Three Types of Real Option Problems

Junxi Liu

February 22, 2023

Here I summarized three main types of questions in real options analysis. This file should not be regarded as an exhaustive summary of all possible problems and all the materials covered in class, but hopefully will be a useful guide for your review.

1 Given WACC Rate (Calculating PV and Option Price)

In this type of question, the WACC is explicitly given, while they might have different names. To calculate the NPV in this case you can simply use the WACC as discount factor (and recall that in the formula for NPV, WACC is just what we needed, as it is the weighted average cost of capital which should be comprehensive in terms of all costs). The unique feature of this kind of question is that no information on potential replicating portfolios is available.

In this case, the price of an option is simply the difference between the NPV with the option and the NPV without the option, calculated using WACC and the NPV formula.

Example: Exam Question 2021 (Part)

You can find the solutions in the seminar slides or on Moodle.

A property developer is considering taking advantage of the current increase in people working from home. It believes that it is possible to create a block of 500 new 'personal distance' offices with high-speed interconnections and 'smart rooms' that will be attractive to employers once the pandemic threat recedes. The current estimate of the rental revenue per office unit per year is £41,250; The cost of servicing each office unit is £20,000 per year. The riskless rate of return – which is used by the project planners to discount all monetary flows - is 5%. If the project is

undertaken this year, it will cost £67.5 million and generate revenue starting now. Once built, the offices are expected to generate the same annual costs and revenues forever.

(a) What is the NPV if the project is undertaken now?

(b) Now suppose that next year, when the current uncertainty is resolved, the rental revenue per unit per year is expected either to rise to £90,000 (the recovery state, with probability 25%) or to fall to £25,000 (the 'long Covid' state, probability 75%) – and to remain at that level forever. If the decision is delayed, the project cost will rise to £75 million in the recovery state or fall to £52 million in the long Covid state, but the annual servicing cost will remain at £20,000 per unit per year. What is the NPV if the project is delayed?

(Note that, here essentially an objective probability is given)

(c) When (if at all) should the project be undertaken and what is the option to delay the project worth to the developer?

2 No WACC, Given Risk-free Interest Rate, Objective Possibility, and a Perfectly Correlating Security (Calculate PV)

In this type of question, the WACC is not explicitly given, and as a result it usually means that you can't calculating NPV simply using the formula we have learned. Instead, usually a twin-security, which perfectly correlates with the project that the real option is about, will be given. The twin-security is being traded at the market, so it has a present value. Since it correlates with the project the real option is concerned with, that means we can measure the project in terms of "X units of the twin-security", and thus getting the present value for the project. After getting the present value, we can use the NPV formula to calculate what is the true WACC here.

The WACC here usually will be larger than the risk-free interest rate, as the risk-free interest rate is only one of the many potential capital costs in the real world, and the trading price of the twin-security may have already taken other costs into account.

The most distinct feature of this type of question is that some "out of nowhere" security will be

given to you. It hints that the risk-free interest rate is not regarded as WACC and you might need to consider utilizing it to conduct the valuation.

Example: Exam Question 2019 (Part)

You can find the solutions on Moodle.

The UK is considering a post-Brexit trade deal. If the deal is signed in Year 0, by Year 1 the expected present value of the deal will change from today's expectation, to £1500 billion if the world economy goes up or £667 billion if the world economy goes down. Between Year 1 and Year 2, the world economy will change again; the expected present value of the deal will be £2250 billion if the economy goes up 2 years running; £1000 billion if it goes down in Year 1 and up in Year 2 or up in Year 1 and Down in year 2; or £444 billion if it goes down 2 years in a row. The probability of the world economy improving in any given year each year is 70%. The initial implementation cost of the deal is £1050 billion. Delaying the decision from Year 0 to Year 1 will cost £175 billion and increase the implementation cost by 10%. Delaying the decision from Year 1 to Year 2 will cost an additional £25 billion and increase the implementation cost by a further 10%. (Implementation costs are incurred when the decision to take the deal is made). There is a tradeable security that currently (in Year 0) costs £100 billion per unit and follows a binomial lattice with $u = 3/2$ and $d = 2/3$ (meaning the price in any given year is u times the previous price if the asset goes up and d times the previous price if it goes down). It is correlated with the world economy. The riskless rate of interest is 5%.

(a) What is the trade deal worth today and should it be done?

(b) What would the one-year delay option be worth if considered as a decision tree? Hint: find WACC that makes the expected present value of the project next year ignoring implementation cost equal to the current expected value (from part a) and compute the option NPV discounted by WACC using the 'objective probabilities'.

Note that, this is the decision-tree-approach, where you will use a uniform WACC to discount everything.

3 No Objective Possibility, Given PV, Risk-free Interest Rate – Risk Neutral Probability (Calculating Option Price)

This type of question is kind of tricky to identify. Essentially, you will have to understand risk-neutral probability. Risk-neutral probability means that, at what probability, the future expected value of a project discounted by the risk-free interest will equal to its present value: then for a risk-neutral person they are indifferent between investing and not investing.

Example: Exam Question 2021 (See above for the problem)

You can find the solutions in the seminar slides or on Moodle.

(d) A university student living in the town points out that the project is risky (at least in the first year). How would you take this into account? (be as specific as you can clear about the method and the implications for valuing the option to delay)

Note that here essentially the objective probabilities from (b) is abandoned. You will need to value the option to delay, knowing the PV and risk-free interest rate. The PV in this question is a bit dodgy – it comes from "The current estimate of the rental revenue per office unit per year".

Example: Exam Question 2019 (See above for the problem)

You can find the solutions on Moodle.

(c) From a real options perspective? Hint: use the risk-neutral probabilities and the riskless discount rate.

This is more of a standard type. You get the PV from (a), and you can calculate the risk-neutral probability and discount using riskless discount rate.

4 No Objective Possibility, Given PV, Risk-free Interest Rate – Replicating Portfolio (Calculating Option Price)

The intuition behind replicating portfolios is that we want to create a portfolio that can eliminate risk. Once risks are eliminated, we can safely use the risk-free interest rate to discount.

Therefore, to create a replicating portfolio, you will need to leverage two assets: the risky project and the riskless interest rate. By holding a certain amount of the two assets, you can replicate the option's return in both the good state and the bad state. After knowing the number for the risky project and riskless asset, the option is just a linear combination of the two assets you know – the project and the risk-free asset. Therefore, you will be able to price the option.

Mathematically, the risk-neutral probability approach is equivalent to replicating portfolio approach. You can verify in the following question.

Example: Exam Question 2019 (See above for the problem)

(c2) From a real options perspective? Hint: use the replicating portfolio approach.

Solution:

The risky asset you have now is a project with a current value $V_0 = 1000$, which can go up to $V_u = 1500$ and can go down to $V_d = 667$. In addition, you have a risk-free asset B that will be 1.05B after one year.

The option you hope to value, whose value is denoted by C, will generate $1500-1050*1.1 = 345$ billion in the good state, and generate 0 in the bad state.

Suppose in the replicating portfolio, you hold x units of the risky asset and y units of the riskless asset. To fully mimic the performance of the option,

(note that the delay cost happens in period 0)

$$x * 1500 + 1.05y = 345$$

$$x * 667 + 1.05y = 0$$

Solve the system of equations, and you will get

$$x = 0.414166, y = -263.094$$

This means that, initially the value of the option will be the linear combination of the risky asset and the riskless asset, i.e.

$$C = 0.414166 * 1000 - 263.094 * 1 = 151.072$$

which is the total value of option C and the 175 billion cost (think about it in this way: if you are an option seller, you know that whoever buys your option will have to pay the 175 cost, and so when you are selling it you have to deduct this fixed cost for potential buyers to make your option desirable). Therefore, the option is worth 151.072-175 = -23.9, which is the same answer as what you get using risk-neutral probability approach.