Things to know

1. Coalitional games:
    a. Instead of individuals, we now have different coalitions (groups) of players (which includes singletons). Group of all players is the grand coalition.
    b. Each coalition has a set of actions which generates payoffs for each player.
    c. In transferable utility games, the sum of payoffs for all actions of a coalition is the same.
    d. Core: set of actions of the grand coalition such that no coalition has an action that all its members will prefer to it (i.e. can isolate themselves from the grand coalition and be better off)
        i. In TU games, actions in the core $\Leftrightarrow$ sum of payoffs exceeds V(S) for every coalition S.
2. Simultaneous Games with incomplete information:
    a. E.g. They do not know types of other players, their preferences, strategies etc.
    b. Cannot condition on others' moves because of simultaneity.
    c. Strategies are conditional on each type (since they know their own type)
    d. Bayes Nash equilibrium: each type is playing their best response given their beliefs about others' types.
        i. Questions usually will ask you to verify a Bayes NE:
        ii. Expected payoffs must be maximized by the specified actions in the BNE, given the other types' actions and beliefs about what other types there are.
    e. Note: when asked to solve for <u>all Bayes NE</u>, one can use the normal form with extended strategies (Types X Actions) expected payoffs for each player calculated using the prior joint probabilities and solve for the NE.

        e.g 2 types and 2 actions (U,L). Then the set of extended strategies are written as (type 1 strategy, type 2 strategy).

	UL		LL	UU	LL
UL	X_1, X_2				
LL					
UU					
LU					

As an example, the entry $X_1 = P(type1, type1)\pi_1(U,U) + P(type1, type2)\pi_1(U,L) + P(type2, type1)\pi_1(L,U) + P(type2, type2)\pi_1(L,L)$.

Maximising payoffs here is equivalent to maximizing payoffs conditional on one's type.

Problem set 3b

1.

Firstly, let us write the game in normal form: there will be 4 different payoff matrices, 1 for each combination of types as follows. Note that Player 2 is the Husband, while Player 1 is the Wife.

(1,2)	$t^2 \rightarrow$	L		NL	
$t^1 \downarrow$	σ^1, σ^2	O	F	O	F
L	O	$3, 2$	$0, 0$	$3, -1$	$0, 3$
	F	$0, 0$	$2, 3$	$0, 0$	$2, 3$
NL	O	$3, 2$	$3, 0$	$3, -1$	$3, 3$
	F	$0, 0$	$-1, 3$	$0, 0$	$-1, 3$

The joint probability distributions for each type are shown here:

(1,2)	L	NL
L	0.1	0.4
NL	0.4	0.1

From this, we can then calculate the probability that one is playing with a particular type given my own type (Using Bayes rule)

$$P(Partner\ type = \ L|Own\ type = L) = 1/5$$

$$P(Partner\ type = \ NL|Own\ type = L) = 4/5$$

$$P(Partner\ type = \ L|Own\ type = NL) = 4/5$$

$$P(Partner\ type = \ NL|Own\ type = NL) = 1/5$$

Our aim here is to show that the strategies for the wife $(L \rightarrow F, NL \rightarrow O)$ and for the husband $(L \rightarrow O, NL \rightarrow F)$ form a Bayes Nash equilibrium. This means that given each one's beliefs about what type their partner is, the given strategies of each husband (wife) type are the best response to the strategies of the wife (husband).

From above, we have all the ingredients to evaluate each type's expected payoffs.

i) Showing that a L Husband will play O

For a type L husband, given the strategy of "wives": $(L \rightarrow F, NL \rightarrow O)$

	Wife is NL (plays O)	Wife is L (plays F)	Expected payoff
O	2	0	$\frac{4}{5} * 2 + \frac{1}{5} * 0 = 1.6$
F	0	3	$\frac{4}{5} * 0 + \frac{1}{5} * 3 = 0.6$

ii) Showing that a NL Husband will play F (actually it is a strictly dominant strategy)

For a type NL husband, given the strategy of "wives": $(L \rightarrow F, NL \rightarrow O)$

	Wife is NL (plays O)	Wife is L (plays F)	Expected payoff
O	-1	3	$\frac{4}{5} * -1 + \frac{1}{5} * 3 = -0.2$
F	3	3	$\frac{4}{5} * 3 + \frac{1}{5} * 3 = 3$

iii) Showing that a L wife will play F

For a type L wife, given the strategy of "husbands": $(L \rightarrow O, NL \rightarrow F)$

	Husband is NL (plays F)	Husband is L (plays O)	Expected payoff
O	0	3	$\frac{4}{5} * 0 + \frac{1}{5} * 3 = 0.6$
F	2	0	$\frac{4}{5} * 2 + \frac{1}{5} * 0 = 1.6$

iv) Showing that a NL wife will play O (actually it is a strictly dominant strategy)

For a type NL wife, given the strategy of "husbands": $(L \rightarrow O, NL \rightarrow F)$

	Husband is NL (plays F)	Husband is L (plays O)	Expected payoff
O	3	3	$\frac{4}{5} * 3 + \frac{1}{5} * 3 = 3$
F	-1	3	$\frac{4}{5} * -1 + \frac{1}{5} * 3 = -0.2$

Thus, we have shown that each type (of husband/wife) is playing their best response given the strategy of their partner in conjunction with their beliefs of their type. Hence, the strategies for the wife $(L \rightarrow F, NL \rightarrow O)$ and for the husband $(L \rightarrow O, NL \rightarrow F)$ form a Bayes Nash equilibrium.

Notice that here, the payoffs are symmetric in that a loving (non-loving) wife gets similar payoffs as a loving (non-loving) husband, with actions switched. Thus intuitively, the best response of each wife type should just be the flipped.

2)

1,2	C	D
C	4,4	0,5
D	5,0	1,1

Nice

1,2	C	D
C	2,2	0,5
D	5,0	1,1

Bad

Refer to the answer sheet when it is released for a more detailed answer. Actually, it is quite simple here because no matter what is my type (or the opponents type), the strictly dominant strategy is still to defect; even with incomplete information about my opponent, the best thing I can do will be to defect. Thus, it should be obvious that the Bayes Nash equilibrium would be to defect no matter what. Being nice here is not enough. It might have been more interesting if the payoffs of defecting (cooperating) were lower (higher) if the other chose to cooperate.

Problem set 3a

2) Landowner-worker game where any group of fewer than $n-1$ workers refuses to work with the landowner. Let player 1 be the landowner. This is supposed to represent a case of a union.

We have 3 classes of coalitions:

The grand coalition: Landowner + $(n-1)$ workers. It has a value of $F(n)$.

Any other coalition with the landowner has a value of $F(1)$: only the landowner works; workers refuse to work.

Any other coalition without the landowner has a value of 0: they need a landowner to have output.

Thus, the core consists of all payoff distributions: $(x_1, \ldots., x_n)$ with $\sum x_i = F(n)$ and satisfying:

i) $x_1 \geq F(1)$: From landowner alone coalition
ii) $x_j \geq 0, j > 1$: From worker alone coalition
iii) $x_1 + \sum_{j \in X} x_j \geq F(1), |X| < n-1$
iv) $\sum_{i=1}^{n} x_i = F(n)$: From grand coalition.

Core is any allocation of $F(n)$, with landowner getting at least $F(1)$: this includes very unequal allocations of output.

What happens if they only refuse to work when fewer than $n-2$ workers ?

Then we have 4 classes of coalitions:

The grand coalition: Landowner + $(n-1)$ workers. It has a value of $F(n)$.

Landowner + $(n-2)$ workers. It has a value of $F(n-1)$.

Any other coalition with the landowner has a value of $F(1)$: only the landowner works; workers refuse to work.

Any other coalition without the landowner has a value of 0: they need a landowner to have output.

Thus, the core has an additional condition:

i) $x_1 \geq F(1)$: From landowner alone coalition

ii) $x_j \geq 0, j > 1$: From worker alone coalition

iii) $x_1 + \sum_{j \in X} x_j \geq F(n-1),\ |X| = n-2$

iv) $x_1 + \sum_{j \in X} x_j \geq F(1), |X| < n-2$

v) $\sum_{i=1}^{n} x_i = F(n)$: From grand coalition.

Anything in the core for second case is in the core for first case: wider range of possible values in first case. Case here excludes more extreme allocations of output for workers; this arises due to competition between possible coalitions of workers.