EC202 Week 8

Junxi Liu

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Externalities and Public Goods

- Does the world in the previous few lectures sound too unrealistic?
- Trade is not always fair and perfect
  - Information asymmetry
  - Questionable utility functions
  - Behavioral choices
  - Waste
- Externalities
  - An externality occurs when a person's well-being or a firm's production capability is directly affected by the actions of other consumers or firms (rather than indirectly affected through changes in price).
- Public Goods
  - A good that is excludable and rivalrous is called a *private good*. A good that is non-excludable and non-rivalrous is called a *public good*.

In-class Question

Q4. In the fireworks example of Lecture 7, Section 2.2, we change Andy's preferences to $u_A = c_A + 2\ln(1+f)$, while Bob still has preferences $u_B = c_B + \ln(1+f)$, where $f = f_A + f_B$. Each still face budget constraint of $c_i + pf_i \leq M_i$, where M_i for each $i \in \{A, B\}$ is high enough to ensure that each consume with $c_i > 0$.

a) Find the Nash Equilibria. What levels of M_A and M_B constitute "high enough" to guarantee that the Nash Equilibria are of this form?

b) Find the socially optimal level of fireworks by maximising the sum of utilities (assuming high M_A and M_B).

c) Derive and draw the demand curves of each player and the "aggregate demand" curve showing their combined marginal willingness to pay for an extra unit. Show the difference between fireworks being considered a public good and a private good.

d) Suppose $p = \frac{1}{2}$ is the market price at which Andy and Bob can buy any number of fireworks. Find the Lindahl Equilibrium.

e) Suppose $p = \frac{1}{2}$. Find the level of subsidy the government could introduce to correct this problem of underprovision of fireworks.

f) Alternatively the government considers giving Andy and Bob some fireworks. Is there a level of fireworks they could give that would give the socially efficient outcome?

g) Alternatively, suppose the government leaves it to Andy and Bob to sort out between them. Assume the conditions for the Coase Theorem holds. Find the set of possible agreements that might result. Is the Lindahl Equilibrium a member of this set? The algebra gets messy here due to the utility functions, so don't need to do the calculations, but explain what calculations you would do to answer this question.

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a) Find the Nash Equilibria. What levels of M_A and M_B constitute "high enough" to guarantee that the Nash Equilibria are of this form?

a) Andy solves

$$\max_{(c_A, f_A) \in \mathbb{R}^2_{\geq 0}} u_A = c_A + 2\ln(1 + f_A + f_B) \text{ s.t. } c_A + pf_A \leq M_A$$

Assuming an interior solution

$$\frac{\frac{du_A}{dc_A}}{1} = \frac{\frac{du_A}{df_A}}{p} \iff 1 = \frac{2}{p(1 + f_A + f_B)}$$

$$\iff f_A = \frac{2}{p} - 1 - f_B$$

Thus we get

$$\text{Best Response of } A \text{ is } f_A = \begin{cases} \frac{2}{p} - 1 - f_B & f_B < \frac{2}{p} - 1 \\ 0 & f_B \geq \frac{2}{p} - 1 \end{cases}$$

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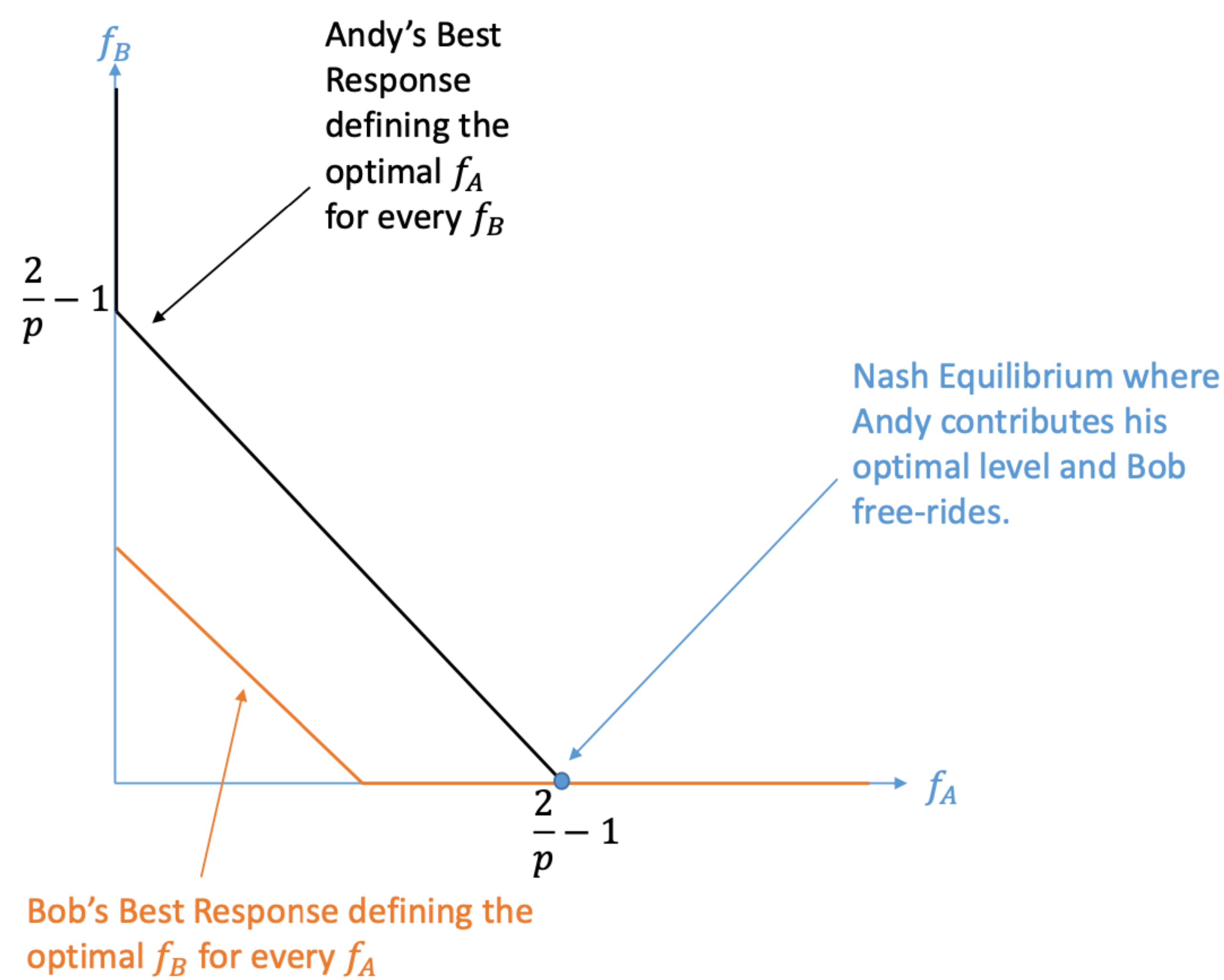
a) Find the Nash Equilibria. What levels of M_A and M_B constitute "high enough" to guarantee that the Nash Equilibria are of this form?

By similar logic or copying from Example 2.4 of Lecture 7 we get

$$\text{Best Response of } B \text{ is } f_B = \begin{cases} \frac{1}{p} - 1 - f_A & f_A < \frac{1}{p} - 1 \\ 0 & f_A \geq \frac{1}{p} - 1 \end{cases}$$



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a) Find the Nash Equilibria. What levels of M_A and M_B constitute "high enough" to guarantee that the Nash Equilibria are of this form?

As we see above, there is a unique Nash Equilibrium where Andy contributes and Bob free-rides. The above is drawn for low p; while for $p \geq 2$, Andy is priced out of buying any fireworks. So in general, the Nash Equilibrium is

$$\text{NE}: f_A = \max\left\{\frac{2}{p} - 1, 0\right\}, \quad f_B = 0$$

To write their best responses in the way we have, it required that M_A and M_B are sufficiently high that $c_A > 0$ and $c_B > 0$, which required:

$$\text{For Andy: } p\left(\frac{2}{p} - 1 - f_B\right) < M_A$$

$$\text{For Bob: } p\left(\frac{1}{p} - 1 - f_A\right) < M_B$$

Since our Nash Equilibrium is of the form above this simplifies to requiring

$M_A > 2 - p$. While the restriction on Bob's income is satisfied for all $M_B > 0$ assuming that Andy's restriction is satisfied so that $f_A = \max\left\{\frac{2}{p} - 1, 0\right\}$.

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b) Find the socially optimal level of fireworks by maximising the sum of utilities (assuming high M_A and M_B).

b) The socially optimal level is found by solving

$$\max_{(c_A,f_A),(c_B,f_B)\in\mathbb{R}^2_{\geq 0}} U = u_A + u_B$$
$$\text{subject to } c_i + f_i \leq M_i \quad i \in \{A, B\}$$

which assuming M_i sufficiently high so that agents want to consume some consumption good simplifies to

$$\max_{c_A,c_B,f\geq 0} U = c_A + 2\ln(1+f) + c_B + \ln(1+f) = 3\ln(1+f) + (c_A + c_B)$$
$$\text{subject to } c_A + c_B + f \leq M_A + M_B$$

This has solution where

$$\frac{\frac{dU}{d(c_A+c_B)}}{1} = \frac{\frac{dU}{df}}{p} \iff 1 = \frac{3}{p(1+f)}$$
$$\iff f = \frac{3}{p} - 1$$

Also allowing for the possibility of $p > 3$, we get the efficient number of fireworks is $f = \max\left(\frac{3}{p} - 1, 0\right)$.

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c) Derive and draw the demand curves of each player and the "aggregate demand" curve showing their combined marginal willingness to pay for an extra unit. Show the difference between fireworks being considered a public good and a private good.

c) If the goods were private goods, the demands for Andy and Bob respectively would be

$$q_A = \begin{cases} \frac{2}{p} - 1 & p < 2 \\ 0 & p \geq 2 \end{cases}$$

By similar logic or copying from Example 2.4 of Lecture 7 we get

$$q_B = \begin{cases} \frac{1}{p} - 1 & p < 1 \\ 0 & p \geq 1 \end{cases}$$

The left hand diagram shows their aggregate demand if fireworks were private goods. The right hand diagram shows their aggregate demand when they are public goods.

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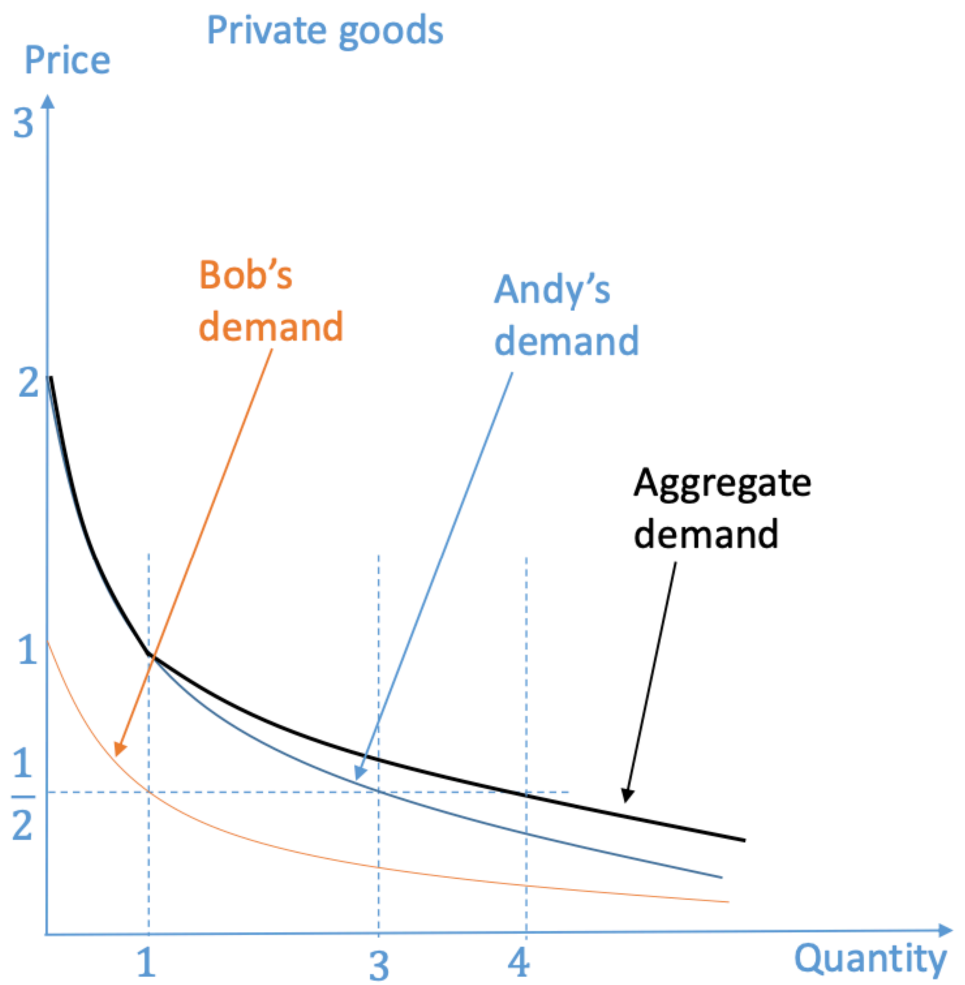
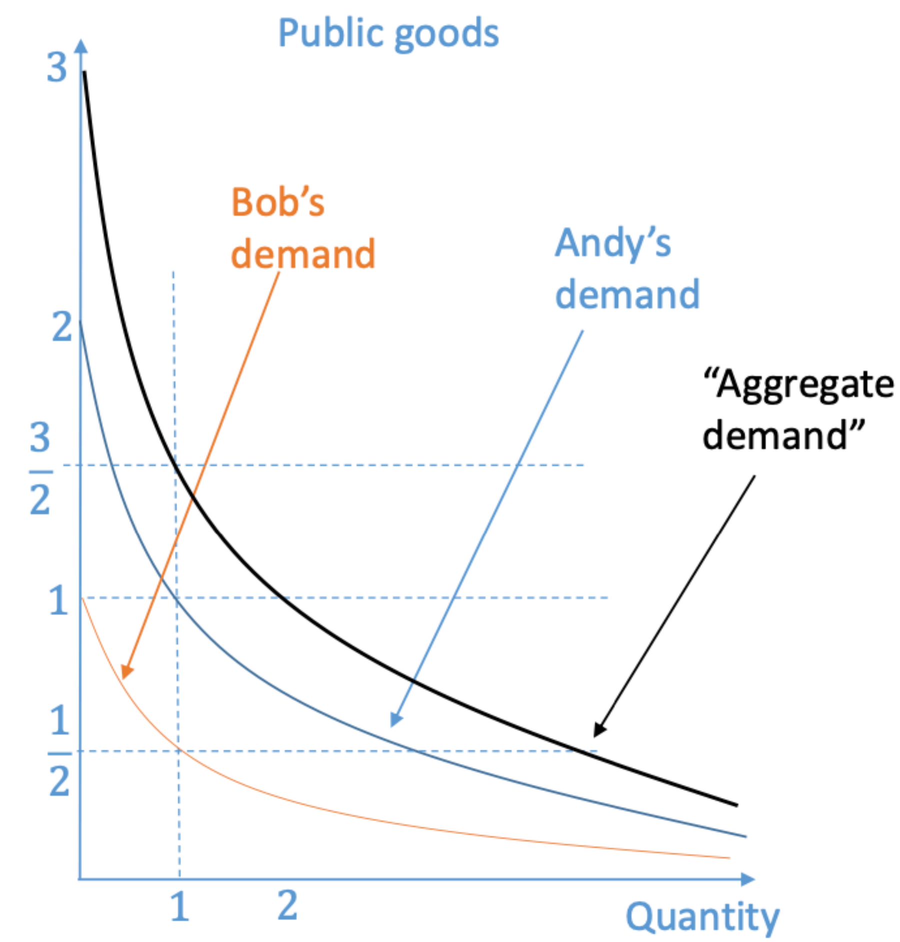


We sum up individual demands horizontally: At price p, Andy is prepared to buy $q_A = \max(2/p - 1, 0)$ and Bob is prepared to buy $q_B = \max(1/p - 1, 0)$. Then aggregate demand is calculated by $q_A + q_B$.



We sum up individual demands vertically: At quantity q, Andy values the marginal extra unit at $p_A = \frac{2}{1+q}$ and Bob values it at $p_B = \frac{1}{1+q}$. So the sum of values for the marginal extra unit is $\frac{3}{1+q}$.

Q4. In the fireworks example of Lecture 7, Section 2.2, we change Andy's preferences to $u_A = c_A + 2\ln(1+f)$, while Bob still has preferences $u_B = c_B + \ln(1+f)$, where $f = f_A + f_B$. Each still face budget constraint of $c_i + pf_i \leq M_i$, where M_i for each $i \in \{A, B\}$ is high enough to ensure that each consume with $c_i > 0$.

c) Derive and draw the demand curves of each player and the "aggregate demand" curve showing their combined marginal willingness to pay for an extra unit. Show the difference between fireworks being considered a public good and a private good.

As seen above in the private goods market, aggregate demand is given by

$$Q^{agg} = \begin{cases} \frac{3}{p} - 2 & p < 1 \\ \frac{2}{p} - 1 & p \in [1, 2] \\ 0 & p > 2 \end{cases}$$

While in the public goods market "aggregate demand", the joint marginal willingness to pay for an extra unit of output is given by

$$P^{agg} = \frac{3}{1 + Q}$$

Q4. In the fireworks example of Lecture 7, Section 2.2, we change Andy's preferences to $u_A = c_A + 2\ln(1+f)$, while Bob still has preferences $u_B = c_B + \ln(1+f)$, where $f = f_A + f_B$. Each still face budget constraint of $c_i + pf_i \leq M_i$, where M_i for each $i \in \{A, B\}$ is high enough to ensure that each consume with $c_i > 0$.

d) Suppose $p = \frac{1}{2}$ is the market price at which Andy and Bob can buy any number of fireworks. Find the Lindahl Equilibrium.

d) The Lindahl Equilibrium quantity occurs where $p = \frac{1}{2}$ intersects the "aggregate demand" curve. That is we find Q such that

$$\frac{1}{2} = \frac{3}{1+Q} \iff Q = 5$$

At this quantity, consumers' marginal willingness to pay and hence what they pay per unit are

$$p_A = \frac{1}{3}, \quad p_B = \frac{1}{6}$$

Thus in the Lindahl Equilibrium a total of 5 fireworks are bought at a total cost of $\frac{5}{2}$. Andy bears $\frac{2}{3}$ of this cost paying $\frac{5}{3}$ and Bob bears $\frac{1}{3}$ of this cost paying $\frac{5}{6}$.

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e) Suppose $p = \frac{1}{2}$. Find the level of subsidy the government could introduce to correct this problem of underprovision of fireworks.

e) The efficient level when $p = \frac{1}{2}$ is $Q = 5$. However in Nash Equilibrium Bob buys none and Andy is only prepared to buy $f_A = \max\left\{\frac{2}{p} - 1, 0\right\}$. So to make this equal to 5 we require Andy to face a price of $p_A = \frac{1}{3}$. Thus we need a subsidy of

$$s = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

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f) Alternatively the government considers giving Andy and Bob some fireworks. Is there a level of fireworks they could give that would give the socially efficient outcome?

f) If the government provides an additional $g < \frac{2}{p} - 1$ units [6]. Then Andy maximises utility by solving

$$\max_{(c_A, f_A) \in \mathbb{R}^2_{\geq 0}} u_A = c_A + 2\ln(1 + g + f_A + f_B) \text{ s.t. } c_A + pf_A \leq M_A$$

We find that

$$\text{Best Response of } A \text{ is } f_A = \begin{cases} \frac{2}{p} - 1 - g - f_B & f_B < \frac{2}{p} - 1 - g \\ 0 & f_B \geq \frac{1}{p} - 1 - g \end{cases}$$

By symmetry

$$\text{Best Response of } B \text{ is } f_B = \begin{cases} \frac{1}{p} - 1 - g - f_A & f_A < \frac{1}{p} - 1 - g \\ 0 & f_A \geq \frac{1}{p} - 1 - g \end{cases}$$

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f) Alternatively the government considers giving Andy and Bob some fireworks. Is there a level of fireworks they could give that would give the socially efficient outcome?

We get a similar best response diagram to what we drew in a). Thus our Nash Equilibria now is $f_B = 0$, $f_A = \max\left\{\frac{2}{p} - 1 - g, 0\right\}$ and so once again the total amount of fireworks is only

$$f = f_A + f_B + g = \frac{2}{p} - 1$$

Thus for $g < \frac{2}{p} - 1$, the government provision has just crowded out private provision and resulted in no more fireworks being bought. If the government wants the level of fireworks to be efficient it needs to provide all $f = \max\left\{\frac{3}{p} - 1, 0\right\}$ of them itself.

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g) Alternatively, suppose the government leaves it to Andy and Bob to sort out between them. Assume the conditions for the Coase Theorem holds. Find the set of possible agreements that might result. Is the Lindahl Equilibrium a member of this set? The algebra gets messy here due to the utility functions, so don't need to do the calculations, but explain what calculations you would do to answer this question.

g) By the Coase Theorem we expect players to negotiate to mutually beneficial and socially efficient level of fireworks. We know the efficient level, $f = \max\left\{\frac{3}{p} - 1, 0\right\}$, but mutually beneficial means each must be at least as well off as they were under the status quo. So we need to calculate each player's utility in the status quo which is the NE found in part a). I assume that $\frac{3}{p} - 1 > 0$ as the opposite case is trivial. The set of agreements will be of the following form for some $c_1, c_2 \in \mathbb{R}$

$$\left\{(f_A, f_B) \in \mathbb{R}^2_{\geq 0} \mid (f_A, f_B) = \left(c, \frac{3}{p} - 1 - c\right), c \in (c_1, c_2)\right\}$$

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To calculate c_2, we need to find the cost Andy would need to incur under $f = \frac{3}{p} - 1$ to be indifferent to the status quo and let c_2 be the number of fireworks to give Andy that cost. To calculate c_1, we need to find the cost Bob would need to incur under $f = \frac{3}{p} - 1$ to be indifferent to the status quo and let c_2 be the number of fireworks to give Bob that cost. In Lindahl Equilibrium, Andy purchases two thirds and Bob purchase one third so we can see if this is a member of that set. Alternatively we can do this more directly by calculating both peoples' utility at the NE and Lindahl Equilibrium and seeing whether both players are at least as well off at the latter. Note that the property of efficiency is already guaranteed at the Lindahl Equilibrium.

Further thoughts?

- To what extend does the existence of externality justify government intervention?
- To what extend does the public goods problem justify government intervention?
- What is a good policy?