Blockholder Disclosure Thresholds and Hedge Fund Activism *

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Abstract

Blockholder disclosure thresholds shape the incentives for hedge fund activism, which are jointly determined with real investment and managerial behavior. The value of lower thresholds (more transparency) to uninformed investors hinges on the marginal cost of trading against an activist versus the marginal benefits of increased managerial discipline. Hedge fund activism can be excessive: if market opacity hurts uninformed investors sufficiently, the improved managerial disciplining fails to offset the reduction in real investment, and society benefits from lower thresholds. Activists may desire disclosure thresholds if their threat of participation discourages managerial misbehavior; then neither dispersed investors nor society support thresholds.

Keywords: Hedge fund activism, blockholder disclosure thresholds, informed trading, investor activism.

JEL codes: G34, G14, G18, K22.

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1 Introduction

Hedge fund activism mitigates agency problems that affect governance in publicly-traded companies with dispersed owners (Brav et al. 2008; Bebchuk et al. 2015). However, their business models and short-termism of their strategies (Brav et al. 2010) generate controversy. Activist hedge funds are, by nature, informed traders that profit from trading on their information advantages at the expense of uninformed shareholders. As a result, activists can impair real investment, destroying value (Leland 1992; Bernhardt et al. 1995). Blockholder disclosure thresholds can limit the profits that activist funds make by trading, but this, in turn, affects their incentives to discipline management, and thus managerial behaviour. Our paper analyzes how disclosure thresholds determine hedge fund activism, real investment and managerial behaviour; and we derive the optimal policies from the perspectives of dispersed uninformed investors, activist funds, and the society.

The driving mechanisms are complex. Uninformed investors benefit from the disciplining effect of hedge fund activism, but incur costs when trading with activists that are privately informed about their own value-enhancing potential. Investors value lower disclosure thresholds when the gains from increased market transparency outweigh the losses from lower managerial discipline. At the other end, activist funds profit from acquiring undervalued stock when their participation has not been revealed; but can be hurt if their trading profits (trading losses for uninformed investors) reduce real investment, or if the possibility of their intervention deters managerial malfeasance. We show that activists never benefit from a disclosure threshold *solely* because it boosts real investment, but they can gain from tighter thresholds that reduce their incentives to intervene, thereby raising the likelihood that managers pursue their own interests at the expense of the firm. We find that, depending on how disclosure thresholds affect managerial actions, either uninformed investors or activist funds may value tighter disclosure thresholds, but socially optimal thresholds always lie weakly between the preferred thresholds of uninformed investors and activist funds. This reflects that society does not internalize trading transfers between uninformed investors and the activist, but it does care about both the real investment and managerial discipline consequences.

In 2011, senior partners at Wachtell, Lipton, Rosen & Katz (WLRK), a prominent law firm specializing in corporate and securities law and corporate governance, submitted a rule-making petition—the WLRK (2011) Petition—to the Securities and Exchange Commission (SEC) advocating that rules governing the disclosure of blocks of stock in publicly traded companies be tightened. WLRK argued that the US disclosure threshold of 5% allows activist investors to secretly accumulate enough stock to create fundamental changes, and they only hold positions for short periods of time (Brav et al. 2010; Becht et al. 2017). This, WLRK argued, damages market transparency and investor confidence. Academics responded, arguing that a crucial incentive for activist funds is the ability to purchase stock at prices that do not yet reflect the value of their actions, and that tighter disclosure rules would discourage hedge fund activism (Bebchuk and Jackson 2012; Bebchuk et al. 2013). In turn, they argued that discouraging activism would harm small investors, who would then not glean the value-enhancing benefits of hedge fund activism on corporate behavior.

Our paper sheds light on when and how the interests of uninformed investors and activist hedge funds conflict. Our model of hedge fund activism reveals how disclosure thresholds affect (i) incentives of activist funds to engage in costly managerial disciplining; (ii) real investment of small uninformed investors; (iii) choices by managers of whether to pursue potentially value-destroying activities. We characterize the factors determining optimal disclosure thresholds from the perspectives of investors, activists and society.

Activist funds profit from secretly acquiring undervalued stock and selling it at higher prices after they intervene. Share prices typically rise sharply when an activist's presence is revealed because the market anticipates subsequent intervention, and Bebchuk et al. (2015) provide evidence that these post-disclosure spikes in share prices reflect the long-term value of intervention. Accordingly, the main source of rents for activist funds is the price change caused by their own interventions, and the value of the shares acquired prior to revealing themselves is key to their profitability (Bebchuk and Jackson 2012; Becht et al. 2017). A disclosure threshold limits the equity position that can be secretly acquired, reducing incentives to intervene. Importantly, the expected levels of activism, and thus of managerial discipline, determine the profitability of real investment by uninformed investors. In turn, this real investment affects the value of activist interventions, creating a feedback effect on the incentives of activists to participate. The optimal disclosure threshold policy for each party reflects the tensions faced with regard to the preferred level of market transparency.

Consider the tradeoffs faced by uninformed investors. Higher transparency (a lower disclosure threshold) reduces their trading losses, but it also reduces the willingness of hedge fund activists to intervene. In turn, this encourages management to pursue its own interests at the expense of shareholders. Uninformed investors value lower disclosure thresholds when the expected trading losses saved outweigh the benefit of free riding on the activist's costly managerial disciplining. They gain from the reduced shares that activists acquire when those shares are not needed to induce activism, but they are harmed when the share limit discourages activism. Their optimal disclosure threshold, when interior, trades these considerations off. In particular, uninformed investors value lower disclosure thresholds whenever activist participation rates are relatively unresponsive to changes in trading transfers. This responsiveness is captured formally in our analysis by the *profit elasticity of activism*.

Despite the long-term value of hedge fund activism (Brav et al. 2015; Bebchuk et al. 2015), researchers have found that activist funds tend to have short investment horizons (Brav et al. 2008, 2010; Boyson and Mooradian 2011), and that they acquire stock after targeting a firm (Bebchuk et al. 2013). We model this by considering a large informed (potential) activist fund that is external to the firm, and whose incentives to incur the cost of intervention are provided by the increase in the value of the stock that he acquires. That is, the activist's sole source of rents is the increase in stock value due to intervention relative to the acquisition price, making activism directly related to block size.

The activist endogenously determines how many shares to acquire. In our static dealership model, a competitive market maker posts prices conditional on the sign of the net order flow. Then the activist trades along with a random, uniformly-distributed measure of shareholders (initial investors) who receive liquidity shocks that force them to sell their shares. The activist's order trades off between the benefits of a larger block size and the costs of the information revealed. Uniformly-distributed liquidity trade allows us to solve for informed trade in closed form while preserving a key tension of informed trading, namely adverse price effects. What matters for our analysis and findings are how disclosure thresholds affect an activist's *ex-ante* expected trading profits, prior to an intervention decision.

The second key tension in our model is that the activist's trading profits depend on the value of intervention, which is directly related to real investment—value-enhancing actions in larger companies have bigger impacts. When the trading losses of initial investors are large relative to the benefits of managerial disciplining, activism diminishes the profitability of investors and in turn real investment, which reduces the value of the activist intervention. The activist does not internalize the investment feedback effect in his trading because he participates only after initial investment has been sunk. A disclosure threshold can serve as a commitment device for an activist to limit his trade, and thereby raise real investment. Surprisingly, we establish the activist never benefits from a disclosure threshold just because it boosts real investment: we prove that investment feedback is a second-order effect relative to trading transfers. For the activist, the benefits of increased trading on his information advantage always outweigh the cost of associated reductions in real investment.

The negative effect of market opacity on real investment captures the original concerns of the Williams Act (1968), which introduced the disclosure thresholds that are the subject of controversy. They were designed to "alert investors in securities markets to potential changes in corporate control and to provide them with an opportunity to evaluate the effect of these potential changes".[1] Uncertainty over managerial behaviour and activist participation accumulated on share transactions from initial equity offerings translate into stock

[1] Quote of the case resolution Wellman v. Dickinson, 682 F.2d 355, 365-66 (2d. Cir. 1982), citing GAF Corp. v. Milstein, 453 F.2d 709, 717 (2d Cir. 1971), cert. denied, 406 U.S. 910 (1972). (External link *here*). Used in "Section 13(d) Reporting Requirements Need Updating" by David A. Katz of WLRK in *Harvard Law School Forum on Corporate Governance and Financial Regulation*, 2012.

price discounts that benefit activist funds, which are informed about the value of their own actions. Trading is a zero-sum game in which the activist's expected trading profits represent expected trading losses to uninformed investors (e.g., Holmström and Tirole 1993). When trading losses outweigh the benefits of monitoring, hedge fund activism harms uninformed investors, causing them to reduce investment. Conversely, activism fosters investment when it benefits uninformed investors. By regulating trading transfers, disclosure thresholds affect real investment. This link between market efficiency and economic efficiency was first made in Bernhardt et al. (1995). Here, we identify twin real effects of informed trade by hedge fund activists: (i) it encourages activists to create value by intervening in underperforming companies, and (ii) it affects real investment.

The third strategic agent is a firm's management. Its manager can take a value-destroying action to obtain private benefits, but she incurs a reputation cost if disciplined by the activist. Improvements in the performance and governance achieved by activists often come at the expense of managers and directors who see sharp reductions in compensation and a higher likelihood of being replaced (Brav et al. 2010; Fos and Tsoutsoura 2014). As a result, the threat of being disciplined by an activist improves managerial performance (Gantchev et al. 2019). We capture this mechanism, recognizing the ex-ante disciplining role of hedge fund activists in discouraging managerial malfeasance. Since higher trading transfers make an activist more willing to act *if* management misbehaves, they also induce better behavior by management.

This managerial feedback benefits uninformed investors, but, paradoxically, by reducing the likelihood that a manager pursues actions that benefit herself at the expense of shareholders, it reduces an activist's opportunities to extract profit from its business of disciplining management. When managers are sensitive to the threat of activism, uninformed shareholders are happy to increase the disclosure threshold, as they only realize trading losses when the activist intervenes, and hence are conditional on managers' malfeasance. Raising the disclosure threshold both increases activists' intervention rates (ex-post disciplining) and discourages malfeasance (ex-ante disciplining). The same mechanism represents a tension for an activist fund, which trades off higher conditional trading profits against a lower probability of profiting. Management's responsiveness to the threat of activism, i.e., the extent of managerial feedback, is captured in our analysis by the *activism elasticity of management*. When this elasticity is sufficiently large, activist funds benefit from a disclosure threshold because it allows them to commit to lower intervention rates that encourage managerial misbehavior. We show that whenever activists value a disclosure threshold, uninformed investors would want greater activism, indeed preferring no threshold. In effect, the willingness of an activist hedge fund to act discourages excessively—from its perspective, but not shareholders—the desire of management to pursue its own interests at the expense of shareholders. Shareholders gain from an activist's willingness to engage without having to pay in terms of trading costs.

We characterize the socially-optimal disclosure threshold and show that it can only coincide with the preferred policies of uninformed investors or activists if they all prefer no threshold. Society (a regulator) does not internalize the transfer of trading profits from uninformed investors to the hedge fund, caring only about the gross expected value of the firm net of the cost of capital and the cost of activism. Intuitively, society is not directly concerned about trading in financial markets, but only the indirect real effects of such trading. We show that the socially-optimal disclosure threshold is always weakly between the thresholds preferred by shareholders and the activist hedge fund.

We next relate the paper to the literature. Section 2 studies a simple model of hedge fund activism in which managerial behavior is exogenous. Section 3 introduces blockholder disclosure thresholds and derives the optimal policies for the different parties. Section 4 endogenizes managerial behavior. A conclusion follows. An Appendix contains all proofs.

1.1 Related Literature

Our paper contributes to a growing body of research on hedge fund activism. Shleifer and Vishny (1986) introduced the role of blockholders as monitors of corporate management. More recent research has focused on the relation between financial markets and the monitoring incentives of blockholders (see Edmans and Holderness, 2017's review). The literature on hedge fund activism underscores that disciplining management often *is* the business of blockholders. The key role of financial markets follows from the strategies of blockholders, who acquire stock in target companies before the price reflects the value of their actions.

Findings from the empirical literature on hedge fund activism motivate our main modelling assumptions. Collin-Dufresne and Fos (2015) and Gantchev and Jotikasthira (2017) provide evidence that hedge fund activists exploit liquidity sales to purchase stock in target companies. A host of papers document that, on average, activist funds enhance the value of companies by disciplining management (Brav et al. 2008; Clifford 2008; Klein and Zur 2009; Boyson and Mooradian 2011; Brav et al. 2015; Bebchuk et al. 2015) through costly interventions (Gantchev 2013). Brav et al. (2010), Fos and Tsoutsoura (2014) and Keusch (2017) provide empirical foundations for our assumption that managers in target companies are penalized when disciplined by activist funds. Our paper endogenizes firm value by assuming that investorment reflects the expected value of the company as determined by corporate governance. While this relation has not been established in the literature on hedge fund activism, La Porta et al. (2006) and Djankov et al. (2008) find evidence of higher investment in markets with more legal investor protection. Some of our predictions have empirical support, while others remain to be tested. The model predicts that the stock price reaction that follows disclosure of an activist fund captures the value of their actions (Bebchuk et al. 2015), and that disclosure thresholds constrain their acquisitions (Bebchuk et al. 2013). Gantchev et al. (2019) provide evidence of the ex-ante disciplining effect of hedge fund activism.

Few papers have formally studied hedge fund activism. Our paper recognizes the role of financial markets on the incentives of activists to take positions in a target company and intervene. This property is shared with Back et al. (2018), who characterize the dynamic trading by an activist investor. They follow Kyle (1985) by introducing stochastic liquidity trade that provides camouflage for a blockholder's trades. Back et al. (2018) revisit the classic question of the relationship between liquidity and economic efficiency, and show how the intervention cost function affects outcomes. We simplify the trading process (static) and the cost of intervention (fixed) in order to endogenize firm value in terms of real investment and managerial behavior, and to study the role of market transparency, i.e., of blockholder disclosure thresholds. In these ways, the two papers complement each other.

Market liquidity plays a key role in our model. The activist fund is initially external to the target company, so liquidity camouflages its purchases of shares and diminishes adverse price impacts, making intervention more profitable. This positive relationship was formalized by Maug (1998) and Kahn and Winton (1998) in the context of general blockholder interventions, and Kyle and Vila (1991) in the context of takeovers.

Other analyses of hedge fund activism share with our paper the core trade-off between the financial benefit of increasing a target company's value (and thus share price) and the cost of intervention. Fos and Kahn (2018) study the relation between ex-ante and ex-post correction effects of hedge fund activism. Our analysis shares with theirs the key distinction between the conditional and unconditional probability of activism, as well as the efficiency gains generated by threats of intervention. Burkart and Lee (2018) compare hedge fund activism with hostile takeovers in a complete information setting, showing that they can be viewed as polar approaches to the free-riding problem of Grossman and Hart (1980). Burkart and Dasgupta (2019) model hedge fund activism as a dual-layered agency model between investors, activists and managers. Activist funds compete for investor flows, which affects their governance as blockholders. In their paper, funds inflate short-term performance by increasing payouts financed by leverage, which discourages value-creating interventions in economic downturns due to debt overhang. Brav et al. (2018) recognize the complementarity of costly interventions by distinct funds in the same target and model the resulting coordination problem.

Our paper is also related to the insider trading literature. A number of papers discuss formally the desirability of insider trading regulations by modelling the insider as an informed trader (e.g., Fishman and Hagerty 1992; Khanna et al. 1994). In our model, the hedge fund activist is an informed trader that profits from trading with uninformed investors. This reduces the profitability of uninformed investors, who then reduce their investments. Leland (1992) and Bernhardt et al. (1995) first incorporate this mechanism in the study of the welfare effects of insider trading. This literature focuses on the informational role of prices and anticipation of future trading by uninformed agents with informed traders; our current paper combines this anticipation of future trading with how such informed trading provides incentives for managerial disciplining. A more direct link to the insider trading literature concerns the impact of mandatory disclosure rules for insiders (see Huddart et al. 2001).

Our paper is motivated by a debate that has been largely overlooked by the finance literature. Many calls for revisions of blockholder disclosure rules have been made by prominent lawyers, hedge funds and academics.[2] Bebchuk and Jackson (2012) provide a comprehensive analysis in corporate law of the law and economics of blockholder disclosure thresholds, and Bebchuk et al. (2013) empirically analyse pre-disclosure accumulations of hedge fund activists. In line with their findings, our paper shows that lower disclosure can increase investor value. This is against a widely-held view that higher transparency *must* provide more investor protection,[3] a view that ignores investor protection from activists.

2 Hedge Fund Activism

In this section we model hedge fund activism and characterize the inter-linkage with real investment. We consider a firm that raises capital for a project whose value depends on the initial investment by uninformed *investors* and a business plan that may be either good or bad. The *manager* can deliberately adopt the bad business plan in order to obtain private rents at the expense of shareholders. The bad plan reduces value for shareholders unless an outside *activist* hedge fund intervenes to discipline management and implement the good plan. All agents are risk neutral. There are four dates, $t = 0, 1, 2, 3$. There is no discounting.

[2] *Harvard Law School Forum on Corporate Governance and Financial Regulation* contains a discussion of interventions by different parties.

[3] See Schouten and Siems (2010) and its references for the corporate law literature; see La Porta et al. (2006) and Djankov et al. (2008) for papers using ownership disclosure rules in indexes of investor protection.

At date $t = 0$ a continuum of dispersed investors invest capital k in a project with an expected date $t = 3$ payoff of

$$V = f(k)\left[1 - \delta \cdot 1_{\{m=0\}}\right]. \tag{1}$$

Here, f is a standard production technology with $f'(\cdot) > 0$, $f''(\cdot) < 0$, $f'(0) \to \infty$. The indicator function accounts for the business plan $m \in \{0, 1\}$ implemented by the manager at $t = 1$. The good plan ($m = 1$) yields cash flows $f(k)$ to investors. The bad plan ($m = 0$) yields nothing with probability $\delta \in [0, 1]$. Equivalently, the bad plan destroys a fraction δ of the project's value. Investors are uninformed, unable to distinguish between good and bad business plans. We initially assume that the manager adopts the bad business plan ($m = 0$) with some exogenous probability z. Section 4 endogenizes managerial malfeasance. The marginal cost of capital is $r > 0$. Initial investors become shareholders who receive claims to terminal project payoffs that they may trade in a market at $t = 2$. We normalize shares outstanding to have measure one.

The market at $t = 2$ features initial investors who receive liquidity shocks that force them to sell their shares, an activist hedge fund when it identifies that the bad business plan was adopted (and by acting can change this plan) and a competitive market maker. The market maker posts prices $\{P_l, P_h\}$ that break even in expectation conditional on the *sign* of the net order flow, and then traders place orders. Orders are executed at the low price P_l if sell orders exceed buy orders, i.e., if net order flow is negative; and they are executed at the high price P_h if buy orders exceed sell orders, i.e., if net order flow is positive. We assume that the amount of liquidity-driven sell orders l is drawn from a uniform distribution on $[0, b]$. We let $x(l)$ denote the associated density, and observe that $b \in [0, 1]$ is a measure of market liquidity—when b is larger, an activist can submit a larger buy order with reduced risk of being uncovered by the market maker. The activist fund is an outsider to the firm that has no initial stake but can acquire shares if it identifies managerial malfeasance that it can address, allowing it to profit. The activist identifies managerial malfeasance when it occurs with probability $\lambda < 1$. The activist can discipline management by incurring a fixed cost c, forcing the firm to shift from the bad business plan to the good one. The activist privately observes this cost c. Other market agents share a common prior that c is distributed on $[0, C]$ according to a strictly positive and weakly decreasing density g and associated cumulative function G. The activist chooses how many shares $\alpha \in [0, 1]$ to acquire, which we term his *position*. We denote net order flow by $\omega = \alpha - l$: ω equals the difference between liquidity sales and the position acquired by the activist. Thus, orders are executed at P_l when $\omega \leq 0$ and at P_h when $\omega > 0$.

To ease presentation, we assume that the activist cannot trade on private information that the manager maximized shareholder value (choosing $m = 1$). That is, we assume that the activist can only intervene if $m = 0$. We relax this assumption in Appendix B, showing that it does not qualitatively affect results.[4] For simplicity we also assume that if an activist takes a position after management misbehaves (takes action $m = 0$) then he disciplines management; i.e., he does not "cut-and-run" by selling shares before engaging with management. Cutting and running becomes unattractive when it impairs the reputation of activist funds, which Johnson and Swem (2017) find to be important for their profitability.[5]

At $t = 3$, the project delivers cash flows $f(k)$ if the manager implemented the good plan or if the activist disciplines the manager. Otherwise, expected cash flows are $(1 - \delta) f(k)$. Figure 1 summarizes the timing of events.

Parameters δ and z capture the severity of the agency problem between management and ownership. If $\delta = 0$, both business plans yield cash flows $f(k)$, so there are no frictions between investors and the manager, and thus no room for managerial disciplining; and if $z = 0$ the manager always implements the good business plan. In contrast, $\delta > 0$ and $z > 0$ imply that the manager may destroy shareholder value to obtain private benefits, creating

[4] Appendix B allows the activist to acquire stock when the business plan is good ($m = 1$). This *increases* the activist's information rents without affecting the net value of the project. This hurts uninformed investors, causing them to reduce investment.

[5] Empirical evidence shows that hedge fund activism increases the value of target companies via costly disciplining (see Gantchev (2013) for the costs of activism, and Bebchuk et al. (2015) for evidence on the value of hedge fund activism).

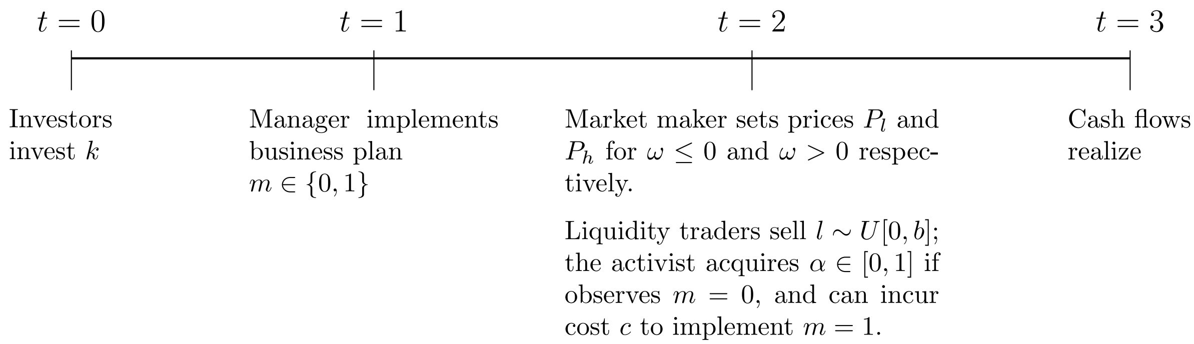

Figure 1: Time line

a potential role for hedge fund activism. In Section 4 we endogenize the probability z that the manager implements the bad business plan.

Parameter λ captures search frictions of activist funds or the visibility of companies that are susceptible of being targeted by an activist fund. For instance, Gantchev et al. (2019) find evidence that industry peers of firms targeted by activists have increased perceptions of their exposure to activism, i.e., higher λ.

We assume that the activist correctly identifies the good business plan, that it can discipline management with certainty, and that it buys shares in the target company at a single time where shareholders (investors) face liquidity shocks. In practice, these processes are dynamic (Collin-Dufresne and Fos 2015; Back et al. 2018), with uncertain costs (Gantchev 2013) and outcomes (Becht et al. 2017). We abstract from these mechanics to study the incentives provided by financial markets. What matters for our analysis are the *expectations* that an activist forms about these costs and outcomes at $t = 2$ when deciding *whether* to try to discipline management. The decision is based on the balance between *expected* financial benefits and engagement costs, and the likely dynamic price impacts of trading—and not the particular paths that can be realized given a decision to move forward. In our setting, the cost of activism c is orthogonal to initial investment. This reflects the increasing returns of activism with respect to firm size on a reduced form that keeps our model tractable.

We represent the interplay between corporate governance and real investment in a static setting. The impact of trading transfers on the incentives of initial investors captures price discounts for uncertainty over managerial behaviour and activist participation accumulated on share transactions from initial equity offerings. The agency problem owners-managers imposes a cost on uninformed shareholders for liquidating their shares, a cost that is transferred over transactions between uninformed investors at any point in time. That is, subsequent uninformed investors who acquire shares from any initial investors inherit their problem—they value the disciplining role of activists, but face an expected trading cost associated with possibly unwinding their positions in a market with activists, and hence would want the same real investment as initial investors. Our model applies to settings where firm value might be affected by managerial behaviour and in turn by the participation of activist funds.

Our trading environment preserves the crucial feature for our analysis that there is an adverse price effect via trading that reveals information to the market (see, e.g., Bebchuk and Jackson 2012). The continuum of random liquidity sales l lets us endogenize the activist's position α. The uniform distribution yields a simple closed-form solution for this position and provides an intuitive interpretation of stock sales and purchases in terms of a firm's shares. The continuum of liquidity shocks differentiates our model from most corporate finance models that feature simple discrete (typically binary) levels of liquidity trade. For tractability, we consider a stock market where the market maker posts two prices similar to a bid and an ask price before receiving orders, and then executes all transactions at the relevant price. In Appendix C we consider an alternative distributional and timing formulation in which the price function of the market maker is continuous over realizations of the net order flow. All qualitative findings extend in that setting.

2.1 Market Equilibrium

We solve recursively for the perfect Bayesian Nash equilibrium. At $t = 2$, real investment has been sunk by uninformed investors and is observable to all parties, the manager adopted the bad business plan with probability z, and the activist observes malfeasance with probability λ. A competitive risk-neutral market maker posts prices P_l and P_h for net sales and net purchases; then uninformed investors receive liquidity shocks and trade simultaneously with the activist. At $t = 0$, investors anticipate the subsequent events and invest capital.

2.1.1 Trading

Proposition 1 summarizes the Bayesian Nash equilibrium in the subgame at date 2. The market maker sets prices that earn it zero expected profits given the activist's decisions, and the activist participation and trade is optimal given the prices posted by the market maker.

Proposition 1 *At $t = 2$ after real investment has been sunk, the market maker posts prices*

$$\begin{aligned} P(\omega) = P_l &\equiv \left[\tfrac{2[1-z(1-\lambda G(c_t^*))\delta]-z\lambda G(c_t^*)}{2-z\lambda G(c_t^*)}\right] f(k) \quad \textit{if} \quad \omega \leq 0 \\ P(\omega) = P_h &\equiv f(k) \quad \textit{if} \quad \omega > 0. \end{aligned} \tag{2}$$

After prices are posted, if the activist observes managerial malfeasance ($m = 0$) and the cost of activism is sufficiently small, $c \leq c_t^*$ where

$$c_t^* = z\left[\frac{1-\lambda G(c_t^*)}{2-z\lambda G(c_t^*)}\right]\frac{b}{2}\delta f(k), \tag{3}$$

then he takes an equity position

$$\alpha^* = \frac{b}{2} \tag{4}$$

in the market with the $l \sim U[0, b]$ initial investors who sell their shares, and then disciplines management. Otherwise the activist does not participate.

A proof is in Appendix A; here we provide the key intuition. The market maker has a conjecture about the size α of the activist's position conditional on participating. In equilibrium, this conjecture is correct. The market maker uses Bayes rule to set prices P_l and P_h. Letting

a_1 denote activism and a_0 denote the absence of activism, the market maker's pricing rule is

$$\begin{aligned} P(sgn(\omega)) &= E\left[V|sgn(\omega)\right] \qquad (5) \\ &= \Big[\Pr\left[a_1|sgn(\omega)\right]\Pr\left[V = f\left(k\right)|a_1\right] + \Pr\left[a_0|sgn(\omega)\right]\Pr\left[V = f\left(k\right)|a_0\right]\Big] f\left(k\right), \end{aligned}$$

where $sgn(\omega)$ denotes the sign of net order flow.

A net buy order $\omega > 0$ reveals with probability one that the activist took a position, in which case the project is sure to pay $f(k)$. That is, $\Pr\left[a_1|\omega > 0\right] = 1$ and $\Pr\left[V = f\left(k\right)|a_1\right] = 1$. In contrast (weakly) net sell orders $\omega \leq 0$ are consistent with both the presence and the absence of activism, and allow the activist to extract information rents from uninformed investors. Conditional on the activist buying α shares when participating, the expected value of the project when there is a net sale of stock is

$$P_{l(\alpha)} = \left[\frac{b - \alpha z\lambda G(c_t) - bz(1 - \lambda G(c_t))\delta}{b - \alpha z\lambda G(c_t)}\right] f(k). \qquad (6)$$

When the activist participates and liquidity shocks outweigh the number of shares that he buys, i.e., when $l \geq \alpha$, there is a net supply of shares and the activist acquires the stock below its true value at $P_l < f\left(k\right)$. If, instead, $l < \alpha$, there is a net demand for stock and the activist pays P_h for each share, making no profit. The probability that the activist camouflages his share purchase with liquidity sales is $\int_\alpha^b \frac{1}{b} dl = \frac{b-\alpha}{b}$. It follows that his expected gross profits conditional on buying α shares are

$$E[\Pi_A|a_1] = \left(\frac{b-\alpha}{b}\right)\alpha\left[f(k) - P_l\right]. \qquad (7)$$

Inspection of (7) reveals that the activist faces a trade-off between the number of undervalued shares that he may acquire α and the expected cost of information revelation $\frac{b-\alpha}{b}$. This captures adverse price effects by which the expected stock price paid by the activist rises as he buys more shares. The activist's expected trading profits in (7) are maximized by a share purchase of $\alpha^* = b/2$; the price P_l in (2) is obtained by evaluating (6) at $\alpha = \alpha^*$. Greater liquidity b makes it easier for the activist to camouflage his trade, encouraging him to acquire a larger position.

If the activist observes managerial malfeasance, he disciplines management when doing so is expected to be profitable, i.e., when $E[\Pi_A|a_1] \geq c$. This relation and the market maker's price policy $P_{l(\alpha)}$ pin down the activist's cost participation cut-off in equilibrium:

$$c_t = (b - \alpha)\alpha z \left[\frac{1 - \lambda G(c_t)}{b - \alpha z \lambda G(c_t)}\right] \delta f(k), \tag{8}$$

which takes the form in (5) when evaluated at the optimal position of $\alpha^* = b/2$, i.e., $c_t^* \equiv c_t(\alpha^*)$. To see that the cut-off c_t^* is unique, observe that the right-hand side of (8) decreases in c_t for $\alpha \leq b$. In equilibrium, the activist employs a threshold strategy such that, conditional on observing malfeasance, he buys α^* shares and disciplines management if and only if $c \leq c_t^*$.

The cut-off c_t^* captures two key equilibrium features. First, it represents the activist participation threshold, and thus the extent of managerial disciplining. The probability that the activist intervenes to discipline the manager after observing the manager taking an action that reduces shareholder value is $G(c_t^*)$. Thus, a higher c_t^* implies superior governance. Second, c_t^* captures the activist's expected conditional trading profits. In equilibrium, the activist's expected trading profits *equal* the expected trading losses of uninformed investors because trading is a zero-sum game in which the market maker expects to break even (e.g., Holmström and Tirole 1993). Thus, c_t^* represents the expected transfer of trading profits from uninformed investors to the activist conditional on the activist intervening.

Conditional trading transfers c_t^* increase with investment k: The greater is real investment, the greater is the project value, and hence (i) the more valuable is managerial disciplining, and (ii) the more profitable it is for the activist to intervene. Two assumptions drive this result. First, the cost of activism is independent of the company's value, so the incentives for disciplining are positively related to stock ownership (Shleifer and Vishny 1986).[6]

[6] Brav et al. (2018) argue that it can be harder for activists to intervene in larger companies due to credit

Second, the value enhanced by the intervention is multiplicative, not additive. Therefore, the relevant measure of incentives is the activist's *dollar* ownership, not its *share* ownership (Edmans and Holderness 2017).[7]

These conditional trading transfers c_t^* also rise with market liquidity b. High liquidity increases activist trading profits and thus the probability $G(c_t^*)$ that the activist finds it profitable to discipline management. In line with Kahn and Winton (1998) and Maug (1998), higher liquidity allows the activist to increase his position with a reduced risk of discovery, thereby encouraging intervention. Back et al. (2018) model the dynamics of position building by activist funds and show the potentially positive effects of liquidity. Consistent with this, Collin-Dufresne and Fos (2015) and Gantchev and Jotikasthira (2017) provide evidence that activist funds camouflage their purchases with liquidity trades by other parties.

2.1.2 Investment

At $t = 0$, uninformed investors anticipate trading outcomes and activism levels, and invest capital so as to maximize expected profits. In addition to the investment decision, Proposition 2 characterizes expected project payoffs and how they are split among market participants in expectation at $t = 0$. This sets the ground for the analysis of the key interacting forces in the model and the introduction of blockholder disclosure thresholds.

Proposition 2 *The expected value at $t = 0$ of the project given investment k is*

$$E[V] = [1 - z(1 - \lambda G(c_t^*))\delta] f(k) \equiv \pi_V f(k). \tag{9}$$

The expected gross profits of the activist are:

$$E[\Pi_A] = z\lambda G(c_t^*) \frac{c_t^*}{f(k)} f(k) \equiv \pi_A f(k). \tag{10}$$

constraints. Our model can be modified to provide a similar prediction in the presence of financial constraints.

[7] In the related context of CEO incentives, Baker and Hall (2004) and Edmans et al. (2009) show theoretically that a CEO's dollar ownership and not percentage ownership is relevant when the CEO has a multiplicative effect on firm value.

The expected gross profits of uninformed investors are:

$$E[\Pi_I] = (\pi_V - \pi_A) f(k) \equiv \pi_I f(k). \tag{11}$$

The investment k by uninformed investors solves

$$\pi_I f'(k) - r = 0. \tag{12}$$

Total expected cash flows are the product of $f(k)$ and the probability $\pi_V \in [0,1]$ that the project succeeds. Proposition 2 reveals that expected total rents are split between the activist and uninformed investors in proportions π_A/π_V and π_I/π_V respectively. This follows because the market maker earns zero expected profits, which means that activist trading profits are extracted one-for-one from uninformed investors. The expected gross profits of the activist equal the product of the unconditional probability that he participates $z\lambda G(c_t^*)$ and the expected trading profits c_t^* from participating. Uninformed investors obtain, in expectation, the rest of the "pie", $(\pi_V - \pi_A) f(k)$. Real investment, characterized by (12), maximizes the ex-ante expected profits of uninformed investors at date 0.

Proposition 2 shows that activism has an impact on real investment via its effect on the expected profits of uninformed investors. Investors face a tension as to their preferred extent of activism, where the extent of activism is captured by $G(c_t^*)$. Higher transfers of trading profits c_t^* increase the proportion of cash flows taken by the activist in expectation π_A, reducing the investors' portion π_I. However, higher trading transfers also incentivize activist participation, and the increased managerial discipline raises total expected cash flows $\pi_V f(k)$. As a result, greater trading transfers c_t^* to activists need not hurt uninformed investors. In particular, activism fosters real investment when investor gains from managerial disciplining outweigh the associated trading losses, and it discourages real investment otherwise.

This mechanism underscores the *investment feedback effect* faced by the activist. The value of activism is directly related to the size of the project—the profitability of the activist grows with real investment, i.e., c_t^* grows with k. But, expected levels of activism affect investment. Therefore, expected activism affects real investment, which, in turn, affects the extent of activism. Crucially, the activist does not internalize this investment feedback in his trading decision at $t = 2$, because real investment has already been sunk. Thus, when the activist participates, he takes a position α^* to maximize *conditional* expected profits (7), i.e., for a given k, rather than *unconditional* expected profits (10).

Our analysis identifies novel strategic interactions between uninformed investors and activist funds. The linkage between investment and trading profit transfers is similar to that found in papers studying the real effects of informed trading (Leland 1992; Bernhardt et al. 1995). We incorporate a new element: the informed trader is an activist fund who can increase investment value by alleviating agency problems between owners and managers (Brav et al. 2008; Klein and Zur 2009, Brav et al. 2015; Bebchuk et al. 2015). The effect of hedge fund activism on real investment is thus twofold: Informed trading reduces the profitability of uninformed investors who respond with lower investment; but it also encourages the intervention of activist funds that discipline management, thereby incentivizing investment.

3 Blockholder Disclosure Thresholds

Blockholder disclosure thresholds require a shareholder to disclose stock holdings once they reach a certain fraction of the overall voting rights in a publicly-traded firm. In recent years, hedge fund activism has led some market participants and commentators to call for an expansion of these rules. We briefly relate our analysis to the discussion and then derive the optimal threshold policies for investors, hedge fund activists and society.

Ownership disclosure rules are one tool of financial regulators to prevent the expropriation of rents by large shareholders at the expense of uninformed investors. Investor protection is at the core of the regulatory framework of most prominent financial systems, which seek to guarantee incentives for real investment (see, e.g., OECD's 2004 Principles of Corporate

Governance, the Williams Act of 1968 for the US, or the 2004 EU Transparency Directive). Consistently, La Porta et al. (2006) and Djankov et al. (2008) provide evidence that greater legal protection of investors is associated with more developed financial markets. Both studies construct protection indices that include ownership disclosure rules. This view aligns with the WLRK (2011) Petition, which argues that ownership disclosure rules no longer serve their purpose because activist funds can gain control of companies with small positions.

Our model captures the link between investor protection and investment, but it challenges the monotone positive relationship between more transparency and protection. Our initial analysis shows how the ability to secretly acquire stock encourages activist funds to discipline management. This mechanism captures the views of many academics that hedge fund activism raises values of target companies, providing positive externalities for other investors.

The debate on the merits of revising blockholder disclosure thresholds also reveals the seemingly arbitrary levels set worldwide. Regulations differ greatly across financial systems. For example, the threshold is 5% in the US and France, it is 10% in Canada, but only 3% in Germany and the UK. Despite the vast potential impacts of small differences in these rules, it remains unclear what brings regulatory bodies to choose one specific level. The following analysis sheds light on the impact of revisions to these rules by capturing the key mechanisms.

3.1 Optimal Policies

We now show how blockholder disclosure thresholds can regulate the level of hedge fund activism, deriving the optimal policies for investors, activist and society. Ownership disclosure rules may limit the number of undervalued shares that the activist can acquire, reducing his incentives to participate. If a legal disclosure threshold $\overline{\alpha}$ is implemented, an activist must publicly announce his position when it crosses the threshold. Then the activist has no incentive to establish a larger position because doing so would reveal his presence causing the stock price to rise to $P_h = f(k)$, which would eliminate his information rents, rendering intervention unprofitable.[8] Corollary 3 follows immediately:

Corollary 3 *A disclosure threshold $\overline{\alpha}$ is binding if and only if $\overline{\alpha} < \alpha^*$. In equilibrium, when a disclosure threshold binds the activist sets $\alpha = \overline{\alpha}$.*

The activist's conditional trading profits $c_t(\alpha)$ in (8) increase with his position for $\alpha < \alpha^*$. Thus, when the activist participates, he acquires a position $\alpha = \min\{\overline{\alpha}, \alpha^*\}$. The mechanism implies that for a given firm characterized by $f(k)$, a binding threshold necessarily reduces both the profits and extent of hedge fund activism. To see this, let $\overline{c}_t \equiv c_t(\overline{\alpha})$ represent the trading profits, and hence participation cut-off, associated with a position determined by a binding threshold $\overline{\alpha} < \alpha^*$. Because trading profits increase in α, activism is now less profitable, i.e., $\overline{c}_t < c_t^*$, making the activist less likely to participate, i.e., $G(\overline{c}_t) < G(c_t^*)$. A direct consequence is that managerial malfeasance is more likely to destroy value. This mechanism is consistent with arguments against expanding ownership disclosure rules (see Section 3.1). However, our paper shows that they only comprise part of the overall effect.

The argument is incomplete because it neglects the effects of a disclosure threshold on real investment. Changes in expected levels of activism at $t = 2$ also alter real investment at $t = 0$, which, in turn, affects the activist's incentives to participate. A binding disclosure threshold reduces the conditional transfer of trading profits from investors to the activist, which may incentivize real investment, creating a positive investment feedback that can increase activism.

Proposition 4 derives the consequences of blockholder disclosure thresholds by characterizing the ordering of the optimal disclosure threshold policies for investors, the activist and a welfare-maximizing regulator representing society. We denote these policies $\overline{\alpha}_I$, $\overline{\alpha}_A$ and $\overline{\alpha}_R$ respectively. We present our results as a function of the *profit elasticity of activism*,

$$\varepsilon_a(c_t) = \frac{\partial G(c_t)}{\partial c_t} \frac{c_t}{G(c_t)}.$$

Here, ε_a captures the responsiveness of activism to informed trading: the higher is ε_a, the

[8] This price reaction is consistent with evidence by Bebchuk et al. (2015) that the stock-price spike that follows disclosure reflects the long-term value of the intervention.

bigger is the increment in the probability that the activist intervenes $G(c_t)$ in response to a marginal increase in expected trading profits c_t. Absent a binding disclosure threshold, when the activist participates he buys α^* shares, earns expected gross profits c_t^*, and the profit elasticity of activism is $\varepsilon_a\left(c_t^*\right) \equiv \varepsilon_a^*$.

Proposition 4 *There exists cutoffs* $\varepsilon_a^{*R} \equiv -\left(\frac{c_t^*}{\delta f(k)-c_t^*}\right)\left[\frac{\frac{df(k)/d\alpha}{f(k)}}{\frac{dc_t^*/d\alpha}{c_t^*}}\right]$ *and* $\varepsilon_a^{*I} \equiv \left(\frac{c_t^*}{\delta f(k)-c_t^*}\right)$, *on the profit elasticity of activism where* $\varepsilon_a^{*R} < \varepsilon_a^{*I}$ *such that*

1. *No one benefits from a binding disclosure threshold if the profit elasticity of activism is sufficiently high:* $\varepsilon_a^* \geq \varepsilon_a^{*I} \Rightarrow \alpha^* \leq \{\overline{\alpha}_I, \overline{\alpha}_A, \overline{\alpha}_R\}$.

2. *Only investors benefit from a binding disclosure threshold if the profit elasticity of activism is intermediate:* $\varepsilon_a^{*R} \leq \varepsilon_a^* < \varepsilon_a^{*I} \Rightarrow 0 < \overline{\alpha}_I < \alpha^* \leq \{\overline{\alpha}_A, \overline{\alpha}_R\}$.

3. *Both investors and society gain from a binding disclosure threshold if the profit elasticity of activism is low, with investors gaining more:* $\varepsilon_a^* < \varepsilon_a^{*R} \Rightarrow 0 < \overline{\alpha}_I < \overline{\alpha}_R < \alpha^* \leq \overline{\alpha}_A$.

Figure 2 illustrates the results; a full proof is in Appendix A. Optimal disclosure threshold policies are characterized by the first order conditions (FOCs) of net profit functions with respect to the activist position α. Corollary 3 implies that when the optimal position is less than α^*, it can be achieved in equilibrium by a binding disclosure threshold.

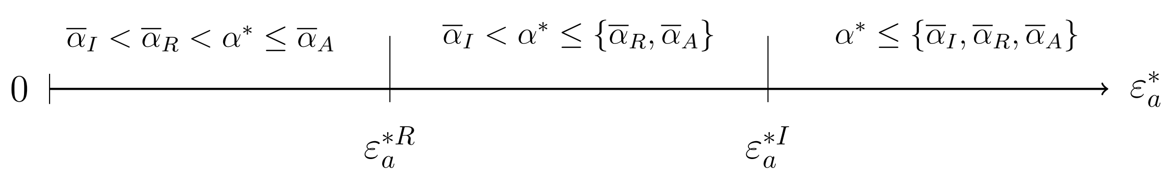

Figure 2: Optimal Disclosure Thresholds

Uninformed investors maximize $\pi_I f(k) - rk$. The associated FOC reveals that they benefit from a binding disclosure threshold if and only if

$$g(c_t^*)\left[\delta f(k) - c_t^*\right] < G(c_t^*), \tag{13}$$

which can be rearranged to $\varepsilon_a^* < \varepsilon_a^{*I}$. The left-hand side (LHS) represents the marginal benefits to uninformed investors of increasing the transfer of trading profits to the activist when $\alpha = \alpha^*$, i.e., for c_t^*. Higher transfers cause the probability that the activist participates conditional on observing managerial malfeasance to rise by $g\left(c_t^*\right)$. The associated benefit for investors is the difference between the total value enhanced by the activist $\delta f(k)$ and their trading losses c_t^*. The right-hand side (RHS) captures the conditional loss from marginally higher transfers: with probability $G(c_t^*)$ the activist would have participated anyway, even if expected trading profits had not increased.

A binding disclosure threshold $\overline{\alpha}$ reduces transfers of trading profits, $c_t(\overline{\alpha}) \equiv \overline{c}_t < c_t^*$. Equation (13) shows that this raises the marginal benefits to investors of activism (LHS) and reduces the associated losses (RHS), increasing marginal profitability. Equivalently, a binding threshold raises the profit elasticity of activism ε_a, and it requires less trading transfers from investors to encourage higher activism. Transfers of trading profits are the cost that investors incur in exchange for managerial discipline, and this cost rises with the extent of activism.

The optimal extent of activism for investors solves this FOC: $\overline{\alpha}_I$ solves $\varepsilon_a = c_t / \left[\delta f(k) - c_t\right]$ when $\varepsilon_a^* < \varepsilon_a^{*I}$. Full disclosure is never optimal. If the activist cannot acquire stock secretly, trading profits and hence transfers vanish, i.e., if $\alpha \to 0$ then $c_t \to 0$. But then the activist never participates. Then, the marginal benefits of discipline for investors outweigh the corresponding trading losses, i.e., $g(c_t)\left[\delta f(k) - c_t\right] > G(c_t)$. Thus, uninformed investors always benefit from some degree of market opacity, i.e., $\overline{\alpha}_I > 0$: the marginal profitability to uninformed investors of activism is always positive whenever $\overline{\alpha}_I$ is sufficiently small.

The optimal extent of activism can be achieved by a disclosure threshold when the corresponding trading transfers are lower than those in the unconstrained equilibrium, i.e., when (13) holds, but not otherwise. The mechanism highlights the asymmetric role of disclosure rules, which can only limit, but not foster, informed trading. If, absent regulation, the marginal profitability of activism for investors is positive, i.e., if (13) is not satisfied, the desired extent of activism cannot be achieved and the optimal policy is non-binding, i.e.,

$\overline{\alpha}_I \geq \alpha^*$. We discuss below the role of market liquidity, which determines α^* and thus whether a particular disclosure threshold binds.

Our argument builds on the result that transfers of trading profits increase with the activist's position, i.e., $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$. It follows that restricting α reduces c_t. *This is not immediate.* We earlier established that the activist faces an investment feedback effect that he does not internalize. In particular, the activist's position at $t = 2$ influences initial investment k, and this determines the trading profits from a given position α. A binding disclosure threshold regulates the number of shares that the activist buys in equilibrium. We have

$$\frac{dc_t}{d\alpha} = \underbrace{\frac{\partial c_t}{\partial \alpha}}_{mg.\ net\ trading\ transfer} + \underbrace{\frac{\partial c_t}{\partial k}\frac{\partial k}{\partial \alpha}}_{investment\ feedback}. \tag{14}$$

Net trading transfers capture the effect of the activist's position on transfers at $t = 2$ for a given investment k; Proposition 1 shows that these transfers increase with α for $\alpha < \alpha^*$ in the absence of a disclosure threshold. The investment feedback effect captures the impact of the activist's position on real investment $\frac{\partial k}{\partial \alpha}$, and hence on trading transfers $\frac{\partial c_t}{\partial k}$. Real investment always raises trading transfers, and thus the extent of activism, i.e., $\frac{\partial c_t}{\partial k} > 0$. However, the activist's position α might be large enough to hurt investors, who respond by reducing investment. That is, if $\alpha > \overline{\alpha}_I$ then $\frac{\partial k}{\partial \alpha} < 0$, and the effect of a larger position on trading transfers is determined by the balance of two opposing forces: positive net transfers and a negative investment feedback. We show that, surprisingly, the tension is always resolved against the investment feedback effect, so $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$.

This result reflects the subordinated nature of investment feedback with respect to the direct impact of trading transfers. Intuitively, these transfers lead the activist to take a position α^*, which, in turn, affects investment. If the reduction of investment from increasing α was strong enough to reduce the activist's trading profits, i.e., if $\frac{dc_t}{d\alpha} < 0$, then it would also increase investor profits because $g(c_t)\,[\delta f(k) - c_t] < G(c_t)$ when $\frac{\partial k}{\partial \alpha} < 0$. But then investors would *increase* investment, not reduce it, benefiting activists. Because trading transfers are the activist's sole source of income, this mechanism explains why he never benefits from a binding disclosure threshold:

Corollary 5 *Negative investment feedback reduces the positive impact of increasing the activist position on trading profits c_t and thus on the extent of activism $G(c_t)$. However, it does not alter the sign of the impact, i.e., $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$: the activist never benefits from a blockholder disclosure threshold just because it boosts investment.*

Thus, when investors seek a binding disclosure threshold $\overline{\alpha}_I < \alpha^*$, a conflict of interest arises between them and the activist. An activist position that exceeds $\overline{\alpha}_I$ harms investors, reducing real investment. This, in turn, reduces the profitability of activism and the levels of managerial discipline (negative investment feedback). Nonetheless, the investment response is never strong enough to outweigh the net positive effect of additional shares on activist profits. Therefore, the activist never wants a binding disclosure threshold to increase investment.

Society maximizes total expected value net of the costs of capital rk and the expected costs of activism $z\lambda G(c_t) E[c|c \leq c_t]$. Society gains from a binding disclosure threshold if

$$z\lambda g(c_t^*)[\delta f(k) - c_t^*]\frac{dc_t^*}{d\alpha} + \pi_A f'(k)\frac{\partial k}{\partial \alpha} < 0, \tag{15}$$

which can be rearranged to $\varepsilon_a^* < \varepsilon_a^{*R}$. The condition reveals that society cares about both the value-enhancing effects of activism and real investment. The first term in (15) captures the impact of the activist's equity position on project value via managerial discipline. This is positive for all $\alpha < \alpha^*$. In particular, Corollary 5 shows that the extent of activism is directly related to the activist's position regardless of the investment feedback, i.e., $\frac{dc_t}{d\alpha} > 0$. Moreover, greater managerial discipline always creates value. Here, $g(c_t)\delta f(k)$ is the conditional increase in gross value, and $g(c_t)c_t$ is the corresponding increase in expected cost of activism. The second term in (15) represents investment feedback that is not internalized by investors. More specifically, real investment solves $\pi_I f'(k) - r = 0$, but the optimal investment for society sets $(\pi_I + \pi_A)f'(k) - r = 0$, Society only benefits from a disclosure threshold if investors gain, but the converse is not true. For $\varepsilon_a^* < \varepsilon_a^{*R}$ to hold, the investment feedback must be negative, i.e., $\frac{\partial k}{\partial \alpha} < 0$, implying that $\varepsilon_a^* < \varepsilon_a^{*I}$. Intuitively, society only cares about the real economy, and not about secondary markets (trading transfers). The only social cost of increasing managerial disciplining is the potential reduction in investment. If this is sufficiently strong, then (15) holds and the regulator wants to set a binding disclosure threshold. Still, this threshold always exceeds the optimal threshold from the perspective of investors who do care about trading transfers.

That society's preferred disclosure threshold lies (weakly) between those preferred by investors and activists also arises in models of insider trading where real investment is endogenous (Leland 1992; Bernhardt et al. 1995). The social cost of increasing activism is a potential reduction of real investment due to lower market transparency. Thus, a regulator only considers implementing a disclosure threshold when this also benefits investors. Yet, while investors incur trading losses society does not, so the socially-optimal level of market transparency is lower. Therefore, when the regulator seeks a binding threshold, it always exceeds the preferred threshold of investors, and sometimes they disagree on the need for a binding policy.

3.1.1 The role of liquidity and the cost of activism

Results in Proposition 4 reflect interactions of two opposing forces: (i) market liquidity and (ii) the cost of activism. We discuss and interpret their roles by studying how market liquidity determines the impact of a reduction in the cost of activism on optimal disclosure policies.

Formally, we consider a *transformation* τ of the cost distribution g that transfers probability mass $\min\{g(c), \tau\}$ from each realization $c \in (0, C]$ to $c = 0$. A transformation $\tau > 0$ scales down density g, reducing both the expected cost and the profit elasticity of activism ε_a. This reduction in the cost of activism weakly increases the desirability of a disclosure threshold for investors and society. Corollary 6 shows how market liquidity affects this force.

Corollary 6 *Let cost distribution function g be such that no one benefits from a binding disclosure threshold, i.e., $\varepsilon_a^* \geq \varepsilon_a^{*I}$. There exist reductions in the cost of activism $\tau^I(b) < \tau^R(b)$*

such that

1. *Investors gain from a binding disclosure threshold if and only if* $\tau > \tau^I(b)$.

2. *Society gains from a binding disclosure threshold if and only if* $\tau > \tau^R(b)$.

Both transformations decrease with market liquidity, i.e., $\tau^{I\prime}(b) < 0$ *and* $\tau^{R\prime}(b) < 0$.

A transformation $\tau > 0$ reduces the marginal profitability of trading transfers for investors at any given c_t, and they benefit from a binding disclosure threshold when the cost reduction is large enough. In particular, $\varepsilon_a^* = \varepsilon_a^{*I}$ when $\tau = \tau^I(b)$, and a larger τ tightens the optimal threshold. An analogous intuition holds for society. At the limit, as τ grows large so that almost all probability mass is transferred to zero, activism becomes almost costless, and investors do not need to incentivize activist participation with higher trading profits. Then, the optimal policy for both investors and society approaches full transparency, i.e., $\{\overline{\alpha}_I, \overline{\alpha}_R\} \rightarrow 0$.

Transformations $\tau^I(b)$ and $\tau^R(b)$ decrease with the extent of market liquidity b because this makes activism more profitable: α^* and c_t^* both rise. All else equal, this reduces the profit elasticity of activism, and may make a binding disclosure beneficial for investors. In (13), the LHS decreases and the RHS increases. It follows that the cost reduction that leads investors to gain from a binding policy falls as market liquidity rises.

These results reflect that activists require market liquidity to establish equity stakes and profit from intervention (Maug 1998; Kahn and Winton 1998). Disclosure thresholds operate against liquidity by increasing market transparency and limiting an activist's position (Bebchuk et al. 2013). Greater liquidity reduces the marginal profitability of activism for investors, making disclosure thresholds more desirable. Costs of activism work in the opposite direction. If disciplining management is likely to be very costly, investors want to concede further trading transfers to incentivize activism, so they do not benefit from a disclosure threshold. That is, with high costs, i.e., large $g(c_t)$, the profit elasticity of activism is large for relatively opaque markets, so investors do not want to limit the potential trading profits of activists.

The cost of activism is often related to managerial entrenchment. Staggered boards make it harder to gain control of a company in a proxy contest, discouraging activism. Our model is consistent with Gompers et al. (2003) and Bebchuk and Cohen (2005), who find evidence of a negative correlation between firm value and management-favouring provisions. In such instances, relaxing disclosure thresholds can benefit investors by alleviating the negative effect of these provisions at the expense of market opacity and investor trading losses.

4 Managerial Feedback

We next endogenize the probability of managerial malfeasance to characterize the complete inter-linkage between investment, hedge fund activism and corporate management, and the consequences for optimal blockholder disclosure threshold policies.

We extend our model by assuming that if the manager implements the good business plan ($m = 1$) at $t = 1$, she receives a payoff that is normalized to zero at $t = 3$. If, instead, the manager adopts the bad plan ($m = 0$), her payoff depends on whether she is disciplined by the activist. If the activist does not intervene, adopting the bad business plan gives the manager a fixed benefit φ. If the activist disciplines the manager, she does not receive the private benefit and incurs a privately-observed reputation cost $\rho > 0$. Other market agents share a common prior that ρ is distributed on $[0, R]$ according to a strictly positive density h and associated cumulative function H. Because the manager only cares about the *net* benefit of malfeasance, one could alternatively assume that the benefits of malfeasance are random, and the reputation cost is fixed. To make it easier to establish that second-order conditions hold in the derivation of optimal blockholder disclosure threshold policies, we assume that private benefits from malfeasance are sufficiently high: $2R < \varphi(1 - \lambda)^2$.

Both private benefits from malfeasance φ and the reputation costs of being disciplined by an activist ρ allow for multiple interpretations. For instance, managerial benefits from acting against shareholders might be related to increasing executive compensation or empirebuilding mergers and acquisitions that managers value but harm firm value. The costs of being disciplined by an activist may reflect career prospects. For example, Fos and Tsoutsoura (2014) report that facing a direct threat of removal is associated with \$1.3-\$2.9 million in foregone income until retirement for the median incumbent director in their sample; and Keusch (2017) finds that in the year after activists intervene, internal CEO turnover rises 7.4%.

4.1 Market Equilibrium

The manager employs a threshold strategy, implementing the bad business plan if and only if $\rho \leq \rho_t$. At the cut-off, the expected private benefits from malfeasance equal the expected loss due to punishment, $(1-\lambda G\left(c_t\right))\varphi = \lambda G\left(c_t\right)\rho_t$, which we solve for:

$$\rho_t = \varphi \left[\frac{1-\lambda G\left(c_t\right)}{\lambda G\left(c_t\right)}\right]. \tag{16}$$

The probability of managerial malfeasance is $H(\rho_t)$. The equilibrium analysis is analogous to Section 2.1, where both Propositions 1 and 2 extend by directly setting $z \equiv H(\rho_t)$.

The solution for ρ_t reveals that malfeasance declines with the *conditional* probability of activism $G(c_t)$: the more likely the activist is to participate after observing malfeasance, the less likely is the manager to misbehave. We call the managers' response to the threat of activism, the *managerial feedback* effect. This effect is negative, reflecting that the threat of activism deters managers from destroying shareholder value. Activism disciplines management through two complementary channels: (i) ex post, the activist intervenes to change the business plan when it is bad; (ii) ex ante, it discourages the adoption of the bad plan.

The mechanism is consistent with anecdotes suggesting that executives of firms that are yet-to-be-targeted by activist funds feel threatened and proactively work to evaluate firm policies that minimize their vulnerability to attacks by activist funds.[9] Gantchev et al.

[9]See, for instance, "Key Issues for Directors in 2014" by Martin Lipton of Wachtell, Lipton, Rosen and Katz, The Harvard Law School Forum on Corporate Governance and Financial Regulation, December 16, 2013; "Boardrooms Rethink Tactics to Defang Activist Investors", The New York Times, November 11, 2013.

(2019) find evidence that non-target firms, observing that their peers are being targeted by activists, perceive a higher risk of becoming a future target, and change their policies to mitigate this risk. Relatedly, Fos and Kahn (2018) develop a model where the threat of intervention by a blockholder discourages managers from destroying shareholder value. They study the incentives for the blockholder to exit and the impact on ex-ante and ex-post correction mechanisms; we analyse how managerial feedback responds to market transparency.

4.2 Optimal Policies

We study optimal blockholder disclosure threshold policies when the probability of managerial malfeasance is endogenous. Managerial feedback raises new policy questions. For example, additional trading profits increase the conditional profitability of activism (Proposition 4) and reduce activists' opportunity to profit (managerial feedback). Do investors still benefit from a disclosure threshold? What are the implications for real investment, and thus for society? How does managerial feedback affect the reluctance of hedge fund activists to support ownership disclosure rules? Could activist funds seek lower thresholds than investors?

Proposition 7 answers these questions, characterizing the ordering of optimal disclosure policies for market participants. We define an elasticity measure that allows us to present results intuitively: the *activism elasticity of management,*

$$\varepsilon_m = \frac{\partial H(\rho_t)}{\partial G\left(c_t\right)} \frac{G\left(c_t\right)}{H(\rho_t)}.$$

Here, $\varepsilon_m < 0$ captures a manager's reaction to the threat of activism. The bigger is ε_m (in absolute value), the larger is the reduction in the probability of managerial malfeasance $H(\rho_t)$ in response to a marginal increase in the conditional probability of activism $G(c_t)$. In the absence of a binding disclosure threshold, when the activist participates he buys α^* shares and has expected gross profits c_t^*. Moreover, the manager adopts the bad business plan if and only if $\rho \leq \rho_t(c_t^*) \equiv \rho_t^*$ and the activism elasticity of management is $\varepsilon_m\left(c_t^*, \rho_t^*\right) \equiv \varepsilon_m^*$. In the proposition below, we assume that second-order conditions are well-behaved for investors and activists; the Appendix shows that this will be so when the costs of intervention for the activist and the reputation costs of management have uniform distributions.

Proposition 7 *Suppose that the net expected profits of investors and activists are quasiconcave in α for $\alpha \leq \alpha^*$. Then there exist cutoffs on the activism elasticity of management,* $\varepsilon_m^{*A} \equiv -\frac{1}{\varepsilon_a^*}\left(\frac{c_t^*}{c_t^*-E[c|c\leq c_t^*]}\right)$, $\varepsilon_m^{*I} \equiv -\left(\frac{\partial\pi_A/\partial H(\rho_t^*)}{\partial\pi_I/\partial H(\rho_t^*)}\right)\left[\frac{\delta f(k)-c_t^*}{c_t^*} - \frac{1}{\varepsilon_a^*}\right]$ *and* $\varepsilon_m^{*R} \equiv \left[\frac{\delta f(k)-c_t^*}{c_t^*} + \frac{1}{\varepsilon_a^*}\left(\frac{\frac{df(k)/d\alpha}{f(k)}}{\frac{dc_t^*/d\alpha}{c_t^*}}\right)\right]\left[-\frac{\partial\pi_I/\partial H(\rho_t^*)}{\partial\pi_A/\partial H(\rho_t^*)} - \frac{c_t^*-E[c|c\leq c_t^*]}{c_t^*}\right]^{-1}$, *with* $\varepsilon_m^{*A} < \varepsilon_m^{*I} < \varepsilon_m^{*R}$ *such that*

1. *If the activism elasticity of management is sufficiently high, then only the activist benefits from a binding disclosure threshold:* $\varepsilon_m^* < \varepsilon_m^{*A} \Rightarrow 0 < \overline{\alpha}_A < \alpha^* \leq \{\overline{\alpha}_I, \overline{\alpha}_R\}$.

2. *If the activism elasticity of management is moderately high, then no one benefits from a binding disclosure threshold:* $\varepsilon_m^{*A} \leq \varepsilon_m^* \leq \varepsilon_m^{*I} \Rightarrow 0 < \alpha^* \leq \{\overline{\alpha}_I, \overline{\alpha}_A, \overline{\alpha}_R\}$.

3. *If the activism elasticity of management is moderately low, then only investors benefit from a binding disclosure threshold:* $\varepsilon_m^{*I} < \varepsilon_m^* \leq \varepsilon_m^{*R} \Rightarrow 0 < \overline{\alpha}_I < \alpha^* \leq \{\overline{\alpha}_A, \overline{\alpha}_R\}$.

4. *If the activism elasticity of management is low enough, then investors and society gain from a binding disclosure threshold, but activists do not:* $\varepsilon_m^{*R} < \varepsilon_m^* \Rightarrow 0 < \overline{\alpha}_I < \overline{\alpha}_R < \alpha^* \leq \overline{\alpha}_A$.

Figure 3 illustrates the results. When the activism elasticity of managerial malfeasance is high, both the regulator and investors want more activism because they gain from deterring malfeasance—neither wants a binding disclosure threshold: $\varepsilon_m^* < \varepsilon_m^{*I}$. In contrast, the activist is harmed by reduced malfeasance and can gain from a threshold that limits his capacity to intervene if management is sensitive enough to the profitability of intervention, i.e., if $\varepsilon_m^* < \varepsilon_m^{*A}$. When, instead, this elasticity is low enough, the ordering of optimal policies is reversed, and the considerations of Proposition 4 dominate for the three parties. Then, investors gain more from a tighter threshold than the regulator ($\varepsilon_m^* > \varepsilon_m^{*I}$) because they incur the trading losses that society does not internalize; the regulator wants a tighter threshold than the activist ($\varepsilon_m^* > \varepsilon_m^{*R}$) because the negative investment feedback harms society; and while lower investment hurts the activist, it does not modify his optimal position (Corollary 5). Investors, activist funds and society can only agree on disclosure thresholds for intermediate activism elasticity levels ϵ_m, where everyone believes that disclosure thresholds should not bind.

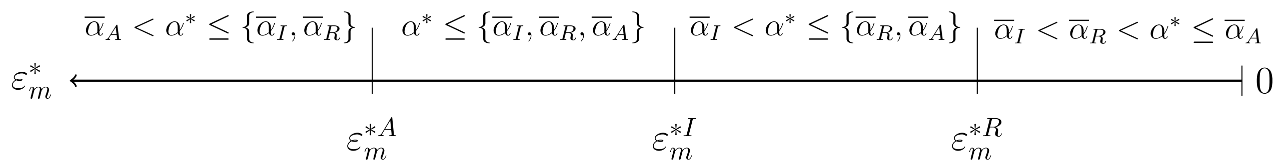

Figure 3: Optimal Disclosure Thresholds with Managerial Feedback

A proof is in the Appendix. Here, we develop the intuition. Setting $\left\{\varepsilon_m^{*A}, \varepsilon_m^{*I}, \varepsilon_m^{*R}\right\} = 0$ and rearranging terms with respect to ε_a^* yields the cutoffs in Proposition 4. Proposition 7 reveals how those findings are altered when management's behavior is sensitive to the possibility of hedge fund activism. The activist benefits from a binding disclosure threshold if

$$\begin{aligned} H(\rho_t)\lambda G\left(c_t^*\right) + M_A &< 0 \qquad (17) \\ \textit{where} \quad M_A \equiv \frac{dH\left(\rho_t^*\right)}{dc_t}\lambda G\left(c_t^*\right)\left[c_t^* - E\left[c|c \leq c_t^*\right]\right] &< 0. \end{aligned}$$

The condition can be rearranged to $\varepsilon_m^* < \varepsilon_m^{*A}$. Here, M_A represents the managerial feedback effect, which hurts the activist—well-behaving management destroys the raison dêtre of activists. Higher trading profits c_t^* increase the *conditional* profitability of activism, and the extent of activism upon managerial malfeasance $G\left(c_t^*\right)$—Proposition 4. However, it also deters management from acting against uninformed investors, reducing the activist's opportunity to profit. As a result, increasing a binding disclosure threshold, $\bar{\alpha}$, and hence increasing *trading* profits, need not increase the activist's *unconditional* expected profits.

Here, $\frac{dH(\rho_t^*)}{dc_t} = h(\rho_t^*)\frac{d\rho_t^*}{dc_t}$ captures the responsiveness of management to the threat of activism. A large h implies a high activism elasticity of management ε_m, and a large reduction in malfeasance in response to a marginal increase in the conditional profitability of activism.

Then, the activist benefits from a disclosure threshold that effectively commits the activist to reducing intervention rates, thereby encouraging managerial malfeasance. In contrast, when h is very small,[10] activism does not meaningfully deter managerial malfeasance, and ε_m goes to zero. With minimal managerial feedback, $M_A \to 0$, so (17) never holds and predictions reduce to those in Proposition 4: the activist is hurt by a binding disclosure threshold.

The cut-off ε_m^{*A} increases with ε_a^*—the higher is the profit elasticity of activism, the more the activist values a disclosure threshold. When higher trading profits greatly increase the extent of activism, they may also strongly deter managerial malfeasance. Then, the responsiveness ε_a^* of the activist to its potential trading profits harms it—so that the activist gains from a binding disclosure threshold that restrains its responsiveness. In those circumstances, neither investors nor the regulator want a binding disclosure threshold. This reflects that the activist's gains from a binding disclosure threshold are due to the increased managerial malfeasance that it causes, malfeasance that destroys surplus directly when the activist does not intervene and indirectly when the activist incurs costs of intervention. But then, investors and the regulator value the extensive discouragement effect of potential activism on managerial malfeasance. In particular, when the marginal value to the activist of tightening the disclosure threshold is positive, it is negative for investors and the regulator; and vice versa.

Proposition 7 shows that there exists a range of values $\varepsilon_m^* \in \left[\varepsilon_m^{*A}, \varepsilon_m^{*I}\right]$ such that no market participant gains from a binding disclosure threshold. If $\varepsilon_m^* \geq \varepsilon_m^{*A}$, managerial feedback is small enough from the activist's perspective not to outweigh the benefits of higher conditional profits from participating. Moreover, if $\varepsilon_m^* \leq \varepsilon_m^{*I}$ then the benefits to uninformed investors from deterring managerial malfeasance exceed the associated trading losses of activism, which they incur only if management misbehaves. Thus, investors, too, do not want to limit an activist's trading profits, even though those profits come at their expense. In

[10] When the second-order conditions hold, local statements about h hold globally for all c_t associated with binding disclosure thresholds.

particular, investors gain from a binding disclosure threshold if

$$\frac{H(\rho_t^*)\lambda}{f(k)}\Big[g\left(c_t^*\right)\left(\delta f\left(k\right)-c_t^*\right)-G\left(c_t^*\right)\Big]+M_I \quad < \quad 0 \tag{18}$$
$$\textit{where} \quad M_I \equiv \frac{dH(\rho_t^*)}{dc_t}\frac{\partial \pi_I}{\partial H(\rho_t^*)} \quad > \quad 0,$$

which can be rearranged to $\varepsilon_m^* > \varepsilon_m^{*I}$.

Comparing equations (13) and (18) reveals the effect of managerial feedback for investors, M_I. Equation (13) in Section 3 shows that, absent managerial feedback, investors' preference over disclosure thresholds only reflects the direct marginal costs and benefits of activism encapsulated in the first term. When management responds to the threat of activism, the positive effects of activism to investors become twofold: it increases managerial discipline, ex post, *and* it deters managerial malfeasance, ex ante. Thus, managerial feedback reduces the desirability of disclosure thresholds to investors. When $h(\rho)$ is tiny for ρ associated with $c_t \leq c_t^*$, feedback vanishes, so $M_I \to 0$, and (18) reduces to (13). When h is higher, management's actions become more sensitive to the extent of activism. As a result, investors may find a binding disclosure threshold undesirable even if the conditional marginal profitability of activism is negative, i.e., even if $g\left(c_t^*\right)\left(\delta f\left(k\right)-c_t^*\right) < G\left(c_t^*\right)$. It follows that if investors find a binding disclosure threshold desirable with managerial feedback, then they also do so in the absence of managerial feedback: $\varepsilon_a^* < \varepsilon_a^{*I}$ is necessary for $\varepsilon_m^{*I} < 0$, and hence for (18) to hold.

Society does not internalize management's private gains from malfeasance, but is affected by the destruction of project value. A regulator wants a binding disclosure threshold when

$$\Big[H(\rho_t^*)\lambda g\left(c_t^*\right)\left(\delta f(k)-c_t^*\right)+M_R\Big]\frac{dc_t^*}{d\alpha}+\pi_A f'(k)\frac{\partial k}{\partial \alpha} \quad < \quad 0 \tag{19}$$
$$\textit{where} \quad M_R = M_I f(k) + M_A \quad > \quad 0,$$

which can be rearranged to $\varepsilon_m^* > \varepsilon_m^{*R}$. M_R captures the social impact of managerial feedback. Society does not care about transfers of profits between investors and the activist caused by managerial feedback; it only cares about the aggregate effect, $M_R = M_I f(k) + M_A$. Formal expansion of M_R (see Appendix A) reveals that the social benefits of managerial feedback consist of the sum of two elements, scaled by the response of management $\frac{dH(\rho_t)}{dc_t}$ to the threat of activism. The regulator wants greater *potential* activism and hence weaker ownership disclosure rules when managers respond by more to the threat of discipline. The first element in the expansion is the value enhanced by deterring malfeasance: $\delta f(k)\,[1 - \lambda G(c_t)]$. Here, $\delta f(k)$ is the difference in firm value under good and bad business plans; and $1 - \lambda G(c_t)$ is the probability that the activist does stop a bad plan when it is implemented. The second element is the expected cost incurred by the activist when it disciplines management, $\lambda G(c_t) E[c|c \le c_t]$. Deterring malfeasance means that those costs are not incurred.

The sole social cost of activism is a potential reduction in investment. Thus, the regulator must gain from a nonbinding disclosure threshold if it benefits investors: we can only have $\varepsilon_m^{*R} < \varepsilon_m^*$ if investment is reduced by the transfer of trading profits from investors to the activist, i.e., if $\frac{df(k)}{d\alpha} = f'(k)\frac{\partial k}{\partial \alpha} < 0$.[11] The profit elasticity of activism ε_a^* reduces the harmful effects of negative investment feedback, raising the optimal disclosure threshold.

5 Concluding Remarks

Hedge fund activism has generated debate about the desirability of revising blockholder disclosure thresholds. These rules were set to protect small investors from abusive tactics of blockholders. We identify the tradeoffs. Disclosure thresholds may discourage activist funds from intervening to protect small investors from corporate managers who take actions that benefit themselves at the expense of firm value; but activist funds are also informed traders who profit from trading on their information advantage about their value-enhancing actions at the expense of uninformed investors. While managerial discipline creates value and in-

[11] To have $\varepsilon_m^{*R} < \varepsilon_m^*$, the cutoff ε_m^{*R} must be negative. The denominator is positive because $-\frac{\partial \pi_I/\partial H(\varphi_t^*)}{\partial \pi_A/\partial H(\varphi_t^*)} > 1$ whereas $\frac{c_t^* - E[c|c \le c_t^*]}{c_t^*} < 1$. Thus, $\varepsilon_m^{*R} < 0$ if and only if $\frac{\delta f(k) - c_t^*}{c_t^*} + \frac{1}{\varepsilon_a^*}\left(\frac{\frac{df(k)/d\alpha}{f(k)}}{\frac{dc_t^*/d\alpha}{c_t^*}}\right) < 0$.

centivizes real investment, the associated trading rents extracted from uninformed investors reduce their profitability and impair investment, destroying value.

We show that the preferences for binding disclosure thresholds of investors, activist funds and society are never aligned. When investors gain from a binding threshold, they benefit more than regulators, and activists are necessarily harmed even though, in this instance, the binding thresholds cause investors to increase their investments. Activists can gain from a disclosure threshold because it acts as a commitment device for intervening less frequently. However, we prove that such commitment is never beneficial for activists when it fosters real investment, but only when it encourages sufficient managerial malfeasance. Thus, activists gain only when investors and society are harmed. The threat of activism disciplines managers and increases investment value without the need for investors to incur further trading losses, and the increased investment benefits society. We only find scope for agreement when all market participants gain from non-binding disclosure thresholds. This requires that the willingness of activists to intervene be sufficiently sensitive to the degree of market opacity, but, in turn, that firm management not be too sensitive to the threat of activism in its choices of whether to take actions that benefit itself at the expense of shareholders.

Our analysis provides insights for policy makers. We characterize how optimal disclosure rules that target activist investors (e.g., 13D filings in the US) are determined by multiple factors that differ across firms, suggesting that a more tailored approach is desirable. Our model links the desirability of disclosure thresholds to market fundamentals (e.g., liquidity), firm characteristics (e.g., market capitalization and managerial entrenchment) and the regulatory framework (e.g., cost of activism). The mechanisms revealed can help regulators setting thresholds contingent on these characteristics or, even further, to evolve towards a framework in which companies have some discretion over their own ownership disclosure rules.

6 Appendix A: Proofs

6.1 Proof of Proposition 1

Market maker. Let $\widehat{\alpha}$ be the market maker's conjecture about the activist trade, which is correct in equilibrium. Denote $\widehat{c}_t \equiv c_t(\widehat{\alpha})$ the corresponding conjecture about his cost participation threshold.

The activist does not participate when either he does not observe the company take the bad business plan, or he observes that the bad business is implemented but it is too costly to intervene. When the activist does not participate, the order flow is negative with certainty, i.e., $Pr[\omega \leq 0|a_0] = \int_0^b \frac{1}{b} dl = 1$. Conversely, the activist participates when he observes that the manager implemented the bad business plan and the cost of intervention is sufficiently small. The probability of a negative net order flow when the activist participates is $Pr[\omega \leq 0|a_1] = \int_\alpha^b \frac{1}{b} dl = \frac{b-\alpha}{b}$. From Bayes rule,

$$Pr[a_1|\omega \leq 0] = \frac{z\lambda G(\widehat{c}_t)\left(\frac{b-\widehat{\alpha}}{b}\right)}{z\lambda G(\widehat{c}_t)\left(\frac{b-\widehat{\alpha}}{b}\right) + [1 - z\lambda G(\widehat{c}_t)]}. \tag{20}$$

and

$$\begin{aligned} Pr[V = f(k)|a_0] &= \frac{(1-z) + z(1-\lambda G(\widehat{c}_t))(1-\delta)}{(1-z) + z(1-\lambda G(\widehat{c}_t))(1-\delta) + z(1-\lambda G(\widehat{c}_t))\delta} \\ &= \frac{1 - z\lambda G(\widehat{c}_t) - z(1-\lambda G(\widehat{c}_t))\delta}{1 - z\lambda G(\widehat{c}_t)}. \end{aligned} \tag{21}$$

Substitute both (20) and (21) into (5) and use $Pr[a_0|\omega \leq 0] = 1 - Pr[a_1|\omega \leq 0]$ to obtain $P_{l(\alpha)}$ in (6). Evaluating $P_l(\alpha)$ at the equilibrium position $\alpha^* = b/2$ yields P_l in (2).

Activist. The activist's position $\alpha^* = b/2$ is derived in the main text, and the market maker's conjecture is correct in equilibrium, i.e., $\widehat{\alpha} = \alpha^*$. Uniqueness of c_t follows because the left-hand side of (8) increases with c_t, while the right hand side decreases with c_t for $\alpha < b$.

6.1.1 An in-depth look at cut-off c_t

Analysis of (8) reveals that c_t is maximized by a position $\alpha^{**} \in (\alpha^*, b)$. This feature is not relevant for the argument in the main text; here we prove the result and derive the formal intuition for completeness.

To study c_t as a function of α and k, define $F \triangleq c_t - (b-\alpha)\alpha z \left[\frac{1-\lambda G(c_t)}{b-\alpha z \lambda G(c_t)}\right] \delta f(k)$. From the Implicit Function Theorem, $\frac{\partial c_t}{\partial q} = -\frac{\partial F/\partial q}{\partial F/\partial c_t}$. Direct calculations yield:

$$\frac{\partial F}{\partial c_t} = 1 + (b-\alpha)\alpha z \lambda g(c_t)\left[\frac{b - z\alpha}{(b-\alpha z\lambda G(c_t))^2}\right]\delta f(k); \tag{22}$$

$$\frac{\partial F}{\partial k} = -(b-\alpha)\alpha z\left[\frac{1-\lambda G(c_t)}{b-\alpha z\lambda G(c_t)}\right]\delta f'(k); \tag{23}$$

$$\frac{\partial F}{\partial \alpha} = -z\left(\frac{1-\lambda G(c_t)}{(b-\alpha z\lambda G(c_t))^2}\right)\Big[(b-2\alpha)(b-\alpha z\lambda G(c_t)) + (b-\alpha)\alpha z\lambda G(c_t)\Big]\delta f(k). \tag{24}$$

For $\alpha < b$, we have $\frac{\partial F}{\partial c_t} > 0$ and $\frac{\partial F}{\partial k} < 0$. Thus, c_t increases with investment, i.e., $\frac{\partial c_t}{\partial k} > 0$.

Inspection of (24) reveals that the sign of $\frac{\partial F}{\partial \alpha}$ is determined by the term in brackets, which can be rearranged to $\alpha^2 z\lambda G(c_t) - \alpha 2b + b^2$. Solving (24)=0 for α yields

$$\alpha^{**} = \frac{b\left[1-\sqrt{1-z\lambda G(c_t)}\right]}{z\lambda G(c_t)} \in \left(\frac{b}{2}, b\right) \quad \text{and} \quad \alpha^{\cancel{**}} = \frac{b\left[1+\sqrt{1-z\lambda G(c_t)}\right]}{z\lambda G(c_t)} > b \tag{25}$$

with $\frac{\partial F}{\partial \alpha} > 0 \Leftrightarrow \alpha \in (\alpha^{**}, \alpha^{\cancel{**}})$. To verify $\alpha^{**} \in \left(\frac{b}{2}, b\right)$ notice that α^{**} increases with $z\lambda G(c_t) \in (0,1)$. Further analysis reveals $lim_{z\lambda G(c_t)\to 1}\alpha^{**} = b$, and applying L'Hôpital's rule yields $lim_{z\lambda G(c_t)\to 0}\alpha^{**} = b/2$. To show $\alpha^{\cancel{**}} > b$ use the same intuition and note that $\alpha^{\cancel{**}}$ decreases with $z\lambda G(c_t)$. It is immediate that $\frac{\partial c_t}{\partial \alpha} > 0$ for $\alpha < \alpha^{**}$, and $\frac{\partial c_t}{\partial \alpha} < 0$ for $\alpha \in (\alpha^{**}, b]$.

Intuitively, the result reveals that for a given initial investment k, the activist would maximize gross expected profits and participation by committing to a larger position $\alpha^{**} \in (\alpha^*, b)$ when participating before prices were posted. This follows from price function $P_{l(\alpha)}$ in (6), which decreases with the activist position reflecting that the bigger the stock purchase conditional on participating, the less likely is activism from the market maker's perspective given a net sale of stock $\omega \leq 0$. *This* market feedback is not internalized by the activist because prices are set before traders place orders. If the activist could commit to a more aggressive trade, he would benefit from both a lower price and a larger number of undervalued shares whenever his stock purchase was concealed by liquidity sales, which would occur with a smaller probability. In equilibrium, the inability to commit leads the activist to acquire a smaller position α^* when he participates.

6.2 Proof of Proposition 2

Gross expected profits. Consider an arbitrary position α. The unconditional project value $E[V]$ in Proposition 2 weighs cash flows $f(k)$ with the probabilities that (i) the manager implements the good business plan, $1-z$; (ii) the manager implements the bad plan but is disciplined by the activist, $z\lambda G(c_t)$; (iii) the manager implements the bad plan and is not disciplined by the activist but the project succeeds anyway, $z[1-\lambda G(c_t)](1-\delta)$.

The activist's gross profits are obtained by weighting his conditional profits $E[\Pi_A|a_1]$ with the probability of participation $z\lambda G(c_t)$,

$$\begin{aligned} E[\Pi_A] &= \pi_A f(k) \\ \textit{with } \pi_A &= z\lambda G(c_t)(b-\alpha)\alpha z \left[\frac{1-\lambda G(c_t)}{b-\alpha z\lambda G(c_t)}\right]\delta \\ &= z\lambda G(c_t)\frac{c_t}{f(k)}. \end{aligned} \tag{26}$$

By construction, expected investors' profits are the residual $E[\Pi_I] = [\pi_V - \pi_A]\, f(k)$,

$$\begin{aligned} E[\Pi_I] &= \pi_I f(k) \\ \textit{with } \pi_I &= [1-z(1-\lambda G(c_t))\delta] - z\lambda G(c_t)(b-\alpha)\alpha z \left[\frac{1-\lambda G(c_t)}{b-\alpha z\lambda G(c_t)}\right]\delta \\ &= [1-z(1-\lambda G(c_t))\delta] - z\lambda G(c_t)\frac{c_t}{f(k)}. \end{aligned} \tag{27}$$

Proposition 2 provides expressions for expected profits in equilibrium, substituting $\alpha =$

$\alpha^* = b/2$. Rearranging π_A as a function of c_t shows that α affects expected profits only through trading transfers c_t and capital, i.e., $E[\Pi_A](c_t(\alpha), k(\alpha))$ and $E[\Pi_I](c_t(\alpha), k(\alpha))$.

Real Investment. The first-order condition for investors' *net* profits $\pi_I f(k) - rk$ characterizes real investment. Note that while π_I is a function of both activism and investment, small investors are price takers who do not internalize the effects of their own investment.

6.3 Proof of Proposition 4

Investors. Investor net expected profits are $\pi_I f(k) - rk$. To derive their optimal disclosure threshold we differentiate with respect to α:

$$\frac{d}{d\alpha}\left\{\pi_I f(k) - rk\right\} = \left[\frac{\partial \pi_I}{\partial c_t}\frac{dc_t}{d\alpha} + \frac{\partial \pi_I}{\partial k}\frac{\partial k}{\partial \alpha}\right] f(k) + \left[\pi_I f'(k) - r\right]\frac{\partial k}{\partial \alpha} \tag{28}$$

where π_I is given by (27) and c_t is characterized in (8). We show that (28) is strictly positive at $\alpha = 0$, implying that investors always benefit from some degree of market opacity, i.e., $\overline{\alpha}_I > 0$. We then prove that (28) decreases in α for $\alpha < \alpha^*$. Therefore, if (28) is negative at $\alpha = \alpha^*$, then the optimal disclosure threshold $\overline{\alpha}_I$ solves (28) $= 0$ and $\overline{\alpha}_I < \alpha^*$. If, instead, (28) is positive at $\alpha = \alpha^*$, then the optimal threshold is non-binding, i.e., $\overline{\alpha}_I \geq \alpha^*$.

Analysis of (28) simplifies because of two properties. First, Proposition 2 shows that in equilibrium $\pi_I f'(k) - r = 0$, so the last term of (28) vanishes. Second, any interior maximum of $\pi_I f(k) - rk$ satisfies $\frac{\partial k}{\partial \alpha} = 0$ because the activist position that maximizes investor profits, also maximizes investment.[12] Using these two features and the expansion $\frac{dc_t}{d\alpha} = \frac{\partial c_t}{\partial \alpha} + \frac{\partial c_t}{\partial k}\frac{\partial k}{\partial \alpha}$, it follows from (28) that any interior solution $\overline{\alpha}_I < \alpha^*$ solves $\frac{\partial \pi_I}{\partial c_t}\frac{\partial c_t}{\partial \alpha} f(k) = 0$. The proof of Proposition 1 shows that $\frac{\partial c_t}{\partial \alpha} > 0$ for $\alpha < \alpha^{**}$ and $\alpha^* < \alpha^{**}$. Therefore, if there is an interior

[12] From the Implicit Function Theorem, $\frac{\partial k}{\partial \alpha} = -\frac{\frac{\partial \pi_I}{\partial c_t}\frac{\partial c_t}{\partial \alpha} f'(k)}{\pi_I f''(k)}$, where the denominator is negative. The proof of Proposition 4 continues by showing that the numerator in this expression characterizes the sign of (28). Therefore, $\frac{\partial k}{\partial \alpha} > 0$ if and only if $\frac{d}{d\alpha}\left\{\pi_I f(k) - rk\right\} > 0$.

solution $\overline{\alpha}_I < \alpha^*$, it must be characterized by $\frac{\partial \pi_I}{\partial c_t} = 0$, where

$$\frac{\partial \pi_I}{\partial c_t} = \frac{z\lambda}{f(k)} \left[g(c_t)(\delta f(k) - c_t) - G(c_t) \right]. \tag{29}$$

Because $g(c)$ is decreasing in c, (29) decreases with c_t.

At $\alpha = 0$, activist trading profits are zero, i.e., $c_t = 0$. It follows that if $\alpha = 0$ then (29) > 0, and thus (28) > 0. Therefore, investors always benefit from some degree of market opacity, i.e., from $\overline{\alpha}_I > 0$. We prove below that trading transfers increase with the activist's position $\alpha < \alpha^*$ despite investment feedback, i.e., that $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$—see the *activist* section of the proof. Hence, (29) decreases with α for $\alpha < \alpha^*$, and the same is true for (28). A binding optimal threshold exists if and only if (28) < 0 for $\alpha = \alpha^*$. Moreover, it satisfies (28) $= 0$. The condition (28) < 0 can be rearranged as $\varepsilon_a^{*I} < \varepsilon_a^*$.

Activist. Net expected activist profits are

$$\pi_A f(k) - z\lambda G(c_t) E[c|c \leq c_t] = z\lambda G(c_t) [c_t - E[c|c \leq c_t]], \tag{30}$$

where the right-hand side uses the solution for π_A in Proposition 2. Here, $z\lambda G(c_t)$ is the probability that the activist participates, i.e., the probability that (i) the manager adopts the bad business plan, (ii) the activist observes it, and (iii) his cost of intervention is sufficiently small. Conditional on intervention being optimal, his expected profits are the difference between trading profits c_t and the cost of disciplining management, which is expected to be $E[c|c \leq c_t] = \left[\int_0^{c_t} cg(c)\,dc\right]/G(c_t)$. To derive the activist's optimal disclosure threshold, we differentiate with respect to α:

$$\frac{d}{d\alpha} \{z\lambda G(c_t) [c_t - E[c|c \leq c_t]]\} = z\lambda G(c_t) \frac{dc_t}{d\alpha}. \tag{31}$$

The result follows because

$$\begin{aligned}\frac{dE\left[c|c\leq c_t\right]}{d\alpha} &= \frac{\partial E\left[c|c\leq c_t\right]}{\partial c_t}\frac{dc_t}{d\alpha} \\ &= \frac{g\left(c_t\right)}{G\left(c_t\right)}\left[\frac{\frac{\partial}{\partial c_t}\left\{\int_0^{c_t} cg\left(c\right)dc\right\}}{g\left(c_t\right)}-E\left[c|c\leq c_t\right]\right]\frac{dc_t}{d\alpha} \\ &= \frac{g\left(c_t\right)}{G\left(c_t\right)}\left[c_t-E\left[c|c\leq c_t\right]\right]\frac{dc_t}{d\alpha}\end{aligned} \tag{32}$$

where the last line uses $\frac{\partial}{\partial c_t}\left\{\int_0^{c_t} cg\left(c\right)dc\right\}=g\left(c_t\right)c_t$.

The sign of (31) is determined by $\frac{dc_t}{d\alpha}=\frac{\partial c_t}{\partial \alpha}+\frac{\partial c_t}{\partial k}\frac{\partial k}{\partial \alpha}$. Proof of Proposition 1 shows $\frac{\partial c_t}{\partial \alpha}>0$ for $\alpha<\alpha^{**}$ with $\alpha^*<\alpha^{**}$, and that $\frac{\partial c_t}{\partial k}>0$. Hence, for the activist to gain from a binding disclosure threshold it must be that $\frac{\partial k}{\partial \alpha}<-\frac{\partial c_t/\partial \alpha}{\partial c_t/\partial k}$ for $\alpha<\alpha^*$, i.e., that the negative investment response to activism by investors is strong enough to outweigh the positive marginal net trading transfers. We prove that this cannot be so by contradiction.

If $\frac{\partial k}{\partial \alpha}<0$, a marginal increase in the activist's position must hurt investors, implying that $\frac{\partial \pi_I}{\partial c_t}<0$. Suppose that the investment feedback satisfies $\frac{\partial k}{\partial \alpha}<-\frac{\partial c_t/\partial \alpha}{\partial c_t/\partial k}$ and thus that $\frac{dc_t}{d\alpha}<0$. By assumption increasing α reduces c_t, so it must increase *investor* profits because $\frac{\partial \pi_I}{\partial c_t}<0$. But this higher profitability leads investors to increase capital when the activist increases his position $\frac{\partial k}{\partial \alpha}>0$, a contradiction. It follows that $\frac{dc_t}{d\alpha}>0$ for $\alpha<\alpha^*$.

This argument yields that $\overline{\alpha}_A\geq\alpha^*$ in the absence of managerial feedback, yielding Corollary 5. This result is used above to solve for the optimal disclosure threshold of investors.

Regulator. The regulator maximizes the project value net of capital costs and expected activism costs, maximizing

$$\pi_V f(k)-rk-z\lambda G\left(c_t\right)E\left[c|c\leq c_t\right], \tag{33}$$

where π_V is given in Proposition 2. To derive the optimal disclosure threshold for society, we differentiate (33) with respect to α:

$$\begin{aligned}
& \frac{d}{d\alpha}\left\{\pi_V f(k) - rk - z\lambda G\left(c_t\right) E\left[c|c \leq c_t\right]\right\} && (34)\\
= \ & \frac{\partial \pi_V}{\partial c_t}\frac{dc_t}{d\alpha} f(k) + \pi_V f'\left(k\right)\frac{\partial k}{\partial \alpha} - r\frac{\partial k}{\partial \alpha} - z\lambda\left[g\left(c_t\right) E\left[c|c \leq c_t\right] + G\left(c_t\right)\frac{dE\left[c|c \leq c_t\right]}{dc_t}\right]\frac{dc_t}{d\alpha} && \\
= \ & \frac{\partial \pi_V}{\partial c_t}\frac{dc_t}{d\alpha} f(k) + \left[\pi_V f'\left(k\right) - r\right]\frac{\partial k}{\partial \alpha} - z\lambda g\left(c_t\right) c_t \frac{dc_t}{d\alpha} && (35)\\
= \ & \left[\frac{\partial \pi_V}{\partial c_t} f(k) - z\lambda g\left(c_t\right) c_t\right]\frac{dc_t}{d\alpha} + \pi_A f'\left(k\right)\frac{\partial k}{\partial \alpha}\ . && (36)
\end{aligned}$$

where the second equality (35) uses (32); the third equality (36) rearranges (35) using both equilibrium relationships $\pi_I f'\left(k\right) - r = 0$ (optimal investment by the investors) and $\pi_V = \pi_I + \pi_A$ from Proposition 2. Next, in (36) substitute for $\frac{\partial \pi_V}{\partial c_t} = z\delta\lambda g\left(c_t\right)$ obtained by differentiating the expression for π_V in Proposition 2, and use $\pi_A = z\lambda G\left(c_t\right)\frac{c_t}{f(k)}$ to obtain

$$\begin{aligned}
(34) \quad & = \quad z\lambda g\left(c_t\right)\left[\delta f(k) - c_t\right]\frac{dc_t}{d\alpha} + z\lambda G\left(c_t\right)\frac{c_t}{f\left(k\right)} f'\left(k\right)\frac{\partial k}{\partial \alpha} && (37)\\
& \underset{sign}{=} \quad g\left(c_t\right)\left[\delta f(k) - c_t\right]\frac{dc_t}{d\alpha} + G\left(c_t\right) c_t \frac{f'\left(k\right)}{f\left(k\right)}\frac{\partial k}{\partial \alpha}.
\end{aligned}$$

The first line of (37) corresponds to the condition in (15). The second line of (37) can be rearranged to obtain the expression for ε_a^{*R} in Proposition 4 by noting that $\frac{df(k)}{d\alpha} = f'\left(k\right)\frac{\partial k}{\partial \alpha}$.

It has been shown that $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$. Moreover $\delta f(k) - c_t > 0$ because c_t increases with $\alpha < \alpha^*$ and from (3) it follows that $\delta f(k) - c_t^* > 0$. Thus, $\frac{\partial k}{\partial \alpha} < 0$ is a necessary condition for a binding disclosure threshold to be optimal for the regulator, and $\varepsilon_a^{*R} < \varepsilon_a^{*I}$.

6.4 Proof of Corollary 6

We prove Corollary 6 in three steps.

1. A transfer $\tau > 0$ always reduces $g(c_t)$ and increases $G(c_t)$. A transfer $\tau > 0$ creates both a direct and an indirect effect on $g(c_t)$. The *direct* effect reduces $g(c_t)$ and increases $G(c_t)$ for any given $c_t \in (0, C]$. The *indirect* effect reduces c_t. In particular, from both (8) and the Implicit Function Theorem, c_t decreases in G. Thus, the increase in G caused by the direct effect diminishes c_t. The two effects have opposite effects on $g(c_t)$, but the direct effect always outweighs the indirect effect, so the transfer unambiguously reduces $g(c_t)$ and increases $G(c_t)$. To see this, suppose that a transfer $\tau > 0$ leads to a bigger $g(c_t)$, so the decrease in c_t outweighs the reduction of g. It follows that $G(c_t)$ is smaller, and therefore c_t is larger, a contradiction.

2. There exist cutoffs τ^I and τ^R such that $\varepsilon_a^* < \varepsilon_a^{*I}$ if and only if $\tau^I > \tau$, and $\varepsilon_a^* < \varepsilon_a^{*R}$ if and only if $\tau^R > \tau$. Moreover, $\tau^I < \tau^R$. We showed that any transfer $\tau > 0$ reduces both c_t and $g(c_t)$, and increases $G(c_t)$. From the characterizations in Proposition 4, it follows that ε_a^*, ε_a^{*I} and ε_a^{*R} decrease with a transfer $\tau > 0$. Consider now the biggest possible transfer $\tau = \sup\{g\}$, so that all probability mass accumulates at $c = 0$. Then, for any $c_t > 0$, we have $g(c_t) = 0$ and $G(c_t) = 1$. From the characterizations of cutoffs ε_a^*, ε_a^{*I} and ε_a^{*R} in Proposition 4, it follows that a transfer $\tau = \sup\{g\}$ yields $\varepsilon_a^* = 0$, and $\varepsilon_a^{*I} > \varepsilon_a^{*R} > 0$. By continuity, there exist cutoffs $\left\{\tau^I, \tau^R\right\} \in (0, \sup\{g\}]$. Since $\varepsilon_a^{*R} < \varepsilon_a^{*I}$, these thresholds satisfy $\tau^I < \tau^R$.

3. Cutoffs τ^I and τ^R decrease with market liquidity b. Ceteris paribus, higher liquidity b raises c_t, and reduces marginal profits of investors $g(c_t)\left[\delta f(k) - c_t\right] - G(c_t)$. Thus, a smaller transfer τ^I is needed for $\varepsilon_a^{*I} < \varepsilon_a^*$. When $g(c_t)\left[\delta f(k) - c_t\right] < G(c_t)$, raising trading transfers c_t makes investors' marginal profits more negative, and raises the negative investment feedback $\frac{\partial k}{\partial \alpha} < 0$. From (37) it follows that marginal profits for society eventually become negative.

6.5 Proof of Proposition 7

We derive the critical cutoffs $\{\varepsilon_m^I, \varepsilon_m^A, \varepsilon_m^R\}$ in an analysis that mirrors the proof of Proposition 4 incorporating $z \equiv H\left(\varphi\left[\frac{1-\lambda G(c_t)}{\lambda G(c_t)}\right]\right)$. The proof then compares the cutoffs and derives the implications for optimal disclosure thresholds. Finally, it shows that second order conditions hold when the costs of activism and reputation costs are uniformly distributed.

We first verify that the partial effects of α and k on trading transfers c_t preserve the same sign. Substituting $H(\rho_t)$ for z in the function F defined in the Proof of Proposition 1 yields

$$\begin{aligned}\frac{\partial F}{\partial c_t} &= 1-\left[\frac{(b-\alpha)\alpha\delta f(k)}{b-\alpha H(\rho_t)\lambda G(c_t)}\right]\left(\frac{dH(\rho_t)}{dc_t}(1-\lambda G(c_t))-H(\rho_t)\lambda g(c_t)\right) \\ &\quad -\left[\frac{(b-\alpha)\alpha\delta f(k)}{(b-\alpha H(\rho_t)\lambda G(c_t))^2}\right]H(\rho_t)(1-\lambda G(c_t))\alpha\lambda\left(\frac{dH(\rho_t)}{dc_t}G(c_t)+H(\rho_t)g(c_t)\right) \\ &= 1-\left[\frac{(b-\alpha)\alpha\delta f(k)}{b-\alpha H(\rho_t)\lambda G(c_t)}\right]\frac{dH(\rho_t)}{dc_t}(1-\lambda G(c_t))\left(1+\frac{\alpha H(\rho_t)\lambda G(c_t)}{b-\alpha H(\rho_t)\lambda G(c_t)}\right) \\ &\quad +\left[\frac{(b-\alpha)\alpha\delta f(k)}{b-\alpha H(\rho_t)\lambda G(c_t)}\right]H(\rho_t)\lambda g(c_t)\left(1-\frac{\alpha H(\rho_t)(1-\lambda G(c_t))}{b-\alpha H(\rho_t)\lambda G(c_t)}\right),\end{aligned} \tag{38}$$

where the second equality follows from rearranging. The expressions for $\frac{\partial F}{\partial k}$ and $\frac{\partial F}{\partial \alpha}$ follow from substituting $H(\rho_t)$ for z in (23) and (24) respectively.

The first line in the second equality of (38) is positive because $\frac{dH(\rho_t)}{dc_t} < 0$. The second line is positive because $\left(1-\frac{\alpha H(\rho_t)(1-\lambda G(c_t))}{b-\alpha H(\rho_t)\lambda G(c_t)}\right) > 0$. Thus, $\frac{\partial F}{\partial c_t} > 0$. Moreover, the Proof of Proposition 1 shows that $\frac{\partial F}{\partial k} < 0$, and that $\frac{\partial F}{\partial \alpha} < 0$ for $\alpha < \alpha^{**}$ and $\frac{\partial F}{\partial \alpha} > 0$ for $\alpha \in (\alpha^{**}, b)$.

From the IFT it follows that $\frac{\partial c_t}{\partial k} > 0$ and $\frac{\partial c_t}{\partial \alpha} > 0$ for $\alpha < \alpha^{**}$ and $\frac{\partial c_t}{\partial \alpha} < 0$ for $\alpha \in (\alpha^{**}, b]$. Therefore, adding managerial feedback to the benchmark setting developed in Section 2 does not alter the signs of the effects of α and k on trading transfers c_t.

Investors. The derivative of investors' net profits with respect to α is given by (28). If no interior solution exists, then investors do not benefit from a disclosure threshold, i.e., $\overline{\alpha}_I \geq \alpha^*$.

In equilibrium, $\pi_I f'(k) - r = 0$ and $\frac{\partial k}{\partial \alpha} = 0$ at an interior maximum, $\overline{\alpha}_I < \alpha^*$. Use $\frac{dc_t}{d\alpha} = \frac{\partial c_t}{\partial \alpha} + \frac{\partial c_t}{\partial k}\frac{\partial k}{\partial \alpha}$ to simplify (28) to $\frac{\partial \pi_I}{\partial c_t}\frac{\partial c_t}{\partial \alpha} f(k)$. We have $\frac{\partial c_t}{\partial \alpha} > 0$ for $\alpha < \alpha^{**}$ with $\alpha^* < \alpha^{**}$, and $\frac{\partial c_t}{\partial k} > 0$. Hence, an interior maximum $\overline{\alpha}_I < \alpha^*$ is characterized by $\frac{\partial \pi_I}{\partial c_t} = 0$, where

$$\frac{\partial \pi_I}{\partial c_t} = \frac{H(\rho_t)\lambda}{f(k)}\left[g(c_t)(\delta f(k) - c_t) - G(c_t)\right] + \underbrace{\frac{dH(\rho_t)}{dc_t}\frac{\partial \pi_I}{\partial H(\rho_t)}}_{M_I} \tag{39}$$

Here, $\frac{dH(\rho_t)}{dc_t} = h(\rho_t)\frac{d\rho_t}{dc_t} = g(c_t)\frac{\partial H(\rho_t)}{\partial G(c_t)}$ and $\frac{\partial \pi_I}{\partial H(\rho_t)} = -\left[\delta(1-\lambda G(c_t)) + \lambda G(c_t)\frac{c_t}{f(k)}\right]$. Because $M_I > 0$, managerial feedback raises the marginal profitability of a higher cutoff to investors. At $\alpha = 0$, activist trading profits are zero, so $c_t = 0$, and hence (39) > 0 and thus (28) > 0: investors always value some market opacity, i.e., $\overline{\alpha}_I > 0$. To characterize ε_m^I, rearrange (39) $= 0$ as:

$$0 = \frac{H(\rho_t)\lambda}{f(k)}\left[g(c_t)(\delta f(k) - c_t) - G(c_t)\right] + g(c_t)\frac{\partial H(\rho_t)}{\partial G(c_t)}\frac{\partial \pi_I}{\partial H(\rho_t)}. \tag{40}$$

Substitute $\varepsilon_a = \frac{g(c_t)}{G(c_t)}c_t$ and $\varepsilon_m = \frac{\partial H(\rho_t)}{\partial G(c_t)}\frac{G(c_t)}{H(\rho_t)}$ into (40), then divide by $H(\rho_t)$ and rearrange:

$$\begin{aligned} 0 &= \frac{\lambda}{f(k)}\left[\varepsilon_a\left(\frac{\delta f(k) - c_t}{c_t}\right) - 1\right] + \varepsilon_m \frac{g(c_t)}{G(c_t)^2}\frac{\partial \pi_I}{\partial H(\rho_t)}; \qquad (41)\\ &= \frac{\lambda}{f(k)}\left[\varepsilon_a\left(\frac{\delta f(k) - c_t}{c_t}\right) - 1\right] + \varepsilon_m\varepsilon_a \frac{1}{G(c_t)c_t}\frac{\partial \pi_I}{\partial H(\rho_t)};\\ \Rightarrow\ & -\left(\frac{\lambda G(c_t)\frac{c_t}{f(k)}}{\partial \pi_I / \partial H(\rho_t)}\right)\left[\frac{\delta f(k) - c_t}{c_t} - \frac{1}{\varepsilon_a}\right] = \varepsilon_m. \end{aligned}$$

The expression for ε_m^I follows directly from $\lambda G(c_t)\frac{c_t}{f(k)} = \frac{\partial \pi_A}{\partial H(\rho_t)}$. When investors value a binding disclosure threshold, i.e., when $\overline{\alpha}_I < \alpha^*$, it satisfies $\varepsilon_m = \varepsilon_m^I$.

Activist. Expected net activist profits are given in (30). Differentiating yields

$$\begin{aligned} &\frac{d}{d\alpha}\left\{H(\rho_t)\lambda G(c_t)\left[c_t - E\left[c|c \leq c_t\right]\right]\right\} \qquad (42)\\ = &\left[H(\rho_t)\lambda G(c_t) + \underbrace{\frac{dH(\rho_t)}{dc_t}\lambda G(c_t)\left[c_t - E\left[c|c \leq c_t\right]\right]}_{M_A}\right]\frac{dc_t}{d\alpha}. \end{aligned}$$

where $\frac{dc_t}{d\alpha} = \frac{\partial c_t}{\partial \alpha} + \frac{\partial c_t}{\partial k}\frac{\partial k}{\partial \alpha}$ was derived in the proof of Proposition 4. Because $M_A < 0$, managerial feedback reduces the marginal profits from increasing α to the activist.

Since (42) > 0 at $\alpha = 0$, the activist always benefits from some market opacity, i.e., $\overline{\alpha}_A > 0$. If no interior solution exists, then the activist does not want a disclosure threshold, i.e., $\overline{\alpha}_A \geq \alpha^*$. Set (42) $= 0$ to derive ε_m^A, and recall that $\frac{dc_t}{d\alpha} > 0$ for $\alpha < \alpha^*$. Thus, (42) $= 0$ implies

$$H(\rho_t) + \frac{dH(\rho_t)}{dc_t}\left[c_t - E\left[c|c \leq c_t\right]\right] = 0. \tag{43}$$

Substitute $\frac{dH(\rho_t)}{dc_t} = g(c_t)\frac{\partial H(\rho_t)}{\partial G(c_t)}$, $\varepsilon_a = \frac{g(c_t)}{G(c_t)}c_t$ and $\varepsilon_m = \frac{\partial H(\rho_t)}{\partial G(c_t)}\frac{G(c_t)}{H(\rho_t)}$ into (43) and divide by $H(\rho_t)$:

$$\begin{aligned} 0 &= 1 + g(c_t)\frac{\partial H(\rho_t)}{\partial G(c_t)}\frac{1}{H(\rho_t)}\left[c_t - E\left[c|c \le c_t\right]\right] \\ &= 1 + \frac{g(c_t)}{G(c_t)}\frac{\partial H(\rho_t)}{\partial G(c_t)}\frac{G(c_t)}{H(\rho_t)}\left[c_t - E\left[c|c \le c_t\right]\right] \\ &= 1 + \varepsilon_m\varepsilon_a\left[\frac{c_t - E\left[c|c \le c_t\right]}{c_t}\right]. \end{aligned} \tag{44}$$

The characterization of ε_m^A follows directly.

Regulator. The regulator's net expected payoff is given by (33). Differentiating with respect to α yields the marginal payoff to the regulator of increasing the activist's position:

$$\begin{aligned} &\frac{d\pi_V}{dc_t}\frac{dc_t}{d\alpha}f(k) + \pi_V f'(k)\frac{\partial k}{\partial\alpha} - r\frac{\partial k}{\partial\alpha} - \frac{dH(\rho_t)}{dc_t}\lambda G(c_t)E\left[c|c \le c_t\right]\frac{dc_t}{d\alpha} \\ &-H(\rho_t)\lambda g(c_t)E\left[c|c \le c_t\right]\frac{dc_t}{d\alpha} - H(\rho_t)\lambda G(c_t)\frac{g(c_t)}{G(c_t)}\left[c_t - E\left[c|c \le c_t\right]\right]\frac{dc_t}{d\alpha}. \end{aligned} \tag{45}$$

Substitute the equilibrium relationship $\pi_I f'(k) - r = 0$ and $\pi_V = \pi_I + \pi_A$ to rearrange the regulator's marginal payoff from increasing α as:

$$\begin{aligned} (45) &= \frac{d\pi_V}{dc_t}\frac{dc_t}{d\alpha}f(k) + \pi_A f'(k)\frac{\partial k}{\partial\alpha} \\ &\quad -\frac{dH(\rho_t)}{dc_t}\lambda G(c_t)E\left[c|c \le c_t\right]\frac{dc_t}{d\alpha} - H(\rho_t)\lambda g(c_t)c_t\frac{dc_t}{d\alpha} \\ &= -\frac{dH(\rho_t)}{dc_t}\Big[\delta f(k)\left[1 - \lambda G(c_t)\right] + \lambda G(c_t)E\left[c|c \le c_t\right]\Big]\frac{dc_t}{d\alpha} \\ &\quad +H(\rho_t)\lambda g(c_t)\left[\delta f(k) - c_t\right]\frac{dc_t}{d\alpha} + \pi_A f'(k)\frac{\partial k}{\partial\alpha}, \end{aligned} \tag{46}$$

where the second equality follows from $\frac{d\pi_V}{dc_t} = H(\rho_t)\delta\lambda g(c_t) - \frac{dH(\rho_t)}{dc_t}\delta\left[1 - \lambda G(c_t)\right]$. Rearranging further yields:

$$
\begin{aligned}
(45) &= \Big[H(\rho_t)\,\lambda g(c_t)\,[\delta f(k)-c_t]+M_R\Big]\frac{dc_t}{d\alpha}+\pi_A f'(k)\,\frac{\partial k}{\partial\alpha}, \qquad (47)\\
\text{where } M_R &\equiv -\frac{dH(\rho_t)}{dc_t}\Big[\delta f(k)[1-\lambda G(c_t)]+\lambda G(c_t)\,E[c|c\le c_t]\Big].
\end{aligned}
$$

This equation corresponds to the characterization in (19). Since $M_R>0$, managerial feedback increases the marginal profitability to the regulator of increasing the activist's position.

To ease exposition, we define $\Psi \equiv -\frac{\partial\pi_V}{\partial H(\rho_t)}f(k)+\lambda G(c_t)\,E[c|c\le c_t]$ so that $M_R = -\frac{dH(\rho_t)}{dc_t}\Psi$. Moreover, recall that $\frac{dH(\rho_t)}{dc_t}=g(c_t)\,\frac{\partial H(\rho_t)}{\partial G(c_t)}$. Substituting, rewrite (47) as:

$$
\begin{aligned}
0 &= \left[-\frac{\partial H(\rho_t)}{\partial G(c_t)}g(c_t)\,\Psi+H(\rho_t)\lambda g(c_t)\,(\delta f(k)-c_t)\right]\frac{dc_t}{d\alpha}+\pi_A\frac{df(k)}{d\alpha} \qquad (48)\\
&= g(c_t)\,(\delta f(k)-c_t)-\frac{g(c_t)}{G(c_t)}\frac{\partial H(\rho_t)}{\partial G(c_t)}\frac{G(c_t)}{H(\rho_t)}\frac{\Psi}{\lambda}+\frac{\pi_A}{H(\rho_t)\,\lambda}\frac{df(k)/d\alpha}{dc_t/d\alpha}\\
&= \varepsilon_a\left(\frac{\delta f(k)-c_t}{c_t}\right)G(c_t)-\varepsilon_m\varepsilon_a\frac{\Psi}{\lambda c_t}+\frac{\pi_A}{H(\rho_t)\,\lambda}\frac{df(k)/d\alpha}{dc_t/d\alpha}\\
&= \varepsilon_a\left(\frac{\delta f(k)-c_t}{c_t}\right)-\varepsilon_m\varepsilon_a\frac{\Psi}{\lambda G(c_t)\,c_t}+\frac{\frac{df(k)/d\alpha}{f(k)}}{\frac{dc_t/d\alpha}{c_t}}\\
&= -\varepsilon_m+\frac{\lambda G(c_t)\,c_t}{\Psi}\left[\frac{\delta f(k)-c_t}{c_t}+\frac{1}{\varepsilon_a}\left(\frac{\frac{df(k)/d\alpha}{f(k)}}{\frac{dc_t/d\alpha}{c_t}}\right)\right],
\end{aligned}
$$

where the third line uses $\varepsilon_a=\frac{g(c_t)}{G(c_t)}c_t$ and $\varepsilon_m=\frac{\partial H(\rho_t)}{\partial G(c_t)}\frac{G(c_t)}{H(\rho_t)}$. To derive ε_m^R note in the last line of (48) that using $\pi_V=\pi_I+\pi_A$ we obtain

$$
\begin{aligned}
\frac{\lambda G(c_t)\,c_t}{\Psi} &= \left[-\frac{\frac{\partial\pi_I}{\partial H(\rho_t)}f(k)}{\lambda G(c_t)\,c_t}-\frac{\frac{\partial\pi_A}{\partial H(\rho_t)}f(k)}{\lambda G(c_t)\,c_t}+\frac{E[c|c\le c_t]}{c_t}\right]^{-1} \qquad (49)\\
&= \left[-\frac{\partial\pi_I/\partial H(\rho_t)}{\partial\pi_A/\partial H(\rho_t)}-\frac{c_t-E[c|c\le c_t]}{c_t}\right]^{-1}.
\end{aligned}
$$

Cutoff relation. The analysis above rearranges the marginal payoffs to market participants of increasing α. When second-order conditions hold, (i) activist marginal profits are decreasing at α^* when $\varepsilon_m^*<\varepsilon_m^{*A}$; (ii) investors' marginal profits are decreasing at α^* if $\varepsilon_m^{*I}<\varepsilon_m^*$;

(iii) the regulator's marginal payoff is decreasing at α^* when $\varepsilon_m^{*R} < \varepsilon_m^*$.

Next, we show that $\varepsilon_m^A < \varepsilon_m^I < \varepsilon_m^R$. Because the marginal profits of all market agents are positive at $\alpha = 0$, $\varepsilon_m \in (\varepsilon_m^A, \varepsilon_m^I)$ when $\alpha = 0$. It follows that if second-order conditions hold, when investors want a binding disclosure threshold, activists do not and vice versa. Moreover, no party wants a binding disclosure threshold if $\varepsilon_m^* \in [\varepsilon_m^{A*}, \varepsilon_m^{I*}]$.

To see that $\varepsilon_m^A < \varepsilon_m^I$, note that the relation is equivalent to

$$\frac{1}{\varepsilon_a}\left(-\frac{\partial\pi_A/\partial H\left(\rho_t\right)}{\partial\pi_I/\partial H\left(\rho_t\right)} - \frac{c_t}{c_t - E\left[c|c \leq c_t\right]}\right) < -\left(\frac{\partial\pi_A/\partial H\left(\rho_t\right)}{\partial\pi_I/\partial H\left(\rho_t\right)}\right)\left[\frac{\delta f\left(k\right) - c_t}{c_t}\right]. \tag{50}$$

The left-hand side of (50) is negative because

$$-\frac{\partial\pi_A/\partial H\left(\rho_t\right)}{\partial\pi_I/\partial H\left(\rho_t\right)} = -\frac{\lambda G\left(c_t\right)\frac{c_t}{f(k)}}{-\left[\left[1-\lambda G\left(c_t\right)\right]\delta + \lambda G\left(c_t\right)\frac{c_t}{f(k)}\right]} \in (0,1),$$

whereas $\frac{c_t}{c_t - E[c|c\leq c_t]} > 1$. The right-hand side of (50) is positive because $\partial\pi_A/\partial H\left(\rho_t\right) > 0$, whereas $\partial\pi_I/\partial H\left(\rho_t\right) < 0$, so we have $\varepsilon_m^A < \varepsilon_m^I$.

To see that $\varepsilon_m^I < \varepsilon_m^R$, note that a necessary condition for $\varepsilon_m^R < 0$ is that investment decrease with α, i.e., $\frac{df(k)}{d\alpha} = f'(k)\frac{\partial k}{\partial\alpha} < 0$, which implies that activist marginal profits decrease and thus $\varepsilon_m^I < \varepsilon_m$. Hence, if $\varepsilon_m^R = 0$, then $\varepsilon_m^I < \varepsilon_m < 0$. That is, the sole cost to society of increasing trading transfers is a reduction in real investment, while the benefits exceed those for investors. Thus, for $\varepsilon_m^R < \varepsilon_m$, it is necessary, but not sufficient, that $\varepsilon_m^I < \varepsilon_m$, which implies $\varepsilon_m^R < \varepsilon_m^I$.

6.5.1 The uniform-uniform case

We show that when both c and ρ are uniformly distributed, second-order conditions hold.

Investors. We rewrite the first-order condition for investors in (40), first substituting in the uniform distribution of the manager's cost of reputation, and then the uniform distribution of the activist's cost of intervention. Substituting $H(\rho_t) = \frac{\rho_t}{R}$ and $h(\rho_t) = \frac{1}{R}$, (40) becomes:

$$
\begin{aligned}
0 \quad = \quad & \underbrace{\frac{\varphi}{R}\left[\frac{1-\lambda G(c_t)}{\lambda G(c_t)}\right]}_{H(\rho_t)} \frac{\lambda}{f(k)}\left[g(c_t)(\delta f(k) - c_t) - G(c_t)\right] \\
& + \underbrace{\frac{\varphi}{R}\left[\frac{g(c_t)}{\lambda G(c_t)^2}\right]}_{-\partial H(\rho_t)/\partial c_t} \underbrace{\left[\delta(1-\lambda G(c_t)) + \lambda G(c_t)\frac{c_t}{f(k)}\right]}_{-\partial \pi_I/\partial H(\rho_t)}.
\end{aligned} \tag{51}
$$

Multiplying (51) by $\frac{R}{\varphi}\left[\frac{\lambda G(c_t)}{1-\lambda G(c_t)}\right]\frac{f(k)}{\lambda}$ yields an equivalent condition

$$
0 = g(c_t)(\delta f(k) - c_t) - G(c_t) + \left[\frac{g(c_t)}{\lambda G(c_t)}\right]\left[\delta f(k) + \frac{\lambda G(c_t)}{1-\lambda G(c_t)}c_t\right], \tag{52}
$$

which we multiply yet again by $\frac{1}{g(c_t)}$ and rearrange to obtain

$$
0 = \delta f(k)\left[\frac{1+\lambda G(c_t)}{\lambda G(c_t)}\right] + c_t\left[\frac{\lambda G(c_t)}{1-\lambda G(c_t)}\right] - \frac{G(c_t)}{g(c_t)}. \tag{53}
$$

Substituting $G(c_t) = \frac{c_t}{C}$ and $g(c_t) = \frac{1}{C}$, the first-order condition (53) for investors becomes

$$
0 = \delta f(k)\left[\frac{C+\lambda c_t}{\lambda c_t}\right] + c_t\left[\frac{\lambda c_t}{C-\lambda c_t}\right] - c_t \tag{54}
$$

We prove that there is a unique solution to the first-order condition for investors by showing that the right-hand side (RHS) of (54) decreases in c_t. Differentiating yields

$$
\begin{aligned}
\frac{d}{dc_t}RHS(54) \quad &= \quad \left(\frac{C}{C-\lambda c_t}\right)^2 - \frac{\delta f(k) C}{\lambda c_t^2} - 2 \\
&= \quad \left(\frac{1}{1-\lambda G(c_t)}\right)^2 - \frac{\delta f(k)}{\lambda G(c_t) c_t} - 2,
\end{aligned} \tag{55}
$$

where the second line uses $G(c_t) = c_t/C$ for $c_t \leq C$. Notice that (55) is negative for $c_t \to 0$ and increasing in c_t. We derive an upper bound for c_t and show that $\frac{d}{dc_t}RHS(54) < 0$ for such trading transfers, establishing that the solution to the first-order condition is unique. Trading profits c_t are maximized by the position α^{**} in (25) and with the highest liquidity shock $b = 1$. Substituting into the expression for c_t in (8) yields an implicit upper bound on c_t:

$$\begin{aligned} c_t &\leq \frac{(1-\lambda G(c_t))\left(1-\sqrt{1-H(\rho_t)\lambda G(c_t)}\right)\left(H(\rho_t)\lambda G(c_t)+\sqrt{1-H(\rho_t)\lambda G(c_t)}-1\right)\delta f(k)}{H(\rho_t)(\lambda G(c_t))^2\sqrt{1-H(\rho_t)\lambda G(c_t)}} \\ &= \frac{(1-\lambda G(c_t))}{H(\rho_t)(\lambda G(c_t))^2}\left[2-2\sqrt{1-H(\rho_t)\lambda G(c_t)}-H(\rho_t)\lambda G(c_t)\right]\delta f(k) \end{aligned} \tag{56}$$

A simpler (and weaker) upper bound on c_t follows from considering the maximum value of the numerator in (56), i.e.,

$$\begin{aligned} c_t &\leq \frac{(1-\lambda G(c_t))}{H(\rho_t)(\lambda G(c_t))^2}2\delta f(k) \\ &= (1-\lambda G(c_t))\underbrace{\frac{R}{\varphi}\left(\frac{\lambda G(c_t)}{1-\lambda G(c_t)}\right)}_{1/H(\rho_t)}\frac{2\delta f(k)}{(\lambda G(c_t))^2}, \end{aligned} \tag{57}$$

or equivalently,

$$\frac{\varphi}{2R} \leq \frac{\delta f(k)}{\lambda G(c_t)c_t}. \tag{58}$$

Plugging (58) in (55) and comparing the first two terms reveals that a sufficient condition for $\frac{d}{dc_t}RHS(54) < 0$ is

$$\left(\frac{1}{1-\lambda G(c_t)}\right)^2 \leq \frac{\varphi}{2R}. \tag{59}$$

The condition $2R < \varphi(1-\lambda)^2$ follows.

Activist. Substitute $H(\rho_t) = \frac{\rho_t}{R}$ and $h(\rho_t) = \frac{1}{R}$ to rewrite the activist's first-order condition (43) as:

$$0 = \underbrace{\frac{\varphi}{R}\left[\frac{1-\lambda G(c_t)}{\lambda G(c_t)}\right]}_{H(\rho_t)} + \underbrace{\left(-\frac{\varphi}{R}\frac{g(c_t)}{\lambda G(c_t)^2}\right)}_{\partial H(\rho_t)/\partial c_t}\left[c_t - E\left[c | c \leq c_t\right]\right] \tag{60}$$

Multiplying (60) by $\frac{R}{\varphi}\left[\frac{\lambda G(c_t)}{1-\lambda G(c_t)}\right]$ yields a simpler, equivalent condition

$$0 = 1 - \left[\frac{g(c_t)}{G(c_t)}\right]\left(\frac{c_t - E\left[c|c \leq c_t\right]}{1 - \lambda G(c_t)}\right). \tag{61}$$

Substitute $G(c_t) = \frac{c_t}{C}$ and $g(c_t) = \frac{1}{C}$ and note that $c_t - E\left[c|c \leq c_t\right] = \frac{c_t}{2}$. It follows that the activist's first-order condition satisfies

$$0 = 1 - \frac{1}{2}\left(\frac{C}{C - \lambda c_t}\right). \tag{62}$$

The right-hand side decreases in c_t, implying a unique solution.

7 Appendix B: Allowing for pure informed trading

We relax the assumption that the activist can only take a position when the manager implements the bad business plan and show that it does not qualitatively alter our results. We reproduce the analysis of the market in Section 2.1 assuming that at $t = 2$, if the activist observes the company, which occurs with probability λ, he can take a position, regardless of the business plan implemented by the manager at $t = 1$.

Proposition 8 *At $t = 2$, the market maker posts prices*

$$\begin{array}{lll} P(\omega) = P_l \equiv \left[\frac{2[1-z(1-\lambda G(c_t^*))\delta]-z\lambda G(c_t^*)-(1-z)\lambda}{2-z\lambda G(c_t^*)-(1-z)\lambda}\right] f(k) & if & \omega \leq 0 \\ P(\omega) = P_h \equiv f(k) & if & \omega > 0. \end{array} \tag{63}$$

If the activist (a) observes managerial malfeasance ($m = 0$) and the activism cost satisfies

$$c \leq c_t^* = z\left[\frac{1-\lambda G(c_t^*)}{2-z\lambda G(c_t^*)-(1-z)\lambda}\right]\frac{b}{2}\delta f(k), \tag{64}$$

or (b) observes that the manager behaves ($m = 1$); then he takes position

$$\alpha^* = \frac{b}{2}, \tag{65}$$

and disciplines management in situation (a). Otherwise the activist does not participate.

We provide the full proof at the end of this section; here we discuss the differences with the model in the main text. Case (a) is equivalent to the setting studied in Section 2.1. The activist intervenes to discipline management if the conditional trading profits of doing so (weakly) outweigh the cost of intervention, i.e., if $c \leq c_t^*$. Case (b) captures the difference from the benchmark model, as the activist can acquire stock when the manager behaves ($m = 1$). It reveals an intuitive result:

Corollary 9 *Pure informed trading occurs if and only if the activist observes that the manager implemented the good business plan. Therefore, it has unconditional probability* $(1-z)\lambda$.

When the activist observes the good plan, he can profit from his information advantage (trading profits) without incurring activism costs. Thus, he always takes a position. Moreover, the activist would never act as a mere informed trader after observing the bad plan. This would imply acquiring overvalued stock, and has negative expected profits.

Notably, when the activist acts as a mere informed trader, he takes the same position $\alpha^* = b/2$. Whether he intends to discipline management or not, a position of $b/2$ maximizes trading profits. If management misbehaves, the activist only participates if these trading profits outweigh the cost of intervention. If management behaves, he always participates (upon observing management's action).

A positive net order flow $\omega > 0$ reveals the activist, and activist participation is associated to certain cash flows $f(k)$ —as in the benchmark model. This is because additional participation only occurs if the business plan is good —case (b). A weakly negative order flow $\omega \leq 0$ is consistent with both activist absence and participation. Corollary 9 highlights that the new assumption increases informed trading, but only when the good plan is implemented. A negative order flow is now associated with smaller project expected cash flows and thus P_l is lower than in the benchmark model. This increases the conditional expected trading profits of the activist c_t^* and in turn his participation after observing managerial malfeasance $G(c_t^*)$.

Proposition 10 *The expected value at $t = 0$ of the project given investment k is*

$$E\left[V\right] = \left[1 - z(1 - \lambda G(c_t^*))\delta\right] f(k) \equiv \pi_V f(k). \tag{66}$$

Expected gross profits of the activist are:

$$E[\Pi_A] = \left[(1 - z)\lambda + z\lambda G(c_t^*)\right] \frac{c_t^*}{f(k)} f(k) \equiv \pi_A f(k). \tag{67}$$

Expected gross profits of uninformed investors are:

$$E[\Pi_I] = (\pi_V - \pi_A) f(k) \equiv \pi_I f(k). \tag{68}$$

Investment by uninformed investors k solves

$$\pi_I f'(k) - r = 0. \tag{69}$$

The proof follows directly from that of Proposition 2 in the text and the subseqent discussion. Given initial investment, the project has the same value $E[V]$ as in the benchmark setting—Proposition 2. The new assumption alters the distribution of revenues between investors and the activist. Equation (67) reveals that the activist, in addition to obtaining larger conditional trading profits (bigger c_t^*), also obtains them with higher probability. In particular, trading transfers are realized if either (a) the activist disciplines management, which occurs with probability $z\lambda G(c_t^*)$; or (b) the activist acts as a mere informed trader, which has probability $(1-z)\lambda$. It follows that π_I is smaller than in Proposition 2 and therefore investment levels captured by (69) are lower, too.

7.1 Proof of Proposition 8

Market maker. Let $\widehat{\alpha}$ be the market maker's conjecture about the size of the activist's trade, which is correct in equilibrium, and let $\widehat{c}_t \equiv c_t(\widehat{\alpha})$ be his conjecture about the cost participation threshold.

The activist does not participate if either he does not observe the business plan, or he observes that the bad plan is implemented but it is too costly to intervene. When the activist does not does not participate, the order flow is negative with certainty, i.e., $Pr[\omega \leq 0|a_1] = \int_0^b \frac{1}{b} dl = 1$. Conversely, the activist participates if he observes the plan and either the manager implemented the good business plan, or she implemented the bad business plan and the cost of intervention is sufficiently small. The probability of a negative net order flow when the activist participates is $Pr[\omega \leq 0|a_1] = \int_\alpha^b \frac{1}{b} dl = \frac{b-\alpha}{b}$. From Bayes rule,

$$Pr[a_1|\omega \leq 0] = \frac{[(1-z)\lambda + z\lambda G(\widehat{c}_t)]\left(\frac{b-\widehat{\alpha}}{b}\right)}{[(1-z)\lambda + z\lambda G(\widehat{c}_t)]\left(\frac{b-\widehat{\alpha}}{b}\right) + (1-z)(1-\lambda) + z(1-\lambda G(\widehat{c}_t))}, \tag{70}$$

and

$$
\begin{aligned}
Pr[V=f(k)|a_0] &= \frac{(1-z)(1-\lambda)+z(1-\lambda G(\widehat{c}_t))(1-\delta)}{(1-z)(1-\lambda)+z(1-\lambda G(\widehat{c}_t))(1-\delta)+z(1-\lambda G(\widehat{c}_t))\delta} \\
&= \frac{(1-z)(1-\lambda)+z(1-\lambda G(\widehat{c}_t))(1-\delta)}{(1-z)(1-\lambda)+z(1-\lambda G(\widehat{c}_t))}. \qquad (71)
\end{aligned}
$$

Substituting (70) and (71) into (5) and using $Pr[a_0|\omega \leq 0] = 1 - Pr[a_1|\omega \leq 0]$ yields

$$
P_{l(\widehat{\alpha})} = \frac{b-\widehat{\alpha}z\lambda G(\widehat{c}_t)-\widehat{\alpha}(1-z)\lambda - bz(1-\lambda G(\widehat{c}_t))\delta}{b-\widehat{\alpha}z\lambda G(\widehat{c}_t)-\widehat{\alpha}(1-z)\lambda}, \qquad (72)
$$

which, evaluated at the equilibrium position $\alpha^* = b/2$, yields P_l in (64).

Activist. The activist's gross expected profits from participating do not depend on whether he intervenes to discipline to discipline management incurring cost c, or if he acts as a mere informed trader after observing the good business plan, which is costless. Gross expected profits are therefore given by (7) and maximized by $\alpha^* = b/2$. Upon observing the bad business plan $m = 0$, the activist participates if and only if $c \leq E[\Pi_A|a_1]$, implying that the *intervention* cost cut-off satisfies $c_t = E[\Pi_A|a_1]$. Substituting $P_{l(\alpha)}$ in (72) into $E[\Pi_A|a_1]$ and noting that the market maker's conjecture is correct in equilibrium yields

$$
c_t = (b-\alpha)\alpha z \left[\frac{1-\lambda G(c_t)}{b-\alpha z\lambda G(c_t)-\alpha(1-z)\lambda}\right]\delta f(k), \qquad (73)
$$

which, evaluated at $\alpha = \alpha^*$, reads as c_t^* in Proposition 8.

8 Appendix C: Alternative Trading Environment

In this section we develop an alternative formulation of the trading market and show that our qualitative results are preserved.

At $t = 2$, initial investors receive liquidity shocks that force them to sell shares. For tractability reasons, we follow Edmans (2009) and assume that liquidity shocks are exponentially distributed: liquidity shock l has density

$$y(l) = \begin{cases} \mu e^{-\mu l} & \text{if} \quad l \geq 0 \\ 0 & \text{if} \quad l < 0 \end{cases} . \tag{74}$$

To translate this to our framework we assume that when investors receive shock l, they must sell collectively a share $\gamma Y(l)$ of the firm, where $Y(l) = 1 - e^{-\mu l}$ is the cumulative distribution of liquidity shock l. If the activist observes managerial malfeasance and decides to participate, it chooses to acquire share $\gamma Y(\alpha)$ of the firm. The market maker observes $\omega = \alpha - l$, but not its components, and sets a price that breaks even in expectation, i.e., setting price equal to the expected project payoffs.

Other than the new trading environment, assumptions remain unchanged. We solve the model recursively, following the steps detailed in Section 2.

Proposition 11 *At $t = 2$, if the activist observes managerial malfeasance ($m = 0$) and the activism cost satisfies*

$$c \leq c_t^* = z \left[\frac{1 - \lambda G(c_t^*)}{2 - z\lambda G(c_t^*)} \right] \frac{\gamma}{2} \delta f(k); \tag{75}$$

then he takes a position $\gamma Y(\alpha^*) = \frac{\gamma}{2}$, which corresponds to

$$\alpha^* = \frac{ln(2)}{\mu}, \tag{76}$$

and disciplines management. Otherwise, the activist does not participate.

The market maker, upon observing $\omega = \alpha - l$, sets prices

$$
\begin{aligned}
P(\omega) = P_l &\equiv \left[\tfrac{2[1-z(1-\lambda G(c_t^*))\delta]-z\lambda G(c_t^*)}{2-z\lambda G(c_t^*)}\right] f(k) \quad if \quad \omega \leq 0 \\
P(\omega) = P_h &\equiv f(k) \qquad\qquad\qquad\qquad\qquad\qquad if \quad \omega > 0.
\end{aligned} \tag{77}
$$

A full proof is provided later; here we provide the intuition and link it to the benchmark model. If $\omega > 0$, the market maker knows with certainty that the activist took a position and the project pays off $f(k)$. If $\omega \leq 0$, the expected project value is

$$P_{l(\alpha)} = \left[\frac{1 - z\lambda G(c_t)Y(\alpha) - z(1-\lambda G(c_t))\delta}{1 - z\lambda G(c_t)Y(\alpha)}\right] f(k). \tag{78}$$

The activist's trade off between the number of undervalued shares that he may acquire $\gamma Y(\alpha)$ and the expected cost of information revelation $\int_\alpha^\infty y(l)dl = 1 - Y(\alpha)$ is analogous to the benchmark setting. His expected gross profits conditional on buying $\gamma Y(\alpha)$ shares are

$$E[\Pi_A|a_1] = [1 - Y(\alpha)]\gamma Y(\alpha)\left[f(k) - P_l\right]. \tag{79}$$

Expected gross profits are maximized by $\alpha^* = ln(2)/\mu$, which corresponds to a position $\gamma Y(\alpha^*) = \gamma/2$. The activist's cost participation cut-off is pinned down by $E[\Pi_A|a_1] = c_t$:

$$c_t = [1 - Y(\alpha)]\gamma Y(\alpha) z \left[\frac{1 - \lambda G(c_t)}{1 - z\lambda G(c_t)Y(\alpha)}\right] \delta f(k), \tag{80}$$

and takes the form in (75) when evaluated at the optimal position of $\alpha^* = ln(2)/\mu$. The cut-off c_t is unique and the activist employs a threshold strategy. Moreover, $Y(\alpha^*) = 1/2$, which plugged in (78) yields P_l in (77).

Note that $P_{l(\alpha)}$ decreases with α: the market feedback described in Proof of Proposition 1 is also present in this environment and the analogue intuition applies, i.e., the activist would benefit from committing to a more aggressive trade before stock prices are posted.

Subsequent analysis . Proposition 2 remains unchanged with respect to the main text: in expectation the project's gross expected profits are split in different proportions between uninformed investors and the activist. These proportions are determined by trading transfers in equilibrium c_t^*, which provide a role for blockholder disclosure thresholds. The analysis of optimal policies follows.

8.1 Proof of Proposition 11.

Let $\widehat{\alpha}$ be the market maker's conjecture about the activist's trade, which is correct in equilibrium. Let $\widehat{c}_t \equiv c_t(\widehat{\alpha})$ be the analogous conjecture about his cost participation threshold. The market maker observes ω. Given ω, either (i) the activist did not take a position and $l = -\omega$; or (ii) the activist participates and $l = -\omega + \widehat{\alpha}$. From our assumptions it follows that the unconditional probability that the activist does not participate is $[1 - z\lambda G(\widehat{c}_t)]y(-\omega)$, and the unconditional probability that he participates is $z\lambda G(\widehat{c}_t)\, y(-\omega + \widehat{\alpha})$. Thus, the expected project value is

$$\begin{aligned} E[V] \;=\; & \left[\frac{y(-\omega)(1-z) + y(-\omega+\widehat{\alpha})z\lambda G(\widehat{c}_t)}{y(-\omega)(1-z) + y(-\omega+\widehat{\alpha})z\lambda G(\widehat{c}_t) + y(-\omega)z[1-\lambda G(\widehat{c}_t)]}\right] f(k) \\ & + \left[\frac{y(-\omega)z[1-\lambda G(\widehat{c}_t)]}{y(-\omega)(1-z) + y(-\omega+\widehat{\alpha})z\lambda G(\widehat{c}_t) + y(-\omega)z[1-\lambda G(\widehat{c}_t)]}\right](1-\delta)f(k). \end{aligned} \tag{81}$$

Suppose the market maker observes $\omega > 0$. Then $y(-\omega) = 0$, and the activist participates with certainty so $P(\omega) = P_h$. If, instead, $\omega \leq 0$, the market maker does not know whether the activist participates, with $y(-\omega + \widehat{\alpha}) = \mu e^{\mu(\omega - \widehat{\alpha})}$ and $y(-\omega) = \mu e^{\mu\omega}$. The term $\mu e^{\mu\omega}$ cancels out of the numerator and denominator. Using $Y(\alpha) = 1 - e^{-\mu\alpha}$ yields (78).

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