EC202 Seminar Week 5
Covering Materials from Week 3

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Recap

- Pure exchange economy
- Utility function
- Edgeworth Box
  - Convex indifference curves
  - Pareto dominating set
  - Pareto optimal set
- Marginal Rate of Substitution (MRS)
- Marginal utility
- Budget set
- Allocation

Notions

- Excess demand and supply

  The excess demand of good j, denoted z_j can be calculated as

$$z_j = \sum_{i \in I} x_{ij} - \sum_{i \in I} e_{ij}$$

  For a 2×2 economy, this means:

  $z_1 = (x_{A1} + x_{B1}) - (e_{A1} + e_{B1}) \quad z_2 = (x_{A2} + x_{B2}) - (e_{A2} + e_{B2})$

- Utility Maximisation Problem

$$\max_{\mathbf{x}_i \in X} u_i(\mathbf{x}_i) \text{ subject to } \mathbf{p} \cdot \mathbf{x}_i \leq \mathbf{p} \cdot \mathbf{e}_i$$

- Walrasian Equilibrium

  Definition A price vector $\mathbf{p}^* = (p_1^*, p_2^*, \ldots, p_J^*) \in \mathbb{R}_{\geq 0}^J$ and an allocation $\mathbf{x}^* = (\mathbf{x}_i^*)_{i \in I} \in \mathbb{R}_{\geq 0}^{I \times J}$ constitute a Walrasian Equilibrium if

  i) For each consumer $i \in I$, bundle $\mathbf{x}_i^*$ solves the consumer's (UMP) at prices p^*.

  ii) All markets clear: $\sum_{i \in I} \mathbf{x}_i^* = \sum_{i \in I} \mathbf{e}_i$

  We call $\mathbf{p}^*$ a Walrasian Equilibrium price vector and $\mathbf{x}^*$ a Walrasian Equilibrium allocation.

  - Existence?
  - Uniqueness?

More Notions...

- What is an Walrasian Equilibrium allocation, **x?**
- Aggregate demand and supply ***function***

  Definition 3.2. The aggregate demand function $\sum_{i \in I} \mathbf{x}_i(\mathbf{p}) \in \mathbb{R}^J_{\geq 0}$ is the total demanded by consumers as a function of prices. This is a J-dimensional vector whose j^{th} element is $\sum_{i \in I} x_{ij}(\mathbf{p})$

- Excess demand and supply ***function***

  Definition 3.3. The excess demand function is $\mathbf{z}(\mathbf{p}) = \sum_{i \in I} \mathbf{x}_i(\mathbf{p}) - \sum_{i \in I} \mathbf{e}_i \in \mathbb{R}^J$. We can also talk about the excess demand for good j as the j^{th} element of this, which is $z_j(\mathbf{p}) = \sum_{i \in I} x_{ij}(\mathbf{p}) - \sum_{i \in I} e_{ij}$. When $z_j(\mathbf{p}) > 0$ we say that there is excess demand of good j and when $z_j(\mathbf{p}) < 0$ we say there is excess supply of good j.

- What is a price vector, v?
- What (the heck) is, Walras' Law?

  Proposition 3.1. Walras' Law: Assuming all individuals have locally nonsatiated preferences, for every price vector $\mathbf{p} \in \mathbb{R}^J_{\geq 0}$ for which individuals' demand functions are finite, $\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = p_1 z_1(\mathbf{p}) + \cdots + p_J z_J(\mathbf{p}) = 0$. In words the value of the excess demand vector equals 0 . This holds for all prices regardless of whether they are equilibrium prices or not.

- ***p*x = w***

In-class Question

Q3. Consider a 2×2 economy where preferences are represented by $u_A, u_B : \mathbb{R}^2_{\geq 0}$ where $u_A = x^{\alpha}_{A1} x^{1-\alpha}_{A2}$ and $u_B = x^{\beta}_{B1} x^{1-\beta}_{B2}$ for some $\alpha, \beta \in (0, 1)$. Assume both goods are in strctly positive supply.

a) Argue that, for any initial endowment, there are no Walrasian Equilibria where one good has price 0 and hence in an Walrasian Equilibrium we must have $(p_1, p_2) \in \mathbb{R}^2_{>0}$.

b) Let prices be $(p_1, p_2) \in \mathbb{R}^2_{>0}$ and incomes be M_A, M_B. Verify that optimal demands are

$$\mathbf{x}_A(\mathbf{p}, M_A) = \left(\frac{\alpha M_A}{p_1}, \frac{(1-\alpha) M_A}{p_2} \right)$$

$$\mathbf{x}_B(\mathbf{p}, M_B) = \left(\frac{\beta M_B}{p_1}, \frac{(1-\beta) M_B}{p_2} \right)$$

In-class Question

c) Consider the initial endowment $\mathbf{e}_A = (0,1), \mathbf{e}_B = (1,0)$:

i) Find optimal demands $\mathbf{x}_A(\mathbf{p})$ and $\mathbf{x}_B(\mathbf{p})$. Show that these demands satisfy Walras' Law.

ii) Find the Walrasian Equilibrium and illustrate it on an Edgeworth box.

iii) Verify on your Edgeworth box and algebraically that both players prefer the Walrasian Equilibrium to their initial allocation.

d) Repeat b) for initial endowment $\mathbf{e}_A = \left(\frac{1}{2}, \frac{1}{2}\right), \mathbf{e}_B = \left(\frac{1}{2}, \frac{1}{2}\right)$ and $\alpha < \beta$