Things to know:

1. Evolutionary stability:
   a. Genes govern behaviour (strategies)
   b. What kinds of genes and behaviour will prevail is governed by fitness.
   c. Polymorphism is when there are several types in the overall population.
      i. Stable vs Unstable polymorphic equilibria.

Problem set 5a.

Q1 Wimp vs Machos.

Payoff table when they meet each other

	Wimp	Macho
Wimp	0,0	-2,4
Macho	4,-2	-8,-8

a) Solving for the ESS.

Step 1: Assume that there is a proportion ρ of Machos and calculate the expected payoffs (Fitness) of each type.

Machos: $\rho \times -8 + (1-\rho)\times 4 = 4 - 12\rho$

Wimp: $\rho \times -2 + (1-\rho)\times 0 = -2\rho$

Step 2: Solve for conditions which cause one type to be fitter than the other.

Macho is fitter when $4 - 12\rho > -2\rho$

This gives $\rho < \frac{2}{5}$.

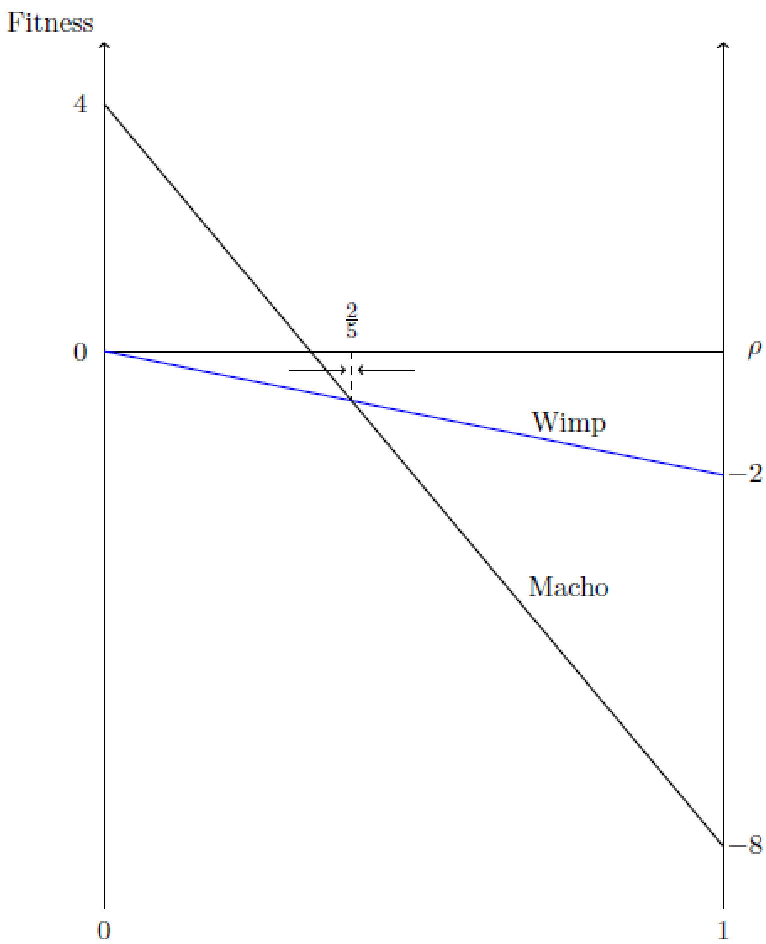
So, the population of Machos rises when $\rho < \frac{2}{5}$ and falls when $\rho > 2/5$. This means that the ESS proportion of Machos is 2/5.

b) This implies (stable) polymorphism. [This contrasts with one of the examples in the lecture]

Note: Intuitively this is because of the high negative payoff which Machos face when their population is too high: their gains are mainly generated from interacting with Wimps and are reduced when the Wimp population falls.

In contrast, for the example in the lecture, there is a benefit to interacting with a majority of one's own population which causes the unstable polymorphic equilibria, but 2 stable single-type equilibria.

c)



Q2) Bertrand competition

i) Symmetric costs c

Profit function from the description in the question:

$$\pi^i(p^i, p^j) = \begin{cases} (p^i - c)\, q^i, & \text{if } p^i < p^j \\ \frac{1}{2}\,(p^i - c)\, q^i, & \text{if } p^i = p^j \\ 0, & \text{if } p^i > p^j \end{cases}$$

In a Nash equilibrium, they will both charge $p = c$.

<u>Sketch of Proof:</u>

First, note that they will never set a price below marginal cost since they will potentially earn negative profits which will be dominated by just setting prices at marginal cost and earning 0 profits. Thus, we can restrict attention to $p_i, p_j \geq c$.

Second, we can then see that in equilibrium, we cannot have any firm setting $P > c$.

If $p_i \neq p_j > c$, then the firm which sets the lower price can gain by setting a slightly higher price.

If $p_i = p_j > c$, then by setting a lower price, a firm can gain all the output.

None of them can benefit from deviating from $p = c$.

If they can cooperate, they would set the same price p^* and earn $(p^* - c)q/2$ each which can be arbitrarily high. But this cannot be sustained as mentioned earlier as each firm has the incentive to deviate by setting price slightly lower.

ii) Asymmetric costs (c_H, c_L)

Let the price which the high (low) cost firm charges be p_H (p_L) .

Let $\overline{p} = \max(p_H, p_L)$, $\underline{p} = \min(p_H, p_L)$

Then, the profit function if a firm charges $p_i$is

If $p_i < \underline{p}$, It always gets all the market share q; so profits = $(p_i - c_i)\, q$.

If $p_i = \underline{p}$, It always gets all the market share when facing the firm which charges $\overline{p}$, but only half the profits when facing the other firm; so profits = $\frac{1}{2} \times \frac{1}{2}(p_i - c_i)\, q + \frac{1}{2}(p_i - c_i)\, q$.

If $\underline{p} < p_i < \overline{p}$, It always gets all the market share when facing the firm which charges $\overline{p}$, but nothing when facing the other firm; so profits = $\frac{1}{2}(p_i - c_i)\, q$.

If $p_i = \overline{p}$, It gets half the market share when facing the firm which charges $\overline{p}$, but nothing when facing the other firm; so profits = $\frac{1}{2} \times \frac{1}{2} (p_i - c_i)\, q$.

If $p_i > \overline{p}$, It never gets any market share; so profits = 0.

Now, we can use this to check whether each firm type charging their marginal cost is a perfect Bayesian equilibrium. I.e. $p_L = c_L < c_H = p_H$.

Substituting values, we have for the <u>low cost</u> firm:

$$E\left(\pi^i\left(p^i, p^j\right) | c^i = c_L\right) = \begin{cases} \left(p^i - c_L\right) q^i, & \text{if } p^i < c_L; \\ 0, & \text{if } p^i = c_L; \\ \frac{1}{2}\left(p^i - c_L\right) q^i, & \text{if } c_L < p^i < c_H; \\ \frac{1}{4}\left(p^i - c_L\right) q^i, & \text{if } p^i = c_H; \\ 0, & \text{if } p^i > c_H. \end{cases}$$

Notice that profits are not maximized since by charging some price in between c_L and c_H, it can earn positive profits (this is from facing the high cost firm). Since this is greater than 0, the low cost firm would want to deviate and thus this is not a PBE.

For additional information on what the Perfect Bayesian Equilibrium looks like in this case, refer to the following paper: Bertrand Competition with Cost Uncertainty, Routledge, R (2010); Economic Letters.

Q3) Exactly the same as Q2. See the posted solutions on Moodle.