Positive profits in competitive credit markets with adverse selection*

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Abstract

We analyze competitive credit markets with asymmetric information in which borrowers seek financing for either positive or negative net present value projects. The striking result is that there always exists an equilibrium where investment is efficient, while competitive lenders make strictly positive profits. When borrowers have zero opportunity costs of investment, this equilibrium co-exists with a market breakdown. With opportunity costs, no matter how small, the market can never break down, while the positive-profits equilibrium is unique if the average project quality is sufficiently low. This equilibrium exists even if borrowers can offer menus of contracts, and it coincides with the unique incentive compatible menu when almost all types have a negative net present value project. As lender profits can be taxed without distortions, we characterize the incentive compatible redistributions.

Keywords: Adverse selection, strictly positive profits, market breakdown

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1 Introduction

In markets with adverse selection, can the participation of certain high-risk or low-return individuals or firms lead to destruction of resources and undermine the possibility for others to trade? Tirole (2010), chapter 6, provides a classic example of a credit market in which firms are either of high quality or low, and seek financing for a risky project from competitive lenders. In the model, high-quality borrowers find it impossible to signal their type to lenders: their indifference curves coincide. As a result, when lenders cannot offer menus of contracts *and* must earn zero profits, the unique equilibrium is pooling. There are two possibilities: if the expected quality of the pool is high, then lenders offer a single interest rate and all types invest; if the expected quality of the project is, instead, low, then the market breaks down and all firms are denied credit. Similar outcomes obtain in models with *soft* budget constraints (see, e.g., Dewatripont and Maskin (1995)).

This paper uncovers fundamental ways in which this market breakdown result is fragile. In particular, we show that a market breakdown requires both (i) the assumption that borrowers have *exactly zero* opportunity cost of investment; and (ii) the exogenous imposition of zero profits for lenders. In contrast, we find that in markets plagued by severe adverse selection, rather than a breakdown, one should expect an active market in which competitive lenders make strictly positive profits. This finding can reconcile the high interest rates set by microcredit institutions in developing economies that target unbanked borrowers to make large profits (see, e.g., Karlan and Zinman (2010)). It is also consistent with the proliferation and success of firms offering payday loans and installment lending, or "buy here, pay here" car dealerships that target poor-credit customers. These firms explicitly identify high-risk populations of customers, charge exorbitant interest rates and make very high profits, despite facing many competitors (see, e.g., Melzer (2011)).[1]

Our first observation is that, even with zero opportunity cost, a market breakdown can be a unique equilibrium *if and only if* lenders are exogenously constrained to make zero profits. Otherwise, even with no opportunity costs, there always exists an alternative equilibrium in which the low types stay out of the market, while high types invest, promising to repay the entire stream of cash flows that the investment generates to the lenders. At this equilibrium, all borrowers get the same payoff as they do with a market breakdown. However, lenders are better off, as they make strictly positive profits and extract the full surplus from the investment opportunity. Thus, this equilibrium Pareto dominates the market-breakdown equilibrium and maximizes net social surplus.

[1] In the fast-growing sector of online installment lending, for instance, Elevate Credit Inc. saw annual revenue soaring 1,000% over the last five years, while Enova grew by 46% in the same period, according to Bloomberg (10/29/2019).

Our second observation is that the complete absence of opportunity costs is not realistic. For instance, potential borrowers might have a few dollars that can be used for down-payment, or they might have to forgo alternative opportunities that have some value, or the investment opportunity might demand some effort. We ask: which equilibrium is robust to the introduction of positive opportunity costs? Is it the market breakdown or the positive-profit separating equilibrium?

Strikingly, we find that when opportunity costs are positive, the market never breaks down in equilibrium. In contrast, the separating equilibrium always exists, regardless of parameter values. Markets do not break down because low types are less productive than high types, so their participation constraint binds at a lower interest rate. The no-investment allocation is broken by a high type offering a feasible interest rate just high enough to keep low types out. This deviation would make low types worse off relative to the status quo, at which they get their outside option payoff and don't invest. In contrast, the deviation is strictly profitable for high types, if lenders assign probability one that they are behind it, and consequently accept the offer. It follows that the market can never break down with positive opportunity costs, no matter how small.

In sharp contrast, the separating equilibrium with positive lender profits exists regardless of parameter values, and it always satisfies the Intuitive Criterion. What sustains the equilibrium is that low types do not invest. This implies that any deviation to a lower interest rate would make them weakly better off regardless of whether lenders accept the offer—making them strictly better off—or they reject it, in which case they are indifferent. Thus, the Intuitive Criterion has no bite.[2] Importantly, at this equilibrium the interest rate does *not* depend on the fraction of each type in the population. Therefore, the equilibrium is robust to the introduction of heterogeneous priors across agents or even ambiguity aversion over the fractions of types.

When the average net present value of borrowers is positive, and opportunity costs are small, there also exists a continuum of pooling equilibria that feature inefficient over investment—i.e., negative-NPV borrower types also invest, alongside high types. Generically, lenders make strictly positive profits at these pooling equilibria, and they all satisfy the Intuitive Criterion because borrowers' indifference curves coincide.

An implication of our equilibrium characterization is that efficient investment obtains as a unique equilibrium only when either there are enough low-type borrowers that the average borrower has a negative net present value project, or when there are absolutely no negative NPV borrowers in the market. In the intermediate region with a positive but small enough fraction of low types, pooling equilibria with inefficient over-investment by low types also exist. Therefore, a regulator seeking to implement the efficient level

[2] For similar reasons, the separating equilibrium would survive under stronger refinements, such as D1.

of investment would need to impose a *floor* on the admissible interest rates charged by lenders that exceeds all possible equilibrium pooling rates. With such a floor, lenders would make strictly positive profits, which could be fully taxed without distortion and redistributed to the agents. We show that there always exists a continuum of incentive compatible, feasible transfers that the regulators can set.

Consistent with most of the literature—this analysis presumes that lenders cannot issue a menu of contracts. But, as is well known, in this setting there is scope for borrowers to offer menus of contracts that give high types incentives to invest and pay low types to stay out of the market. Given that such menus are rarely—if ever—observed empirically, Tirole (2010) outlines an argument for ruling them out. He observes that menus risk attracting "fake entrepreneurs, who do not even have a project". He conjectures that, as a result, the fraction of low-quality firms "could quickly become very close to one, leading to a market breakdown after all". This observation also naturally applies to any transfers of lender profits a regulator may seek to implement.

These considerations lead us to extend our model by introducing menus of contracts and allowing for entry of 'fake entrepreneurs', drawn by the possibility of collecting a positive payment from lenders in return for not investing. We obtain three main results. First, regardless of the extent of entry by 'fake entrepreneurs', markets never break down when opportunity costs are positive. Second, the separating equilibrium with positive profits and all pooling equilibria exist even when borrowers can deviate to a menu. Third, with unlimited entry of 'fake entrepreneurs', the unique incentive compatible menu is equivalent to the single contract which sustains the separating equilibrium with positive lender profits.

We complete the characterization by showing that there exists a continuum of equilibria in which all borrowers offer a non-degenerate menu. In such equilibria, the incentive compatibility constraint for the low type must bind. This means that while incentive compatible tax transfer schemes can achieve the maximal menu payoff to high types, but no better, tax transfer schemes can provide greater payments to low types beyond what they can obtain with equilibrium menus alone.

2 Baseline model

Our two date ($t = 0, 1$) economy features two types of borrowers that must obtain external credit to finance a project. A type $\theta \in \{\theta_H, \theta_L\}$ borrower has a project that requires an investment of \$1 at $t = 0$ to generate cash flows at $t = 1$ that equal x with probability θ, and are zero otherwise. A fraction $p \in (0, 1)$ of borrowers are high types with $\theta = \theta_H$. Thus, the mean borrower type is $\theta_0 := p\theta_H + (1 - p)\theta_L$. Each borrower type has an *opportunity cost* $c \geq 0$ of taking on a project. Here, c could capture the cost of effort, time or other resources that must be devoted to a project, or the value of an alternative project that cannot be simultaneously undertaken. While we assume that the cost is the same for both types, our analysis trivially extends to encompass asymmetric costs, and qualitative results are unchanged. Importantly, we assume:

Assumption 1. *Only the high type has a positive NPV project:* $\theta_H x \geq 1 + c > \theta_L x$.

Borrowers can raise funds from competitive lenders. The risk-free rate is normalized to zero, and all agents are risk neutral. Our baseline model assumes that borrowers cannot offer a menu of contracts to lenders. Thus, under full information, the competitive lenders lend to high types at a rate $R_H = \frac{1}{\theta_H}$, while low types are denied credit and don't invest.

The game. We analyze a two-stage signaling game. In the first stage, a borrower proposes an interest rate R to lenders, who then form a belief $p'(R) := \Pr.[\theta = H|R]$ about the type of borrower that offered the contract. In the second stage, depending on this belief, lenders accept or reject the contract. If lenders accept, then investment, which is observable and verifiable, occurs and payoffs realize. If, instead, lenders reject the proposed funding terms, then the borrower consumes its endowment, which we normalize to zero.

We focus on Perfect Bayesian Equilibria that satisfy the Intuitive Criterion of Cho and Kreps (1987). The expected payoff of a type θ borrower that offers a rate R is $U_\theta(R) := i[\theta(x - R) - c]$, where $i \in \{0, 1\}$ denotes the decision of a lender. When $i = 1$, the lender funds the project. When $i = 0$, there is no financing. An equilibrium is defined as follows:

Definition 1. *A Perfect Bayesian Equilibrium of the game that survives the Intuitive Criterion must satisfy the following properties:*

1. *Sequential rationality: borrower types propose interest rates R^*_H and R^*_L optimally (the interest rates could be the same), given the lenders' beliefs and their associated optimal acceptance decision function $i^*(R)$;*
2. *Belief consistency: $p'(R^*_H)$ and $p'(R^*_L)$ are derived from Bayes' Rule;*
3. *Intuitive criterion: there does not exist a feasible $R \neq R^*_H$ such that: (i) $U_L(R|p') < U_L(R^*_L)$ for any $p' \in [0, 1]$; (ii) $U_H(R|p' = 1) > U_H(R^*_H)$.*

We say that an equilibrium *exists* if it satisfies the conditions detailed in Definition 1. We first consider the benchmark case of $c = 0$ that both Dewatripont and Maskin (1995) and Tirole (2010) emphasize. We then allow for $c > 0$.

2.1 Benchmark case: no opportunity costs

At a separating equilibrium, high types could obtain funding at the full-information rate $R_H = \frac{1}{\theta_H}$ at which lenders earn zero profits. However, because the indifference curves of the two types coincide, single crossing fails: the expected payoff to a low type from mimicking the high type and borrowing at R_H is $\theta_L(x - \frac{1}{\theta_H}) = \frac{\theta_L}{\theta_H}(\theta_H x - 1) \geq 0$, where the inequality follows from Assumption 1 that the high type has a positive net present value project, and the absence of an opportunity cost (i.e., $c = 0$). Therefore, the low types mimic and the contract becomes loss making for lenders.

In a pooling equilibrium with investment, both types of borrowers propose the same interest rate. Lenders form consistent beliefs and accept to fund the project. Investment by both types takes place, and lenders earn non-negative profits. Lenders accept the borrower's offer only if the gross interest rate R exceeds $R_P := \frac{1}{\theta_0}$, which is the rate at which lenders break even under the prior belief p. When $R > R_P$, the Intuitive Criterion does not have a bite, because at a lower proposed interest rate R' with $R > R' \geq R_P$, both high and low types always benefit whenever the off-equilibrium offer is accepted by a lender, and are worse off otherwise. Limited liability imposes a ceiling on the interest rate: $R_P \leq x$. As a result, a pooling equilibrium with investment exists only if $\theta_0 x \geq 1$.

The more interesting case is when $\theta_0 x < 1$, where the celebrated market breakdown result obtains: there is no investment because at any feasible rate $R \leq x$, both types would enter the market, causing the pooling contract to be loss making for lenders.

Our first observation is that this equilibrium is unique only if one *imposes exogenously* that lenders must make zero profits, and, in particular, that lenders cannot earn strictly positive profits in equilibrium. However, there is also an equilibrium in which the high type proposes $R = x$ and invests, while low types stay out of the market. Equivalently, in equilibrium, the low types may offer any $R < x$, which would subsequently be rejected by the lenders. A low type would earn strictly positive profits were that offer accepted so that any feasible $R < x$ could be offered by the low type on-the-equilibrium path. Both low and high-type borrowers make zero expected returns regardless of whether they offer to borrow at $R = x$ or not, leaving them indifferent between pursuing the investment and staying out. As a result, there is an equilibrium in which only high types enter, lenders make strictly positive profits of $\theta_H x - 1 > 0$, and net social surplus is maximized.

Perhaps surprisingly, both this equilibrium with positive lender profits and the market breakdown exist *regardless* of parameter values. To see why, consider the separating equilibrium first. We established that it is an equilibrium when $\theta_0 x < 1$, so it remains to consider $\theta_0 x \geq 1$. In this region, there exist pooling equilibria with interest rates $R \in [R_P, x]$. Suppose that there is also a separating equilibrium with $R = x$, and consider deviating from it to a lower R. The low types always weakly benefit, as they obtain a non-negative payoff at the deviant contract, so the Intuitive Criterion again has no bite. A similar argument applies to the market breakdown. Proposition 1 summarizes.

Proposition 1 (No opportunity costs)**.** *When $c = 0$, the Perfect Bayesian Equilibria of* *the two-stage signaling game that satisfy the Intuitive Criterion are as follows:*

Separating: *High types borrow at $R = x$, low types do not invest. Lenders make strictly positive profits. The equilibrium exists for all parameter values.*

Pooling: *All types borrow at a pooling rate $R \in [R_P, x]$, where $R_P = \frac{1}{\theta_0}$, and they subsequently invest. Lenders make zero profits if $R = R_P$, and strictly positive profits otherwise. A pooling equilibrium exists if only if the average project has a positive net present value at the prior: $\theta_0 x - 1 \geq 0$.*

Breakdown: *There is also an equilibrium where no credit is extended.*

Proposition 1 clarifies that, in the absence of opportunity costs, a separating equilibrium always exists in which the low types are kept out through their participation constraint. In a model such as ours in which indifference curves across types coincide, this is the only form of separation that can obtain.

The co-existence of the market breakdown equilibrium with this alternative equilibrium underscores two observations. First, lenders can make strictly positive profits in equilibrium even though they have no bargaining power. Second, maximized equilibrium net social surplus is non-monotone in the fraction of low types. It has one peak at $p = 1$, i.e., when there are no low types, and another at $\hat{p} \equiv \frac{1-\theta_L x}{(\theta_H - \theta_L)x}$, where:

$$(\hat{p}\theta_H + (1 - \hat{p})\theta_L)x - 1 = 0. \tag{1}$$

This raises the question as to whether the properties of the market breakdown equilibrium, or the positive profit equilibrium, or both, are knife-edged. To investigate, we extend the analysis to the realistic setting of strictly positive opportunity costs: $c > 0$.

2.2 Positive opportunity costs

When opportunity costs are large enough, the full-information contract becomes feasible, making the problem uninteresting. This occurs whenever c is so high that a low type prefers to stay out rather than mimic the high type and obtain financing at the full-information rate $R_H = \frac{1}{\theta_H}$, incurring the opportunity cost c. This happens whenever:

$$c \geq \bar{c} \equiv \theta_L \left(x - \frac{1}{\theta_H} \right) = \frac{\theta_L}{\theta_H}(\theta_H x - 1) > 0. \tag{2}$$

The last inequality holds as the high type has a positive NPV project (Assumption 1). We henceforth restrict attention to the interesting range of costs $c \in (0, \bar{c})$. Our first result is that whenever opportunity costs are strictly positive, markets do not breakdown.

Lemma 1. *When $c > 0$, the market can never break down.*

Proof. Suppose that the market breaks down in equilibrium. Consider the deviation by the high type in which it offers an interest rate R that yields the low type a negative payoff from mimicking, i.e., offering an R that satisfies $\theta_L(x - R) - c < 0$. If such an interest rate is accepted by lenders, low types would be strictly worse off than at the market breakdown, while high types would be strictly better off if lenders hold a belief that $p'(R) = 1$: the lenders would accept the offer because the high type's project has a positive net present value (Assumption 1). Thus, by the Intuitive Criterion we must have $p'(R) = 1$ and the market does not break down. □

Lemma 1 highlights the fragility of the market breakdown equilibrium. Not only it is *one* of multiple equilibria when $c = 0$, but it *only exists in this case*: when $c > 0$ the credit market never breaks down, regardless of the degree of informational asymmetries. Our second Lemma shows that, in sharp contrast, the positive-profits separating equilibrium exists regardless of the opportunity cost $c \in [0, \bar{c})$.

Lemma 2. *For every opportunity cost $c < \bar{c}$, there exists a unique separating equilibrium in which high types propose interest rate $R_B := \frac{\theta_L x - c}{\theta_L}$ but low types either propose $R \neq R_B$ or do not seek funding. Lenders fund projects if and only if R_B is proposed, earning strictly positive profits.*

Proof. **Existence.** Suppose the high types offer an interest rate R_B that makes the participation constraint of low types bind:

$$\theta_L(x - R_B) - c = 0 \iff R_B = \frac{\theta_L x - c}{\theta_L}. \tag{3}$$

This rate can be sustained in equilibrium by the belief that R_B is offered by a high type, while any other R observed off-equilibrium is believed to be offered by a low type. A deviation to a lower interest rate can only benefit a type if lenders accept with positive probability. A low type would always deviate if this acceptance probability is positive, and a high type would only deviate if the acceptance probability is high enough.[3] Therefore, this separating equilibrium with positive profits satisfies the Intuitive Criterion.

Uniqueness. A separating equilibrium exists only if $R \geq R_B$. Were $R < R_B$, a low type would also pool on R, as it could earn strictly positive profits if funded. Now, consider any equilibrium with interest rate $R \in (R_B, x)$. In such an equilibrium, a low type would stay out, but a high type would be willing to propose it. Low types would be made strictly

[3]Indeed, if the low type proposed an interest rate $R \in (0, R_B)$, it would earn strictly positive profits if accepted, so such an offer can be also observed on the equilibrium path.

worse off by deviating to $R' \in (R_B, R)$ were the offer accepted because their expected payoff from offering this rate would be strictly negative, and thus below their status quo payoff from staying out. In contrast, high types would be strictly better off offering an $R' \in (R_B, R)$ so lenders must believe that $p'(R) = 1$, and with this belief, the lenders would accept because the high type's project has a strictly positive net present value. Thus, any separating equilibrium with $R > R_B$ does not satisfy the Intuitive Criterion. □

Lemma 2 shows that there is always a unique separating equilibrium in which low types stay out, while high type invest at a rate such that lenders make strictly positive profits. Importantly, at this equilibrium the interest rate does not depend on the fraction of each type in the population. Thus, the equilibrium is robust to the introduction of heterogeneous priors across agents or even ambiguity aversion over the fractions of types.

If $\theta_0 x \geq 1$, then a pooling equilibrium might also exist in which both types offer an interest rate of $R \geq R_P = \frac{1}{\theta_0}$, at which lenders make non-negative profits given that all types invest. A type-θ borrower participates only if $\theta(x - \frac{1}{\theta_0}) \geq c$. Because low-type borrowers are less productive, they exit the pool at a lower opportunity cost threshold, $\underline{c}$, where:

$$\theta_H(x - \frac{1}{\theta_0}) > \theta_L(x - \frac{1}{\theta_0}) = \frac{\theta_L}{\theta_0}(\theta_0 x - 1) \equiv \underline{c}. \tag{4}$$

Thus, a pooling equilibrium with investment can only exist if opportunity costs are low enough that $c \in [0, \underline{c}]$. This interval is non-empty only if the project's ex-ante expected net present value is positive—i.e., only if $\theta_0 x \geq 1$. Also, note that $R_P > R_H$ implies that $\bar{c} > \underline{c}$. Therefore, there is an interval of opportunity costs $(max\{0, \underline{c}\}, \bar{c})$ such that no pooling equilibrium exists *and* a unique separating equilibrium exists. Lenders necessarily make strictly positive profits in this separating equilibrium. Thus, if one exogenously imposed zero profits, no equilibrium would satisfy the Intuitive Criterion. Lemma 3 characterizes the necessary and sufficient conditions for pooling to arise in equilibrium.

Lemma 3. *When $\theta_0 x \geq 1$ and $c \in [0, \underline{c}]$, there exists a continuum of pooling equilibria where all types invest at a rate $R^* \in [R_P, R_B]$. Lenders make zero profits if $R^* = R_P$, and they make strictly positive profits otherwise.*

Proof. Whenever pooling is feasible, it must feature an interest rate that is weakly lower than R_B, because $R_B(\underline{c}) = \frac{1}{\theta_0}$ and $\partial R_B / \partial c < 0$. Therefore, deviating to any feasible pooling interest rate strictly benefits both types relative to the positive-profits equilibrium. To see that pooling with strictly positive profits survives the Intuitive criterion, start from a pooling equilibrium with $R > R_P$ and consider the deviation to $R' \in [R_P, R)$. This deviation can only benefit a type if a lender accepts with positive probability. A low type would always deviate if this acceptance probability is positive, and a high type would only deviate if the acceptance probability is high enough. Therefore, any pooling equilibrium with $R \in [R_P, R_B]$ satisfies the Intuitive Criterion. Unless $R = R_P$, at all other pooling equilibria lenders make strictly positive profits. □

Combining the contents of these Lemmas, we now provide a full characterization in Proposition 2 of the set of Perfect Bayesian Equilibria of the game.

Proposition 2 (Positive opportunity costs)**.** *When $c \in (0, \bar{c})$, the Perfect Bayesian Equilibria of the two-stage signaling game that satisfy the Intuitive Criterion are as follows:*

Separating: High types borrow at $R_B = \frac{\theta_L x - c}{\theta_L}$ and invest, while low types do not invest. Lenders make strictly positive expected profits by funding the high types, generically. The equilibrium exists for all parameter values.

Pooling: All types borrow at a pooling rate $R \in [R_P, R_B]$, where $R_P = \frac{1}{\theta_0}$, and they invest. Lenders make zero profits if $R = R_P$, and strictly positive profits otherwise. The equilibrium exists only if the following two conditions hold:

(a) The ex-ante project has a positive NPV: $\theta_0 x - 1 > 0$;

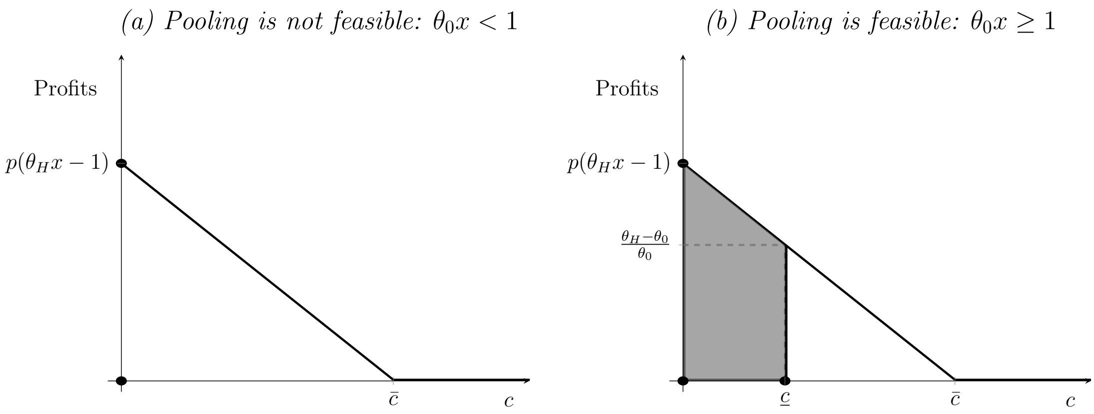
(b) The opportunity cost is not too large: $c \leq \underline{c}$.

Under these conditions, we always have $R_P \leq R_B$, with equality if $c = \underline{c}$.

The analysis thus far implies that starting from a market breakdown equilibrium and increasing opportunity costs from zero to some tiny $\epsilon > 0$ leads to a discontinuous jump in lender expected profits from 0 to $p(\frac{\theta_H}{\theta_L}[\theta_L x - \epsilon] - 1) = p(\theta_H x - 1) - p\frac{\theta_H}{\theta_L}\epsilon > 0$. In sharp contrast, starting at some $c > 0$, the effect of increasing c on lender profits is continuous and decreasing: $\frac{\partial}{\partial c}\left[p(\frac{\theta_H}{\theta_L}[\theta_L x - c] - 1)\right] = -p\frac{\theta_H}{\theta_L} < 0$. This is shown in Figure 1, Panel (a), for the case in which the average project has a negative net present value. When $c = 0$ there can be two levels of lender profits in equilibrium: zero or the full net present value of the high project. However, strikingly, it is sufficient to increase the opportunity costs from zero to a small $\epsilon > 0$, to eliminate the zero-profit equilibrium, and obtain strictly positive profits at the unique equilibrium.

Panel (b) of Figure 1 considers the case in which the average project has positive net present value. When $c \leq \underline{c}$, any level of lender profits between zero and the profits made at the separating contract with interest rate R_B can be sustained in the (pooling) equilibria of the game. Interestingly, however, there is another discontinuity in lender profits at $\underline{c}$, above which the separating contract with positive profits again becomes the unique equilibrium that satisfies the Intuitive Criterion. Because any equilibrium pooling interest rate exceeds the full-information rate for high types, it is often argued that interest rates are 'high' due to adverse selection. Here, however, all pooling equilibria feature interest

Figure 1: Competitive lender profits in equilibrium



rates that are *strictly below* the interest rate in the positive-profits separating equilibrium. While the full-information rate is not implementable, this alternative contract is always feasible and it maximizes net social surplus. In a real sense, interest rates are 'too low' due to adverse selection. A regulator seeking to maximize social surplus would find it optimal to impose a *floor* of R_B on admissible interest rates. With such a floor, low types do not pool and lenders make strictly positive profits, which could be fully taxed without introducing any distortion.

A possible caveat to this analysis is that it presumes that borrowers cannot offer menus of contracts. However, as the literature has established (see, e.g., Tirole (2010), chapter 6), the planner's solution might involve all borrowers offering a pooling menu of contracts in which lenders pay low types not to invest. We now extend the model to allow for menus of contracts. We also introduce entry of 'fake entrepreneurs' who are drawn by the possibility of collecting payments from lenders in return for not investing.

3 Menus and 'fake entrepreneurs'

In full generality, borrowers can offer menus of contracts to lenders that consist of a tuple $\kappa = \{R_\theta, q_\theta, y_\theta\}_{\theta \in \{H,L\}}$, where R_θ denotes the interest rate charged for a type-θ borrowers if it invests, y_θ denotes a payment from the lender to the borrower when it does not invest, and $q_\theta \in \{0, 1\}$ denotes whether or not investment occurs. In the first stage of the game, borrowers offer a menu. In the second stage, lenders either accept or reject given their beliefs. If a lender accepts, then the borrower selects its preferred option from the menu: either the H- or L- tuple (i.e., κ_H or κ_L). By the revelation principle, the equilibrium menu can be designed without loss of generality so that L-types prefer the L option, and H-types prefer the H option. Thus, the following incentive compatibility conditions must hold:

$$\underbrace{q_H[\theta_H(x-R_H)-c]+(1-q_H)y_H}_{\equiv U_H(\kappa_H)} \geq \underbrace{q_L[\theta_H(x-R_L)-c]+(1-q_L)y_L}_{\equiv U_H(\kappa_L)} \quad (IC_H)$$

$$\underbrace{q_L[\theta_L(x-R_L)-c]+(1-q_L)y_L}_{\equiv U_L(\kappa_L)} \geq \underbrace{q_H[\theta_L(x-R_H)-c]+(1-q_H)y_H}_{\equiv U_L(\kappa_H)}. \quad (IC_L)$$

We also suppose that if lenders offer to pay low types to stay out, then this might attract *'fake entrepreneurs'*, effectively increasing the share of low types in the population. We model this in the simplest possible way, by assuming that whenever $y_L > 0$, the proportion of high types in the population falls to γp, for some $\gamma \in [0,1]$. This extension nests our baseline model as the special case of $\gamma = 1$, and it allows for free entry of bad types (i.e., $\gamma = 0$), as suggested by Tirole (2010).

Because high types are more productive than low types, we always have $U_H(\kappa_H) > U_L(\kappa_H)$. Therefore, there exists a continuum of menus that satisfy both ICs. Our first Lemma shows that (i) only menus at which IC_L binds can satisfy the Intuitive Criterion, (ii) low types must not invest (i.e., $q_L = 0$), and (iii) high types invest ($q_H = 1$). Henceforth, when we refer to a menu we implicitly mean a *non-degenerate* menu—i.e., a menu with two different options—as a degenerate menu is equivalent to a single contract.

Lemma 4. *For an equilibrium in which all types offer a non-degenerate menu of contracts to exist and satisfy the Intuitive Criterion, IC_L must bind, $q_L = 0$, and $q_H = 1$.*

Proof. First, suppose that an equilibrium exists in which IC_L is slack. Then deviating to an incentive compatible, feasible menu with a lower y_L (or a higher R_L) together with a slightly lower R_H makes high types better off whenever it is accepted by the lender, but it would make low types worse off. The Intuitive Criterion then implies that the deviation must come from a high type, leading to its acceptance and making it profitable. Thus, IC_L must bind.

Next, suppose that an equilibrium exists with $q_H = 0$. Lenders accept the menu only if $-\gamma p y_H + (1-\gamma p)(q_L\theta_L R_L - (1-q_L)y_L) \geq 1$. If $q_L = 0$, this reads $-\gamma p y_H - (1-\gamma p)y_L \geq 1$, which is always violated and so the menu is rejected. If $q_L = 1$, we have $-\gamma p y_H + (1-\gamma p)\theta_L R_L \geq 1$. This can be rewritten as $(1-\gamma p)\theta_L R_L \geq 1 + \gamma p y_H$. From the participation constraint of a low type, we know that $R_L \leq R_B$. Plugging $R_L = R_B$ yields $\theta_L x - 1 - c \geq \gamma p[y_H + (\theta_L x - c)]$. Because low types have a negative NPV project (Assumption 1), the left hand side of the inequality is strictly negative: $\theta_L x - 1 - c < 0$. In contrast, the right hand side is weakly positive whenever $c \leq \bar{c}$. Thus, we cannot have $q_H = 0$.

Finally, suppose that $q_L = 1$. It is immediate that IC_L and IC_H jointly hold if and only if $R_H = R_L$, which is equivalent to a pooling equilibrium with a single contract. □

In light of Lemma 4, without loss of generality, we can set $y_H = 0$. IC$_L$, which binds, reads $y_L = \theta_L(x - R_H) - c$. At such a menu, lender profits are $\gamma p \theta_H R_H - (1-\gamma p) y_L$ or, substituting IC$_L$ inside, $\gamma p \theta_H R_H - (1-\gamma p)(\theta_L(x - R_H) - c)$. Thus, profits are non-negative if:

$$R_H \geq \frac{(1-\gamma p)[\theta_L x - c] + \gamma p}{\gamma p \theta_H + (1-\gamma p)\theta_L} \equiv R_M. \tag{5}$$

The next Lemma establishes that with the measure one of 'fake entrepreneurs' suggested by Tirole (2010), the only incentive compatible menu offers a subsidy to low types of $y_L = 0$ and an interest rate for high types of $R_H = R_B$. That is, the unique incentive compatible menu is equivalent to the unique separating equilibrium with positive profits.

Lemma 5. *When $\gamma = 0$, the only incentive compatible menu is equivalent to the separating contract with positive profits: high types borrow at R_B, low types stay out and $y_L = 0$.*

Proof. From equation (5), $R_M(\gamma = 0) = R_B$. At any $R < R_B$, low types would prefer the H-option within the menu, violating incentive compatibility. □

When, instead, $\gamma > 0$, equation (5) implies that $R_M < R_B$. It follows that there exists a continuum of equilibria that satisfy the Intuitive Criterion in which menus are offered, with a positive subsidy to low types and an interest rate $R_H \in [R_M, R_B]$.

Proposition 3. *If one allows for menus, all the single-contract allocations described in Proposition 2 remain equilibria and satisfy the Intuitive Criterion.*

In addition, when $\gamma > 0$ there exist equilibria that satisfy the Intuitive Criterion in which all types offer a menu with interest rate $R_H \geq \frac{(1-\gamma p)[\theta_L x - c] + \gamma p}{\gamma p \theta_H + (1-\gamma p)\theta_L}$ and subsidy $y_L = \theta_L(x - R_H) - c$, where $R_H \leq R_B$. Lenders make non-negative profits and only high types invest. As the measure of 'fake entrepreneurs' increases, R_B is unchanged, while $\partial R_M / \partial \gamma < 0$. Therefore, the range of possible menu equilibria shrinks.

Proof. First, we establish that our separating equilibrium with a single contract and positive lender profits remains an equilibrium in the presence of menus. At this equilibrium, high types offer R_B, low types stay out and, therefore, they consume their outside option which we normalized to zero. Thus, they would be weakly better off offering any incentive compatible menu κ at which $y_L \geq 0$. High types would only benefit if the menu is accepted, as $R_H \leq R_B$. Therefore, the Intuitive Criterion does not bite.

Now, consider a pooling equilibrium. High types gain from deviating to a menu only if $R_H \leq R'$, where R' is an equilibrium pooling rate, and lenders accept the off-equilibrium offer. The utility of low types at the pooling equilibrium is $\theta_L(x - R') - c$. At the menu, their utility is $y_L = \theta_L(x - R_H) - c$. Thus, if high types gain from deviating to a menu, so do low types. Thus, the Intuitive Criterion has no bite and pooling remains an equilibrium.

Finally, to see that there exists a continuum of menu equilibria, consider one such equilibrium with interest rate $R_H \in (R_M, R_B)$. Deviating to a menu with a lower interest rate $R' \in (R_M, R_H)$ benefits both types if and only if the deviation is accepted by the lenders. Thus, the Intuitive Criterion does not bite. That $\partial R_M/\partial\gamma < 0$ is immediate from (5). $\square$

Because inefficient pooling equilibria with over-investment can exist, a regulator seeking to maximize net social surplus would find it optimal to set a *floor* of the interest rate of R_B. At such interest rates, only high types invest and lender profits are strictly positive. The regulator could tax these lender profits by up to 100% and redistribute. The set of feasible redistributions is characterized by two incentive constraints. First, low types must be better off not investing and not pretending to be a high type. Second, high types must prefer to invest. Because high types are more likely to succeed, the optimal scheme involves a transfer of t_H conditional on a project succeeding, a transfer of zero to investors with failed projects, and a transfer of t_L to the types that do not invest.

At such transfers, the indirect utilities are $\theta_H(x - R_B) - c + \theta_H t_H = \frac{c(\theta_H - \theta_L)}{\theta_L} + \theta_H t_H$ for high types, and t_L for low types. If high types pretend to be low types, they only consume the transfer t_L. Therefore, their incentive constraint reads: $t_L \leq \frac{c(\theta_H - \theta_L)}{\theta_L} + \theta_H t_H$. Moreover, if low types pretend to be high types and invest, their expected payoff is $\theta_L t_H$.

Therefore, the incentive constraint for low types reads $\theta_L t_H \leq t_L$. At $t_L = \theta_L t_H$ the incentive constraint for the low type binds, but that for the high type is slack: $\theta_L t_H < \frac{c(\theta_H - \theta_L)}{\theta_L} + \theta_H t_H \iff t_H + \frac{c}{\theta_L} > 0$. One cannot further raise the transfer to the high type, as the low type breaks even on investment given interest rate R_B, but one can increase t_L above $\theta_L t_H$ due to the efficiency gains associated with the high type investing. Expected lender profits at the separating equilibrium with interest rate R_B are $\pi := \gamma p(\theta_H R_B - 1) = \gamma p(\theta_H x - \frac{\theta_H}{\theta_L} c) - 1$, where the fraction of high types is reduced to $\gamma p \leq p$ because redistributions draw 'fake entrepreneurs'. Transfers must satisfy the participation constraint of lenders so that $\gamma p \theta_H t_H + (1 - \gamma p) t_L \leq \pi$. Combining this with $\theta_L t_H = t_L$ yields the highest transfer to high types that can be supported in the equilibrium t_H^*, and the associated transfer to low types t_L^*:

$$t_H^* = \frac{\gamma p \left(\theta_H x - 1 - \frac{\theta_H}{\theta_L} c\right)}{\gamma p \theta_H + (1 - \gamma p)\theta_L} \quad \text{and} \quad t_L^* = \frac{\theta_L \gamma p \left(\theta_H x - 1 - \frac{\theta_H}{\theta_L} c\right)}{\gamma p \theta_H + (1 - \gamma p)\theta_L}. \tag{6}$$

Corollary 1. *A regulator seeking to maximize net social surplus would optimally impose a floor on the admissible interest rate $R \geq R_B$. Under this regulation, investment is efficient, and lenders make strictly positive profits that can be taxed at no distortion up to*

100%, and redistributed through incentive compatible transfers (t_H, t_L) *such that* $t_H \leq t_H^*$.

It is easy to check that, for every γ, the menu implements the same allocation as our positive-profits equilibrium with transfers (t_H^*, t_L^*) given in equation (6). In particular, no equilibrium menu satisfying the Intuitive Criterion implements higher transfers than t_L^* to low types. This follows from Lemma 4, which states that IC_L must bind in equilibrium, i.e., $\theta_H t_H = t_L$. Therefore, low types can never obtain more than t_L^* using menus in equilibrium. To give low types more than t_L^*, one needs to employ taxes and transfers.

We conclude by noting that as γ falls—i.e., as more 'fake entrepreneurs' enter—so do feasible redistributions, and they collapse to zero as the fraction of low types goes to one.

4 Conclusions

Our paper shows that in markets plagued by severe adverse selection, a market breakdown is *never* a unique equilibrium: there exists an alternative equilibrium where high types invest, low types stay out, and lenders make strictly positive profits. Strikingly, this equilibrium exists for every parameter values, and it implements efficient investments. This positive-profit equilibrium is robust to the introduction of menus of contracts, and it is unique in the realistic scenario where opportunity costs are positive, no matter how small, and there is a large number of low types. At this equilibrium, lender profits can be taxed up to 100% and redistributed, without introducing any market distortion.

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