International Business and Finance Seminar 3

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Road Map

- In the first two seminars, we've mainly covered
  - Bank essentials: balance sheet; how banks make money; measurements of banks' performance
  - Bond essentials: interest rate; compounding; present value and future value

- Today's seminar will be a bridge between the first two seminars and Friday's seminar, when we will talk extensively about derivatives

More on Future Value and Present Value

- Formal way of defining intra-year compounding

$$FV = PV\left(1+\frac{r}{m}\right)^{m.t}$$

And if $m \to +\infty$, using $\lim_{m\to\infty}\left(1+\frac{1}{m}\right)^{m} = e$, then

$$FV = PV.e^{r.t}$$

- The present value of a multiple cash inflows

If an investment project has a series of expected cash flows $C_1, C_2, \cdots, C_t$ extending over t years and if we also assume that the interest (or discount) rate r has no significant change in this period, the present value of all expected returns will be calculated by :

$$PV = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_t}{(1+r)^t} = \sum_{i=1}^{t}\frac{C_i}{(1+r)^i}$$

Net Present Value

- Net present value, is a measure of "is it a good deal?"
  - NPV is essentially all your future earnings discounted to today, minus all costs discounted to today
  - Positive NPV usually indicates good investment, because — well, why not?

$$NPV = \sum_{i=1}^{t} \frac{C_i}{(1+r)^i} - I_0 > 0$$

$$NPV = \sum_{i=1}^{t} \frac{C_i}{(1+r)^i} - \sum_{j=0}^{m} \frac{I_j}{(1+r)^j} > 0 \quad (m \leq t)$$

Some Very Important Algebra

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$= \frac{n}{2}[a_1 + a_n]$$

$$d = a_n - a_{n-1}$$

where,

- → a_n = n^{th} term
- → a_1 = 1^{st} term
- → n = number of terms
- → S_n = Sum of n terms
- → a_{n-1} = $n\text{-}1^{th}$ term

n^{th} term of a geometric sequence a , ar , ar^2,........ is,

$$a_n = ar^{n-1}$$

Sum of n term of a finite geometric sequence is,

$$a + ar^2 + ar^3 + ... + ar^{n-1}$$

$$= \frac{a(1 - r^n)}{1 - r}$$, when $r \neq 1$

Sum of infinite geometric sequence is,

$$a + ar^2 + ar^3 + ... = \frac{a}{1 - r}$$, when $r < 1$

When Sequences Meet Infinity

$$\lim_{n\to\infty}\sum_{k=1}^{n} a * r^k = \lim_{n\to\infty} a * \frac{r(1-r^n)}{1-r} = a * \frac{r}{1-r} = \frac{a}{\frac{1}{r}-1}$$

The PV of a perpetuity:

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots = \sum_{t=1}^{\infty} \frac{C}{(1+r)^t}$$

So, $PV = \frac{C}{r}$ and the rate of return for a perpetuity can be obtained as: $r = \frac{C}{PV}$

Present Value of Bond

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^t} + \frac{Face\,Value}{(1+r)^t} = \sum_{i=1}^{t} \frac{C}{(1+r)^i} + \frac{Face\,Value}{(1+r)^t}$$

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots = \sum_{i=1}^{\infty} \frac{C}{(1+r)^i} = \frac{C}{r}$$

Growing Perpetuity

Growing Perpetuity:

Let's consider the situation that the stream of a cash flow growing at a constant rate g, so, the PV for growing perpetuity can be written as follows:

$$PV = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots$$

$$= \frac{C_1}{(1+r)} + \frac{C_1(1+g)}{(1+r)^2} + \frac{C_1(1+g)^2}{(1+r)^3} + \cdots$$

$$= \sum_{i=1}^{\infty} \frac{C_1(1+g)^{i-1}}{(1+r)^i} = \frac{C_1}{r-g}$$

The Present Value of Equity: Valuation of Common Stocks

The Present Value of Equity:

There are two sources of payoff for the shareowners (if they stay with their shares for a year): a) cash dividends and b) capital gain or loss.

If P_0, P_1 are the current and expected price (after a year) of a share, respectively, and Div_1 is the expected dividend at the end of a year; the expected rate of return r at the end of the year is:

$$r = \frac{Div_1 + (P_1 - P_0)}{P_0}$$

If dividends grow at a constant rate g each year (like growing perpetuity), the current share price can be calculated by:

$$P_0 = \frac{Div_1}{r - g} \qquad (if\ r > g)$$

We can re-write the formula to show r (expected return) as the subject. In this case;

$$r = \frac{Div_1}{P_0} + g$$

Quiz Questions

Question 5: In March **2012**, the French bank, RCI Banque, issued an **18**-month bond with a face value of **€10,000**, and an annual coupon rate of **2%**, paid quarterly. The issue price was **€9,984.50**. What was it yield to maturity (YTM)?

Question 5: Quarterly coupon interest rate = 0.02/4 = 0.005, so:

$$€9{,}984.50 \; = \; €50\,.\,A^{6}_{R/4} + \frac{10{,}000}{\left(1 + R/4\right)^{6}}$$

We can assume $R/4 = X$ and use the Binomial expansion of $(1+X)^6$, we have:

$$(1+X)^6 = 1 + 6X + 15X^2 + 20X^3 + \cdots + X^6$$

As

$$|X| < 1 \Longrightarrow (1+X)^6 \approx 1 + 6X$$

So,

$$9{,}984.50 = 50\left[\frac{1}{X} - \frac{1}{X(1+X)^6}\right] + \frac{10{,}000}{(1+X)^6}$$

Changes to:

$$9{,}984.50 = \frac{300 + 10{,}000}{1 + 6X} \Longrightarrow X = 0.0052 \Longrightarrow R = 0.02106 = 2.106\%$$

Text

Question 6: In **2019**, Daimler Chrysler had just paid a dividend of **€2** per share on its equity. The dividends are expected to grow at a constant rate of **5%** per year indefinitely. If investors require an **11%** return on the company's equity:

a. What is the current price?
b. What will be the price in 2022?
c. What will be the price in 2034?

Question 6:

a. Current price at a constant growth of 5 per cent per year indefinitely

$P = Div_1 / (R - g)$, whereas $Div_1 = D_0 (1+g)$ i.e. $Div_1 = €2\ (1.05) = €2.1$
$P = €2.1 / (0.11 - 0.05) = €35$

b. Price in 3 years: $P = P_0(1+g)^3 = €35(1.05)^3 = €40.52$

c. Price in 15 years: $P = P_0(1+g)^{15} = €35(1.05)^{15} = €72.76$