Seminar 3 – Expected Utility Theory & Heuristics

Related to choice theory and the experimental designs which people have used to test the standard theory. In general, the experiments here are not of the treatment-control design, but rather are specifically constructed choices for participants which allows us to test how well the theory can predict their choices. In some cases, experimental data can be used to structurally estimate the utility parameters of individuals by letting them make slightly different choices repeatedly.

1) What is the independence axiom? Describe how Allais' experiment showed violations of independence.

In words, two gambles mixed with an irrelevant third one will maintain the same order of preference as when the two are presented independently of the third one.

As an axiom (Usually hold for strict preferences as well): Let A, B, and C be three lotteries with $A \succsim B$, and let (1-q) be the probability that a third choice C is present, then if $qA+(1-q)C \succsim qB+(1-q)C$ then the third choice C is irrelevant, and the preference relation $A \succsim B$ is preserved independently of the presence of C.

Note that the axiom also implies the converse $(A \rightarrow B \textit{ implies } \neg B \rightarrow \neg A)$

This means that if $qB+(1-q)C \succsim qA+(1-q)C$, then $B \succsim A$. Applying the independence axiom again implies that $qB+(1-q)D \succsim qA+(1-q)D$. I.e. replacing a common lottery C with a different lottery D will preserve the preference order.

Allais posed a question like the following (this is the simple version from lectures):

Choice 1: A or B?

- A: $1m for sure.
- B: $5m, 10%; $1m, 89%; 0, 1%.

Choice 2: C or D?

- C: $1m, 11%; 0, 89%.
- D: $5m, 10%; 0, 90%.

He found that most people choose the sure amount under choice 1: option A ($1m for sure) and choose D in choice 2, $5m, 10% of the time, 0, 90% of the time. However, EUT (independence) predicts that for consistency with rationality people should either opt for A-C or B-D. Essentially A can be transformed into C and B can be transformed into D so the independence axiom is clear that the transformation should leave the choice unchanged, but that is not how most people act. This is also discussed with reference to the "Act-State Framework" in lectures. *Notice that by replacing the 89% probability of winning 1 million with 0 in Choice 1, we get Choice 2. In the context of the independence axiom 1-q is 89% and C is the event of winning 1 million for sure.*

Results indicate that individuals usually prefer A and D: They overweigh the 1 % change of losing in B, while less so in D.

Notice that this isn't a standard kind of control-treatment design to test the impact of some kind of factor X. Rather, it is a very specific experiment which is used to test whether outcomes are consistent with some specific theory: it's aim was to prove Expected utility theory as inaccurate.

Here Allais, particularly designed 2 sets of choices which should have consistent predictions under a standard theory. He probably had some theory in mind about how individuals think about very small probability changes from certainty (i.e. 0 % to 1% chance of getting 0) differently when designing Choice 1: thus the choice of 1% which would violate EUT.

To show that the relevant factor X is indeed the overweighing of small changes from certainty, one could have another treatment like:

Choice 1': A or B?

- A: $1m, 80% 0, 20%.
- B: $5m, 10%; $1m, 69%; 0, 21%.

Choice 2': C or D?

- C: $1m, 11%; 0, 89%.
- D: $5m, 10%; 0, 90%.

Repeating choices for many different probability values would also allow for an estimate of an individuals' curvature in probability judgement under the new theory.

Subsequently, this leads to proposals for new kinds of theory. Here, the Allais Paradox is commonly solved in prospect theory by allowing probability weights to become non-linear: there is a "heuristic/bias" where individuals overweigh small changes of probability from certainty. It is a modification of the act-state framework where probabilities were taken to be the objective ones: i.e. a non-linear transformation.

2) Describe the "law of small numbers" and give an example of how it might work.

The general idea is that people often focus too much on interpreting results from small sample sizes. This is somewhat related to the representativeness bias in that we are taking a small sample as representative.

Kahneman & Tversky put forward the idea of the law of small numbers: people look for mean reversion even in very small sample sizes. In other words, people felt a small sample was representative of the whole. For instance, if you flip a head 3 times, you might feel that a tails was more likely to occur on the 4th coin flip. This is related to the gambler's fallacy. Also, when asked to construct random sequences of coin flips, it is observed that people try to balance out observations and seem to underestimate the chances of observing long chains of the same observation. Another real-life finding consistent with the law of small numbers is that lottery numbers which come out recently tend to have higher expected earnings.

The mistaken idea here is that small samples should be similar to what we see in large samples; there is an underestimation of the variability which we see in small samples.

Note the contrast to the hot hand fallacy in gambling: the mistaken idea that what we saw in our small sample will continue to occur. This could be due to different salience of which sample is representative: in the former the large sample is salient, while in the latter the small sample is salient. Both however try to generalise from small samples, but in different manners.

The consequence here is that this could lead to suboptimal decisions: overconfidence in the face of loss/ overconfidence in the face of past success. Overthinking coincidences, over-focusing on surprising anecdotes.

Like before, this is sort of like a test of the "rationality" of individuals in making predictions: are they able to closely estimate fixed probabilities unconsciously? The observations above seem to indicate otherwise. If not, how do we explain people's estimates? Here we see again that theory is tested using experiments and subsequently we create new theories based upon the observations we have and try to test them using new experiments.

With regards to experiments, it might also be useful to keep this and other biases in mind when we analyse people's actions/ stated beliefs as these might be conflated with the true treatment effects.

3) Think of examples (in your life or otherwise) where the anchoring and representativeness heuristics might apply.

Note that heuristics are like certain mental shortcuts (often unconsciously) which we use to make decisions or evaluations.

Anchoring heuristic: earlier information sets a reference frame for what we look at in future.

Representativeness heuristic (Wikipedia): when judging statements, one looks at the degree which it i) is similar in essential characteristics to its parent population, and (ii) reflects the salient features of the process by which it is generated. Individuals assess similarity of objects and organize them based around the category prototype.

Anchoring

In lectures, for anchoring, we looked at a simple example involving multiplying numbers as follows:

Subjects were given 5 seconds to guess the answer to a maths question.

• One group got: 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 (series 1)

• The other got: 1 x 2 x 3 x 4 x 5 x 6 x 7 x 8 (series 2)

It turned out that the group who got series 1 tended to give higher guesses.

Other examples of anchoring?

1. The regular price is listed first so as to anchor people's expectations of a fair price there: e.g. ebay, auctions .etc)
2. Order in which bids come in could matter for your reservation value if there is uncertainty in true value.
3. When reading research papers, your idea of what is a normal is effect depends on what papers you read; a problem here in conjunction with publishing bias is that it is harder (for the audience) to revise our probabilities of a finding downward when faced with a negative replication which exacerbates the problem.

Representativeness heuristic

In lectures, for representativeness we looked at the "Linda example" as follows:

Subjects were given the following description of "Linda":

"Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations."

Next, subjects were given 4 of 8 descriptions and asked to assign probabilities to their likelihood of being correct.

The 8 descriptions were:

- Linda is a teacher in elementary school.

- Linda works in a bookstore and takes Yoga classes.
- Linda is active in the feminist movement.
- Linda is a psychiatric social worker.
- Linda is a member of the League of Women voters.
- Linda is a bank teller.
- Linda is an insurance salesperson.
- Linda is a bank teller and is active in the feminist movement.

Group 1 were given a set that included 6 (Linda is a bank teller) and group 2 were given a set that included 8 (Linda is a bank teller and is active in the feminist movement). We know of course that 8 is a special case of 6 so must be less likely but people assigned higher probability to 6 than 8!

Usually $P(BT,F|info)= P(info|BT,F)\, P(BT,F)/ P(INFO)$. The idea is that they look at how well data resembles what a representative person from the hypothesis would look like: i.e. they focus on $P(info \mid BT,F)$. Comparisons between statements are then made by comparing $P(info|BT,F)$ to $P(info|BT)$.

A correct comparison however requires comparing $P(info|BT,F)\, P(BT,F)/ P(INFO)$ to $P(info|BT)P(BT)/P(INFO)$.

Even if the hypothesis best represents the data [$P(info|BT,F) > P(info|BT)$] , it doesn't mean it is more likely as base rates $P(BT,F)$, $P(BT)$ need to be taken into account.

Students may of course offer different examples. With regards to medicine, people often overweigh the probability of having a disease given a positive test with a known positive test rate. Juries decide whether someone is guilty based on whether someone has a stereotypical look of a criminal (e.g. mean looking).

Other comments

Heuristics could also be relevant as tool for experiments! Suppose that when people are forced to make quick decisions, they often rely on certain shortcuts (seemingly common amongst people). This might allow smaller time periods for decision making to elicit things which come naturally to people's minds. (inner feelings?).

For example, implicit association test: participants are required to rapidly categorise two target concepts with an attribute. E.g. Random Name, Black vs White.

It is also related to studies on reaction times of individuals when they make decisions (how much thought they are putting into a task; longer times means indifference?)

4) How might the frame of a gamble in terms of losses rather than gains change decisions. What does this suggest about risk aversion?

Kahneman and Tversky proposed the following experiment:

In additional to everything else you won you have been given the gift of $2000. You are now required to choose between the following two options:

- A: 50% chance of losing $1000
- B: The certainty of losing $500.

In additional to everything else you won you have been given the gift of $1000. You are now required to choose between the following two options:

- C: 50% chance of winning $1000.
- D: The certainty of winning $500.

The idea here is that the two lotteries are exactly the same in terms of the combined final income, however only the framing is different. By expected utility theory, choices depend on final income, so this should not occur. Again, this is testing another aspect of EUT like in question 1.

They found that A & D are by far the most common choices. So, people often take the sure thing in the first set of choices (A). Moreover, the also found that on average you have to lower the certain amount to around $370 to get indifference. So typically, people take the uncertain gamble (A) over the sure thing when the gamble is over losses.

They generalised this to think of people as risk averse in the domain of gains but not necessarily in the domain of losses. This is present in the S shaped prospect theory curve.

Application: insurance is more attractive when framed as an elimination of risk versus a reduction of risk. This is because people are risk seeking in losses. For example: vaccinations (?), safety equipment; e.g. bicycle helmets (?).

Besides behaviour with respect to risk, it is might also be related to loss aversion where people weigh similar losses heavier than gains. This is related to experiments on mugs about the endowment effect, where willingness to pay is often less to willingness to sell.

Experimentally, the lottery design above has similar characteristics to the others we have gone through: there is no explicit treatment. Rather it is again trying to prove that EUT is not sufficiently accurate in some situations.

Food for thought:

Note that given loss aversion (and other biases), they might be useful as tools in experiments to randomly influence people's decisions in a particular way without using deception. Suppose in a sequential game, we want to pseudo-randomly make participants choose an option, one could use random assignments of framing to nudge people into making certain choices. These random assignments act like an instrumental variable for participants choices of options.

(This is related to the idea of IV regressions in econometrics)