EC202 Week 6
Covering Materials from Week 4

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Production

Definition 1.1. A firm has *technology* or *production set* $Y \subseteq \mathbb{R}^J$ describing the feasible net outputs of the firm. Note that with this notation, for $\mathbf{y} = (y_1, \ldots, y_J) \in Y$, if $y_j < 0$, it means that the firm is using good j as as input; while if $y_j > 0$ then the firm is producing good j as an output.

The above convention that negative amounts of goods mean inputs is made so that we can define the profit that a firm makes when producing $\mathbf{y}$ at price vector $\mathbf{p}$ as:

$$\text{Firm's profit} = \mathbf{p}.\mathbf{y} = p_1 y_1 + p_2 y_2 + \ldots p_J y_J$$

To understand this better, we could decompose this into revenue and cost[1]:

$$\text{Revenue} = \sum_{i:y_i>0} p_i y_i \qquad \text{Cost} = -\sum_{i:y_i<0} p_i y_i$$

$$\implies \text{Revenue} - \text{Cost} = \sum_{i:y_i>0} p_i y_i + \sum_{i:y_i<0} p_i y_i = \mathbf{p}.\mathbf{y}$$

Production

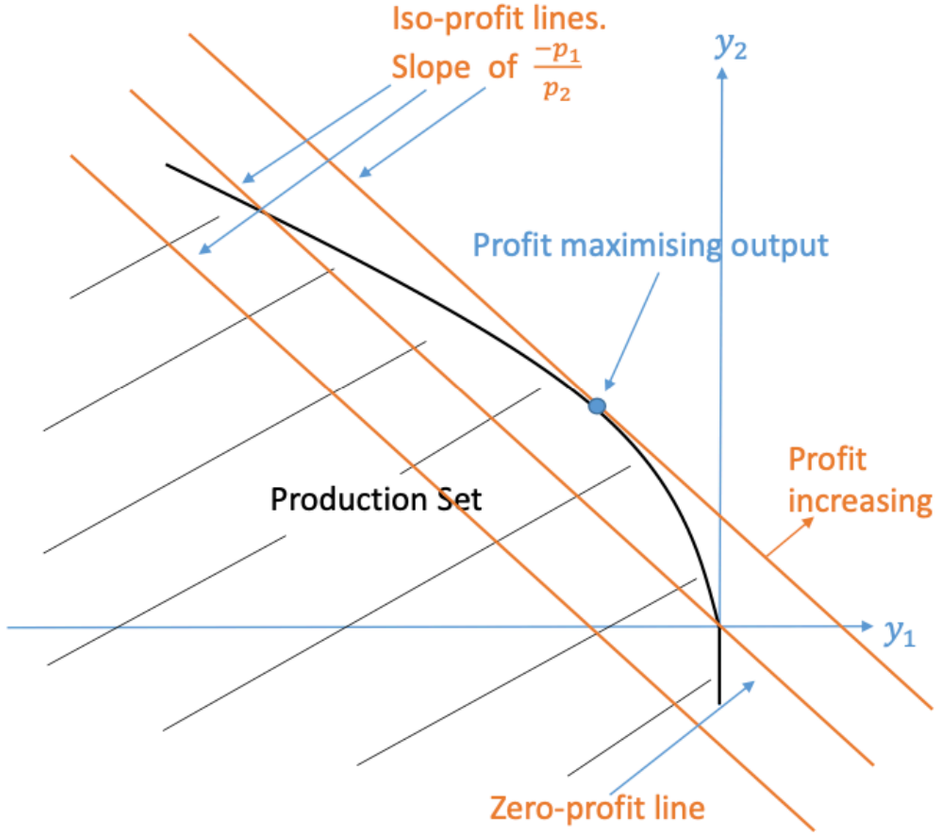
- Walrasian Equilibrium with production
  - Recall Walrasian Equilibrium
  - Formal Setup

Profit Maximization

Given the above defintion of Walrasian Equilibrium, we would like to know what it means for a firm to be profit maximising. The firm is profit maximising with respect to production set Y if it solves

$$\max_{\mathbf{y} \in Y} \mathbf{p}.\mathbf{y}$$

We show graphically what this means for a nonincreasing returns to scale, convex production set where the firm uses good 1 as an input to produce good 2 as an output:



Robinson Crusoe Economies

- The Model
  - Two goods
  - Constraint
  - Marginal output
  - Pareto efficiency

In-class Question

- Q3. Crusoe has 12 units of time (good 1) to allocate between work and leisure. If he works for k hours he can produce 2k units of the consumption good (good 2) and can freely dispose of each good. Crusoe has utility function:

$u : \mathbb{R}^2_{\geq 0} \to \mathbb{R}$ where

$$u(x_1, x_2) = \frac{e^{\sqrt{x_1 x_2}}}{17} + 2\pi(x_1 x_2)^5 - 6$$

a) Find the Pareto efficient bundle and draw a diagram to illustrate it.
b) We model this situation using Walrasian Equilibrium:
i) Assuming free-disposal, write down the Production set and find the firm's profit maximising output vector as a function of prices $\mathbf{y}(\mathbf{p})$ and the profits the firm makes. Draw a diagram to illustrate this.
ii) Find Crusoe's optimal bundle $\mathbf{x}(\mathbf{p})$.
iii) Write down the market clearing conditions and verify that Walras' law holds.
iv) Find the Walrasian equilibrium prices and allocation. Illustrate it on a diagram.
v) 1 Consider the case where Crusoe has some good 2 in the initial endowment, that is $\mathbf{e} = (12, c)$ for $c > 0$. Describe what would happen to equilibrium prices as c increases and give some intuition for this.