EC334 Topics in Financial Economics
Second Seminar

Junxi Liu

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Welcome!

- I am Junxi Liu, a PhD student in the department of economics。
- There are a total of four seminar classes for EC334, one every two weeks. We will be mostly focus on real options and agent problems in the seminars, and also covering other topics.
- As I have classes both on odd weeks and even weeks, from now on I will simply use “First Seminar”, etc, to refer to classes.
- You are from a wide range of majors: Economics, PPE, MORSE, Politics, Math, Exchange student, and many more…
  - The main focus of the seminars will be giving you hands-on experience to use techniques and skills to solve exercises and problems
  - The underlying theme of the seminars will be making sure you understand the intuition and logic of concepts and models

Logistics

- You will find all information and materials of seminar classes on my website: https://warwick.ac.uk/fac/soc/economics/staff/jliu/teaching/
- I will also send materials to you via email
- My office hours (week 3-10 of term 2):
  - Fridays, 12:00-14:00, S2.80
  - Book on my website: https://warwick.ac.uk/fac/soc/economics/staff/jliu/teaching/officehour/
- I will be obliged to take attendances
  - If you couldn't make your seminar class, the easiest way is to ***go to another seminar of mine***, and I will record your attendance then.
  - Information on the time and location of all classes I teach can be found on my website.
  - If none of them suits you, please contact the UG office to mitigate your absence.

Logistics

Junxi Liu ▶ **Teaching**

Teaching

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Office hours for Term 2, 2023-2024: Fridays 12:00-14:00, S2.80, Week 1 - 10.

Please book using this link before you come.

2023-2024

EC334 (Seminars): Topics in Financial Economics: Corporate Finance and Markets, Term 2

EC104 (Seminars): The World Economy: History & Theory, Term 2

EC104 (Seminars): The World Economy: History & Theory, Term 1; Teaching Evaluation

2022-2023

Warwick Summer School (seminars): International Business and Finance, Summer School

EC334 (seminars): Topics in Financial Economics: Corporate Finance and Markets, Term 2; Teaching Evaluation

EC202 (seminars): Microeconomics 2, Term 1; Teaching Evaluation

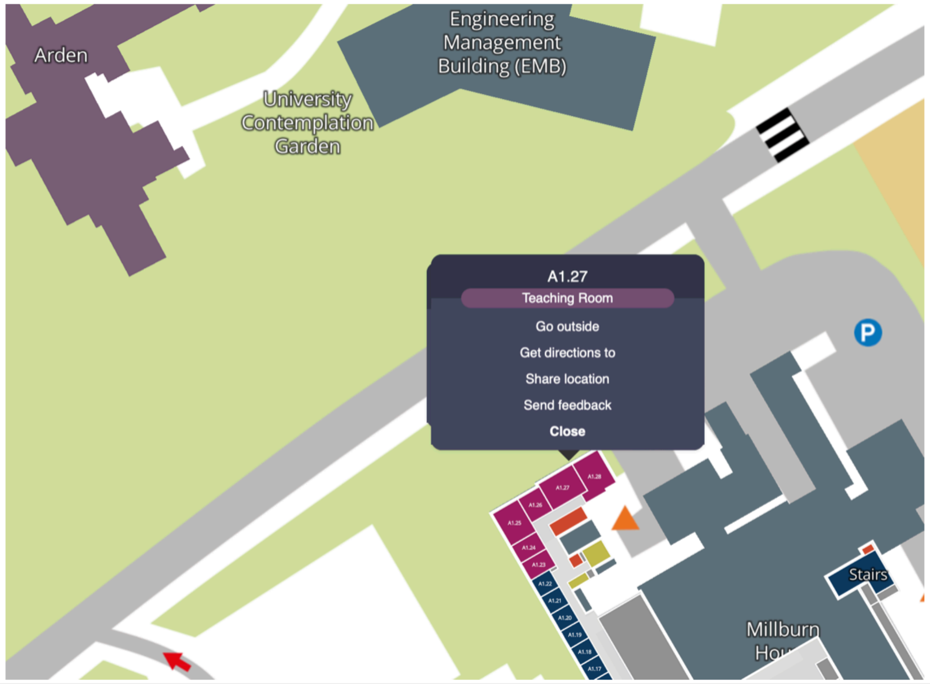
Junxi Liu ▶ Teaching ▶ **EC334 - 2024**

EC334 - 2024

All My Seminars Take Place on Mondays:

- Week 17; 19; 21; 23 (the 3rd, 5th, 7th, 9th week of term 2):
  - 16:00 - 17:00, A1.27 (Millburn)
  - 17:00 - 18:00, F25b (Millburn)
- Week 18, 20, 22, 24 (the 4th, 6th, 8th, 10th week of term 2):
  - 16:00 - 17:00, A1.27 (Millburn)

Location Map (please use the orange triangle as entrance)



NPV

$$NPV = -Investment_0 + \sum_{t=1}^{N} \frac{E(FreeCashFlow_t)}{(1+WACC)^t}$$

When $-1 < r < 1$

$$\lim_{n\to\infty} \sum_{k=1}^{n} a * r^k = \lim_{n\to\infty} a * \frac{r(1-r^n)}{1-r} = a * \frac{r}{1-r} = \frac{a}{\frac{1}{r} - 1}$$

$$\sum_{t=1}^{\infty} \frac{a}{(1+WACC)^t} = \frac{a}{WACC}$$

Seminar Problem 1 Q1

1. Irreversible investment

It costs a risk neutral firm £800 to set up a factory (fixed cost). The factory can produce one unit of output per year forever. The current price of a unit of output is £100. The product price is currently uncertain. In the next period the price will increase or decrease by 50% (with equal probability) and remain fixed forever at the level it has reached. The interest rate is 10%.

a) According to NPV, should the firm invest now? This means spending £800 in the current period and selling the first unit of output for £100 today and either £150 forever afterwards or £50 forever afterwards (with equal probability).

The expected price of output is $0.5 * £150 + 0.5 * £50 = £100$

$$NPV = -£800 + \sum_{t=0}^{\infty} \frac{£100}{(1+10\%)^t} = -£800 + £1100 = £300 > 0$$

So the firm should invest now.

Seminar Problem 1 Q1

b) What is the real option value of waiting to see whether the price goes up or down? This means investing £800 next period and getting the appropriate price forever.

Suppose the firm decides to wait and invest only once it knows the price.

If the price goes down, the firm won't invest; the future (period 1) value is

$$-£800 + \sum_{t=1}^{\infty} \frac{£50}{(1+10\%)^{t-1}} = -£800 + £550 = -£250 < 0$$

If the price goes up, the firm will invest; the future (period 1) value is

$$-£800 + \sum_{t=1}^{\infty} \frac{£150}{(1+10\%)^{t-1}} = -£800 + £1650 = £850 > 0$$

The expected present (period 0) value of waiting and the option value are

$$NPV = 0.5 * 0 + 0.5 * \frac{£850}{1.1} = £386.36$$

$$Option\ value = £386.36 - £300 = £86.36$$

Seminar Problem 1 Q1

c) If the price for deferred investment is higher (the firm has to invest £I > £800 at the beginning of next period) how high can it be before the firm will choose to invest now?

First, work out whether the firm will invest if the price goes up or down

If the price goes down, the future (period 1) value of revenue is

$$\sum_{t=1}^{\infty} \frac{£50}{(1+10\%)^{t-1}} = £550$$

So the firm will invest after a price fall only if $I \leq £550$. If the price goes up, the future value of revenue is

$$\sum_{t=1}^{\infty} \frac{£150}{(1+10\%)^{t-1}} = £1650$$

So the firm will invest after a price rise only if $I \leq £1650$. If price is any larger, it will never invest tomorrow and will certainly invest today, but this ceiling price is too high. If the firm invests tomorrow only if price goes up, its *present* value of waiting is

$$0.5 * \left[\frac{-I}{1.1} + \sum_{t=1}^{\infty} \frac{£150}{(1+10\%)^{t}}\right] = 0.5 * \left[£1500 - \frac{I}{1.1}\right] = £750 - \frac{I}{2.2}$$

The firm will prefer to invest now if £300 exceeds this amount, i.e. $I \geq £990$.

Deferral has value, but if after delay, the cost is ~~too~~ high, you still don't delay.

Don't delay: $NPV = 300$

Delay: (assume the strategy is: exercise if up, otherwise not)

- From (b) even with 800, won't invest when down. $I > 800$ since in (b) delay is more valuable

$$\underbrace{0.5 \times \left(\frac{-I}{1.1} + \frac{150}{0.1} \right)}_{\text{wait}} \leq 300$$

$I \geq 990$ So the highest is 990

Seminar Problem 1 Q1

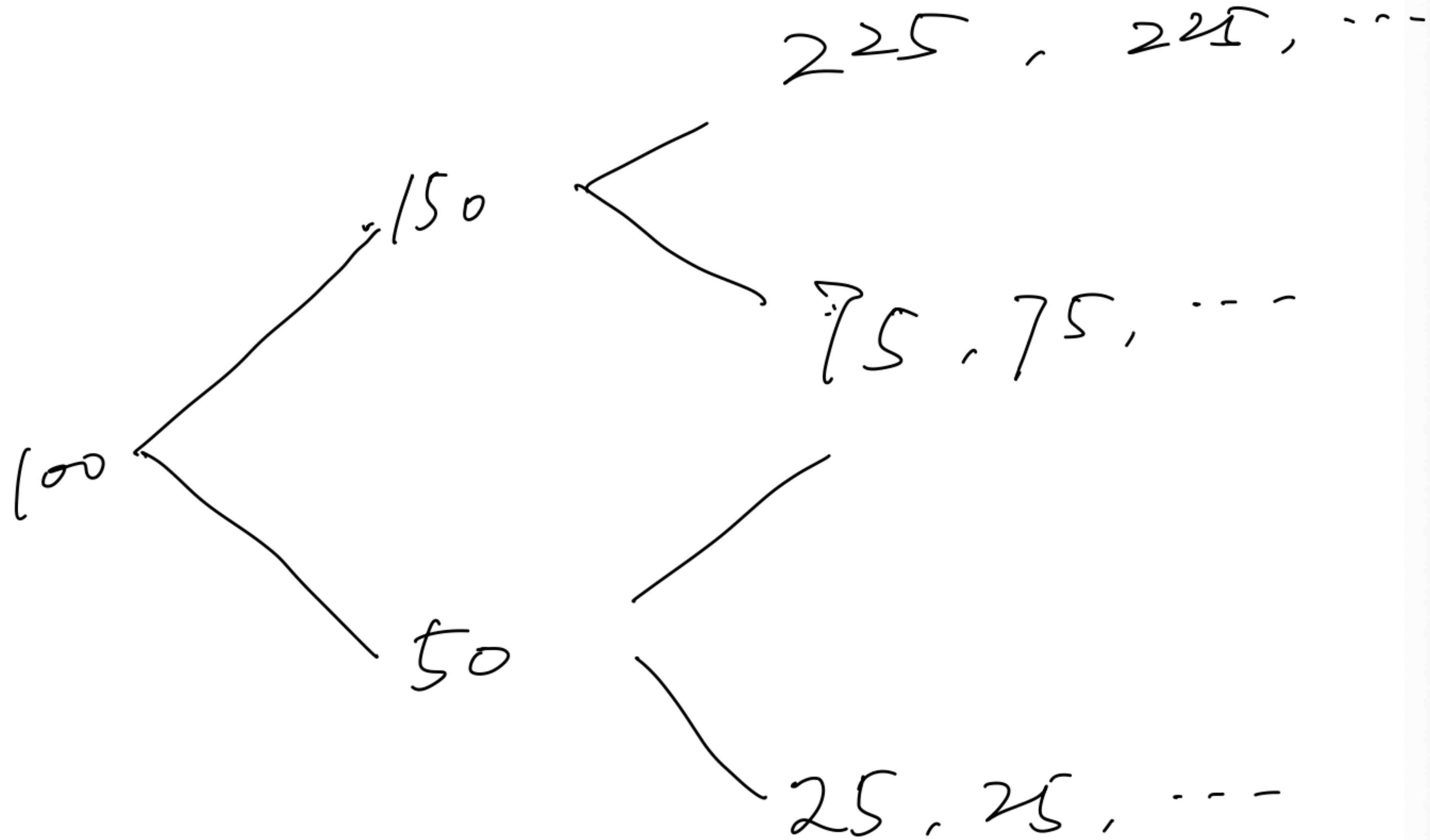
d) How would your answers change if the firm could wait for two periods, and if the price could change at the end of period 1 and again at the end of 2 (in each case going up or down by 50% with equal probability) after which it would stay the same forever?

This is a more complex problem, for you to think about as revision. The changes are;

- Price trajectories are different (up or down tomorrow and the next day)
  - Up, up gives prices: £100, £150, £225, £225…: PV=£2281
  - Up, down gives prices £100, £150, £75, £75…: PV=£918
  - Down, up gives prices £100, £50, $75, £75…: PV=£827
  - Down, Down gives prices £100, £50, £25, £25…: PV=£372
  - Expected present value of revenue is still £1100
- by waiting the firm has to discount the cost of investment and the resulting revenues twice. The optimal decision can be found by considering whether the firm would invest in period 2 if the price were £225 (yes, future value in period 2 = £1675), £75 (still yes, though FV is only £25) or £25 (no). The expected value of waiting (in period 0) is therefore

$$PVW = .25 * £1675 + .5 * £25 + .25 * £0 = \frac{£431.25}{(1.1)^2} = £356$$

So the value of this option = £356 – £300 = £56.



Without any option:

$$PV = 100 + 0.5 \times \left(\frac{150}{1.1} + \frac{50}{1.1}\right) + 0.25 \times \left(\frac{225}{0.1}\right) \div 1.1 + 0.25 \times \left(\frac{25}{0.1}\right) \div 1.1 + 0.5 \times \left(\frac{75}{0.1}\right) \div 1.1$$

$$= 1100$$

Standing at period 2.

$p = 0.25$: 225, 225, ... $PV_2 = 225 + \frac{225}{0.1} = 2475$

$NPV_2 = 1675$

$p = 0.5$: 75, 75, ... $PV_2 = 75 + \frac{75}{0.1} = 825$

$NPV_2 = 25$

$p = 0.25$: 25, 25, ... $PV_2 = 25 + \frac{25}{0.1} = 275$

$NPV_2 < 0$

$$NPV_0 = \frac{1675 \times 0.25 + 25 \times 0.5 + 0}{(1.1)^2} = 356$$

Seminar Problem 1 Q2

2. HS2 – a more complex option

The price of steel is currently $P_0 = £250$ per ton. Next year it will either increase by 50% to $P_g = £375$ or fall by 25% to $P_b = £187.50$ – these changes are equally likely, and the new price (and the plant) will last forever. Firm A is considering investing in a plant making steel girders for high-rise buildings. The investment must be made today or never, and will cost $I_0^A = £1.05\ billion$. The annual value of the project – starting in the period when the investment is made - is always $\mu = 175000$ times the price of steel. The risk free interest rate is $r_f = 5\%$.

a) Should the firm invest in the plant?

$$NPV = \mu P_0 - I_0^A + \frac{0.5*\mu*(P_g + P_b)}{r_f} = -£21.875\ million;\ \text{the investment shouldn't be made.}$$

Seminar Problem 1 Q2

b) Suppose that management has the option to expand the scale of the operation after one year. At that time, it can double the gross value of the project by investing an additional $I_1^A = £800\ million$. What is the value of this option? Should the firm invest in the smelter today?

The decision to make the first investment is taken at $t = 0$. The option must be agreed at $t = 0$ but exercised (expand or not) at $t = 1$. Expected annual revenue after this year is $R^E = 50\% * \mu * 2 * (P_g + P_b) = 49.22\ million = 50\% * 2(R_g + R_b)$.

Strategy: if up, exercise; if down, don't exercise

If Up	If down	Formula	Value
No	No	$\mu P_0 - I_0^A + \frac{R^E}{r_f}$	-£21.875 m
Yes	No	$NPV = \mu P_0 - I_0^A + 0.5\left[\frac{2 \times R_g}{r_f} - \frac{I_1^A}{(1+r_f)}\right] + 0.5\left[\frac{R_b}{r_f}\right] = £253.4225M > 0$	
No	Yes	$NPV = \mu P_0 - I_0^A + 0.5\left[\frac{2 \times R_b}{r_f} - \frac{I_1^A}{(1+r_f)}\right] + 0.5\left[\frac{R_g}{r_f}\right] = -£74.702M < 0$	
Yes	Yes	$\mu P_0 - I_0^A + \frac{2R^E - r_f I_1^A}{r_f(1+r_f)}$	£200.6 m

The investment should be undertaken. The value of the expansion option is the difference between the NPVs; $£253.42\ million - 0$ as the project would not be pursued without this option.

Seminar Problem 1 Q2

Now suppose that Firm B has entered a partnership with a foreign steel producer who offers the following opportunity: Firm B can invest in a rail plant using the partner's steel for a current investment of $I_0 = £20\ million$, but must invest a further $I_1 = £85\ million$ next year. The value of the investment depends on whether the firm wins an HS2 contract - it will learn the outcome next year. If the firm wins the contract, the value of the plant will be $V^G = £7.5\ million$ per year from then on but if another firm wins the contract or HS2 is abandoned, the plant will only be worth $V^B = £3.25\ million$ £3.25 million per year. The firm assesses the chances of a successful bid for the HS2 contract at $\pi = 40\%$.

c) Should the firm invest in the plant?

$$PV(cost) = 20 + \frac{85}{1.05} = 100.9524$$

$$EPV(revenue) = \frac{.4 * 7.5 + .6 * 3.25}{0.05} = 99$$

$$ENPV = 99 - 111.9524 = -1.952380952$$

Or:

$$NPV = \frac{0.4 * £7.5m + 0.6 * £3.25m - 5\% * £85m}{5 * 1.05} - £30m = -£1.95\ million$$

This is negative, so the answer is again no.

Seminar Problem 1 Q2

Suppose that management has the option to abandon the project if its HS2 bid is unsuccessful, losing its initial investment but avoiding the further investment of £75 million.

d) Will the firm invest today, and what is the value of the option to abandon?

Abandon If win	Abandon If lose	Formula	Value
No	No	$\frac{\pi V^G + (1-\pi)V^B}{r_f} - \frac{I_1}{1+r_f} - I_0$	£-1.95 m
No	Yes	$\pi\left(\frac{V^G}{r_f} - \frac{I_1}{1+r_f}\right) - I_0$	£7.62 m
Yes	No	$(1-\pi)\left(\frac{V^B}{r_f} - \frac{I_1}{1+r_f}\right) - I_0$	£-29.57 m
Yes	Yes	$-I_0$	£-20 m

The value of the optimal exercise of the option (£7.62 m) is the value of the option because the investment would not be made otherwise.

Real Option: Question Types

- 1. Directly giving the WACC
- 2. No WACC, but giving a perfectly correlating security
- 3.1. No WACC, using risk-neutral probabilities
- 3.2. No WACC, using replicating portfolio

- You might need to calculate
  - The worth (NPV) of a project, with or without an option
  - WACC, if not given
  - The worth of an option

Real Option: Perfectly Correlating (Twin) Security

- The task is to evaluate a project properly
- But it is hard to know the "true" WACC
- If, there is a security that trades on an efficient market, and the security perfectly correlates with the project, then we can use this security to ***price*** the project
  - While seemingly impractical, this could actually be a good approximation
  - Think of HS2: while it is hard to just know the true WACC, suppose the project's cost is a linear combination of concrete, steel, labor costs, then you can build a security using these factors' market price, and it would be reasonable to argue that this security will correlates well with the actual cost of HS2

Example 1.4: computing WACC

Consider a situation like example 1.3 where WACC is unknown.

Firm can invest $I_0 = £125$ in period 1
Risk-free rate is$r_f = 5\%$
$V^G = £200$ *(good state) or* $V^B = £80$ *(bad state) with equal probability* (π^G*=50%)*

- Using e.g. CAPM, you might look for a firm with a beta with systematic risk like that of the project, making assumptions about the tax rate, market risk premium, capital structure and risk-adjusted cost of debt
- Could also look for an *'equivalent' traded security* with returns perfectly correlated with those of the project

Real Option: Perfectly Correlating Security

- Example: Exam Question 2019

1. The UK is considering a post-Brexit trade deal. If the deal is signed in Year 0, by Year 1 the expected present value of the deal will change from today's expectation, to £1500 billion if the world economy goes up or £667 billion if the world economy goes down. Between Year 1 and Year 2, the world economy will change again; the expected present value of the deal will be £2250 billion if the economy goes up 2 years running; £1000 billion if it goes down in Year 1 and up in Year 2 or up in Year 1 and Down in year 2; or £444 billion if it goes down 2 years in a row. The probability of the world economy improving in any given year each year is 70%. The initial implementation cost of the deal is £1050 billion. Delaying decision from Year 0 to Year 1 will cost £175 billion and increase the implementation cost by 10%. Delaying decision from Year 1 to Year 2 will cost an additional £25 billion and increase the implementation cost by a further 10%. (Implementation costs are incurred when the decision to take the deal is made). There is a tradeable security that currently (in Year 0) costs £100 billion per unit and follows a binomial lattice with $u = \frac{3}{2}$ and $d = \frac{2}{3}$ (meaning the price in any given year is u times the previous price if the asset goes up and d times the previous price if it goes down). It is correlated with the world economy. The riskless rate of interest is 5%.

Real Option: Perfectly Correlating Security

a. What is the trade deal worth today and should it be done?

The value of the deal D is perfectly correlated with the value of the security S (D=10S) so the current PV is $PV(D) = £1000\, billion$. This is less than the implementation cost, so (in billions)

$$NPV = D - I_0 = -50 < 0$$

And the deal should *not* be undertaken

Additional question: what is the implicit WACC from a decision tree approach?

From the DTA perspective, the WACC is 25% since

$$\frac{70\% * 1500 + 30\% * 667}{1 + WACC} = PV(D) = 1000, \text{ so}$$

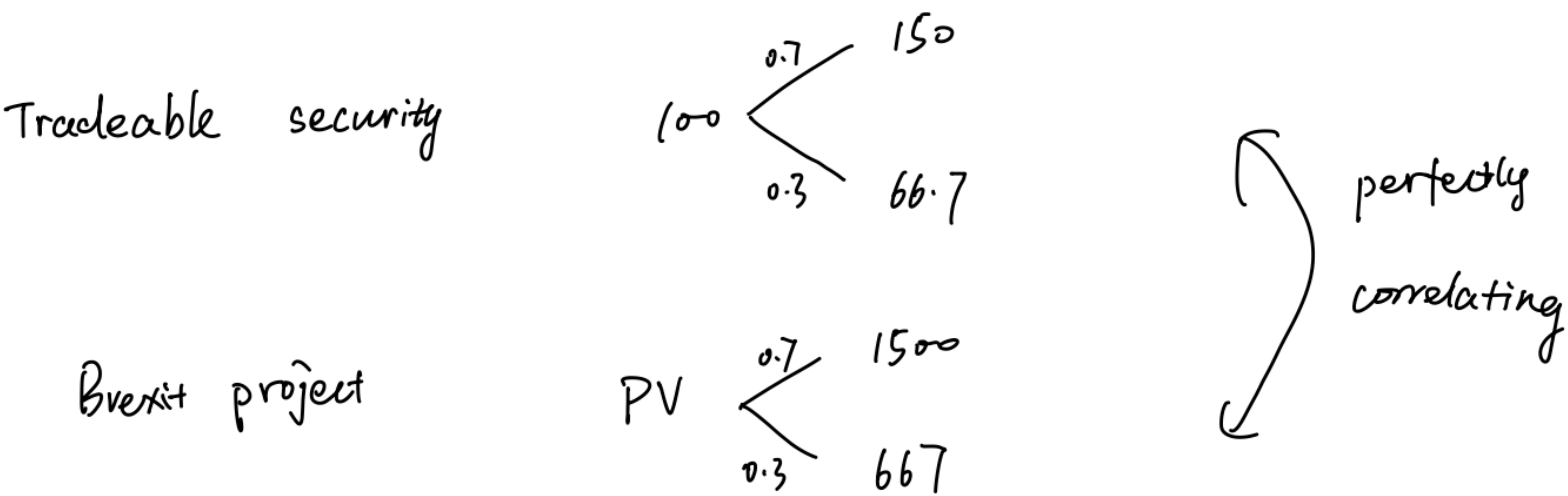
$$WACC = \frac{70\% * 1500 + 30\% * 667}{PV(D)} - 100 = 1.25 - 1 = 25\%$$

Real Option: Replicating portfolio

- If we don’t know the true WACC, but we know the risk-free interest rate, we can create a risk-neutral portfolio, by leveraging two assets, to make a two-equation-two-unknowns problem
- More specifically,
  - by holding a certain amount of the two assets, you can replicate an option’s return in both the good state and the bad state;
  - after knowing the number for the risky project and riskless asset you need, the option is just a linear combination of the two assets you know – the project and the risk-free asset.
  - therefore, you will be able to price the option.
- If the created portfolio is risk-neutral, then we can confidently use the risk-free interest rate to discount between different periods

Real Option: Replicating portfolio

- (continued, with edits) b. what would the one-year delay option be worth from a real option perspective?



Brexit project <=> 10 × Tradeable security

PV = 10 · 100 = 1000 billion pounds

Real Option: Replicating portfolio

No option

1500

1000

667

– 1050 investment

Will not be done,
Project worth 0

With option

$1500 - (1.1 \times 1050) = 345$

P

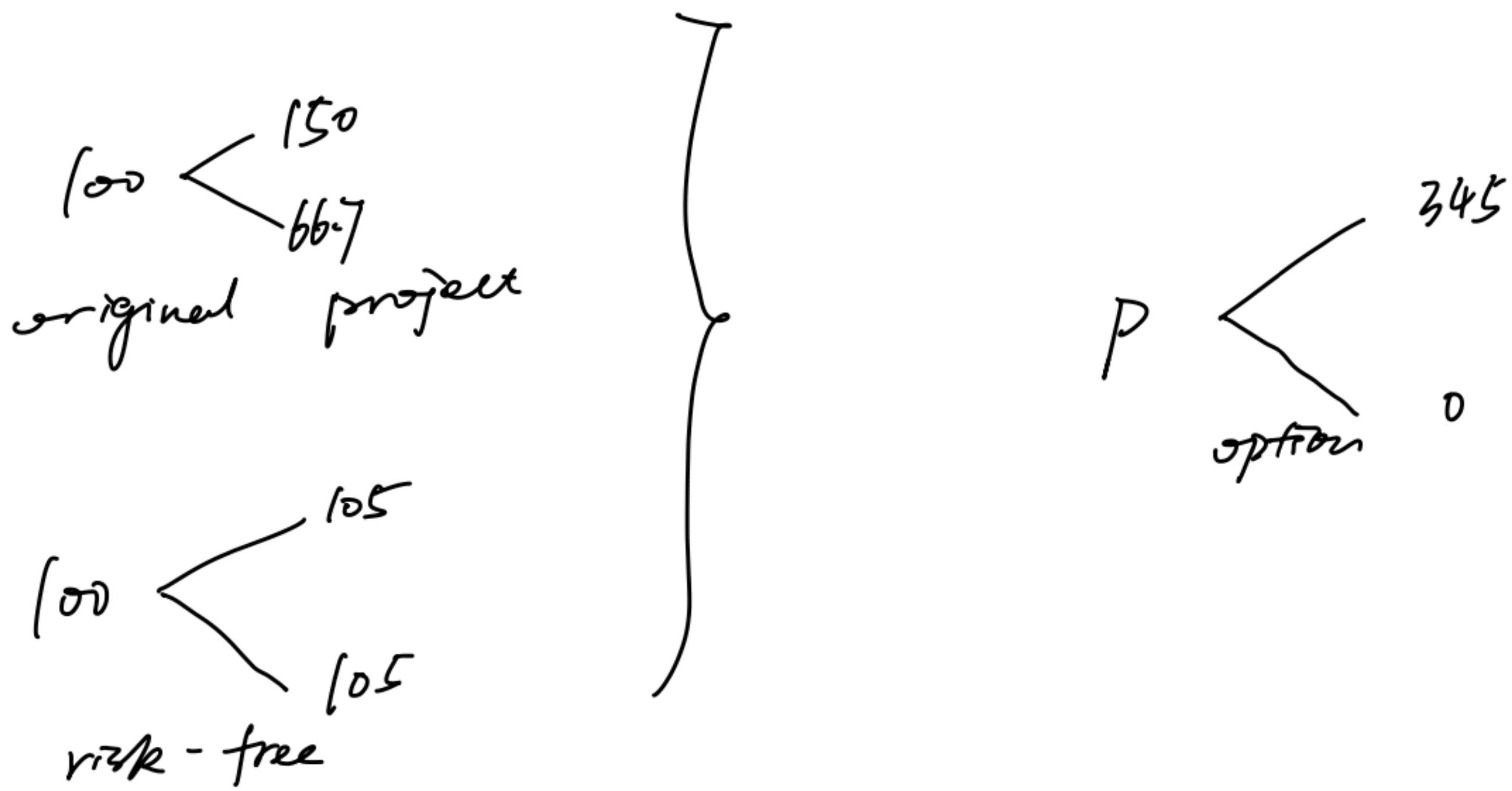
0

– 175 delay cost

Any value with the option for the whole project will be the value of the option

Real Option: Replicating portfolio

Replicating portfolio — pricing the option



$$\begin{cases} X \cdot 150 + B \cdot 105 = 345 \\ X \cdot 66.7 + B \cdot 105 = 0 \end{cases} \Rightarrow \begin{cases} X = 4.1416566627 \\ B = -2.6309380895 \end{cases}$$

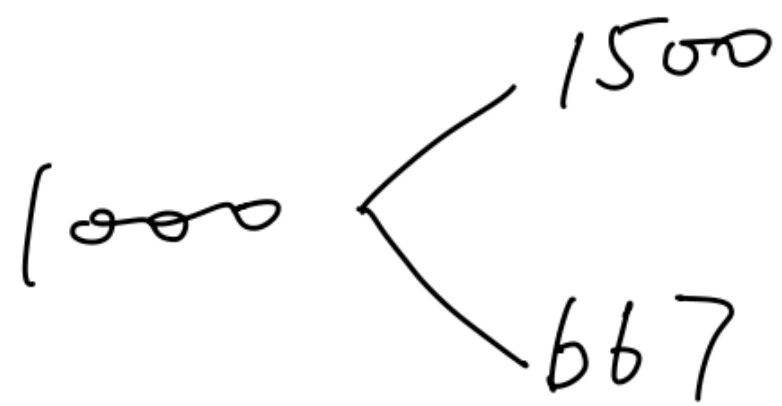
$$NPV = 100X + 100B - 175 = -23.93$$

Real Option: Risk-neutral probabilities

- Risk-neutral probability means that, at what probability, the future expected value of a project discounted by the risk-free interest will equal to its present value: then for a risk-neutral person they are indifferent between investing and not investing
- This is essentially just thinking from another direction: we don’t calculate a portfolio to be risk neutral; we assume things are risk neutral and calculate the implicit probabilities it implies
- Replicating portfolios and risk-neutral probabilities are mathematically equivalent to replicating portfolios — always choose only one of the two!

Real Option: Risk-neutral probabilities

- (continued) b2. what would the one-year delay option be worth from a real option perspective?



At what probabilities, this is equivalent to a risk-free asset, which has an expected payoff of 5%?

Real Option: Risk-neutral probabilities

$$1500 \times p^{ru} + 667 \times (1-p^{ru}) = 1000 \times 1.05$$

$$p^{ru} = 0.459783913 6$$

$$NPV = \frac{345 \times p^{ru} + 0 \times (1-p^{ru})}{1.05} - 175 = -23.93$$