EC202 Seminar Week 9

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A Bit of Logistics

- There will be no seminars next week, nor office hours.

- Per James' request, another seminar and corresponding office hours will be held in term 3. Related information will be updated via email (and also Tabula, I suppose).

Some Key Concepts

- Different voting systems: plurality, alternative, plurality runoff

- Voting preference profile and Social Welfare Function

- Dictatorship and Non-Dictatorship

- Unanimity

- Independence of Irrelevant Alternatives

- Arrow's Impossibility Theorem

Theorem 2.1. *Arrow's Impossibility Theorem*

Let the number of voters be $N \geq 2$ and the number of alternatives be $|\mathcal{A}| \geq 3$ then assuming (UD), there is no SWF satisfying (U), (IIA) and (ND)

- Condorcet winner and loser; Condorcet Cycle

- Single peaked preference

Some Key Concepts

- Black's median voter Theorem

Theorem 2.2. *Black's median voter Theorem*

If preferences are single peaked then the median voter's ideal point is the Condorcet winner.

- Violation of Arrow's Impossibility Theorem

- Strategy Proofness

- Gibbard-Satterthwaite Theorem

Theorem 3.1. *The Gibbard-Satterthwaite Theorem:*

Let the number of voters be $N \geq 2$ and the number of alternatives be $|\mathcal{A}| \geq 3$ then assuming (UD), there is no surjective SCF satisfying (SP) and (ND).

In-class Question

Q3. Recall that for Arrow's Impossibility Theorem to hold, we need the following 6 things:

i) At least 3 alternatives, $|\mathcal{A}| \geq 3$.

ii) At least two voters, $N \geq 2$.

iii) Unrestricted Domain, (UD)

iv) Unanimity, (U)

v) Independence of Irrelevant Alternatives, (IIA)

vi) Non-dictatorship, (ND)

For each of these six, show why they are needed for Arrow's Impossibility Theorem to hold. In other words find an example of an SWF satisfying the other five.

Q4. This question is about the merits of the plurality voting system compared to Alternative Vote. We show some pros and cons of each.

Let there be 3 alternatives and a large number of voters. Each voter has a strict preference ordering so fits into one of the following 6 categories:

	I	II	III	IV	V	VI
1st Preference	a	a	b	b	c	c
2nd Preference	b	c	a	c	a	b
3rd Preference	c	b	c	a	b	a

By varying the percentages of voters in each category, we demonstrate how we can get examples of some of the disadvantages of each system:[1]

a) Consider the voting profiles below.

Profile 1	I 20%	II 15%	III 20%	IV 12%	V 13%	VI 20%
1st Preference	a	a	b	b	c	c
2nd Preference	b	c	a	c	a	b
3rd Preference	c	b	c	a	b	a

Profile 2	I 20%	II 17%	III 20%	IV 12%	V 11%	VI 20%
1st Preference	a	a	b	b	c	c
2nd Preference	b	c	a	c	a	b
3rd Preference	c	b	c	a	b	a

a1) Show that under Profile 1 Alternative vote elects a. However, if 2% of the population change their preferences over (a, c) in favour of a, to give Profile 2 then Alternative Vote will elect b instead of a.

a2) To summarise what happened in i), a was winning under Alternative Vote. We changed how voters felt about (a, c) in favour of a without changing how voters feel about (a, b) or (b, c) and as a result a is no longer elected. Explain why plurality could never fall victim to this peculiarity.[2]

b1) Show an example where assuming people vote truthfully, plurality elects a, however if some voters change their preferences over how they rank b compared to c, then plurality would elect b instead.

b2) Comment on your findings in a1) and how this links to the (IIA) axiom for SWFs and (SP) axiom for SCFs.

c) How does Alternative Vote fare with these two axioms?

d) Show that when a Condorcet winner exists, plurality might fail to elect it. And furthermore it could elect a Condorcet loser.

e) Show that when a Condorcet winner exists, Alternative Vote might fail to elect it. However it cannot elect a Condorcet loser.