

# **EC334 Third Seminar**

# **Corporate Policy**

Junxi Liu

# Corporate Policy

- Main goal for this part: apply the financial theories into corporate policy (especially those not covered in A)
- CWS13: The Role of the CFO, Performance Measurement, and Incentive Design
  - Read the book
- CWS15: Capital Structure and the Cost of Capital: Theory and Evidence
  - Equity versus Debt to fund a firm
  - Value of a Firm
  - Modigliani-Miller propositions
- CWS16: Dividend Policy: Theory and Empirical Evidence
  - How to pay dividend without and with taxes
  - Repurchases

# Imagine You Are Running a Firm...

- Your firm needs money.
  - (Unless you are from a billionaire family)
- You can borrow money.
  - Debt: requires repayment; often with interests; usually tax-free
- You can also sell some shares of your firm
  - Equity: an ownership in the firm; no need to repay; if your firm skyrockets then you are losing a LOT of money...
- Gearing (or Leverage, in the rest of the world)
  - In simple words: borrowing money to invest
  - For corporate policy: the relationship, or ratio, of a company's debt-to-equity
  - For example, a gearing ratio of 70% shows that a company's debt levels are 70% of its equity.

# Imagine You Are Running a Firm...

- You have to keep a balance of debt and equity
- If you know your business is going to be a great success but the investors in the market don't think so
  - you will only be able to sell your shares (equity) at a very low price — which you believe can be a LOT of money in the future!
- So you want to borrow money instead — you don't have to “sell yourself”.
  - However, borrowing means that you will have to repay the money within a certain time frame, and also pay quite a lot interests — you are under pressure!



# Let's See an Successful Story of Equity Investment

- BYD was the world's largest plug-in hybrids and pure electric vehicles maker in 2022, with a total of 1.86 million cars sold.



## ADL BYD to build 130-strong ZE fleet for National Express in Coventry

By routeone Team - February 17, 2022



Careers

### Arriva - Network Manager

routeone Team - February 22, 2023

Arriva are looking to recruit a Network Manager to be a part of its North Midlands operation. Network Manager North Midlands Arriva's vision is to shape a...

East Yorkshire Buses announces plans for new Scarborough depot  
February 22, 2023

First Bus leaves Southampton as Go-Ahead steps in

# Let's See an Successful Story of Equity Investment

- In late 2008, Berkshire Hathaway ponied up the aforementioned \$232 million for a roughly 10% stake in BYD.
- As Buffett recalled, Berkshire initially tried to buy 25% of the company, but Wang (CEO of BYD) refused to release more than 10% of BYD's stock.
- Currently, BYD's market capitalization is ~\$120 billion, which means a 10% stake would worth \$12 billion
- What if Wang used debt to finance, instead of selling the shares to Buffett?

# Modigliani-Miller Propositions

- The market value of a company is correctly calculated as the present value of its future earnings and its underlying assets, and ***is independent of its capital structure.***
- Assumptions:
  - Market efficiency and no asymmetric information
  - No taxes
  - No transaction or bankruptcy costs
  - Hold constant the firm's investment policies
- MM is not to be taken literally, but has implications
  - See Johnathan's slides 22-28

# When There Are Tax and Bankruptcy Costs...

- In many countries, interest is *deductible as a cost* of doing business while dividends are taxed as income – obviously favours debt financing
  - Tax shield of debt (affect Earnings Before Interest and Taxes EBIT)

Interest Tax Shield Calculation			
(\$ in 000s)		Company A	Company B
Revenue		\$50,000	\$50,000
Less: Cost of Goods Sold (COGS)		(10,000)	(10,000)
<b>Gross Profit</b>		<b>\$40,000</b>	<b>\$40,000</b>
Less: Operating Expenses (OpEx)		(5,000)	(5,000)
<b>EBIT</b>		<b>\$35,000</b>	<b>\$35,000</b>
Less: Interest Expense		--	(4,000)
<b>Pre-Tax Income (EBT)</b>		<b>\$35,000</b>	<b>\$31,000</b>
Less: Taxes	Tax Rate	21.0%	
		(7,350)	(6,510)
<b>Net Income</b>		<b>\$27,650</b>	<b>\$24,490</b>
		<b>Tax Shield</b>	<b>\$840</b>

- $PV = PV(\text{equity}) + PV(\text{tax shield}) - PV(\text{distress costs})$ 
  - If you borrow way too much, investors will lose confidence and your firm might go into bankruptcy

# And Here Comes Income Tax...

$I$  is income,  $D$  is Debt,  $V(D)$  is value of firm with debt  $D$

$\tau_i^E$  is personal tax rate on equity (what you got from the stock)

$\tau_i^B$  is personal tax rate on interest (from investment on bonds)

$\tau_C$  is corporate tax rate

$r_D, r_E$  are returns to debt (interest rate) and equity (ROE), respectively.

For an all-equity firm:

$$V_U = \frac{E(EBIT)(1 - \tau_c)(1 - \tau_i^E)}{\rho}$$

. where  $\rho$  is the discount rate for an all-equity firm.

Alternatively, if the firm has outstanding bonds, payment to shareholders will be

$$(EBIT - r_d D)(1 - \tau_c)(1 - \tau_i^E)$$

, where  $D$  is the number of bonds shareholders hold;

and payment to bondholders after personal taxes are

$$r_d D(1 - \tau_B)$$

**Tax Shield!**

Discount the sum of the two, we get

$$V_L = V_U + \left[ 1 - \frac{(1 - \tau_i^E)(1 - \tau_c)}{1 - \tau_i^B} \right] B$$

, where  $B$  is the market value of Bond,  $\frac{r_d D(1 - \tau_i^B)}{r_E}$



# Discussion on the Firm Value with Personal Tax

- Essentially, the gain from having debt is  $\left[1 - \frac{(1 - \tau_i^E)(1 - \tau_c)}{1 - \tau_i^B}\right]B$ 
  - When it is positive, firm has the incentive to hold debts
  - In addition, investor's demand for bonds changes with tax rate on interest income, and investors will be indifferent between bonds and equity if

$$r_D(1 - \tau_i^D) = r_E(1 - \tau_i^E)$$

- Firms, who supply bonds, will be indifferent between supplying and not supplying if

$$r_D = \frac{r_E}{(1 - \tau_C)}$$

- Connecting the above two equations, you will get

$$(1 - \tau_i^E)(1 - \tau_c) = 1 - \tau_i^B$$

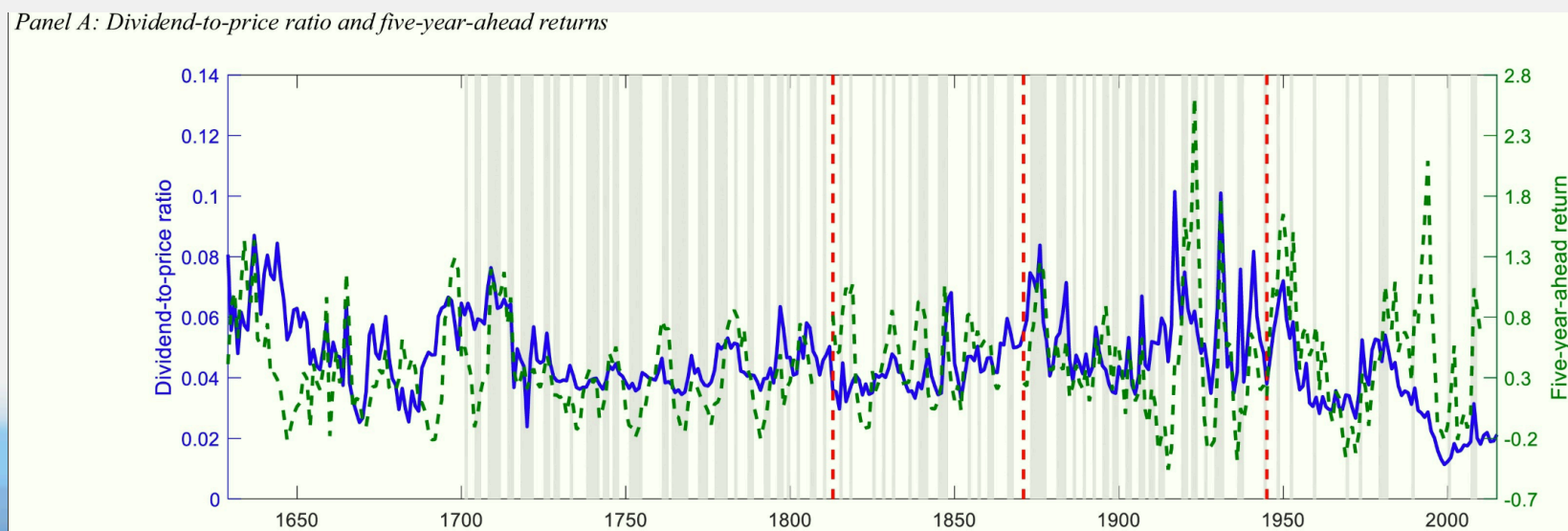
- Which means tax shield is 0!

- Think about the real world...

- Which is higher,  $\tau_c$ ,  $\tau_i^E$ , or  $\tau_i^B$
- In reality, there are not that many investors for whom *post-tax return on debt* is less than *post-tax return on equity*

# Dividend Policy: Why Pay Dividends?

- Historically and currently, this has been a very robust trend
  - Theoretically, in perfect markets dividend policies cannot affect value, and with higher tax dividends are almost always a bad deal compared with repurchases
- Many, many research papers look into this problem...
  - We combine annual stock market data for the most important equity markets of the last four centuries: the Netherlands and UK (1629–1812), UK (1813–1870), and US (1871–2015). We show that dividend yields are stationary and consistently forecast returns. (Golez and Koudijs, 2018)
  - Dividend payments remained prevalent even though repurchases were legal (Turner et al., 2013) and dividend taxation was present (Moortgat et al., 2017).



# Repurchases

- A share repurchase is a transaction whereby a company buys back its own shares from the marketplace. A company might buy back its shares because management considers them undervalued.
- Buyback payments to investors may be tax-efficient if treated as capital gain/loss
- Ownership re-concentrated, shareholder alignment with management *may* be improved
- Used when companies have lots of cash, want to increase leverage.



# Starting on Game Theory

- Define a problem
  - A finite set of players
    - Who have their own strategy space
  - A strategy space (also called action space)
    - Contains all possible strategies for each player
    - The vector of strategies for all players is a strategy profile
  - A payoff function (also called rewards function)
    - A mapping from a strategy profile to a real number
- Example

	<i>H</i>	<i>T</i>
<i>H</i>	1, -1	-1, 1
<i>T</i>	-1, 1	1, -1

Figure 2.18: The matching pennies

# Cheap Talk Model

- Games with incomplete information: Perfect Bayesian Equilibrium
  - Both players choose their best responses
  - Their beliefs follow Bayes' rule where possible:  $\mathbb{P}(B | A) \equiv \frac{\mathbb{P}(B \cap A)}{\mathbb{P}(A)}$
  - Another two components:
    - $\Theta_i$  is player  $i$ 's type space and every element  $\theta_i \in \Theta_i$  is a type
    - $v_i: A \times \Theta \rightarrow \mathbb{R}$  is  $i$ 's payoff function; where  $A = A_1 \times \dots \times A_n$  is the set of action profiles and  $\Theta = \Theta_1 \times \dots \times \Theta_n$  is the set of type profiles
- Player 1 has private information and the payoffs exhibit common values, so that both players' payoffs depend on player 1's private information. Player 1's action is a message that has no direct effect on payoffs.

1. Nature selects a type of player 1  $\theta \in \Theta$  from some common-knowledge distribution  $p$ .
2. Player 1 learns  $\theta$  and chooses some message (action)  $a_1 \in A_1$ .
3. Player 2 observes message  $a_1$  and chooses action  $a_2 \in A_2$ .
4. Payoffs  $v_1(a_2, \theta)$  and  $v_2(a_2, \theta)$  are realized.

# A Motivating Example

To formalize the game we can think of me as player 1, who is the sender of information, and my friend as player 2, who is the receiver of information, and we can imagine that expected traffic conditions are given by  $\theta \in \{1, 3, 5\}$ , where 1 is bad, 3 is mediocre, and 5 is good. Player 1 knows the true value of  $\theta$ , but player 2 knows only the prior distribution of  $\theta$ . Player 1 transmits a message (his action) to player 2 about the traffic conditions. Player 2 then chooses an action (where to live)  $a_2 \in A_2 = \{1, 2, 3, 4, 5\}$ , where 1 is San Francisco, 5 is Palo Alto, and 2, 3, and 4 are towns in between the two cities in that order. The preferences of player 2 are described by the following payoff function: <sup>1</sup>

$$v_2(a_2, \theta) = 5 - (\theta - a_2)^2.$$

Notice that player 2 has a clear best response: given any level of traffic, he wants to choose his residence location equal to the traffic level. That is, his optimal choice is  $a^*(\theta) = \theta$ . To capture the fact that player 1 is biased toward having player 2 live closer to location 5, the preferences of player 1 are given by the following payoff function:

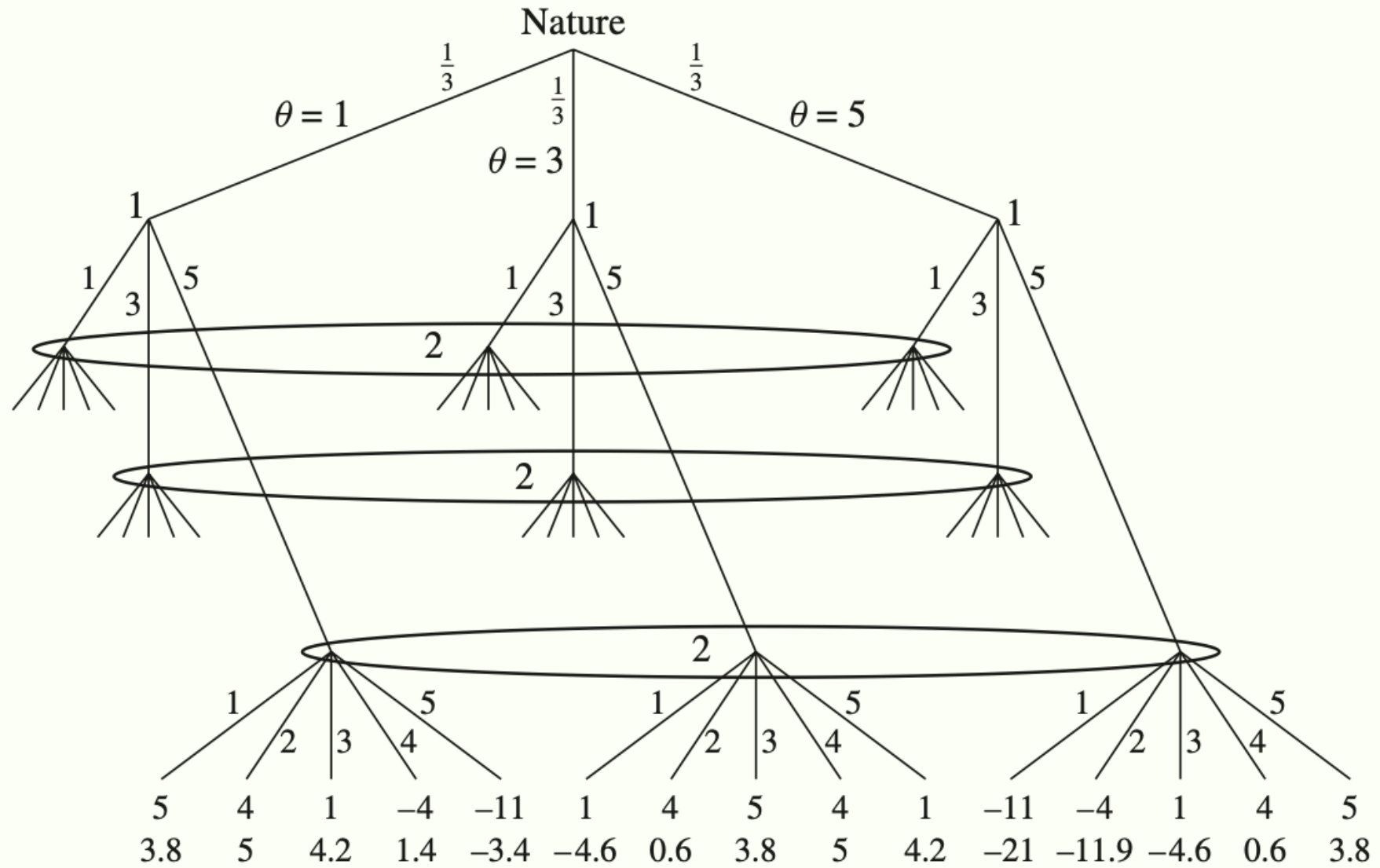
$$v_1(a_2, \theta) = 5 - (\theta + b - a_2)^2,$$

where  $b > 0$  is the bias of player 1.

This is a dynamic game of incomplete information: player 1's type, or the state of the world  $\theta$ , is known only to player 1, while player 2 knows only the distribution of  $\theta$ . Player 1's type affects both his payoff and the payoff of player 2, making this a common-values game. Player 1's action set includes messages that he can transmit to player 2, and player 2's action set includes choosing where to live. To further fix ideas, imagine that player 1 is restricted to sending only one of three messages corresponding to one of these states of nature:

$$a_1 \in A_1 = \{1, 3, 5\}.$$
<sup>2</sup>

# A Motivating Example



**FIGURE 18.1** The commuting conditions information-transmission game.

# A Motivating Example

**Claim 18.1** *There is no perfect Bayesian equilibrium in which player 1 reports the true state of the world.*

**Proof** Assume in negation that player 1 truthfully reporting  $a_1 = \theta$  is part of a perfect Bayesian equilibrium. It therefore must follow that when player 1 sends the message  $a_1 \in \{1, 3, 5\}$  player 2 chooses  $a_2 = a_1$ . We saw that when  $\theta = 3$  player 1 prefers  $a_2 = 5$  over  $a_2 = 3$ . But if player 1 believes that player 2 will follow his advice then when  $\theta = 3$  player 1's best response is  $a_1 = 5$ , a contradiction. ■

The intuition behind this result is simple, and easily generalizes to all such information-transmission games in which there is a bias between the sender and receiver with respect to the receiver's optimal choice. If it is indeed the case that the sender is reporting information truthfully, then it is in the receiver's best interest to take the sender's information at face value. But then if the receiver is acting in this way the sender has an incentive to lie.<sup>5</sup> The next observation is also quite straightforward:

# A Motivating Example

**Claim 18.2** *There exists a babbling equilibrium in which player 1's message reveals no information and player 2 chooses an action to maximize his expected utility given his prior belief.*

**Proof** To construct the babbling (perfect Bayesian) equilibrium let player 1's strategy be to send a message  $a_1 \in \{1, 3, 5\}$  with equal probability of  $\frac{1}{3}$  each regardless of  $\theta$ . This means that the message is completely uninformative: player 2 knows that regardless of the message,  $\Pr\{\theta\} = \frac{1}{3}$  for all  $\theta \in \{1, 3, 5\}$ . This implies that player 2 maximizes his expected payoff,

$$\max_{a_2 \in \{1, 2, 3, 4, 5\}} Ev_2(a_2, \theta) = 5 - \frac{1}{3}(-(1 - a_2)^2) + \frac{1}{3}(-(3 - a_2)^2) + \frac{1}{3}(-(5 - a_2)^2),$$

which is maximized when  $a_2 = 3$ .<sup>6</sup> Because each of player 2's information sets is reached with positive probability, player 2's beliefs are well defined by Bayes' rule everywhere, and player 1 cannot change these beliefs by changing his strategy.<sup>7</sup> Hence player 1 is indifferent between each of the three messages and is therefore playing a best response. ■

Claims 18.1 and 18.2 paint a rather disappointing picture for our simple game. Not only will truthful messages never be part of an equilibrium (claim 18.1), but there is an equilibrium in which player 1's valuable private information will have no effect on player 2's choice. The remaining question is whether there are other equilibria in which there is *some* valuable information transmitted from the sender to the receiver.



## A Motivating Example — Continuous Cheap Talk

The game is basically the same as the one described in the previous section, with the exception of  $\theta \in \Theta = [0, 1]$  and the assumption that the state of the world  $\theta$  is uniformly distributed on  $[0, 1]$ . Let player 2's action set include all real numbers,  $a_2 \in \mathbb{R}$ . Player 1's action set can be any arbitrary set of messages  $A_1$ , but it will be convenient to let  $A_1 = [0, 1]$  so that the message space conforms with the state space  $\Theta$ . Player 2's payoff is  $v_2(a_2, \theta) = -(a_2 - \theta)^2$ , while player 1's payoff is  $v_1(a_2, \theta) = -(a_2 + b - \theta)^2$ , which implies that for any given value of  $\theta \in [0, 1]$ , player 2's optimal choice is  $a_2 = \theta$ , while player 1's is  $a_2 = \theta + b$ . The payoff functions of the two players are depicted in Figure 18.2.

As in the finite example, both players would prefer a higher action to be taken when the state  $\theta$  is higher, but player 1 has a constant bias, making him prefer even higher actions than player 2. This immediately implies that claim 18.1 generalizes to the continuous setting because of the same reasoning: If player 2 believes that player 1 is reporting  $\theta$  truthfully, then player 2's best response is to choose  $a_2 = \theta$ . But if this is player 2's strategy then player 1 will report  $a_1 = \theta + b$  for any  $b \neq 0$ . Hence there can never be a fully truthful equilibrium. Not surprisingly a babbling equilibrium still exists:

**Claim 18.4** *There exists a babbling perfect Bayesian equilibrium in which player 1's message reveals no information and player 2 chooses an action to maximize his expected utility given his prior belief.*

**Proof** We construct the equilibrium in a similar way to the finite case. Let player 1's strategy be to send a message  $a_1 = a_1^B \in [0, 1]$  regardless of  $\theta$ . This means that the message is completely uninformative and player 2 believes that  $\theta$  is distributed uniformly on  $[0, 1]$ . This implies that, conditional on receiving the message  $a_1^B$ , player 2 maximizes his expected payoff,

$$\max_{a_2 \in \mathbb{R}} E v_2(a_2, \theta) = \int_0^1 -(\theta - a_2)^2 d\theta = -\frac{1}{3} + 2a_2 - a_2^2,$$

which is maximized when  $a_2 = \frac{1}{2}$ . Let player 2's off-equilibrium-path beliefs be  $\Pr\{\theta = \frac{1}{2} | a_1 \neq a_1^B\} = 1$  so that his off-the-equilibrium-path best response to any other

message is  $a_2 = \frac{1}{2}$  as well. It is easy to see that player 1 is indifferent between any of his messages and hence choosing  $a_1 = a_1^B$  is a best response. ■



We see that the continuous-space cheap-talk model has the same two extreme results demonstrated for the discrete-space game: there is no truthful equilibrium and there is always a babbling equilibrium. The question then is, how much information can the sender, player 1, credibly transmit to the receiver, player 2? We begin by constructing a perfect Bayesian equilibrium in which player 1 uses one of two messages,  $a'_1$  and  $a''_1$ , and player 2 chooses a different action following each message,  $a_2(a'_1) < a_2(a''_1)$ .<sup>8</sup>

**Claim 18.5** *In a two-message equilibrium player 1 must use a threshold strategy as follows: if  $0 \leq \theta \leq \theta^*$  he chooses  $a'_1$ , whereas if  $\theta^* \leq \theta \leq 1$  he chooses  $a''_1$ .*

**Proof** For any  $\theta$  player 1's payoffs from  $a'_1$  and  $a''_1$  are as follows:

$$v_1(a_2(a'_1), \theta) = -(a_2(a'_1) + b - \theta)^2$$

$$v_1(a_2(a''_1), \theta) = -(a_2(a''_1) + b - \theta)^2,$$

which implies that the extra gain from choosing  $a''_1$  over  $a'_1$  is equal to

$$\Delta v_1(\theta) = -(a_2(a''_1) + b - \theta)^2 + (a_2(a'_1) + b - \theta)^2.$$

The derivative of  $\Delta v_1(\theta)$  is equal to  $2(a_2(a''_1) - a_2(a'_1)) > 0$  because  $a_2(a'_1) < a_2(a''_1)$ . This implies that if type  $\theta$  prefers to send message  $a''_1$  over  $a'_1$  then every type  $\theta' > \theta$  will also prefer  $a''_1$ . Similarly if type  $\theta$  prefers to send message  $a'_1$  over  $a''_1$  then so will every type  $\theta' < \theta$ . This in turn implies that if two messages are sent in equilibrium then there must be some threshold type  $\theta^*$  as defined in claim 18.5. It follows that when  $\theta = \theta^*$  player 1 must be indifferent between sending the two messages. ■

**Claim 18.6** *In any two-message perfect Bayesian equilibrium in which player 1 is using a threshold  $\theta^*$  strategy as described in claim 18.5, player 2's equilibrium best response is  $a_2(a'_1) = \frac{\theta^*}{2}$  and  $a_2(a''_1) = \frac{1-\theta^*}{2}$ .*

**Proof** This follows from player 2's posterior belief and from him playing a best response. In equilibrium player 2's posterior following a message  $a'_1$  is that  $\theta$  is uniformly distributed on the interval  $[0, \theta^*]$ , and his posterior following a message  $a''_1$  is that  $\theta$  is uniformly distributed on the interval  $[\theta^*, 1]$ . Player 2 plays a best response if and only if he sets  $a_2(a_1) = E[\theta|a_1]$ , which proves the result. ■

**Claim 18.7** *A two-message perfect Bayesian equilibrium exists if and only if  $b < \frac{1}{4}$ .*

**Proof** From claim 18.5 we know that when  $\theta = \theta^*$  player 1 must be indifferent between his two messages so that

$$v_1(a_2(a'_1), \theta^*) = v_1(a_2(a''_1), \theta^*),$$

which from claim 18.6 and from the fact that  $\frac{\theta^*}{2} < \theta^* < \frac{1-\theta^*}{2}$  is equivalent to

$$\theta^* + b - \frac{\theta^*}{2} = - \left( \theta^* + b - \frac{1-\theta^*}{2} \right). \quad (18.1)$$

The solution to (18.1) is  $\theta^* = \frac{1}{4} - b$ , which can result in a positive value of  $\theta^*$  only if  $b < \frac{1}{4}$ . To complete the specification of off-the-equilibrium-path beliefs, let player 2's beliefs be  $\Pr\{\theta = \frac{\theta^*}{2} | a_1 \notin \{a'_1, a''_1\}\} = 1$ , so that he chooses  $a_2 = \frac{\theta^*}{2}$ , which causes player 1 to be indifferent between sending the message  $a'_1$  and any other message  $a_1 \notin \{a'_1, a''_1\}$ , implying that his threshold strategy is a best response. ■

## Example from 2021 Exam

A government procurement officer is trying to decide how many doses of a new coronavirus vaccine to order. This decision will depend on the effectiveness of the vaccine, which will be determined by clinical trials conducted by a scientific advisor. You may assume that the effectiveness of the vaccine is given by a random variable  $\varepsilon$ , uniformly distributed on the interval  $[E_0, E_0 + 1]$ . The scientific advisor believes the utility of ordering a quantity  $Q$  is  $U_A(Q|\varepsilon) = 1 + \varepsilon Q - Q^2$ ; if perfectly-informed about effectiveness, the procurement officer would value  $Q$  at  $U_G(Q|\varepsilon) = 1 + (\varepsilon + \beta)Q - Q^2$  where  $\beta$  is a non-negative constant. After the trials, the government officer asks the scientific advisor to report on the vaccine's effectiveness and purchases the quantity that maximises his expected utility. The scientific advisor is not paid for his efforts.

- a. How would you set up this problem? Can the government advisor be sure of purchasing the optimal quantity (according to his preferences)? If so, how? If not, why not? How does your answer depend on the size of  $\beta$ ? **(15 marks)**
- b. Suppose that the minimum effectiveness is  $E_0 = 25\%$  and that  $\beta = 5\%$ . Find the 'babbling equilibrium' for this situation – how much will the government order and what expected utilities will the two parties get? **(7 marks)**
- c. Now construct a two-part equilibrium – depending on the advice they receive, the government will place either a small order  $Q^S$  or a large order  $Q^L$ . At what reported level of effectiveness will the government switch its order size, and what are the values of  $Q^S$  and  $Q^L$ ? **(10 marks)**
- d. How would you find the most efficient equilibrium (you do not have to compute it explicitly, but should say how it could be identified)? **(10 marks)**

a

This is a cheap talk problem; should note that first-best can be achieved only if  $\beta = 0$ . They should note that there is always an equilibrium in which the government ignores the advisor and purchases the a priori optimal amount  $Q_0(E_0, E_0 + 1)$ , which they compute in the next part. The optimal strategy is to partition the range of effectiveness into intervals  $[x, y]$  and associate to each interval the order that maximises expected utility  $Q^*(x, y) = \underset{Q}{\operatorname{argmax}} \int_x^y U_G(Q|\varepsilon) d\varepsilon$ . The more intervals, the more efficient is the outcome, but the number (and thus the efficiency) are bounded above by a decreasing function of  $\beta$ . Finally, they should note that for any two adjacent intervals  $[x, y]$  and  $[y, z]$ , the scientific adviser would be indifferent between the purchase levels for both intervals if she was convinced that the true effectiveness was exactly  $y$  – in other words  $U_A(Q^*(x, y)|y) = U_A(Q^*(y, z)|y)$ .

b

In this case, there is only one purchase level regardless of report. If the government believes that the true state is uniformly distributed on  $[a, b]$ , its expected utility for purchasing  $Q$  is

$$EU_G(Q) = 1 + \left( \frac{\int_a^b \varepsilon d\varepsilon}{b-a} + \beta \right) Q - Q^2 = 1 + \left( \frac{a+b+2\beta}{2} \right) Q - Q^2$$

Optimal  $Q$  is  $\frac{a+b+2\beta}{4}$ . In this case,  $Q = \frac{.25+1.25+.1}{4} = 0.4$ ,  $U_G = 1.16$ ,  $U_A = 1.14$ .



c

Denote the critical report level by  $\varepsilon^*$ . The two order sizes are

$$Q^S = \frac{.25 + \varepsilon^* + 2 * .05}{4} = \frac{.35 + \varepsilon^*}{4}$$

$$Q^L = \frac{1.25 + \varepsilon^* + 2 * .05}{4} = \frac{1.35 + \varepsilon^*}{4}$$

$\varepsilon^*$  is defined by the condition that the advisor should be indifferent between  $Q^S$  and  $Q^L$  when the true state is  $\varepsilon^*$ . Solving  $U_A\left(\frac{.35+\varepsilon^*}{4} \mid \varepsilon^*\right) = U_A\left(\frac{1.35+\varepsilon^*}{4} \mid \varepsilon^*\right)$  for  $\varepsilon^*$  gives  $\varepsilon^*$  (in general, for any level of  $\beta$ ),  $\varepsilon^* = 0.75 + 2\beta$ ; in this case,

$$\varepsilon^* = 0.85$$

$$Q^S = 0.3$$

$$Q^L = 0.55$$

d

The most efficient equilibrium is the one with the greatest number of intervals, so they should look for the largest  $n$  s.t. there exists a sequence  $0.25 = \varepsilon^1, \dots, \varepsilon^n = 1.25$  (or .26 for the 1% case) where

$$Q^i = \frac{\varepsilon^i + \varepsilon^{i+1} + 0.1}{4}$$

And for each  $i = 1, \dots, n - 1$

$$1 + \varepsilon^{i+1} \left[ \frac{\varepsilon^i + \varepsilon^{i+1} + 0.1}{4} \right] - \left[ \frac{\varepsilon^i + \varepsilon^{i+1} + 0.1}{4} \right]^2 = 1 + \varepsilon^{i+1} \left[ \frac{\varepsilon^{i+1} + \varepsilon^{i+2} + 0.1}{4} \right] - \left[ \frac{\varepsilon^{i+1} + \varepsilon^{i+2} + 0.1}{4} \right]^2$$