EC202 Term 1 Problem set 6

November 17, 2022

Pre-class Questions

Not covered in seminar unless time permits.

Q1. Alice and Barbara are considering planting flowers in their garden. Their demand curves, describing their marginal willingness to pay for an extra unit, are given respectively by $p_A = \max\{10 - Q, 0\}$ and $p_B = \max\left\{8 - \frac{Q}{2}, 0\right\}$ where Q is the number of flowers planted in their garden. (I assume Q can be any real number, not just integers.)

a1) Assume Alice and Barbara have their own separate gardens and so for each, flowers are a private good. Derive and draw their aggregate demand $Q^{agg}(p)$ for flowers.

a2) Assume the market is willing to supply any number of flowers at a market price of $p = 4$. Represent the market Equilibrium on a diagram. Is the market Equilibrium efficient?

b) Now suppose that Alice and Barbara share a communal garden. So in the demand equations above $Q = q_A + q_B$. Derive and draw their "aggregate demand", displaying for each level of quantity, their joint marginal benefit for an extra unit of flowers.

c) Find the Lindahl Equilibrium and represent on a diagram. Explain why this is efficient.

d) Assuming no coordination between Alice and Barbara, how many units of flowers will be purchased in total? Why is this fewer than the efficient quantity?

e) For each $i \in \{A, B\}$ are they better off living together or apart from the perspective of how many flowers they have and pay for?

f) Their friend Sarah is training to be an economist. She suspects that the current number of flowers than would be socially optimal. So she steps in and tries to assume the role of a benevolent social planner. She asks each to report their demand function to her, so that she can implement the Lindahl Equilibrium. Alice is very honest and so truthfully reports hers. However Barbara is a bit more strategic and wonders if she can play the system. Assuming Barbara knows that Sarah will implement the Lindahl Equilibrium based on their reported demand functions, should Barbara report her true demand function?

Solution:

a1) Their aggregate demand is given by the horizontal sum of their demand curves. We can solve this algebraically by rewriting them as follows:

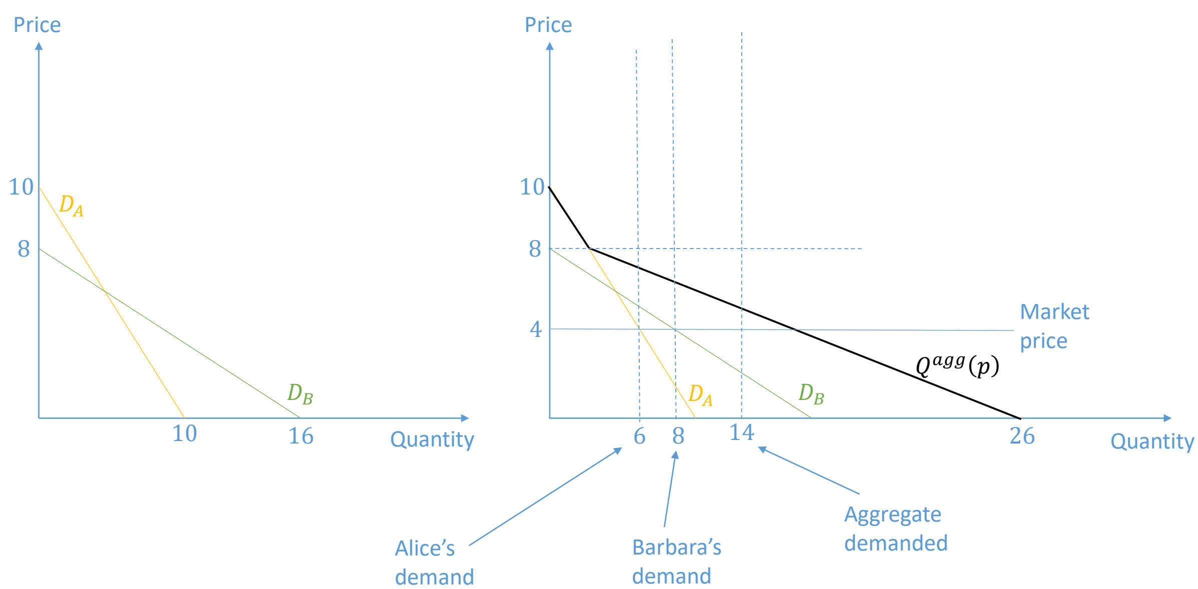
$$\text{Alice: } Q_A = \begin{cases} 10 - p & p < 10 \\ 0 & p \geq 10 \end{cases}$$

$$\text{Barbara: } Q_B = \begin{cases} 16 - 2p & p < 8 \\ 0 & p \geq 8 \end{cases}$$

So their aggregate demand as a function of price is

$$Q^{agg} = \begin{cases} 26 - 3p & p < 8 \\ 10 - p & p \in [8, 10) \\ 0 & p \geq 10 \end{cases}$$

(Tip: you can check your answer by checking the above is continuous at $p \in \{8, 10\}$). The diagram below shows this plus the Equilibrium

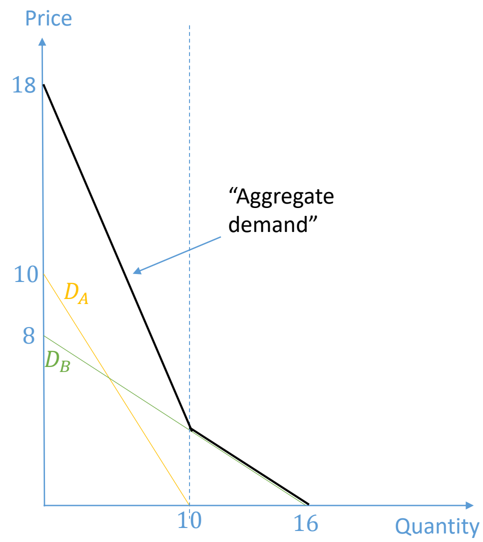


a2) We find the equilibrium by substituting $p = 4$ into the equations derived in a1) and find $q_A = 6$, $q_B = 8$ and so a total quantity of $Q = q_A + q_B = 14$ are purchased. The diagram above illustrates this. It is Pareto efficient, since there are no positive externalities or anything else to disturb efficiency.

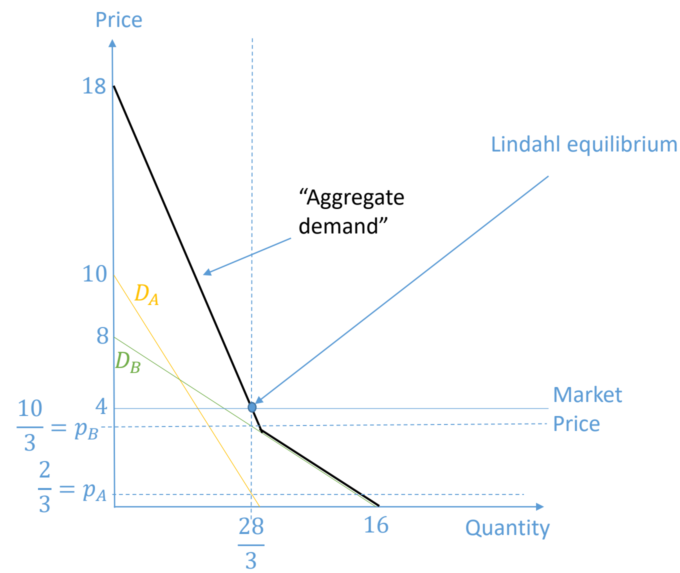
b) For $Q < 10$, both are willing to pay a positive amount and so we sum their marginal willingness to pay. For $Q \in [10, 16)$ only Barbara has a positive marginal willingness to pay. For $Q \geq 16$ neither are willing to pay anything. So we get

$$P^{agg} = \begin{cases} 18 - \frac{3Q}{2} & Q < 10 \\ 8 - \frac{Q}{2} & Q \in [10, 16) \\ 0 & Q \geq 16 \end{cases}$$

We can represent this in the left hand diagram below:



We vertically sum consumers’ marginal willingness to pay to find the total marginal willingness to pay for an extra unit.



Lindahl Equilibrium is where they purchase $\frac{28}{3}$ flowers all together. Alice pays $\frac{2}{3}\left(\frac{28}{3}\right)$; Barbara pays $\frac{10}{3}\left(\frac{28}{3}\right)$.

c) The Lindahl Equilibrium is represented in the right hand diagram above. It occurs where “Aggregate demand” intersects market price. We find this by finding Q such that

$$4 = \begin{cases} 18 - \frac{3Q}{2} & Q < 10 \\ 8 - \frac{Q}{2} & Q \in [10, 16) \\ 0 & Q \geq 16 \end{cases}$$

This occurs at $Q = \frac{28}{3}$. Alice pays a price of $\frac{2}{3}$ per unit, obtained by finding p such that

$$\frac{28}{3} = \begin{cases} 10 - p & p < 10 \\ 0 & p \geq 10 \end{cases}$$

Barbara pays a price of $\frac{10}{3}$ per unit, obtained by finding p such that

$$\frac{28}{3} = \begin{cases} 16 - 2p & p < 8 \\ 0 & p \geq 8 \end{cases}$$

(Tip: you can check your answer by checking these sum to the market price). This is efficient since the combined marginal benefit to Alice and Barbara is above the marginal cost for $Q < \frac{28}{3}$ flowers; and below the marginal cost for $Q > \frac{28}{3}$ flowers.

d) Assuming no coordination, we expect just 8 flowers purchased in total. The reason for this is looking at their demand curves drawn in a1. If less than 6 flowers have currently been purchased then both would have an incentive to purchase a few more. If $Q \in (6, 8)$ have been currently purchased, only Barbara has an incentive to purchase more. But once $Q = 8$ has been reached, neither has an incentive to purchase more. The reason this is inefficient is that they don't take into account the positive externality on the other. [1]

e) First looking at the inefficient equilibrium of d), it depends how much better off depends on who pays for the 8 units of flowers. The last 2 must be paid by Barbara, but the first 6 could have been bought by either. Even if Alice paid for the first 6, she is still strictly better off as Barbara has contributed two more which Alice has not contributed towards but does derive utility from. Barbara on the other hand is strictly better if and only if Alice has bought any of the flowers; while if Alice totally free-rides on Barbara then Barbara is indifferent between living with Alice or not. However if they could find a way of implementing the Lindahl equilibrium of c) then both are strictly better off and the sum of Alice and Barbara's surplus[2] will be at its highest.

f) At the Lindahl Equilibrium Barbara's surplus is the area of the triangle above the market price line, below her demand curve. Using the standard formula for the area of a right-angled triangle we get:

$$\text{Barbara's surplus } = \frac{1}{2}\left(8 - \frac{10}{3}\right)\frac{28}{3} = \frac{196}{9} \simeq 21.78$$

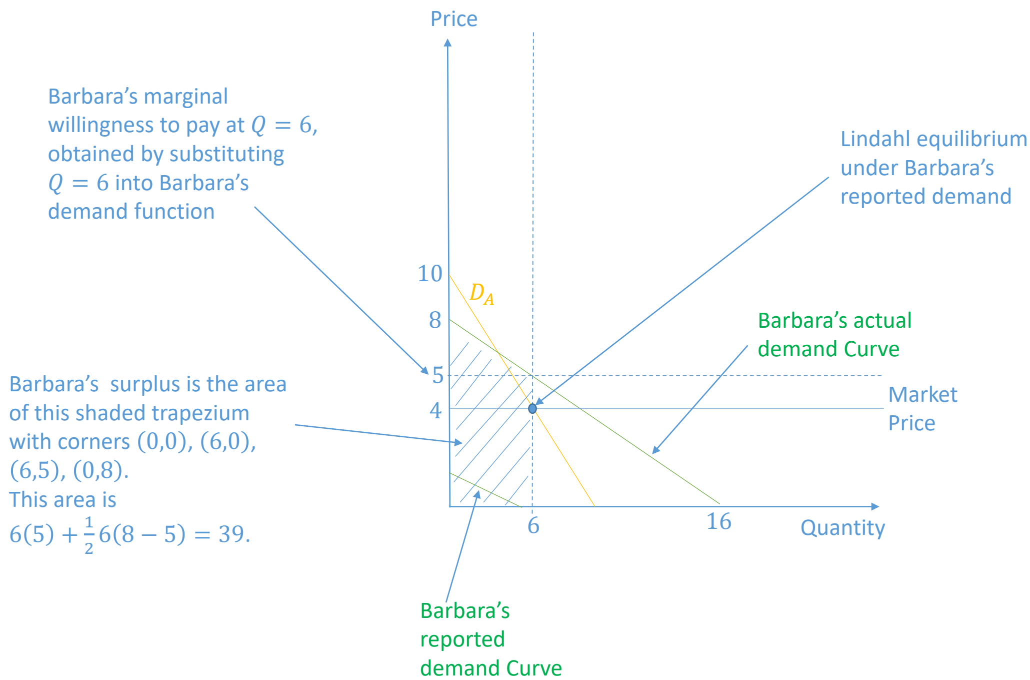
One possible deviation is for her to say she doesn't like flowers at all, that is she isn't willing to pay anything. Or equivalently another low demand such that her marginal willingness to pay drops to 0 after 4 flowers. If she reports this to Sarah then Sarah implements

$$Q = 6, \quad p_A = 4, \quad p_B = 0$$

Here Barbara completely free-rides on Alice and gets surplus as shown below:

[1] If they knew each other's demands then we could invoke the Coase Theorem, since at $Q = 8$ their joint marginal willingness to pay is greater than the cost of purchasing a little extra. So if the assumptions of the Coase Theorem hold then the two could negotiate a price they each pay at which they would both prefer purchasing a few more flowers. The Lindahl Equilibrium can be seen as a solution to that Coasean bargaining problem.

[2] Their surplus is their consumer surplus, the difference between what they would be willing to pay and what they actually pay



If Barbara reports this instead of her true preferences she gets surplus of 36 instead of 21.78, hence she has considerable incentive to lie. I do not claim that here that this is Barbara's best possible report. Finding her best possible report is beyond the scope of this course.

Q2. Suppose in the household public goods model of Lecture 7, Section 2.1, suppose it's average work per person that counts instead of total work. That is person i has utility $v_i(x_i, y) = x_i y$ where y is the average of everybody's y_i. Their resource constraints are still $x_i + y_i \leq 10$.

a) Consider two housemates Andy and Bob, so that $y = \frac{y_A + y_B}{2}$. Find the Nash Equilibrium and efficient level of public good provision.

b) Describe what happens to the Nash Equilibrium as we add more housemates. As more housemates are added, do people become better or worse off?

c) Which model do you think is a better reflection of reality: the model studied in the lecture notes where it's the sum of contributions that matters or here where it is the average contribution?

Solution:

a) We can write Andy's utility function as

$$v_A = x_A \left(\frac{y_A + y_B}{2} \right) = \frac{1}{2} \left[x_A (y_A + y_B) \right] = \frac{1}{2} u_A$$

Where u_A is Andy's utility in the lecture notes. Solving

$$\max_{(x_i,y_i)} u_i(x_i, y_i) \text{ subject to resource constraint}$$

has the same solution in (x_i, y_i) as

$$\max_{(x_i,y_i)} v_i(x_i, y) = \frac{1}{2} u_i(x_i, y_i) \text{ subject to resource constraint}$$

So all of the same analysis carries forward and we find the Nash Equilibrium is

$$(x_A, y_A) = (x_B, y_B) = \left(\frac{20}{3}, \frac{10}{3}\right)$$

While the efficient level of public goods provision is $y_A + y_B = 10$. The only thing that has changed is the utility of each player. We now get $(u_A, u_B) = \left(\frac{200}{9}, \frac{200}{9}\right)$, which is half of what it was previously.

b) As more and more housemates are added, the same applies. With N housemates, we are just dividing utility by N compared to the model in the notes. Thus the Nash Equilibrium and efficient level of public good provision remain the same. The only thing that alters is we are now dividing all utilities by N. This does make one key difference: In the lecture notes, as the number of housemates went up, everybody got better off. Despite the distance between the Nash Equilibrium and the Pareto optimum growing, each benefits from the overall provision going up and tending towards 10 as the number of housemates increases. However in this model where it's the average public good provision that matters, agents are worse off as the number of housemates grows. Indeed as $N \to \infty$, the average provision $y \to 0$ in Nash Equilibrium. The efficient public goods provision is $5N$, thus for each N the average public goods provision remains at $y = 5$, meaning each housemate provides 5 units of the public good on average. Notice that at the symmetric Pareto optimum where each housemate i sets $(x_i, y_i) = (5, 5)$ everyone is equally well off for any number N of housemates, but in Nash Equilibrium, they all become worse off as the number N grows because the distance of the Nash from the Pareto optimum increases.

c) There are elements of the household public good provision that are best described by the model in the lecture notes. For example if a housemate buys a beautiful painting and hangs it up in the hall which everyone appreciates. However there are also elements that are best described by the model here such as housemates willingness to spend time cleaning the communal areas of the house like kitchen and bathroom. This model predicts that as the number of housemates grows, the kitchen will become increasingly messy - which is perhaps a phenomenon that students of this module may have observed in their real lives. This model suggests that as the number of participants grow, it becomes increasingly necessary to introduce some mechanism to compel people to provide the public good if one would like an outcome that is anywhere near being socially efficient.

Q3. In Lecture 6, Example 2.2 we considered a Robinson Crusoe example as follows: Crusoe has initial endowment $\mathbf{e} = (10, 0)$ and preferences

$$u(x_1, x_2) = x_1 x_2 - y_2$$

While the firm he owns has production set

$$Y = \left\{ \mathbf{y} \in \mathbb{R}^2 \mid y_1 \leq 0,\ y_2 \leq 4\sqrt{-y_1} \right\}$$

We showed mathematically that due to the introduction of the pollution negative externality, that the Walrasian Equilibrium was not Pareto efficient .

Now we Change Crusoe's preferences to $u(x_1, x_2) = \frac{1}{4} x_1 x_2 - y_2$

a) What is the Walrasian Equilibrium?

b) Find the Pareto efficient allocation and explain why it is different from both the Walrasian Equilibrium and what we found in Example 2.2.

c) Show diagrammatically why the Walrasian Equilibrium is not Pareto efficient.

d) What prices would be needed so that the firm produces at the Pareto optimal quantity? Why can't we have a Walrasian Equilibrium at these prices?

Solution:

a) The Walrasian Equilibrium is precisely the same as in Example 2.2. This is because, when solving his UMP, Andy can only control (x_1, x_2) not y_2 and so solving

$$\max_{(x_1,x_2)\in\mathbb{R}^2_{\geq 0}} u(x_1, x_2) = \frac{1}{4} x_1 x_2 - y_2 \text{ s.t. budget constraint}$$

is equivalent to solving

$$\max_{(x_1,x_2)\in\mathbb{R}^2_{\geq 0}} \frac{1}{4} x_1 x_2 \text{ s.t. budget constraint}$$

is equivalent to solving

$$\max_{(x_1,x_2)\in\mathbb{R}^2_{\geq 0}} x_1 x_2 \text{ s.t. budget constraint}$$

b) Since $x_2 = y_2$,the Pareto efficient allocation solves

$$\max_{(x_1,x_2)\in[0,10]\times\mathbb{R}_{\geq 0}} u(x_1, x_2) = \frac{1}{4} x_1 x_2 - x_2 \text{ s.t. } x_2 = 4\sqrt{10 - x_1}$$

which is equivalent to

$$\max_{x_1\in[0,10]} u(x_1, x_2) = x_1\sqrt{10 - x_1} - 4\sqrt{10 - x_1}$$

This has solution where $\frac{du}{dx_1} = 0$. This gives $x_1 = 8$. Thus Crusoe works for $10 - 8 = 2$ hours, creating $x_2 = 4\sqrt{2}$ and the Pareto optimal allocation is

$$\mathbf{x} = \left(8, 4\sqrt{2}\right) \quad \mathbf{y} = \left(-2, 4\sqrt{2}\right)$$

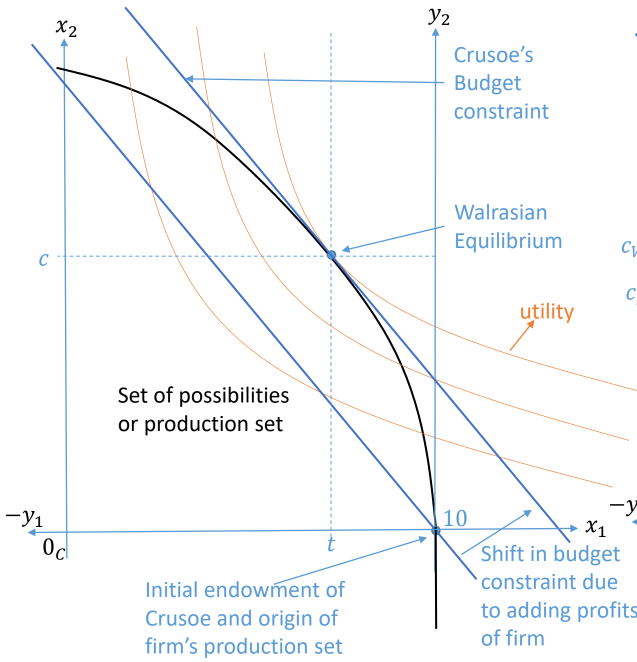
The Walrasian Equilibrium had consumption $\mathbf{x} = \left(\frac{20}{3}, 4\sqrt{\frac{10}{3}}\right)$. You can check that Cruose prefers $\mathbf{x} = (8, 4\sqrt{2})$ by checking that

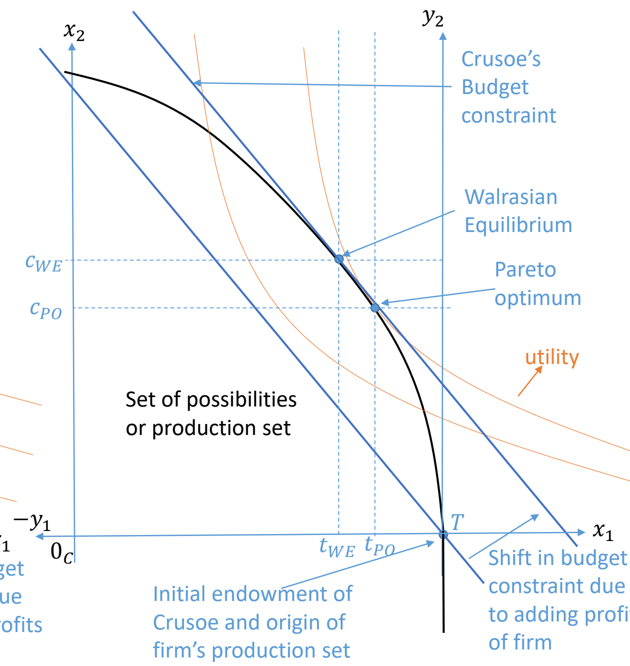
$$u\left(8, 4\sqrt{2}\right) > u\left(\frac{20}{3}, 4\sqrt{\frac{10}{3}}\right)$$

The reason for the difference is the negative externality. When finding the Pareto optimal allocation Crusoe accounts for the negative externality. But when finding Walrasian Equilibrium both Crusoe and firm just optimise utility and profit subject to prices and there is no market for the negative externality so it is not taken into account.

The Pareto optimal allocation is also different from the one in Example 2.2 because here Crusoe's utility from consumption is scaled down relative to the size of the externality, meaning that the externality is relatively bigger. This encourages Crusoe to spend less time working, thus shrinking the size of the externality relative to what was found in Example 2.2.

c) The left hand diagram depicts the no externality case (this is an exact copy of the diagram in Lecture 4); the right hand diagram shows what happens when we introduce externalities





In the left hand diagram, without externalities, the Walrasian Equilibrium is at point (t, c). When we introduce externalities, in the right hand diagram, the firm is still maximising profit subject to price vector (does not care about the negative externality it inflicts on Crusoe). While Crusoe is maximising utility subject to budget constraint, but can only choose his consumption (x_1, x_2). He can't choose the firm's production. Therefore he continues to act as if his indifference curves are as in the left hand diagram. Thus the Walrasian Equilibrium is exactly the same as we found in the left hand diagram.

But when adding the externality, this changes Crusoe's indifference curves. As we can see in the right hand diagram, Crusoe is no longer maximising utility subject to the feasible set. The point that would do this is the Pareto optimum and as we can see, this lies on a higher indifference curve than the Walrasian Equilibrium. In finding this Walrasian Equilibrium, you can think of it like Crusoe finding the $(\mathbf{x}, \mathbf{y})$ feasible allocation (in this case (t_{PO}, c_{PO})) that is best for him and he tells the firm to adjust. But note that he cannot do this in the Walrasian Equilibrium model, because here agents and firms maximise utility and profit subject to prices. And it's only as a result of those prices that markets clear. In order for (t_{PO}, c_{PO}) we would need a steeper price vector, where p_1 is higher relative to p_2.

d) The Pareto optimum is

$$\mathbf{x} = \left(8, 4\sqrt{2}\right) \quad \mathbf{y} = \left(-2, 4\sqrt{2}\right)$$

The firm, maximising profit subject to prices sets[3]

$$\mathbf{y}(\mathbf{p}) = \left(-\frac{4p_2^2}{p_1^2}, 8\frac{p_2}{p_1}\right)$$

So we need to set

$$-2 = -\frac{4p_2^2}{p_1^2} \text{ and } 4\sqrt{2} = 8\frac{p_2}{p_1}$$

As expected (something is wrong if not) these two equations both give the same relationship:

$$p_1 = \sqrt{2}p_2 \text{ so we can set } \mathbf{p} = \left(\sqrt{2}, 1\right)$$

This gives profits of

$$\mathbf{p}.\mathbf{y} = \sqrt{2}(-2) + 1\left(4\sqrt{2}\right) = 2\sqrt{2}$$

So Crusoe faces the following UMP:

$$\max_{(x_1,x_2)\in\mathbb{R}^2_{\geq 0}} u(x_1, x_2) = \frac{1}{4}x_1x_2 - y_2 \text{ s.t. } p_1x_1 + p_2x_2 \leq 10p_1 + 2\sqrt{2}$$

This is equivalent to[4] solving

$$\max_{(x_1,x_2)\in\mathbb{R}^2_{\geq 0}} x_1x_2 \text{ s.t. } p_1x_1 + p_2x_2 \leq 10p_1 + 2\sqrt{2}$$

This has solution where

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2} \Longleftrightarrow p_1x_1 = p_2x_2$$

Putting this back into the budget constraint which at the optimal bundle holds with equality, we get

$$\mathbf{x} = \left(\frac{10p_1 + 2\sqrt{2}}{2p_1}, \frac{10p_1 + 2\sqrt{2}}{2p_2}\right) \mid \mathbf{p} = \left(\sqrt{2}, 1\right)$$
$$= \left(6, 6\sqrt{2}\right)$$

So in summary, at these prices we have optimal demands and production of

$$\mathbf{x} = \left(6, 6\sqrt{2}\right) \quad \mathbf{y} = \left(-2, 4\sqrt{2}\right)$$

But this violates market clearing. Here we have excess demand of good 2 and excess supply of good 1, which is to be expected, since $\frac{p_1}{p_2}$ is higher than in the Walrasian Equilibrium.

[3] I copied this from Lecture 4 as it's the same production set. Or you could work it out again, by setting $-y_1 = k$, $y_2 = 4\sqrt{k}$, write firm's profit as a function of k and prices and maximise with respect to k.

[4] y_2 is just a constant that can be discarded and I have multiplied by 4.

In-class question

Q4. In the fireworks example of Lecture 7, Section 2.2, we change Andy's preferences to $u_A = c_A + 2\ln(1+f)$, while Bob still has preferences $u_B = c_B + \ln(1+f)$, where $f = f_A + f_B$. Each still face budget constraint of $c_i + pf_i \leq M_i$, where M_i for each $i \in \{A, B\}$ is high enough to ensure that each consume with $c_i > 0$.

a) Find the Nash Equilibria. What levels of M_A and M_B constitute "high enough" to guarantee that the Nash Equilibria are of this form?

b) Find the socially optimal level of fireworks by maximising the sum of utilities (assuming high M_A and M_B).

c) Derive and draw the demand curves of each player and the "aggregate demand" curve showing their combined marginal willingness to pay for an extra unit. Show the difference between fireworks being considered a public good and a private good.

d) Suppose $p = \frac{1}{2}$ is the market price at which Andy and Bob can buy any number of fireworks. Find the Lindahl Equilibrium.

e) Suppose $p = \frac{1}{2}$. Find the level of subsidy the government could introduce to correct this problem of underprovision of fireworks.

f) Alternatively the government considers giving Andy and Bob some fireworks. Is there a level of fireworks they could give that would give the socially efficient outcome?

g) Alternatively, suppose the government leaves it to Andy and Bob to sort out between them. Assume the conditions for the Coase Theorem holds. Find the set of possible agreements that might result. Is the Lindahl Equilibrium a member of this set? The algebra gets messy here due to the utility functions, so don't need to do the calculations, but explain what calculations you would do to answer this question.

Solution: Similar to lecture notes

a) Andy solves

$$\max_{(c_A, f_A) \in \mathbb{R}^2_{\geq 0}} u_A = c_A + 2\ln(1 + f_A + f_B) \text{ s.t. } c_A + pf_A \leq M_A$$

Assuming an interior solution

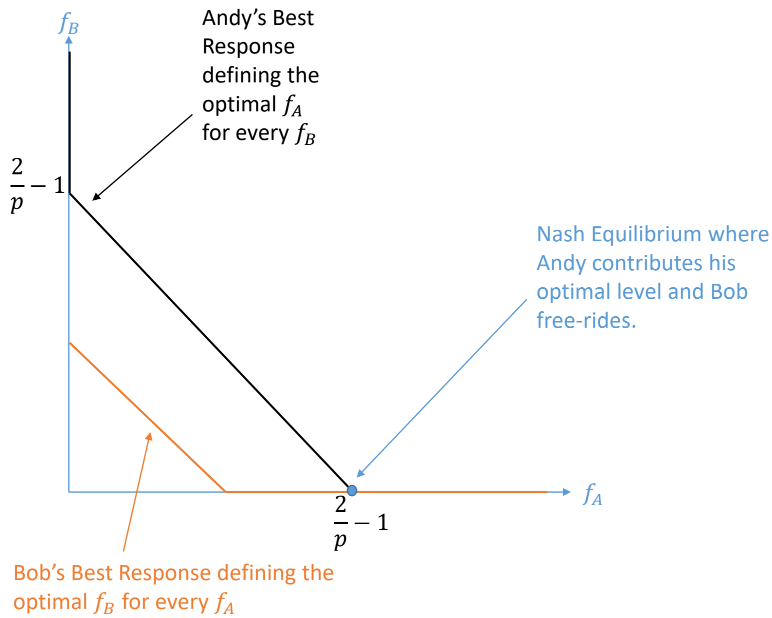
$$\begin{aligned}\frac{\frac{du_A}{dc_A}}{1} = \frac{\frac{du_A}{df_A}}{p} &\iff 1 = \frac{2}{p(1 + f_A + f_B)} \\ &\iff f_A = \frac{2}{p} - 1 - f_B\end{aligned}$$

Thus we get

$$\text{Best Response of } A \text{ is } f_A = \begin{cases} \frac{2}{p} - 1 - f_B & f_B < \frac{2}{p} - 1 \\ 0 & f_B \geq \frac{2}{p} - 1 \end{cases}$$

By similar logic or copying from Example 2.4 of Lecture 7 we get

$$\text{Best Response of } B \text{ is } f_B = \begin{cases} \frac{1}{p} - 1 - f_A & f_A < \frac{1}{p} - 1 \\ 0 & f_A \geq \frac{1}{p} - 1 \end{cases}$$



As we see above, there is a unique Nash Equilibrium where Andy contributes and Bob free-rides. The above is drawn for low p; while for $p \geq 2$, Andy is priced out of buying any fireworks. So in general, the Nash Equilibrium is

$$\text{NE: } f_A = \max\left\{\frac{2}{p} - 1, 0\right\}, \quad f_B = 0$$

To write their best responses in the way we have, it required that M_A and M_B are sufficiently high that $c_A > 0$ and $c_B > 0$, which required:

$$\text{For Andy: } p\left(\frac{2}{p} - 1 - f_B\right) < M_A$$

$$\text{For Bob: } p\left(\frac{1}{p} - 1 - f_A\right) < M_B$$

Since our Nash Equilibrium is of the form above this simplifies to requiring[5]

[5] For $M_A = 2 - p$ we also get the same thing. But if Andy's income drops below this, he is at a corner solution where he spends all his income on fireworks and so the solution depends on how much income he has.

$M_A > 2 - p$. While the restriction on Bob's income is satisfied for all $M_B > 0$ assuming that Andy's restriction is satisfied so that $f_A = \max\left\{\frac{2}{p} - 1, 0\right\}$.

b) The socially optimal level is found by solving

$$\max_{(c_A, f_A),(c_B, f_B) \in \mathbb{R}^2_{\geq 0}} U = u_A + u_B$$
$$\text{subject to } c_i + f_i \leq M_i \quad i \in \{A, B\}$$

which assuming M_i sufficiently high so that agents want to consume some consumption good simplifies to

$$\max_{c_A, c_B, f \geq 0} U = c_A + 2\ln(1+f) + c_B + \ln(1+f) = 3\ln(1+f) + (c_A + c_B)$$
$$\text{subject to } c_A + c_B + f \leq M_A + M_B$$

This has solution where

$$\frac{\frac{dU}{d(c_A + c_B)}}{1} = \frac{\frac{dU}{df}}{p} \iff 1 = \frac{3}{p(1+f)}$$
$$\iff f = \frac{3}{p} - 1$$

Also allowing for the possibility of $p > 3$, we get the efficient number of fireworks is $f = \max\left(\frac{3}{p} - 1, 0\right)$.

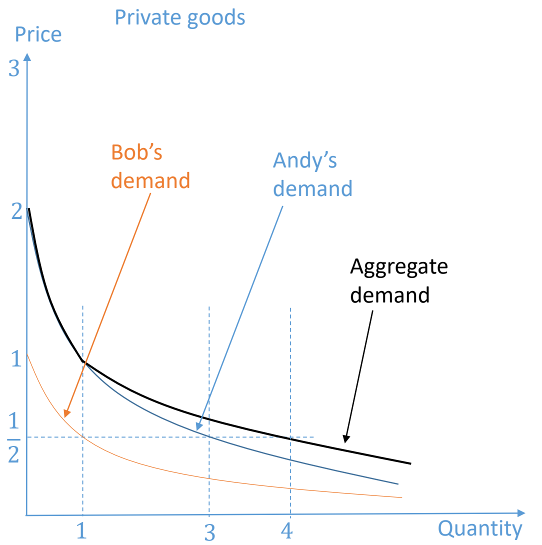
c) If the goods were private goods, the demands for Andy and Bob respectively would be

$$q_A = \begin{cases} \frac{2}{p} - 1 & p < 2 \\ 0 & p \geq 2 \end{cases}$$

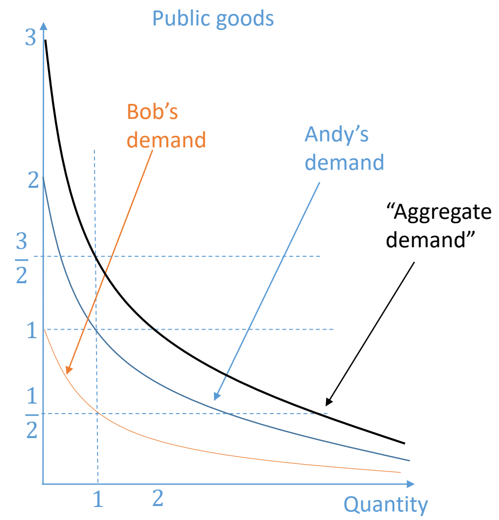
By similar logic or copying from Example 2.4 of Lecture 7 we get

$$q_B = \begin{cases} \frac{1}{p} - 1 & p < 1 \\ 0 & p \geq 1 \end{cases}$$

The left hand diagram shows their aggregate demand if fireworks were private goods. The right hand diagram shows their aggregate demand when they are public goods.



We sum up individual demands horizontally: At price p, Andy is prepared to buy $q_A = \max(2/p - 1, 0)$ and Bob is prepared to buy $q_B = \max(1/p - 1, 0)$. Then aggregate demand is calculated by $q_A + q_B$.



We sum up individual demands vertically: At quantity q, Andy values the marginal extra unit at $p_A = \frac{2}{1+q}$ and Bob values it at $p_B = \frac{1}{1+q}$. So the sum of values for the marginal extra unit is $\frac{3}{1+q}$.

As seen above in the private goods market, aggregate demand is given by

$$Q^{agg} = \begin{cases} \frac{3}{p} - 2 & p < 1 \\ \frac{2}{p} - 1 & p \in [1, 2] \\ 0 & p > 2 \end{cases}$$

While in the public goods market "aggregate demand", the joint marginal willingness to pay for an extra unit of output is given by

$$P^{agg} = \frac{3}{1 + Q}$$

d) The Lindahl Equilibrium quantity occurs where $p = \frac{1}{2}$ intersects the "aggregate demand" curve. That is we find Q such that

$$\frac{1}{2} = \frac{3}{1 + Q} \iff Q = 5$$

At this quantity, consumers' marginal willingness to pay and hence what they pay per unit are

$$p_A = \frac{1}{3}, \qquad p_B = \frac{1}{6}$$

Thus in the Lindahl Equilibrium a total of 5 fireworks are bought at a total cost of $\frac{5}{2}$. Andy bears $\frac{2}{3}$ of this cost paying $\frac{5}{3}$ and Bob bears $\frac{1}{3}$ of this cost paying $\frac{5}{6}$.

e) The efficient level when $p = \frac{1}{2}$ is $Q = 5$. However in Nash Equilibrium Bob buys none and Andy is only prepared to buy $f_A = \max\left\{\frac{2}{p} - 1, 0\right\}$. So to make this equal to 5 we require Andy to face a price of $p_A = \frac{1}{3}$. Thus we need a subsidy of

$$s = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

f) If the government provides an additional $g < \frac{2}{p} - 1$ units[6]. Then Andy maximises utility by solving

$$\max_{(c_A, f_A) \in \mathbb{R}^2_{\geq 0}} u_A = c_A + 2\ln(1 + g + f_A + f_B) \text{ s.t. } c_A + pf_A \leq M_A$$

We find that

$$\text{Best Response of } A \text{ is } f_A = \begin{cases} \frac{2}{p} - 1 - g - f_B & f_B < \frac{2}{p} - 1 - g \\ 0 & f_B \geq \frac{1}{p} - 1 - g \end{cases}$$

By symmetry

$$\text{Best Response of } B \text{ is } f_B = \begin{cases} \frac{1}{p} - 1 - g - f_A & f_A < \frac{1}{p} - 1 - g \\ 0 & f_A \geq \frac{1}{p} - 1 - g \end{cases}$$

We get a similar best response diagram to what we drew in a). Thus our Nash Equilibria now is $f_B = 0$, $f_A = \max\left\{\frac{2}{p} - 1 - g, 0\right\}$ and so once again the total amount of fireworks is only

$$f = f_A + f_B + g = \frac{2}{p} - 1$$

Thus for $g < \frac{2}{p} - 1$, the government provision has just crowded out private provision and resulted in no more fireworks being bought. If the government wants the level of fireworks to be efficient it needs to provide all $f = \max\left\{\frac{3}{p} - 1, 0\right\}$ of them itself.

g) By the Coase Theorem we expect players to negotiate to mutually beneficial and socially efficient level of fireworks. We know the efficient level, $f = \max\left\{\frac{3}{p} - 1, 0\right\}$, but mutually beneficial means each must be at least as well off as they were under the status quo. So we need to calculate each player's utility in the status quo which is the NE found in part a). I assume that $\frac{3}{p} - 1 > 0$ as the opposite case is trivial. The set of agreements will be of the following form for some $c_1, c_2 \in \mathbb{R}$

$$\left\{(f_A, f_B) \in \mathbb{R}^2_{\geq 0} \mid (f_A, f_B) = \left(c, \frac{3}{p} - 1 - c\right), c \in (c_1, c_2)\right\}$$

[6] the working below justifies this threshold

To calculate c_2, we need to find the cost Andy would need to incur under $f = \frac{3}{p} - 1$ to be indifferent to the status quo and let c_2 be the number of fireworks to give Andy that cost. To calculate c_1, we need to find the cost Bob would need to incur under $f = \frac{3}{p} - 1$ to be indifferent to the status quo and let c_2 be the number of fireworks to give Bob that cost. In Lindahl Equilibrium, Andy purchases two thirds and Bob purchase one third so we can see if this is a member of that set. Alternatively we can do this more directly by calculating both peoples' utility at the NE and Lindahl Equilibrium and seeing whether both players are at least as well off at the latter. Note that the property of efficiency is already guaranteed at the Lindahl Equilibrium.

Post-class question

Short essay question 1: Read Section 2.4 of Lecture 7 and (preferably 27A.3.2 and 27A.3.3 of Nechyba or passages in other texts on this topic). Then watch this video <https://www.youtube.com/watch?v=d6DBKoWbtjE> about Alt-Erlaa, a luxury social housing complex in Vienna. As well as providing housing at an affordable price, residents also get a lot of other goods like tennis courts, children's play areas, indoor and outdoor swimming pools, saunas. All of this included in the price of their rent. Discuss whether it is socially efficient for the Austrian government to lump these goods in with the rental price. Would these goods be provided if left to the free market? Could private property developers have done this equally well?

Sketch Solution: Generally the public goods listed here like tennis courts, swimming pools would be under provided or not provided at all if it was left to the residents to raise the money to pay for these things themselves. The total cost needed for each resident would be very large and so if the creation of these facilities relied upon private donations, then each person would have an incentive to free ride. So assuming that the efficient outcome is for these facilities to be built then the government is solving what would otherwise be a case of market failure. This is acting on the same idea as the provision of lighthouses discussed in Section 27A.3.3. The provider, here the government, funds these goods by bundling them in with the price of accommodation. However, there is no guarantee that government is providing consumers with what they want. There might be some people who don't like swimming pools or don't have kids and so would resent being forced to pay towards the creation and upkeep of these facilities. This is where the Tiebout Equilibrium idea of Sections 27A.3.2 comes in. If there are multiple housing complexes like this, each free to provide a slightly different mix of public goods bundled in with the price of accommodation, then consumers are free to choose whichever they prefer. So we get competition between housing providers who will maximise profit by responding to what consumers are demanding.

In this particular case, the government is providing this as social housing instead of a private developer, however the principles listed above would work perfectly well with private firms renting accommodation to people, bundling private goods in with the rental price in this way. Although there may be a few practical reasons why it is the government and not firm doing this:

i) To build something of this scale would require a lot of investment and so a very large capital outlay for any firms.

ii) As was mentioned in the video, tenants feel secure in their tenancy renting from the government, with the belief they could live there their whole life if desired. While if they were renting from a private provider they might fear getting evicted if the firm went out of business or found an alternative way to make more profit.

iii) The government in Austria wants to keep the rents low so that people can afford it. Whereas a private provider would charge whichever price maximises its profits.

Short essay question 2: In 2020 the world was hit by the coronavirus. In coronavirus times, explain some of the associated externalities when people travel and socialise. Discuss why government action is needed to help curtail the spread of this virus.

Sketch Solution: When people with coronavirus travel, they risk infecting other people they bump into, inflicting a direct negative externality on any people they infect and a large society-wide negative externality due to their being more carriers of the virus and so making it harder to contain. However, when making travel decisions, people are more likely to just think about their own needs and wants. For example if you know you have coronavirus, your priority might be to reach a hospital and get treated as quickly as possible rather than getting there in a way that limits how easily the virus is spread to other people. Or imagine you are in a virus stronghold and don't think you have the virus. Then for your own health, you might want to exit the city as soon as possible and move to somewhere safer. However, due to the risk of you having the virus and spreading it to those not yet affected, the wider societal good dictates that you should not be allowed to leave.

There are also more everyday issues: under normal circumstances people visit friends and relatives; or go to pubs, bars and other entertainment facilities. For some people who take the threat of coronavirus seriously, the private cost from the increased threat of getting coronavirus might be enough to dissuade them from engaging in such activities. However for a young, healthy person who is at little personal threat from the virus, it is likely to be in their personal interests to carry on living life normally. Although, for every extra interaction between any 2 people there is a small externality incurred on the rest of society from the chance that one of these 2 people have the virus and the associated impact on the other person and on societal transmission rates. When the size of the negative externality is larger than the difference between private benefits and private costs to the individuals involved, it is in the societal interest that such interactions do not happen.

Due to the mis-alignment between what's in the individual's interests and what is in society's interests, there is a role for government to step in to ensure that less of the negative externality associated with the spread of the virus is inflicted on society. In this case, financial incentives are not practical and are too weak, hence governments take more draconian measures like issuing lockdowns and other travel restrictions. Although when governments impose such draconain measures this has large negative effects on the population's welfare due to reduction in freedom, harm to the economy, mental health issues from isolation etc... If government's lockdowns makes it harder for people to get exercise or harms the economy and increases government debt meaning less money for healthcare in the future then such moves could be very counter-productive for the nation's health, let alone overall welfare. Thus the government needs to ensure the action it takes is proportionate to the scale of the problem. This is not an easy balancing act and government's might change their mind as more evidence comes in. For example in the UK's first lockdown of spring 2020, schools were shutdown, while in the second lockdown, schools and universities have been kept open.