International Business and Finance
Seminar 9

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Game Theory

- You had a quarrel with your boyfriend/girlfriend/partner; the relationship is tense right now
  - You can admit it's your fault, though you might not think so. So if you do this, you feel not that good, but still the relationship is back on track
  - You can wait for your partner to admit it's their fault, and if they do that you will feel really good, and the relationship is back on track
  - But, there is also a possibility that neither of you break the ice, and you two break up. You feel so bad.

Game Theory

- Normal form game: three key aspects
  - Payoffs
  - Players
  - Actions
- In the previous example
  - Payoffs: you feel good or not
  - Players: you and your partner
  - Actions: admit or wait

(your payoff, his/her payoff)		Your partner	
		Admit	Wait
You	Admit	2,2	1,3
	Wait	3,1	-100,-100

Definition 3.3 A **normal-form game** includes three components as follows:

1. A finite **set of players,** $N = \{1, 2, \ldots, n\}$.
2. A **collection of sets of pure strategies,** $\{S_1, S_2, \ldots, S_n\}$.
3. A **set of payoff functions,** $\{v_1, v_2, \ldots, v_n\}$, each assigning a payoff value to each combination of chosen strategies, that is, a set of functions $v_i : S_1 \times S_2 \times \cdots \times S_n \to \mathbb{R}$ for each $i \in N$.

Dominance: It's Always Better To Do So

		Player 2	
		M	F
Player 1	M	$-2, -2$	$-5, -1$
	F	$-1, -5$	$-4, -4$

Definition 4.2 $s_i \in S_i$ is a **strictly dominant strategy** for i if every other strategy of i is strictly dominated by it, that is,

$$v_i(s_i, s_{-i}) > v_i(s_i', s_{-i}) \quad \text{for all } s_i' \in S_i, \quad s_i' \neq s_i, \quad \text{and all } s_{-i} \in S_{-i}.$$

Best Response: It's Good Enough Given Your Information

		Chris	
		O	F
Alex	O	2, 1	0, 0
	F	0, 0	1, 2

As the matrix demonstrates, the best choice of Alex depends on what Chris will do. If Chris goes to the opera then Alex would rather go to the opera instead of going to the football game. If, however, Chris goes to the football game then Alex's optimal action is switched around.

Definition 4.5 The strategy $s_i \in S_i$ is player i's **best response** to his opponents' strategies $s_{-i} \in S_{-i}$ if

$$v_i(s_i, s_{-i}) \geq v_i(s_i', s_{-i}) \quad \forall s_i' \in S_i.$$

Solution: Dominant Strategy Equilibrium

- Every one chooses dominate strategy

		Firm B's strategies	
		Low	*High*
Firm A's strategies	**Low**	**4** *4*	**2** *3*
	High	**3** *2*	**1** *1*

Solution: Nash Equilibrium

- Every one choose best response

Player 2

		action C	action D
Player 1	action A	5, 3	1, 0
	action B	0, 1	2, 4

Solution: Nash Equilibrium

Pure-Strategy Nash Equilibrium in a Matrix

This short section presents a simple method to find all the pure-strategy Nash equilibria in a matrix game if at least one exists. Consider the following two-person finite game in matrix form:

		Player 2		
		L	*C*	*R*
	U	7, 7	4, 2	1, 8
Player 1	*M*	2, 4	5, 5	2, 3
	D	8, 1	3, 2	0, 0

Solution: Nash Equilibrium

Step 1: For every *column,* which is a strategy of player 2, find the highest payoff entry for player 1. By definition this entry must be in the row that is a best response for the particular column being considered. Underline the pair of payoffs in this row under this column:

		Player 2		
		L	C	R
	U	7, 7	4, 2	1, 8
Player 1	M	2, 4	$\underline{5, 5}$	$\underline{2, 3}$
	D	$\underline{8, 1}$	3, 2	0, 0

Step 1 identifies the best response of player 1 *for each of the pure strategies* (columns) of player 2. For instance, if player 2 is playing L, then player 1's best response is D, and we underline the payoffs associated with this row in column 1. After performing this step we see that there are three pairs of pure strategies at which player 1 is playing a best response: (D, L), (M, C), and (M, R).

Solution: Nash Equilibrium

Step 2: For every *row*, which is a strategy of player 1, find the highest payoff entry for player 2. By definition this entry must be in the column that is a best response for the particular row being considered. Overline the pair of payoffs in this entry:

		Player 2		
		L	*C*	*R*
	U	7, 7	4, 2	$\overline{1, 8}$
Player 1	*M*	2, 4	$\underline{\overline{5, 5}}$	$\underline{2, 3}$
	D	$\underline{8, 1}$	$\overline{3, 2}$	0, 0

Step 2 similarly identifies the pairs of strategies at which player 2 is playing a best response. For instance, if player 1 is playing *D*, then player 2's best response is *C*, and we overline the payoffs associated with this column in row 3. We can continue to conclude that player 2 is playing a best response at three strategy pairs: (*D*, *C*), (*M*, *C*), and (*U*, *R*).

Solution: Nash Equilibrium

Step 3: If any matrix entry has both an under- and an overline, it is the outcome of a Nash equilibrium in pure strategies.

This follows immediately from the fact that both players are playing a best response at any such pair of strategies. In this example we find that (M, C) is the unique pure-strategy Nash equilibrium—it is the only pair of pure strategies for which both players are playing a best response. If you apply

this approach to the Battle of the Sexes, for example, you will find both pure-strategy Nash equilibria, (O, O) and (F, F). For the Prisoner's Dilemma only (F, F) will be identified.

		Chris	
		O	F
Alex	O	2, 1	0, 0
	F	0, 0	1, 2

		Player 2	
		M	F
Player 1	M	−2, −2	−5, −1
	F	−1, −5	−4, −4