General Steps

- Know the definition of Nash equilibria as being each player best responding (possibly in mixed strategies) to each other.
- Obtain best response curves for each player, which gives the best reply (the one which maximises profits or payoffs) for any strategy in the set of opponent's strategies (sometimes restricted to pure strategies as in the firm case).
- The intersections of the best response curves give the Nash equilibria for the game.

Recognising that $\gamma = 1$ is like having perfect substitutes.

From the indirect demand function $q_1 = \frac{a(1-\gamma)-p_1+\gamma p_2}{1-\gamma^2} = \frac{a}{1+\gamma} - \frac{p_1-\gamma p_2}{1-\gamma^2}$

We see that as $\gamma \to 1$,

$$q_1 \to \begin{cases} +\infty \; for \; p_1 < p_2 \\ \dfrac{a}{1+\gamma} - \dfrac{p}{1+\gamma} \; for \; p_1 = p_2 = p \\ -\infty \; for \; p_1 > p_2 \end{cases}$$

So intuitively this means that by setting a price close to the others price (but lower), the firm can capture the whole market: This is what we can expect if the products are perfect substitutes. Of course there is an additional condition that $p_i \geq c$, otherwise selling is not profitable. This means that there is a lower bound to the prices which each firm charges.

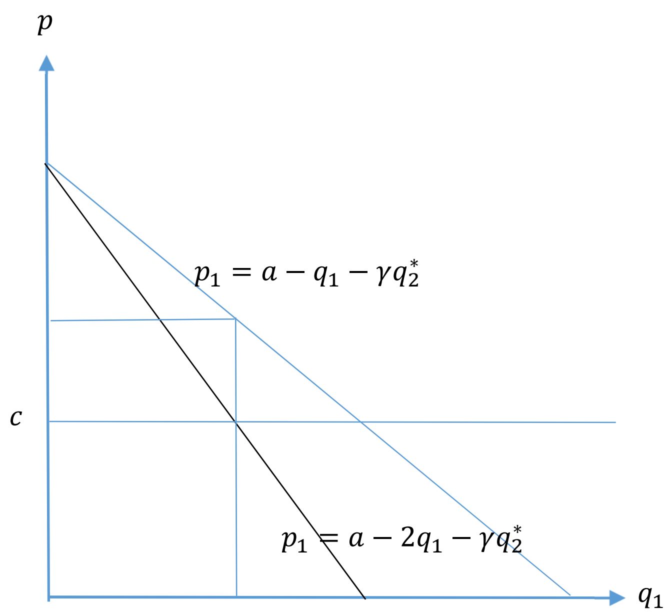
If costs of each firm are different, the market equilibrium will then be $p \cong \max(c_1, c_2)$ and the lower cost firm will capture the whole market.

Also, another way to see it is that $p_1 = p_2 = a - q_1 - q_2$ when $\gamma \to 1$. This means that in (a quantity competition) equilibrium, the prices charged are the same in each market, which is what we would expect when the goods are the same (perfect substitutes).

How do we draw the "supply" and demand curves to show equilibrium?

Quantity competition

Notice that when $0 \leq \gamma < 1$, we have a duopoly market, so each firm has some market power: so they can charge their own prices and determine their own quantities in their own product market. So, the relevant kind of drawing in the quantity competition case is a monopoly style one for <u>each firm</u>, treating the quantity produced by the other firm in equilibrium as exogenous.



To illustrate quantity competition, we would use the standard monopoly competition diagram where they use MC= MR in deciding quantity. MC here is something like the supply curve. In equilibrium we have that $q_2^* = \frac{a-c}{2+\gamma}$.

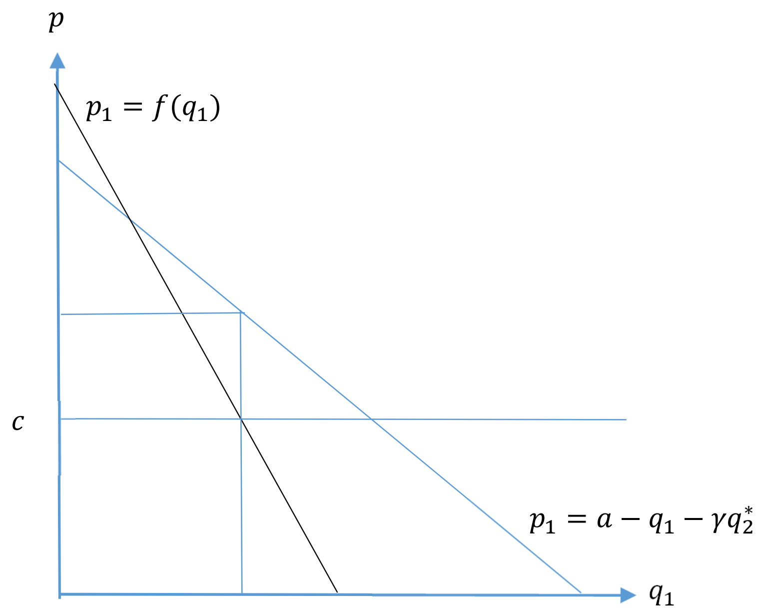
Price Competition

In price competition, the profit maximising condition is slightly different.

The firm maximises $(p_1 - c) * q_1(p_1, p_2)\ wrt\ p_1$ taking p_2 as exogenous, where $q_1 = \frac{a}{1+\gamma} - \frac{p_1}{1-\gamma^2} + \frac{\gamma p_2}{1-\gamma^2}$. This gives the condition $q_1(p_1, p_2) + (p_1 - c) * \frac{\partial q_1}{\partial p_1} = 0$ or $q_1(p_1, p_2) - \frac{(p_1 - c)}{1-\gamma^2} = 0$. Since we assume $\gamma < 1$, we can write the condition as $c = p_1 - (1-\gamma^2)q_1(p_1, p_2)$.

If we put this in terms of quantities instead, we have:

$$c = a - q_1 - \gamma q_2 - (1-\gamma^2)q_1 = a - (2-\gamma^2)q_1 - \gamma q_2 = f(q_1)$$



In equilibrium, $q_2^* = \frac{a-c}{(2-\gamma)(1+\gamma)}$. When γ rises, the y intercept falls and the $f(q_1)$ line becomes gentler.

As $\gamma \to 1$, the equilibrium condition is $c = a - q_1 - q_2^*$, notice that the RHS is exactly now the demand curve when $\gamma \to 1$ and the LHS being the "supply" curve. This is shown in the above figure.

Black line illustrates $f(q_1)$ for some $\gamma \ll 1$. As γ tends to 1, under price competition, the firm loses ability to mark up as $f(q_1)$ and the demand curve become the same thing.

In both cases, we can look at the welfare consequences from these firm diagrams as in a standard monopoly analysis (consumer/producer surplus).