B_{1,2} = -\frac{MU_1}{MU_2} = -\frac{x_{B2}}{x_{B1}}$$

Evaluated at the initial endowment, we get $MRS^A_{1,2} = -\frac{3}{4}$ and $MRS^B_{1,2} = -\frac{1}{2}$.

Q3. Consider a 2×2 pure exchange economy where the initial endowment is $\mathbf{e}_A = (3, 2), \mathbf{e}_B = (2, 1)$ and preferences represented by $u_A, u_B : \mathbb{R}^2_{\geq 0} \to \mathbb{R}$ given by

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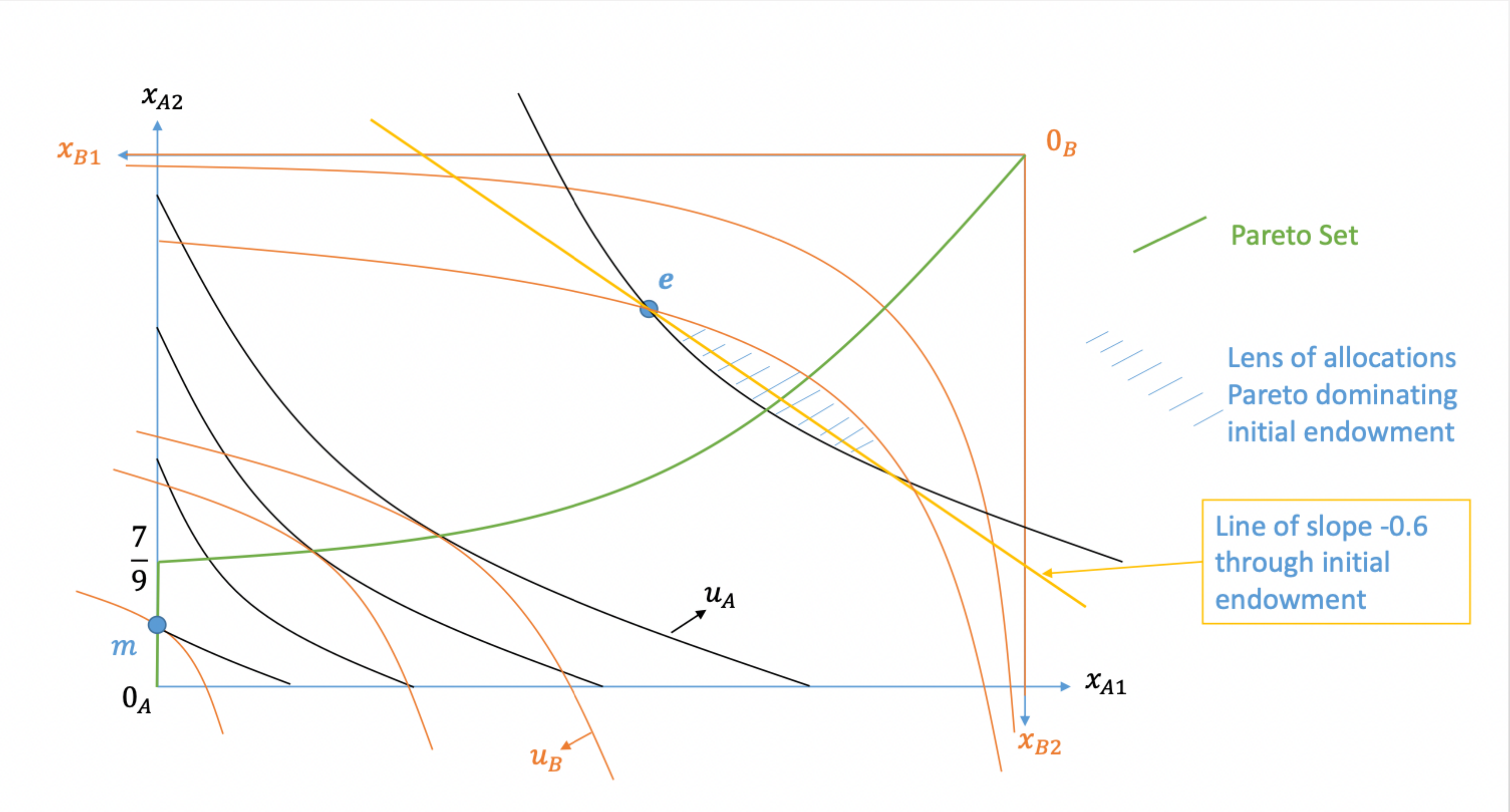
c) Use b) to draw indifference curves through the initial endowment and on your Edgeworth box depict a lens of allocation that Pareto dominates the initial endowment.



Q3. Consider a 2×2 pure exchange economy where the initial endowment is $\mathbf{e}_A = (3,2), \mathbf{e}_B = (2,1)$ and preferences represented by $u_A, u_B : \mathbb{R}^2_{\geq 0} \to \mathbb{R}$ given by

$$u_A(x_{A1}, x_{A2}) = \frac{x_{A1}}{4} + \ln(1 + x_{A2}) \quad u_B(x_{B1}, x_{B2}) = x_{B1}x_{B2}$$

d) Argue that we can find an allocation that Pareto dominates the initial endowment by having Andy give Bob 0.6ε good 2 in exchange for ε more good 1 for some small $\varepsilon > 0$. Test this by setting $\varepsilon = 0.1$ and calculating utilities at $\mathbf{x}_A = (3.1, 1.94), \mathbf{x}_B = (1.9, 1.06)$. You should find this Pareto dominates the initial endowment and give some intuition for this.

d) Observing the MRS of each agent, at the margin (ie for small $\varepsilon > 0$) Andy should be better off as long as he has to give up less than $\frac{3}{4}\varepsilon$ of good 2 for ε more good 1 . While Bob is better off as long as he gets more than $\frac{1}{2}\varepsilon$ more good 2 for giving up ε of good 1 . So exchanging in the ratio of 0.6 should make both better off for small $\varepsilon > 0$. This can be seen on the Edgeworth box, that there is section of allocations along the line with slope -0.6 going through the initial endowment that make both people better off. At initial endowment we calculate $(u_A, u_B) = (1.849, 2)$. At $\mathbf{x}_A = (3.1, 1.94), \mathbf{x}_B = (1.9, 1.06)$ we calculate $(u_A, u_B) = (1.853, 2.014)$.

Q3. Consider a 2×2 pure exchange economy where the initial endowment is $\mathbf{e}_A = (3, 2), \mathbf{e}_B = (2, 1)$ and preferences represented by $u_A, u_B : \mathbb{R}^2_{\geq 0} \to \mathbb{R}$ given by

$$u_A(x_{A1}, x_{A2}) = \frac{x_{A1}}{4} + \ln(1 + x_{A2}) \quad u_B(x_{B1}, x_{B2}) = x_{B1}x_{B2}$$

e) Find the Pareto Set and depict it on your Edgeworth box.

e) Lets start by finding interior Pareto optima. Since both preferences are convex any allocation where we have tangency of indifference curves will be Pareto efficient. Incorporating this with $\mathbf{x}_A + \mathbf{x}_B = (5,3)$ gives

$$MRS^A_{1,2} = MRS^B_{1,2} \iff -\frac{1+x_{A2}}{4} = -\frac{x_{B2}}{x_{B1}}$$

$$\iff \frac{4-x_{B2}}{4} = \frac{x_{B2}}{x_{B1}}$$

$$\iff x_{B2} = \frac{4x_{B1}}{4+x_{B1}}$$

This equation gives us a line of points stretching from 0_B to $(x_{B1}, x_{B2}) = \left(5, \frac{20}{9}\right)$. We can also see that 0_A is Pareto efficient since this is the unique feasible allocation maximising Bob's utility. So intuitively we would expect allocation on the western edge of the form $(x_{B1}, x_{B2}) = (5, c)$ for $c \in \left(\frac{20}{9}, 3\right)$ to also be Pareto efficient. My Edgeworth box considers an example of such a point, labelled m where we can see that Andy's indifference curve is shallower than Bob's and so this is Pareto efficient. Formally we can describe the Pareto Set as

$$\left\{(\mathbf{x}_A, \mathbf{x}_B) \in \mathbb{R}^{2\times 2}_{\geq 0} \mid x_{B2} = \tfrac{4x_{B1}}{4+x_{B1}}, \mathbf{x}_A + \mathbf{x}_B = (5,3)\right\}$$

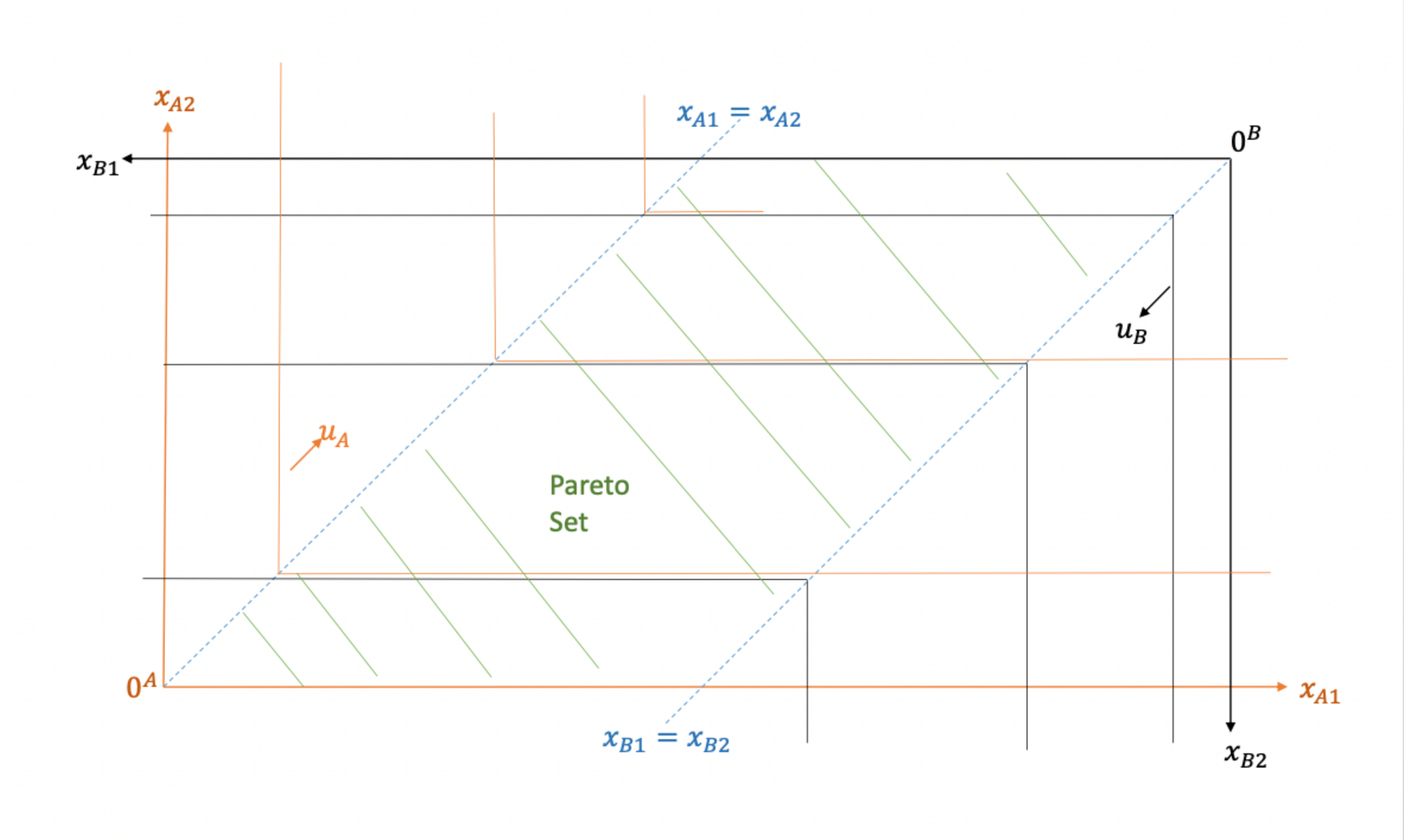
$$\left\{(\mathbf{x}_A, \mathbf{x}_B) \in \mathbb{R}^{2\times 2}_{\geq 0} \mid \mathbf{x}_A = (0,c), c \in \left[0, \tfrac{7}{9}\right], \mathbf{x}_A + \mathbf{x}_B = (5,3)\right\}$$

(Although for purposes of an exam, if set notation is intimidating, I would accept a sketch of the Pareto set, with kink at $\mathbf{x}_A = \left(0, \frac{7}{9}\right)$ identified plus the formula $x_{B2} = \frac{4x_{B1}}{4+x_{B1}}$ given for the curve.)

Q4. Consider a 2×2 pure exchange economy where there is 2 units of good 1 and 1 unit of good 2 in the economy. Let preferences be represented by $u_A, u_B : \mathbb{R}^2_{\geq 0} \to \mathbb{R}$ given by

$$u_A = \min\{x_{A1}, x_{A2}\} \quad u_B = \min\{x_{B1}, x_{B2}\}$$

a) Find the Pareto Set and illustrate it on an Edgeworth box.
b) Discuss why the Pareto Set takes the form it does.



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a) Find the Pareto Set and illustrate it on an Edgeworth box.
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As can be seen above, the Pareto Set is the plane of points

$$\{(\mathbf{x}_A, \mathbf{x}_B) \in \mathbb{R}^{2\times 2}_{\geq 0} \mid x_{A2} \leq x_{A1} \leq x_{A2} + 1, x_{A2} \in [0, 1], \mathbf{x}_A + \mathbf{x}_B = (2, 1)\}$$

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b) Discuss why the Pareto Set takes the form it does.

b) Both people only value units of one good when combined with the equal amount of the other good. Since there is more good 1 than good in our economy, this means that at any feasible allocation there will be some units of good 1 which provide no extra benefit to the person holding them. To illustrate this, consider the allocation $\mathbf{x}_A = \mathbf{x}_B = \left(1, \frac{1}{2}\right)$. This is Pareto efficient because the only way to make one person better off would be to give them more good 2 which would make the other person worse off. At this allocation each player has half a unit surplus of good 1. So for example Andy could give Bob up to half a unit of good 1 without being worse off, but this doesn't help Bob and so doesn't Pareto dominate the previous allocation. Indeed all allocations of the form $\mathbf{x}_A = \left(c, \frac{1}{2}\right), \mathbf{x}_B = \left(2 - c, \frac{1}{2}\right)$ for any $c \in \left[\frac{1}{2}, \frac{3}{2}\right]$ give the same utilities as each other, namely $(u_A, u_B) = \left(\frac{1}{2}, \frac{1}{2}\right)$. Similar logic would hold at any other given share of good 2: For any split $x_{A2} = k, x_{B2} = 1 - k$, there is a line of allocations of length 1 satisfying $x_{A1} \geq k$ and $x_{B1} \geq 1 - k$ at which utilities are $(u_A, u_B) = (k, 1 - k)$. All such points are Pareto efficient because to make one person better off would require them to be given more good 2 which would make the other person worse off.