Positive and Negative Campaigns in Primary and General Elections

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Abstract

We analyze positive and negative campaigning in primary and general elections. Positive campaigning builds a candidate's reputation, while negative campaigning damages a rival's. We provide explanations for why general campaigns are more negative: in the general election, winning primary candidates benefit only from positive primary campaigning; and negative campaigning by a primary loser impairs his intra-party rival's chances. We derive how the relative strengths of candidates interact with the campaigning technology to affect the composition of campaigns. In a primary contest between a strong and weak challenger, the strong challenger's campaign is largely negative, while his weak opponent's is largely positive. In contrast, if challengers are similar, improving one challenger's reputation causes his primary rival to campaign more aggressively, but the effect on the better challenger's own primary campaigning hinges on the strength of the general election opponent.

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1 Introduction

The literature on political campaigning has highlighted the extent to which negative campaigning has come to dominate the political debate between candidates prior to an election, and the consequences for electoral outcomes. For example, in 2012, 85 percent of the $404 million spent by President Obama on advertising was negative in nature, while challenger Mitt Romney devoted 91 percent of his $492 million budget to negative ads.[1] What has received vastly less attention are the sharp differences in the natures of campaigning in primary versus general elections. In particular, campaigning is *far* less negative in primaries, especially when a primary winner faces an established incumbent candidate, and primary underdogs run more positive campaigns than front runners (Peterson and Djupe 2005). For example, in the 2004 Democratic presidential primary, only 38% of ads were negative, whereas in the general election 61% were (CMAG); and, by mid-February in the 2012 Republican primary, the strong favorite Mitt Romney ran 93% more negative ads than positive ads, while Gingrich, Paul and Santorum collectively ran 27% more positive ads than negative ones.[2]

We build a model that delivers these patterns in positive and negative campaigning. We consider a setting in which two challengers compete against each other in a primary, with the winner advancing to face an established incumbent from the opposing party in a general election.[3] Candidates only care about who wins the general election. Obviously, a challenging candidate hopes to win the general election; but failing that, he prefers that his party primary opponent win; and his least preferred outcome is for the incumbent to be re-elected. The incumbent seeks to be re-elected.

In addition to his preferences over who wins the general election, a candidate is described by his initial reputational stock, and a resource budget. Each candidate can devote his resources both to developing his own reputational stock via positive campaigning and to damaging an opponent's reputational stock via negative campaigning. Election outcomes are determined by a contest success function, where the probability a candidate wins depends on his post-campaigning reputational stock and that of his opponent.[4] A candidate chooses

[1] Kantar Media's Campaign Media Analysis Group (CMAG).

[2] http://www.washingtonpost.com/politics/study-negative-campaign-ads-much-more-frequent-vicious-than-in-primaries-past/2012/02/14/gIQAR7ifPR_story.html

[3] The "incumbent" could alternatively be a winner of a primary for the opposing party.

[4] Contest functions have been used to model positive and negative campaigning in a static two candidate

how extensively to campaign in each election, and the positive and negative composition of that campaign. We allow negative and positive campaigning to have different impacts on reputations—the preponderance of negative ads suggest that it is easier to damage a reputation than to build one up. We also allow the effects of primary campaigning on a winning challenger's reputation to decay prior to a general election—between elections, voters may forget some of a primary campaign, and the moderate voters who determine the general election winner may also pay less attention to primary campaigns than party partisans.

We first analyze general election campaigns. In the general election, campaigning choices only reflect their relative effectiveness in influencing who *wins* the election. As a result, in the general election, a candidate with sufficient resources equates marginal benefits by allocating more resources to negative campaigning than to positive campaigning; and a candidate with very limited resources only campaigns negatively.

In contrast, as long as the effects of primary campaigning do not fully decay prior to the general election, a challenger campaigns relatively more positively (less negatively) in a primary than in a general election—a greater share of primary expenditures is devoted toward positive campaigning. Most starkly, we prove that when there is no decay in the effects of primary campaigning on general election outcomes and primary candidates are ex-ante symmetric (identical reputations, resources and preferences), primaries feature more positive campaigning than negative—the exact *opposite* of general elections.

The heightened focus by challengers on positive campaigning in primaries reflects two forces. First, when a candidate wins a primary election, the benefits of positive primary campaigning enhance his reputational stock in the general election. In contrast, a candidate benefits from a negative primary campaign only to the extent that it raises his chance of winning the primary, giving him the opportunity to compete in the general election. Second, when a candidate loses a primary, a positive primary campaign does not tar a primary opponent in the general election, but a negative campaign does. That is, when a candidate loses the primary, any negative campaigning damages his primary rival's reputation, reducing the probability that he wins the general election. Since challengers want the incumbent to lose the general election regardless of who wins the primary, this force causes them to reduce negative campaigning in primaries. Thus, our theory reconciles the empirical observation

game (Soubeyran 2009).

that primary campaigns are less negative than general campaigns.

The shift away from negative campaigning in a primary when more of the effects of primary campaigning persist to the general election may lead to a conjecture that reduced decay *must* raise the probability that the primary winner defeats the incumbent. In fact, this need not be so. When negative campaigning is only slightly more effective than positive campaigning, reduced decay also causes challengers to increase primary spending; and when decay is substantial, the reduced resources a primary winner has for the general election can swamp the gains from reduced negative primary campaigning in determining general election outcomes. As a result, the probability the incumbent loses can be a hump-shaped function of the extent to which the effects of primary campaigning carry over to the general election.

How challengers campaign in a primary hinge on the strength of their general election opponent. If a primary winner will face a well-regarded incumbent who has extensive resources, a bruising primary battle would weaken both challengers, making them unlikely to win the general election. As a result, we predict that when an incumbent is stronger, primary campaigns are more limited and less negative. Conversely, if the challengers have good reputations and extensive resources, or the incumbent is weak and scandal-ridden, the primary winner is likely to win the general election. This causes the challengers to shift their focus toward improving their own chances of getting elected, encouraging them to increase primary spending and to shift the composition toward negative campaigning. Thus, paradoxically, we predict that higher quality primary challengers engage in more negative primary campaigning. These predictions can reconcile Peterson and Djupe's (2005) findings that (a) the greater are per capita primary expenditures, the more negative is the composition of that spending; (b) open seat primaries for a dominant party (weaker general election candidate, typically stronger primary candidates) are more negative, and (c) having more high quality primary candidates leads to more negative primary campaigns.

We conclude our analysis by showing how differences between challengers affect how they campaign against each other. Perhaps counter-intuitively, when challengers are similar, increasing *one* challenger's resources encourages *both* challengers to become more negative and spend more in the primary, as long as each cares enough more about personally winning rather than just defeating the incumbent. The effect on challenger i of giving him a little more resources than his intra-party rival is straightforward—he campaigns more aggressively in the primary both because he has more to spend, and because he will be the party's stronger candidate in the general election. The effect on his primary rival j is less clear: (1) challenger i is now more likely to defeat the incumbent in the general election, but (2) j wants to win personally. As long challengers care enough more about personally winning and the challengers have almost equal chances, the latter effect dominates, so that when his rival grows stronger, the weaker challenger also increases his primary campaigning.

For identical reasons, increasing one challenger's reputation causes his intra-party rival to campaign more aggressively in the primary, when they are similarly situated. However, paradoxically, improving i's reputation causes him to reduce both positive and negative primary campaigning whenever the incumbent is strong. This is because i's better reputation helps him in the primary, encouraging him to conserve resources to raise his chances of defeating the strong incumbent in the general election. Improving i's reputation only causes him to campaign more aggressively in the primary when he faces a weak incumbent (with a poor reputation or limited resources).[5]

The effects of increased asymmetries between challengers are very different when one challenger is far stronger than the other: making a strong challenger even stronger causes his intra-party rival to *reduce* primary campaigning, especially his negative campaigning. A very weak rival internalizes that his stronger primary opponent is much more likely than he to win the general election, that negative primary campaigning reduces that probability, and that greater primary campaigning reduces his own low chance of winning the general election. As a result, a desire for the party's nominee to defeat the incumbent causes the far weaker primary rival to reduce primary spending, and to campaign more positively. This, in turn, induces the stronger challenger to reduce his primary campaigning, as his primary rival's less aggressive campaign reinforces his already extensive advantage. The stronger challenger still spends far more, and is far more negative than his weaker rival in order to ensure victory.

The opposing nature of the predictions concerning campaigning in primary versus general elections when one candidate is far stronger than the other is sharp. In general elections, candidates with more resources, devote greater shares to positive campaigning; and an especially weak challenger only campaigns negatively against an incumbent. In contrast, in

[5]The opposing "incumbent" could also be the winner of the opposing party's primary—such an "incumbent" could be weak if he has limited resources, or a low initial reputation.

primaries, a strong candidate campaigns more negatively than a weak candidate; and especially weak candidate's campaign may be entirely positive. Thus, our model delivers the pattern of primary campaigning in Republican presidential primaries—in 2012, Romney had a stronger reputation and far more resources, and hence he spent far more, and was far more negative than his primary rivals; so, too, in the 2000 South Carolina primary, the weaker primary candidate, John McCain, decided against "battling negative with negative...[in response to] the sheer volume of [Bush's] negative assaults."[6] It is also consistent with the Peterson and Djupe's finding that, in *primaries*, incumbents face less negative campaigning from their (typically weaker) primary opponents.

The literature on positive and negative campaigns dates back to Skaperdas and Groffman (1995) and Harrington and Hess (1996). Skaperdas and Groffman (1995) predict that in two candidate elections, the front-runner engages in more positive and less negative campaigning than his opponent; and in three-candidate contests, no candidate engages in negative campaigning against the weakest opponent, so that to the extent there is negative campaigning, it is either directed against the front-runner or it comes from the front-runner himself. Harrington and Hess (1996) explore negative and positive campaigning in a spatial setting in which (a) agents begin with initial locations but can engage in costly relocation, and (b) an agent's relocation is affected by her rival's actions as well. They predict that a candidate who is perceived as having less attractive personal attributes runs a more negative campaign.

Chakrabarti (2007) extends Harrington and Hess (1995) by introducing a valence dimension that captures personal traits such as integrity. Candidates can now influence both ideological and valance factors via negative advertising: ideological spending shifts an opponent's policy position away from the median and valence spending reduces the opponent's valence index. Candidates campaign more negatively on the issue in which they have an advantage.

Polborn and Yi (2006) develop a model of negative and positive campaigning wherein each candidate can reveal a (good) attribute about himself, or a (bad) attribute about a competitor, and voters update rationally about the information that is not transmitted. They predict that positive and negative campaigning are equally likely.

Brueckner and Kangoh (2013) explore negative campaigning in a probabilistic voting

[6] www.nytimes.com/2000/02/16/us/the-2000-campaigh-the-arizona-senator-mccain-catches-mud-then-parades-it.html.

model, wherein individual vote outcomes are stochastic due to the presence of a random, idiosyncratic valence effect along with other shocks that affect all voters in common. A relatively centrist candidates campaign more negatively than a relatively extreme candidate.

Peterson and Djupe (2005) empirically study the timing and the electoral context in which primary races are likely to become negative. Using a content analysis of newspaper coverage of contested Senate primaries, they find that negativity is an interdependent function of the timing during the race, the status of the Senate seat (whether the seat is open, whether the incumbent is in the primary, etc.), and the number and quality of the challengers in the primary (based on whether challengers previously held office).

Our paper relates more broadly to the literature on contests in which contestants exert both positive and negative efforts. In a work-place setting, Lazear (1989) argues that interactions between workers, whether it be cooperation or sabotage, affect the productivity of co-workers, making the firm's organization and its structure of relative compensation important. He argues that when rewards are based on relative comparisons, wage compression that leads to more equitable pay may reduce uncooperative behavior, but it may also act as a disincentive for better workers. Konrad (2000) explores the interaction of standard rent-seeking efforts that improve a contestant's own performance and sabotage efforts that reduce a rival's performance in lobbying contests. He argues that since sabotage against a group results in a positive externality for all other groups, greater numbers of lobbying groups make sabotage less attractive. Krakel (2005) analyzes two-stage, two-person tournaments in which each player can first help or sabotage a rival; and then players choose efforts. Helping or sabotaging a co-player affects both (1) the likelihood of winning and (2) equilibrium effort and, hence, effort costs. If effort costs dominate the likelihood effect on actions, asymmetric equilibria exist in which one player helps his opponent, and the other sabotages. Soubeyran (2009) proposes a general model of two player contests with two types of effort—attack and defense—and provides sufficient conditions for the existence and uniqueness of a symmetric Nash equilibrium. He then analyzes the effect of attack, i.e., negative campaigning, on voter turnout, and shows that it hinges on the distribution of voters' sensitivity to defense and attack.

Our paper is structured as follows. We next present the model and central results. Section 3 characterizes how the primitives of the environment affect outcomes. Section 4 concludes. Proofs are collected in an Appendix.

2 Model

There are three candidates, i, j and I. Candidates i and j belong to the same party, while I belongs to a different party and is presently in office. Challengers i and j first compete in a primary election, with the winner advancing to face the incumbent I in a general election. Candidates only care about who wins the general election—challenger $k \in \{i, j\}$ receives a payoff U_k from winning and a payoff V_k if his primary opponent wins the general election, and a normalized payoff of 0 if the incumbent wins, where $U_k > V_k > 0$. The incumbent receives a positive payoff if he is re-elected, and none if he loses.

Electoral outcomes are determined by a contest, where the probability a candidate wins rises with his reputation, and declines with his opponent's reputation. Specifically, in a primary election, if $\bar{Z}_{i0}$ and $\bar{Z}_{j0}$ are the candidates' respective reputational stocks just prior to the election, then candidate i wins with probability $\frac{\bar{Z}_{i0}}{\bar{Z}_{i0}+\bar{Z}_{j0}}$, and candidate j wins with residual probability. An analogous contest determines the winner of the general election.[7]

Candidate $k \in \{i, j, I\}$ starts out with an initial reputational stock of $\bar{X}_k$, and a resource budget $\bar{B}_k$. A candidate can devote his resources both to developing his own reputational stock via positive campaigning and to destroying his opponent's reputational stock via negative campaigning. In the primary election, if candidate $k \in \{i, j\}$ invests p_{k0} into positive campaigning to boost his own reputation, and his intra-party rival $\tilde{k}$ spends $n_{\tilde{k}0}$ on negatively campaigning to reduce k's reputation, then candidate k's reputational stock in the primary becomes $\bar{Z}_{k0} = \bar{X}_k \frac{(1+p_{k0})^\alpha}{(1+\rho n_{\tilde{k}0})^\alpha}$. Here $\alpha > 0$ captures the sensitivity of a candidate's reputational stock to campaigning and $\rho > 1$ captures the greater effectiveness of negative campaigning than positive campaigning on influencing candidate reputations. This structure allows us to reconcile *simultaneously* the preponderance of negative campaigning in general elections, and *positive* campaigning in primaries.

If candidate k wins the primary, he enters the general election with a reputational stock of $\bar{Z}_{k1} = \bar{X}_k \frac{(1+\beta p_{k0})^\alpha}{(1+\beta\rho n_{\tilde{k}0})^\alpha}$. Here, $\beta \in [0, 1]$ captures any decay in the effects of primary campaigns on his reputation prior to a general election. A small β, i.e., extensive decay, may reflect that voters largely forget primary campaigns by the time of the general election, or that primary campaigns appeal narrowly to party partisans, and the more moderate voters who

[7]Other papers using contest functions to model political competition include Klumpp and Polborn (2006) and Soubeyran (2009). See Konrad and Kovenock (2009) for other settings with multi-contest models.

determine the general election outcome pay less attention to primary campaigns. The extent of decay may vary with the electoral context. For example, only party partisans follow developments in their party primary campaigns for the House, but more voters follow presidential primary campaigns. In the general election, challenger k's final reputational stock is $\bar{Z}_{k2} = \bar{Z}_{k1}\frac{(1+p_{k1})^\alpha}{(1+\rho n_{I1})^\alpha}$, and the incumbent's final reputational stock is $\bar{Z}_{I2} = \bar{X}_I\frac{(1+p_{I1})^\alpha}{(1+\rho n_{k1})^\alpha}$, $k \in \{i, j\}$.

Challenger k's total electoral resource constraint is $\sum_t(p_{kt}+n_{kt}) \leq \bar{B}_k$, where $p_{kt}, n_{kt} \geq 0$. Thus, when challenger k wins the primary, he has funds $\bar{B}_k-(p^*_{k0}+n^*_{k0}) \equiv \bar{B}_{k1}$ at his disposal in the general election. The incumbent's resource constraint is $p_{I1} + n_{I1} \leq \bar{B}_I \equiv \bar{B}_{I1}$.

Without loss of generality, we write challenger $k \in \{i, j\}$'s ex ante expected payoff as

$$\pi_k = (M_k Pr_{k1} - Pr_{\tilde{k}1})Pr_{k0} + Pr_{\tilde{k}1},$$

where $M_k = \frac{U_k}{V_k}$ captures the relative payoff challenger k receives from personally winning the general election versus having his primary rival win, and Pr_{kt} is the probability that k wins the primary ($t = 0$) or general ($t = 1$) election. When we compare how differences in challengers affect how they campaign, it eases characterizations to assume that $M_k \geq 3$, i.e., challengers strongly prefer personally winning the general election to having a primary rival win.[8]

We pose our analysis in a setting where two challengers face off in a primary with the winner facing an incumbent in the general election. However, it follows directly that our analysis describes equilibrium outcomes when, rather than facing an incumbent in the general election, the two possibly heterogeneous challengers will face the winner of a primary in the opposing party, and the other party's candidates are symmetric in all regards. In this setting, the challengers care only about the equilibrium reputational stock and resources of their general election rival, and not about who wins the opposing party's primary.

We begin by characterizing equilibrium campaigning in a general election.

Proposition 1. *In equilibrium, in the general election, as long as candidate $k \in \{i, j, I\}$ has sufficient resources at his disposal so that $B_{k1} > \frac{\rho-1}{\rho}$, then candidate k engages in both*

[8]We also make the implicit premise that when one challenger is far stronger than his intra-party rival, the weak rival cares enough about personally winning that he prefers to enter the primary, even though this means that the incumbent is more likely to win re-election. Otherwise, the weaker rival would not enter the primary.

positive and negative campaigning, albeit allocating more resources to negative campaigning:

$$n_{k1}^* = \frac{B_{k1}}{2} + \frac{\rho - 1}{2\rho} \quad \text{and} \quad p_{k1}^* = \frac{B_{k1}}{2} - \frac{\rho - 1}{2\rho}.$$

If, instead, candidate $k \in \{i, j, I\}$ has only modest funds at his disposal, $B_{k1} < \frac{\rho-1}{\rho}$, then k only campaigns negatively, $n_{k1}^* = B_{k1}$ and $p_{k1}^* = 0$.

The proof follows directly. The probability challenger $k \in i, j$ defeats the incumbent is

$$Pr_{k1} = \frac{\bar{Z}_{k2}}{\bar{Z}_{k2} + \bar{Z}_{I2}} = \frac{\bar{Z}_{k1} \frac{(1+p_{k1})^\alpha}{(1+\rho n_{I1})^\alpha}}{\bar{Z}_{k1} \frac{(1+p_{k1})^\alpha}{(1+\rho n_{I1})^\alpha} + \bar{X}_I \frac{(1+p_{I1})^\alpha}{(1+\rho n_{k1})^\alpha}},$$

which, written as a function of what k controls in the general election, takes the form

$$Pr_{k1} = \frac{a(1 + p_{k1})^\alpha}{a(1 + p_{k1})^\alpha + b/(1 + \rho n_{k1})^\alpha},$$

where a and b are positive constants (see the Appendix). Multiplying the numerator and denominator by $(1 + \rho n_{k1})^\alpha / b$, and simplifying yields

$$Pr_{k1} = \frac{a[(1 + p_{k1})^\alpha (1 + \rho n_{k1})^\alpha]/b}{a[(1 + p_{k1})^\alpha (1 + \rho n_{k1})^\alpha]/b + 1},$$

implying that challenger k maximizes $(1+p_{k1})^\alpha(1+\rho n_{k1})^\alpha \equiv [P_{k1}N_{k1}]^\alpha$ in the general election.

In the general election, only the relative effectiveness of each form of campaigning in influencing election outcomes matters for how candidates allocate their resources. Because $\rho > 1$ means that negative campaigning is more effective than positive campaigning—it is easier to tear down a reputation than build one up—candidates spend more on negative campaigns than positive ones, and whenever their resources are sufficiently limited, they only negatively campaign. That is, especially weak/underfunded challengers only campaign "against" an incumbent in the general election, and the greater are a candidate's resources, the greater is the share devoted to positive campaigning.

Proposition 2 establishes that candidates campaign relatively more positively in the primary than in the general election.

Proposition 2. *In equilibrium, provided candidate $k \in \{i, j\}$ devotes any resources to positive campaigning, he campaigns relatively more positively in the primary than in the general election: $n_{k0}^* - p_{k0}^* \leq n_{k1}^* - p_{k1}^* = \frac{\rho-1}{\rho}$, where the inequality is strict unless $\beta = 0$.*

As long as the effects of primary campaigns do not fully decay before the general election, a candidate campaigns relatively more positively in the primary than in the general election for two reasons: (1) there is a lingering beneficial effect of positive campaigning in the primary on a challenger's reputation in the general election, should he survive the primary election; and (2) the adverse effects of negative campaigning in the primary against a party rival also carry over to the general election, should the latter emerge from the primary victoriously. Both effects induce a challenger to allocate relatively more resources in a primary toward positive campaigns, and away from negative campaigns.

In much of the analysis that follows we will assume that candidates have sufficient resources, so that equilibrium is characterized by first-order conditions. We will also often assume that challengers are symmetrically situated:

Assumption A1 (sufficient resources): Candidates have sufficient resources that they devote positive resources to both positive and negative campaigning.

Assumption A2 (symmetry): Challengers i and j have identical reputations, $\bar{X}_i = \bar{X}_j \equiv \bar{X}_C$, resources, $\bar{B}_i = \bar{B}_j \equiv \bar{B}_C$ and preferences, $U_i = U_j \equiv U_C, V_i = V_j \equiv V_C$.

We next characterize how primary campaigns are affected when more of the effects of primary campaigning on reputations persist to the general election, i.e., when β is larger.

Proposition 3. *Under* **(A1)** *and* **(A2)** *there exists a* $\hat{\beta} \in (0, 1]$*, such that if* $\beta < \hat{\beta}$*, marginal increases in* β

1. *Reduce negative campaigning in the primary election.*
2. *Increase positive campaigning in the primary as long as the resources* $\bar{B}_C$ *of challengers are not too great.*
3. *Increase total primary campaigning expenditures when the effectiveness* ρ *of negative campaigning is not too high.*

That is, when the beneficial effects of primary campaigns persist more strongly, candidates campaign relatively more positively and less negatively in the primary. Moreover, if negative campaigning is not too much more effective than positive campaigning (i.e., ρ is close enough to one), the end result is that the increase in positive campaigning exceeds the decrease in negative campaigning, causing total primary campaigning expenditures to rise.

We next show that if enough of the effects of primary campaigning persist to the general election, challengers campaign more positively than negatively in the primary, regardless of the relative effectiveness of negative campaigning:

Proposition 4. *Under* **(A1)** *and* **(A2)** *there exists a* $\beta^* \in (0, 1]$ *such that if* $\beta \geq \beta^*$, *then* $p^*_{k0} > n^*_{k0}$. *That is, when enough of the effects of primary campaigns persist, challengers campaign more positively than negatively in the primary.*

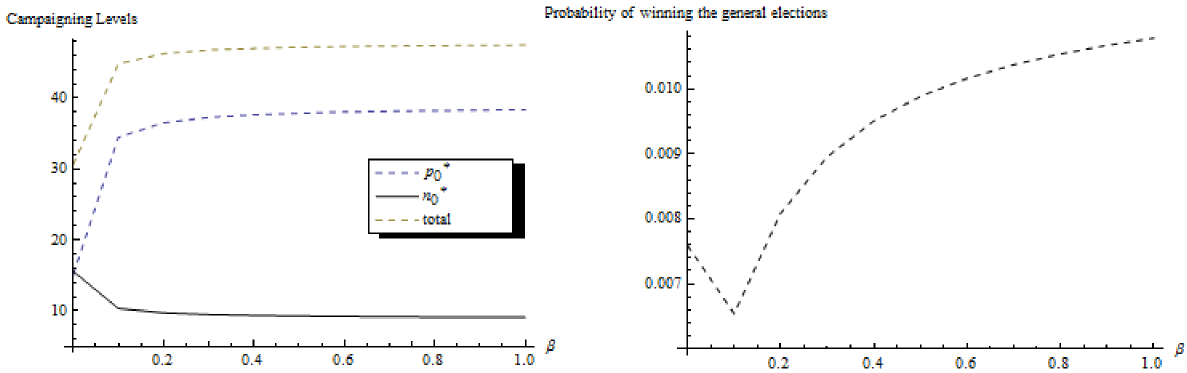

Figure 1: Primary campaigning and probability that a challenger wins in the general election as a function of the extent to which primary campaigning decays when challengers have limited resources. Parameters: $\bar{X}_I = 100, \bar{B}_I = 200, \bar{X}_k = 50, \bar{B}_k = 100, M_k = 10, \rho = 2, \alpha = 2, k \in \{i, j\}$.

The analytical characterizations in Propositions 3 and 4 only obtain when β is sufficiently small or large. However, a numerical analysis indicates that these results extend to intermediate levels of β. That is, as more of the effects of primary campaigning persist to affect general election reputations, positive campaigning in the primary rises while negative campaigning falls. Consistent with Proposition 3, Figure 1 illustrates that when there is nearly complete decay in effects of primary campaigning prior to the general election, total primary campaign expenditures *rise* with β reflecting the strategic complementarities in campaigning. A surprising consequence is that as β first rises from zero, the probability a challenger actually wins the general election *falls*. That is, when challengers internalize greater persistent effects of primary campaigning on general election outcomes, their probability of winning the general election first falls because they have less resources remaining to devote to the general election, and the modest persistence in the effects of primary campaigning means that little of those effects carry over. It is only when enough of the effects of primary campaigning persist that the probability a challenger wins rises with further increases in β. Figure 1 illustrates a scenario where the incumbent is far stronger than the challengers; but similar qualitative patterns hold when challengers are stronger.

3 Comparative statics

We next derive how the primitives describing the electoral environment affect campaigning when challengers are almost symmetric, and β is small enough that primary campaigning has modest effects on a candidate's reputation in the general election. The latter may be expected to hold in elections for the House or Senate where only party partisans follow developments in their party primaries, but moderate voters follow general election campaigning closely.

We first note that when β is close to zero, the direction of the impact of changes in primitives is the same for n_{i0}^* and p_{i0}^* at an interior optimum, since $n_{i0}^* - p_{i0}^* \to \frac{\rho-1}{\rho}$ (save possibly for a change in ρ). This means that one can derive the effect on the direction of changes in n_{i1}^* and p_{i1}^* via the impact on the resource constraint, i.e., n_{i1}^* and p_{i1}^* decline if and only if n_{i0}^* (and hence p_{i0}^*) rise. At an equilibrium $(n_{i0}^*, p_{i0}^*, n_{j0}^*, p_{j0}^*)$,

$$\underbrace{\frac{\partial^2 \pi^i}{\partial \theta \partial n_{i0}}}_{\text{direct effect of a change in } \theta} + \overbrace{\frac{\partial^2 \pi^i}{\partial n_{i0}{}^2}\frac{dn_{i0}}{d\theta} + \frac{\partial^2 \pi^i}{\partial p_{i0} \partial n_{i0}}\frac{dp_{i0}}{d\theta}}^{\text{indirect effect via change in } i\text{'s actions}} + \underbrace{\frac{\partial^2 \pi^i}{\partial p_{j0} \partial n_{i0}}\frac{dp_{j0}}{d\theta} + \frac{\partial^2 \pi^i}{\partial n_{j0} \partial n_{i0}}\frac{dn_{j0}}{d\theta}}_{\text{indirect effect via change in } j\text{'s actions}} = 0.$$

for $\theta \in \bar{X}_I, \bar{B}_I, \alpha$. Also, when challengers are symmetric, a change in their resources, preferences or reputations involves equal changes in both $\bar{B}_i, \bar{B}_j$, or M_i, M_j, or $\bar{X}_i, \bar{X}_j$ so that

$$\underbrace{\frac{\partial^2 \pi^i}{\partial \theta_i \partial n_{i0}} + \frac{\partial^2 \pi^i}{\partial \theta_j \partial n_{i0}}}_{\text{direct effect of a change in } \theta_i, \theta_j} + \overbrace{\frac{\partial^2 \pi^i}{\partial n_{i0}{}^2}\left(\frac{dn_{i0}}{d\theta_i} + \frac{dn_{i0}}{d\theta_j}\right) + \frac{\partial^2 \pi^i}{\partial p_{i0} \partial n_{i0}}\left(\frac{dp_{i0}}{d\theta_i} + \frac{dp_{i0}}{d\theta_j}\right)}^{\text{indirect effect via change in } i\text{'s actions}}$$

$$+ \underbrace{\frac{\partial^2 \pi^i}{\partial p_{j0} \partial n_{i0}}\left(\frac{dp_{j0}}{d\theta_i} + \frac{dp_{j0}}{d\theta_j}\right) + \frac{\partial^2 \pi^i}{\partial n_{j0} \partial n_{i0}}\left(\frac{dn_{j0}}{d\theta_i} + \frac{dn_{j0}}{d\theta_j}\right)}_{\text{indirect effect via change in } j\text{'s actions}} = 0.$$

Lemma 1. *Under* **(A1)** *and* **(A2)** *when* $\beta = 0$, $sign\{\frac{dn_{i0}}{d\theta}\} = sign\{\frac{\partial^2 \pi^i}{\partial\theta\partial n_{i0}}\}$ *and* $sign\{\frac{dn_{i0}}{d\theta_i} + \frac{dn_{i0}}{d\theta_j}\} = sign\{\frac{\partial^2 \pi^i}{\partial\theta_i\partial n_{i0}} + \frac{\partial^2 \pi^i}{\partial\theta_j\partial n_{i0}}\}$.

The lemma states that the indirect effect of a change in a parameter on n^*_{i0}, p^*_{i0} in a symmetric equilibrium, reinforces, or if in the opposite direction, is outweighed by the direct effect. This means that we can derive the signs of changes in n^*_{i0} and p^*_{i0} from the signs of the partial derivatives, $\frac{\partial^2 \pi}{\partial\theta\partial n_{i0}}$ and $\frac{\partial^2 \pi^i}{\partial\theta_i\partial n_{i0}} + \frac{\partial^2 \pi^i}{\partial\theta_j\partial n_{i0}}$, alone. Proposition 5 derives the impacts of changes in the challengers' resources, reputations and preferences on primary campaigning when challengers are symmetric.[9]

Proposition 5. *Under* **(A0)** *and* **(A1)**, *for all* β *sufficiently small,*[10]

1. *Improving challenger reputations,* $\bar{X}_C$, *causes challengers to increase both positive and negative primary campaigning. Their campaigning expenditures in the general election fall, but the probability they defeat the incumbent rises.*

2. *Greater challenger resources,* $\bar{B}_C$, *cause challengers to increase positive and negative campaigning in both elections. The probability a challenger defeats the incumbent rises.*

3. *Increasing challenger payoffs,* U_C, *from winning office, or reducing payoffs,* V_C, *when a party rival wins office, causes challengers to raise both positive and negative primary campaigning. The probability that a challenger defeats the incumbent falls.*

When challengers have better reputations, whoever wins the party primary is more likely to defeat the incumbent. As a result, challengers campaign more aggressively in the primary. Reinforcing the direct effect, when i campaigns more negatively in the primary, this causes j to campaign more negatively, too. This is both because i's negative campaign reduces j's chances of winning (and j prefers to personally win); and, with less funds for the general election, i's chances of defeating the incumbent also fall, making j more worried i's chances of winning the general election if and when he defeats j in the primary.

Greater resources allow challengers to spend more in both elections, improving their chances of winning both races. The effects of greater resources differ from those of better

[9]The analytical results in Propositions 5–6 extend if challengers are *close enough* to being identical.

[10] An extensive numerical analysis suggests that the results in Propositions 5–7 hold regardless of the extent of persistence β in the effects of primary campaigning on general election reputations.

reputations: improving challenger reputations raises primary campaigning expenditures, but reduces general election expenditures; in contrast, increased resources are spread across both elections. This difference reflects the fungibility of campaign resources, but not reputations—resources can be divided arbitrarily between campaigns, but reputations cannot.

Finally, increases in $M_C = U_C/V_C$ mean that challengers care more about winning than just about ousting the incumbent. As a result, the challengers campaign more aggressively against each other, reducing the chances that their party's nominee wins the general election.

Proposition 6. *Under* **(A0)** *and* **(A1)***, for all sufficiently small β, increasing an incumbent's resources, $\bar{B}_I$, or improving his reputation, $\bar{X}_I$, causes both challengers to reduce both positive and negative campaigning in the primary. Their campaigning expenditures in the general election rise, but the probability they defeat the stronger incumbent falls.*

Facing a stronger incumbent, challengers reduce primary campaigning in order to save more for the tougher general election. Further, the strategic complementarities in campaigning mean that when i spends less on campaigning in the primary, so does j. Again this reflects that since i saves more funds for the general election, i is more likely to defeat the incumbent. As a result, j is less worried about losing the primary to i.

Proposition 7. *Under* **(A0)** *and* **(A1)***, for all sufficiently small β, there exists a $\bar{B}_i^*(\beta)$, such that if and only if challenger resources exceed $\bar{B}^*(\beta)$, increasing the sensitivity, α, of reputations to campaigning causes challengers to increase primary campaigning and reduce general election campaigning, and the probability that they defeat the incumbent rises.*

When reputations are more sensitive to campaigning, negative campaigning in the primary falls if and only if challengers have sufficiently limited resources that they will have less funds in the general election than the incumbent. This is because the challengers, when crippled by low budgets, are unable to campaign as intensely as the incumbent in the general election. Increasing the sensitivity of reputations to campaigning aggravates this disadvantage, causing them to reduce their primary campaigning. Even though the challengers devote more of their resources to the general election, they still fail to match the incumbent's spending, so their chances of defeating the incumbent fall as α rises. The opposite occurs if the challengers will have more resources at their disposal in the general election than the incumbent; and if $\bar{B}_i = \bar{B}_i^*$, increasing the sensitivity of reputations to campaigning has no effect on outcomes.

We conclude our analysis by investigating how differences between the two challengers affect how they campaign. We first derive how *small* differences affect challenger choices.

Proposition 8. *Under* **(A0)** *and* **(A1)***, for all β sufficiently small,*

1. *Improving challenger i's reputation, $\bar{X}_i$, causes challenger j to increase his primary campaigning, reducing his chances of winning the general election. Improving challenger i's reputation causes i to campaign more aggressively in the primary if and only if the incumbent is sufficiently weak.*
2. *Increasing challenger i's resources, $\bar{B}_i$, causes both challengers to increase both positive and negative primary campaigning.*
3. *Raising challenger i's payoff, U_i, from being elected to office, or reducing his payoff, V_i, if his party opponent wins office, causes both challengers to spend more on primary campaigns, reducing their chances of winning the general election.*

When the challangers have nearly equal chances of defeating the incumbent, improving challenger i's reputation causes his rival to increase positive and negative primary campaigning. There are offsetting considerations. On the one hand, challenger i's better reputation improves the party's chances in the general election should i win the primary, providing j an incentive to campaign less aggressively in the primary. On the other hand, challenger i's stronger reputation also helps him in the primary, and his rival prefers to personally win the general election than just to have i defeat the incumbent from the other party. As long as $M_j = U_j/V_j \geq 3$, the desire to personally win the general election dominates, so that making challenger i is a little stronger causes his rival to step up his primary campaigning.

One might think that improving challenger i's reputation always encourages him to coast on it and reduce his primary expenditures, to conserve more resources for the general election. In fact, i does this only if the incumbent is sufficiently strong. When the incumbent is relatively weak, i.e., when i is likely to win the general election if he manages to win the primary, then improving i's reputation causes him to devote *more* resources to the primary. With a weak incumbent (e.g., with a scandal-ridden incumbent, or a weak challenger from the opposing party's primary), challenger i's toughest battle becomes the primary, and he responds to the increased campaigning intensity by challenger j by raising his primary campaigning.

In contrast, to the ambiguous effects of improving a challenger i's reputation on his primary campaigning, increasing his resources causes *both* challengers to campaign more aggressively in the primary. That challenger j raises his primary campaigning expenditures reflects the same considerations that drive him when his primary opponent's reputation improves—as long as j cares enough more about winning than about just having his party's nominee win, he devotes more resources to defeating his stronger primary rival. However, the impact of giving i a little more resources no longer depends on the strength of the incumbent—he always campaigns more aggressively in both the primary and the general election. The difference between greater reputation and more resources reflects that campaign resources are fungible, and can be allocated freely across both elections, but reputations are not.

Finally, Proposition 8 shows that when challenger i cares more about personally winning, he steps up his primary campaigning. This raises his chances of winning, but lowers the probability that, conditional on winning the primary, he defeats the incumbent in the general election. In turn, challenger j campaigns more aggressively in the primary, both because j wants to win personally, and because i's greater primary expenditures reduce his chances of defeating the incumbent.

These analytical characterizations describe how small differences between challengers affect how they campaign. We now show that when one challenger is far stronger than the other—with a greater reputation or resources—then the effects of increased differences between challenges change radically, hinging on both the incumbent's strength and the extent to which the effects of primary campaigning persist to affect general election reputations.

Figures 2 and 3 illustrate that if the incumbent is strong, then i's resource expenditures in the primary are a single-peaked function of his reputation. This reflects that if i has a far better reputation than j, then i would rather save most of his resources for the general election, and his stronger reputation allows him to do so, while remaining reasonably confident of winning the primary. Now, as challenger i's advantage grows further, challenger j responds not by increasing his primary campaign expenditures, but by *reducing* them. This is because challenger j internalizes the fact that i is far more likely than he to win the general election,

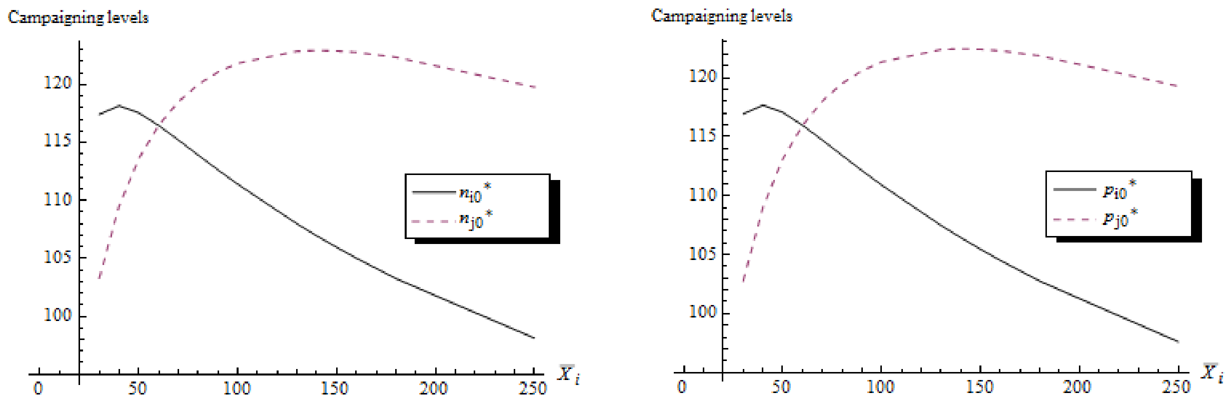

Figure 2: Primary campaigning as a function of challenger i's reputation when the incumbent is strong and the effects of primary campaigning decay. Parameters: $\beta = 0, \bar{B}_I = 900, \bar{X}_I = 150, \bar{X}_j = 60, \bar{B}_j = \bar{B}_i = 700, M_j = M_i = 10, \alpha = 1, \rho = 2$.

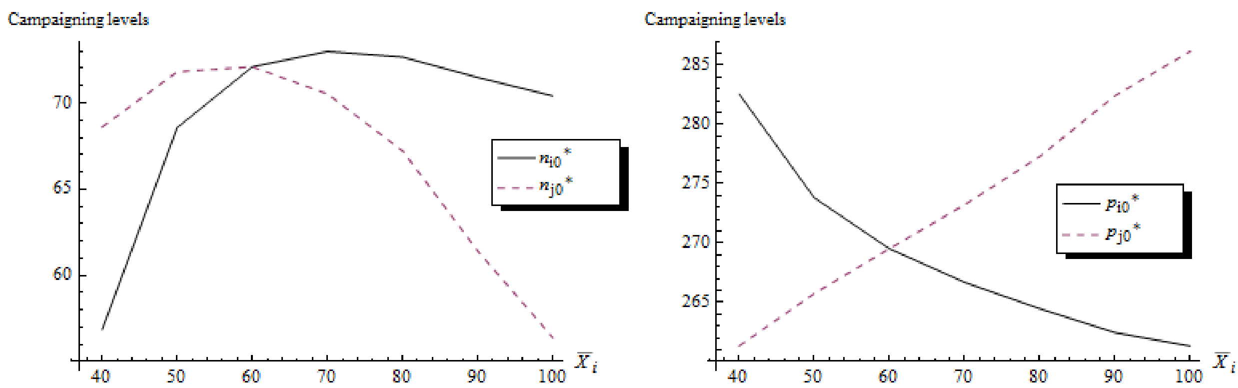

Figure 3: Primary campaigning as a function of challenger i's reputation when the incumbent is strong and effects of primary campaigning persist. Parameters: $\beta = 1, \bar{B}_I = 900, \bar{X}_I = 150, \bar{X}_j = 60, \bar{B}_j = \bar{B}_i = 700, M_j = M_i = 10, \alpha = 1, \rho = 2$.

that negative primary campaigning reduces that probability, and that greater primary campaigning reduces his own already low chances of winning the general election. In contrast, when i is far weaker than j, slight improvements in i's reputation cause j to campaign more aggressively in the primary, as j now worries more that i may win the primary.

Contrasting Figures 2 and 3 reveals that the composition of primary campaigning hinges on the extent to which the effects of primary campaigning persist. When the effects of primary campaigning persist, candidates campaign less negatively in the primary, but *far* more positively. When $\beta = 1$, and challenger i's reputation is low, initial increases in his rep-

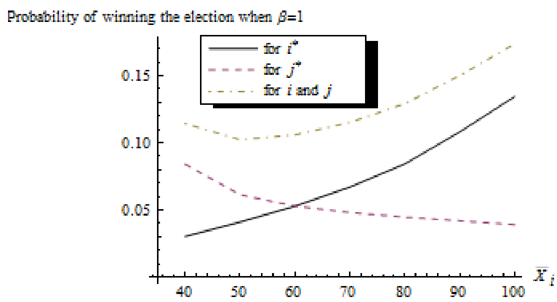

Figure 4: Probability of winning general election as a function of challenger i's reputation when the incumbent is strong and effects of primary campaigning persist. Parameters: $\beta = 1, \bar{B}_I = 900, \bar{X}_I = 150, \bar{X}_j = 60, \bar{B}_j = \bar{B}_i = 700, M_j = M_i = 10, \alpha = 1, \rho = 2$.

utational stock cause him to increase his negative primary campaigning (which raises his chances of winning the primary at his rival's expense in the general election), accompanied by a decline in positive campaigning expenditures that allows him to conserve resources for the general election against a strong incumbent. However, once i is sufficiently stronger than j, further increases in i's reputation lead him to reduce his negative primary campaigning, as his strong reputation is now more likely to be enough to secure a primary win. The effects of further increases in i's reputation on j's campaigning now change: when i is far stronger, j responds by *sharply* reducing his negative campaigning so as to not adversely affect his strong party rival's chances in the general election; and when the effects of primary campaigning persist, j partially compensates for his reduced negative primary campaigning by stepping up his positive primary campaigning. By doing so, j improves his own chances of winning without harming his strong rival's chances against the incumbent.

Figure 4 shows that making one challenger stronger can *lower* the probability that they defeat a strong incumbent. Increasing challenger i's reputation obviously raises his own chances of winning. What Figure 4 shows is that the challengers' collective probability of winning can be a U-shaped function of i's reputation. In particular, when their reputations are similar, making i a little stronger leads to such enhanced primary competition that their collective probability of defeating the incumbent falls. Their probability of defeating the incumbent only begins to rise with challenger i's reputation once i is sufficiently stronger than j, as j then internalizes that his chances are low enough that he should compete less negatively against i. In turn, this makes i more confident of a primary victory, encouraging him to preserve more resources for the general election contest against the incumbent.

In sharp contrast, when the incumbent is weak, then as Figure 5 illustrates, increasing one candidate's reputation or resources causes both challengers to increase their primary campaigning. With a weak incumbent, the primary winner is very likely to win the general election, so when challenger i becomes stronger, with a better reputation or greater resources, challenger j steps up his primary campaigning to raise the probability that he defeats the now stronger primary opponent. Strategic complementarities in campaigning reinforce this effect, causing both challengers to increase primary campaigning, even though this can adversely reduce the probability that their party's nominee wins the general election.

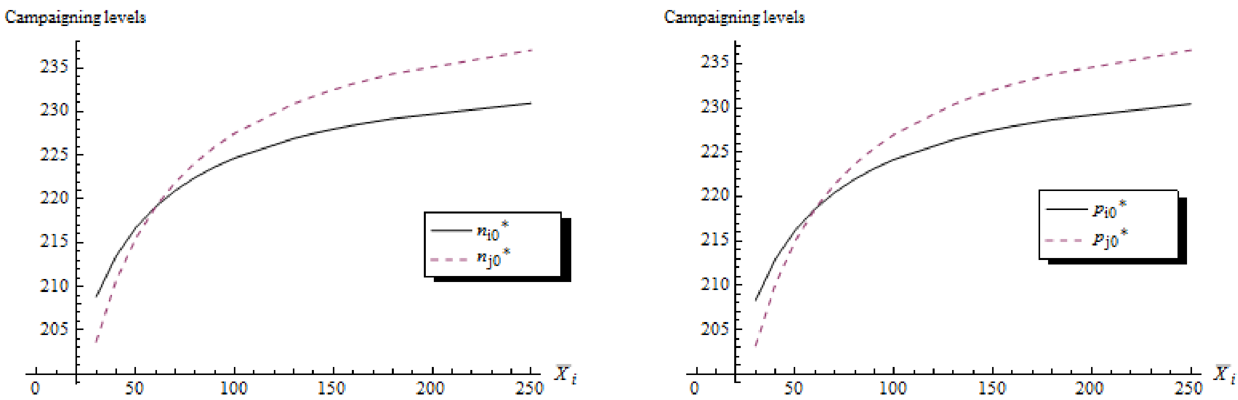

Figure 5: Primary campaigning as a function of challenger i's reputation when the incumbent is weak and the effects of primary campaigning decay. Parameters: $\beta = 0, \bar{B}_I = 100, \bar{X}_I = 150, \bar{X}_j = 60, \bar{B}_j = \bar{B}_i = 700, M_j = M_i = 10, \alpha = 1, \rho = 2$.

Numerical investigations confirm that the qualitative impacts of a stronger challenger are similar regardless of whether a challenger is stronger due to having more resources or a better reputation, subject to the caveat regarding the fungibility of resources. This reflects that the key strategic force is the strength of one challenger relative to the other challenger and to the incumbent, and not the source of the strength. Concretely, when one challenger is far stronger than the other, then when the winner will face a strong incumbent, further increases in that challenger's strength cause his intra-party rival to sharply reduce his negative primary campaigning and to raise his positive campaigning. Thus, for example, our model can reconcile the composition of campaigning in the 2012 Republican primary by Romney and his rivals—the far stronger Romney campaigned far more negatively than positively, whereas his weaker rivals did the opposite. Moreover, when Gingrich gained in reputation following the South

Carolina primary, Romney massively increased his negative campaigning against Gingrich.

4 Conclusion

Negative campaigning has come to dominate the airwaves in general elections throughout the United States. What has received far less attention is that campaigning in primaries tends to be more positive in nature. Our paper provides an explanation for these observations and others. The negative general election campaigns reflect that candidates only care about winning the general election—and it is easier to damage an opponent's reputation via negative campaigning than to build up one's own reputation via positive campaigning. The more positive nature of primary campaigns reflects that candidates internalize both that (a) a primary winner benefits in the general election only from his positive primary campaigning; and (b) a primary loser, impairs his party rival's chances in the general election by campaigning negatively.

More generally, we characterize how the primitives of the electoral environment—relative candidate strengths (initial resources and reputations), the campaigning technology (effectiveness, extent of decay in the general election effects of primary campaigning), and candidate preferences over electoral outcomes (personally winning vs. having the party's nominee win)—affect the magnitudes and composition of campaigning in both elections, and thus electoral outcomes. Thus, if challengers are similarly situated, improving one challenger's reputation causes his primary rival to campaign more aggressively; but the challenger with the enhanced reputation only campaigns more aggressively if the incumbent is sufficiently weak, as then the primary winner is also the likely general election winner. In contrast, making a far stronger challenger even stronger causes his weak intra-party rival to reduce his negative campaigning, due to a desire for the party's nominee to defeat the incumbent. This, in turn, encourages the stronger challenger to reduce his primary campaigning, although the stronger challenger still spends far more, and is far more negative than his weak rival. Thus, our model reconciles the pattern of primary campaigning in the 2012 Republican primary—Romney had a stronger reputation and far more resources, and hence he spent far more, and was far more negative than his primary rivals. This contrasts sharply with our predictions for general elections, where we predict that candidates with more resources campaign relatively more positively, and that especially weak challengers may only campaign negatively.

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6 Appendix

Proof of Propositions 1 and 2. Conditional on winning the primary, candidate i defeats the incumbent in the general election with probability

$$Pr_{i1} = \frac{\bar{Z}_{i2}}{\bar{Z}_{i2} + \bar{Z}_{I2}} = \frac{\bar{Z}_{i1}\frac{(1+p_{i1})^\alpha}{(1+\rho n_{I1})^\alpha}}{\bar{Z}_{i1}\frac{(1+p_{i1})^\alpha}{(1+\rho n_{I1})^\alpha} + \bar{X}_I\frac{(1+p_{I1})^\alpha}{(1+\rho n_{i1})^\alpha}} = \frac{a[(1+p_{i1})^\alpha(1+\rho n_{i1})^\alpha]/b}{a[(1+p_{i1})^\alpha(1+\rho n_{i1})^\alpha]/b+1}.$$

where a and b are positive constants outside of i's control obtained by dividing numerator and denominator by $\bar{X}_I\frac{(1+p_{I1})^\alpha}{(1+\rho n_{i1})^\alpha}$. Candidate i maximizes this probability by maximizing

$$\max_{p_{i1},n_{i1}} (1+p_{i1})^\alpha(1+\rho n_{i1})^\alpha \text{ subject to } p_{i1}+n_{i1} \leq \bar{B}_i - (p_{i0}+n_{i0}) \equiv B_{i1},\ p_{i1}, n_{i1} \geq 0.$$

Since the probability of winning increases in both p_{i1}, n_{i1}, the resource constraint binds, i.e., $p_{i1} + n_{i1} = B_{i1}$. Thus, i's maximization problem reduces to

$$\max_{p_{i1},n_{i1}} (1+p_{i1})^\alpha(1+\rho n_{i1})^\alpha \text{ subject to } p_{i1}+n_{i1} = B_{i1},\ p_{i1}, n_{i1} \geq 0.$$

Hence, the challenger's optimal choices of negative and positive campaigning are

$$p^*_{i1} = \begin{cases} \frac{B_{i1}}{2} - \frac{\rho-1}{2\rho}, & \text{if } \rho-1-\rho B_{i1} \leq 0 \\ 0, & \text{if } \rho-1-\rho B_{i1} > 0 \end{cases} \quad \text{and} \quad n^*_{i1} = \begin{cases} \frac{B_{i1}}{2} + \frac{\rho-1}{2\rho}, & \text{if } \rho-1-\rho B_{i1} \leq 0 \\ B_{i1}, & \text{if } \rho-1-\rho B_{i1} > 0. \end{cases}$$

Similarly, the incumbent's campaigning solves

$$\max_{p_{I1},n_{I1}} (1+p_{I1})^\alpha(1+\rho n_{I1})^\alpha \text{ subject to } p_{I1}+n_{I1} = \bar{B}_I,\ p_{I1}, n_{I1} \geq 0,$$

with solution

$$n^*_{I1} = \frac{\rho-1}{2\rho} + \frac{\bar{B}_I}{2} \quad \text{and} \quad p^*_{I1} = \frac{\bar{B}_I}{2} - \frac{\rho-1}{2\rho}$$

if $\bar{B}_I > \frac{\rho-1}{\rho}$ and $n^*_{I1} = \bar{B}_I$ if $\bar{B}_I \leq \frac{\rho-1}{\rho}$.

The probability candidate i wins the primary is $Pr_{i0} = \frac{\bar{Z}_{i0}}{\bar{Z}_{i0}+\bar{Z}_{j0}}$. Since i only cares about winning the general election, he chooses p_{i0} and n_{i0} to maximize

$$\pi^i = (M_i Pr_{i1} - Pr_{j1})Pr_{i0} + Pr_{j1}.$$

To ease presentation, define $P_{it} = 1 + p_{it}$ and $N_{it} = 1 + \rho n_{it}$ where $t \in \{0, 1\}$, and let

$Q^*_{i0} = \frac{\bar{X}_j (P^*_{j0} N^*_{j0})^\alpha}{\bar{X}_i (P^*_{i0} N^*_{i0})^\alpha}, Q^*_{i1} = \frac{\bar{X}_I (P^*_I N^*_I (1-\beta+\beta N^*_{j0}))^\alpha}{\bar{X}_i (P^*_{i1} N^*_{i1} (1-\beta+\beta P^*_{i0}))^\alpha}$, with analogous expressions for j. Suppose that i has enough resources that he campaigns positively in the general election. Then, at an interior optimum, his primary campaigning choices (N^*_{i0}, P^*_{i0}) satisfy the first-order conditions:

$$\begin{aligned}
\frac{\partial \pi^i}{\partial N_{i0}} &= (M_i Pr_{i1} - Pr_{j1}) \frac{\partial Pr_{i0}}{\partial N_{i0}} + Pr_{i0} \left(M_i \frac{\partial Pr_{i1}}{\partial N_{i0}} - \frac{\partial Pr_{j1}}{\partial N_{i0}} \right) + \frac{\partial Pr_{j1}}{\partial N_{i0}} \\
&= \Big(\frac{M_i}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}} \Big) \frac{\alpha Q^*_{i0}}{N^*_{i0}(1+Q^*_{i0})^2} - \frac{1}{1+Q^*_{i0}} \Big(\frac{\alpha M_i Q^*_{i1}}{N^*_{i1}(1+Q^*_{i1})^2} - \frac{\alpha \beta Q^*_{j1}}{(1-\beta+\beta N^*_{i0})(1+Q^*_{j1})^2} \Big) \\
&\qquad - \frac{\alpha \beta Q^*_{j1}}{(1-\beta+\beta N^*_{i0})(1+Q^*_{j1})^2} = 0, \\
\frac{\partial \pi^i}{\partial P_{i0}} &= (M_i Pr_{i1} - Pr_{j1}) \frac{\partial Pr_{i0}}{\partial P_{i0}} + M_i Pr_{i0} \frac{\partial Pr_{i1}}{\partial P_{i0}} \\
&= \Big(\frac{M_i}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}} \Big) \frac{\alpha Q^*_{i0}}{P^*_{i0}(1+Q^*_{i0})^2} - \frac{\alpha M_i Q^*_{i1}}{(1+Q^*_{i0})(1+Q^*_{i1})^2} \Big(\frac{1}{P^*_{i1}} - \frac{\beta}{1-\beta+\beta P^*_{i0}} \Big) = 0.
\end{aligned}$$

After canceling terms on both sides, these first-order conditions simplify to:

$$\Big(\frac{M_i}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}} \Big) \frac{Q^*_{i0}}{N^*_{i0}(1+Q^*_{i0})} = \frac{M_i Q^*_{i1}}{N^*_{i1}(1+Q^*_{i1})^2} + \frac{\beta Q^*_{j1} Q^*_{i0}}{(1-\beta+\beta N^*_{i0})(1+Q^*_{j1})^2} \tag{1}$$

and

$$\Big(\frac{M_i}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}} \Big) \frac{Q^*_{i0}}{P^*_{i0}(1+Q^*_{i0})} = \frac{M_i Q^*_{i1}}{(1+Q^*_{i1})^2} \Big(\frac{1}{P^*_{i1}} - \frac{\beta}{1-\beta+\beta P^*_{i0}} \Big). \tag{2}$$

Dividing equation (2) by (1), and then rearranging terms, yields

$$\frac{N^*_{i0}}{P^*_{i0}} \Big(\frac{M_i Q^*_{i1}}{N^*_{i1}(1+Q^*_{i1})^2} + \frac{\beta Q^*_{j1} Q^*_{i0}}{(1-\beta+\beta N^*_{i0})(1+Q^*_{j1})^2} \Big) = \frac{M_i Q^*_{i1}}{P^*_{i1}(1+Q^*_{i1})^2} - \frac{\beta M_i Q^*_{i1}}{(1-\beta+\beta P^*_{i0})(1+Q^*_{i1})^2}.$$

Multiplying both sides by P^*_{i0} yields

$$\frac{M_i Q^*_{i1}}{(1+Q^*_{i1})^2} \Big(\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} + \frac{\beta P^*_{i0}}{1-\beta+\beta P^*_{i0}} \Big) + \frac{\beta N^*_{i0} Q^*_{j1} Q^*_{i0}}{(1-\beta+\beta N^*_{i0})(1+Q^*_{j1})^2} = 0. \tag{3}$$

At an interior optimum, $n^*_{i0}, p^*_{i0} > 0$, or equivalently $N^*_{i0}, P^*_{i0} > 1$. The second term of equation (3) is positive, so the first grouped term must be negative, and hence

$$\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} < 0 \Longleftrightarrow \frac{P^*_{i0}}{N^*_{i0}} > \frac{P^*_{i1}}{N^*_{i1}} = \frac{1}{\rho},$$

or equivalently,

$$n^*_{i0} - p^*_{i0} < n^*_{i1} - p^*_{i1} = \frac{\rho - 1}{\rho}.$$

When $\beta = 0$, equations (1) and (2) reduce to

$$\left(\frac{M_i}{1+Q_{i1}^*} - \frac{1}{1+Q_{j1}^*}\right)\frac{Q_{i0}^*}{N_{i0}^*(1+Q_{i0}^*)} - \frac{M_i Q_{i1}^*}{N_{i1}^*(1+Q_{i1}^*)^2} = 0 \tag{4}$$

and

$$\left(\frac{M_i}{1+Q_{i1}^*} - \frac{1}{1+Q_{j1}^*}\right)\frac{Q_{i0}^*}{P_{i0}^*(1+Q_{i0}^*)} - \frac{M_i Q_{i1}^*}{P_{i1}^*(1+Q_{i1}^*)^2} = 0. \tag{5}$$

Then, as before, it follows that

$$\frac{P_{i0}^*}{P_{i1}^*} - \frac{N_{i0}^*}{N_{i1}^*} = 0 \Longleftrightarrow \frac{P_{i0}^*}{N_{i0}^*} = \frac{P_{i1}^*}{N_{i1}^*} = \frac{1}{\rho} \Longleftrightarrow n_{i0}^* - p_{i0}^* = n_{i1}^* - p_{i1}^* = \frac{\rho-1}{\rho}. \tag{6}$$

One can show that even if challengers have less resources so they only campaign negatively in the general election, at an interior optimum in the primary, $\rho P_{i0}^* = N_{i0}^*$ or $n_{i0}^* - p_{i0}^* = \frac{\rho-1}{\rho}$. ■

Proof of Proposition 3. Omitting the i index on π, the following equations hold at a symmetric equilibrium since $\frac{dN_{i0}}{d\beta} = \frac{dN_{j0}}{d\beta}$ and $\frac{dP_{i0}}{d\beta} = \frac{dP_{j0}}{d\beta}$:

$$\frac{\partial^2\pi}{\partial\beta\partial N_{i0}} + \left(\frac{\partial^2\pi}{\partial {N_{i0}}^2} + \frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right)\frac{dN_{i0}}{d\beta} + \left(\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}}\right)\frac{dP_{i0}}{d\beta} = 0 \tag{7}$$

and

$$\frac{\partial^2\pi}{\partial\beta\partial P_{i0}} + \left(\frac{\partial^2\pi}{\partial N_{i0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}}\right)\frac{dN_{i0}}{d\beta} + \left(\frac{\partial^2\pi}{\partial P_{i0}^2} + \frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}}\right)\frac{dP_{i0}}{d\beta} = 0. \tag{8}$$

Then, from equations (7) and (8),

$$\frac{dN_{i0}}{d\beta} = \frac{\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right) - \frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)}{\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) - \left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right)\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right)}. \tag{9}$$

and

$$\frac{dP_{i0}}{d\beta} = \frac{\frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left(\frac{\partial^2\pi}{\partial {N_{i0}}^2} + \frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right) - \frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)}{\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) - \left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right)\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right)}. \tag{10}$$

The proof consists of the following steps.

Step 1. We show that as $\beta \to 0$, $\frac{\partial^2 \pi}{\partial \beta \partial N_{i0}} < 0$. To see this, note that

$$\frac{\partial^2 \pi}{\partial \beta \partial N_{i0}} = \frac{\partial Pr_{i0}}{\partial N_{i0}} \left(M_i \frac{\partial Pr_{i1}}{\partial \beta} - \frac{\partial Pr_{j1}}{\partial \beta} \right) + Pr_{i0} \left(M_i \frac{\partial^2 Pr_{i1}}{\partial \beta \partial N_{i0}} - \frac{\partial^2 Pr_{j1}}{\partial \beta \partial N_{i0}} \right) + \frac{\partial^2 Pr_{j1}}{\partial \beta \partial N_{i0}}.$$

As $\beta \to 0$, (i) $N^*_{i0} \to \rho P^*_{i0}$ from equation (6), and (ii) under symmetry, $\frac{P^*_{i0}}{P^*_{i1}} \to \frac{(M_i-1)(1+Q^*_{i1})}{2M_i Q^*_{i1}}$ (this follows from equation (5) by substituting $Q^*_{i0} = Q^*_{j0} = 1, Q^*_{i1} = Q^*_{j1}$). Substituting yields

$$\begin{aligned} \lim_{\beta \to 0} \frac{\partial^2 \pi}{\partial \beta \partial N_{i0}} &= \frac{\alpha Q^*_{i1}}{2(1+Q^*_{i1})^2} \left(-\frac{\alpha(M_i-1)(N^*_{i0}-P^*_{i0})}{2N^*_{i0}} - \frac{\alpha M_i (N^*_{i0}-P^*_{i0})(1-Q^*_{i1})}{N^*_{i1}(1+Q^*_{i1})} - 1 \right) \\ &= -\frac{\alpha^2 (M_i-1)(\rho-1)}{4\rho(1+Q^*_{i1})^2} - \frac{\alpha Q^*_{i1}}{2(1+Q^*_{i1})^2} < 0. \end{aligned} \tag{11}$$

Step 2. We show that the coefficient of $\frac{\partial P_{i0}}{\partial \beta}$ in equation (7) and that of $\frac{\partial N_{i0}}{\partial \beta}$ in equation (8) are equal as $\beta \to 0$. Note that this follows, if in the limit, $\frac{\partial^2 \pi}{\partial P_{j0} \partial N_{i0}} = \frac{\partial^2 \pi}{\partial N_{j0} \partial P_{i0}}$. Now,

$$\begin{aligned} \lim_{\beta \to 0} \frac{\partial^2 \pi}{\partial N_{j0} \partial P_{i0}} = (M_i Pr_{i1} - Pr_{j1}) \frac{\partial^2 Pr_{i0}}{\partial N_{j0} \partial P_{i0}} + \frac{\partial Pr_{i0}}{\partial P_{i0}} \left(M_i \frac{\partial Pr_{i1}}{\partial N_{j0}} - \frac{\partial Pr_{j1}}{\partial N_{j0}} \right) \\ + M_i \left(\frac{\partial Pr_{i1}}{\partial P_{i0}} \frac{\partial Pr_{i0}}{\partial N_{j0}} + Pr_{i0} \frac{\partial^2 Pr_{i1}}{\partial N_{j0} \partial P_{i0}} \right). \end{aligned}$$

With symmetry, $\frac{\partial^2 Pr_{i0}}{\partial N_{j0} \partial P_{i0}} = \frac{\alpha^2 Q^*_{i0}(1-Q^*_{i0})}{N^*_{i0} P^*_{i0} (1+Q^*_{i0})^3} = 0$, and $\frac{\partial Pr_{i1}}{\partial N_{j0}}, \frac{\partial Pr_{i1}}{\partial N_{j0} \partial P_{i0}} = 0$ as $\beta \to 0$. It follows that

$$\lim_{\beta \to 0} \frac{\partial^2 \pi}{\partial N_{j0} \partial P_{i0}} = \frac{\alpha^2 Q^*_{i1} (M_i+1)}{4\rho P^*_{i0} P^*_{i1} (1+Q^*_{i1})^2}. \tag{12}$$

Similarly, as $\beta \to 0$,

$$\frac{\partial^2 \pi}{\partial P_{j0} N_{i0}} = (M_i Pr_{i1} - Pr_{j1}) \frac{\partial^2 Pr_{i0}}{\partial P_{j0} \partial N_{i0}} - \frac{\partial Pr_{i0}}{\partial N_{i0}} \frac{\partial Pr_{j1}}{\partial P_{j0}} + M_i \frac{\partial Pr_{i0}}{\partial P_{j0}} \frac{\partial Pr_{i1}}{\partial N_{i0}} = \frac{(M_i+1)\alpha^2 Q^*_{i1}}{4\rho P^*_{i1} P^*_{i0} (1+Q_{i1})^2}. \tag{13}$$

We now show that the coefficients $\frac{\partial P_{i0}}{\partial \beta}$ and $\frac{\partial N_{i0}}{\partial \beta}$ are negative. To see this, note that

$$\begin{aligned} \frac{\partial^2 \pi}{\partial N_{i0} \partial P_{i0}} &= (M_i Pr_{i1} - Pr_{j1}) \frac{\partial^2 Pr_{i0}}{\partial N_{i0} \partial P_{i0}} + M_i \frac{\partial Pr_{i0}}{\partial N_{i0}} \frac{\partial Pr_{i1}}{\partial P_{i0}} + M_i \frac{\partial Pr_{i0}}{\partial P_{i0}} \frac{\partial Pr_{i1}}{\partial N_{i0}} + M_i Pr_{i0} \frac{\partial^2 Pr_{i1}}{\partial N_{i0} \partial P_{i0}} \\ &= -\frac{\alpha^2 M_i Q^*_{i1}}{4P^*_{i0} N^*_{i1} (1+Q^*_{i1})^2} - \frac{\alpha^2 M_i Q^*_{i1}}{4N^*_{i0} P^*_{i1} (1+Q^*_{i1})^2} - \frac{\alpha M_i Q^*_{i1} (1+2\alpha-(2\alpha-1)Q^*_{i1})}{4N^*_{i1} P^*_{i1} (1+Q^*_{i1})^3} \\ &= -\frac{\alpha^2 M_i Q^*_{i1}}{2\rho P^*_{i0} P^*_{i1} (1+Q^*_{i1})^2} - \frac{\alpha M_i Q^*_{i1} (1+2\alpha-(2\alpha-1)Q^*_{i1})}{4\rho P^{*2}_{i1} (1+Q^*_{i1})^3}, \end{aligned} \tag{14}$$

as $\frac{\partial^2 Pr_{i0}}{\partial N_{i0}\partial P_{i0}} = -\frac{\alpha^2 Q^*_{i0}(1-Q^*_{i0})}{N^*_{i0}P^*_{i0}(1+Q^*_{i0})^3} = 0$ under symmetry. Thus as $\beta \to 0$ from equations (12)–(14),

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0}\partial N_{i0}} &= \frac{\partial^2 \pi^i}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2 \pi^i}{\partial P_{i0}\partial N_{i0}} \\
&= \frac{-(M_i-1)\alpha^2 Q^*_{i1}}{4\rho P^*_{i0}P^*_{i1}(1+Q^*_{i1})^2} - \frac{\alpha M_i Q^*_{i1}(1+2\alpha-(2\alpha-1)Q^*_{i1})}{4\rho P^{*2}_{i1}(1+Q^*_{i1})^3} \\
&= -\frac{(M_i-1)\alpha(1+2\alpha+Q^*_{i1})}{8\rho P^*_{i0}P^*_{i1}(1+Q^*_{i1})^2} < 0. \qquad (15)
\end{aligned}$$

Next compare the coefficient of $\frac{\partial N_{i0}}{\partial \beta}$ in (7) with that of $\frac{\partial P_{i0}}{\partial \beta}$ in (8). To do this, note that

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial N^2_{i0}} &= (M_i Pr_{i1} - Pr_{j1})\frac{\partial^2 Pr_{i0}}{\partial N^2_{i0}} + 2M_i \frac{\partial Pr_{i0}}{\partial N_{i0}}\frac{\partial Pr_{i1}}{\partial N_{i0}} + M_i Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial N^2_{i0}} \\
&= \left(\frac{M_i}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}}\right)\left[\frac{-\alpha Q^*_{i0}(1+\alpha-(\alpha-1)Q^*_{i0})}{N^{2*}_{i0}(1+Q^*_{i0})^3}\right] - 2M_i\left(\frac{\alpha Q^*_{i0}}{N^*_{i0}(1+Q^*_{i0})^2}\right)\left(\frac{\alpha Q^*_{i1}}{N^*_{i1}(1+Q^*_{i1})^2}\right) \\
&\qquad - \frac{M_i}{1+Q^*_{i0}}\left(\frac{\alpha Q^*_{i1}(1+2\alpha-(2\alpha-1)Q^*_{i1})}{2N^{*2}_{i1}(1+Q^*_{i1})^3}\right) \\
&= -\frac{(M_i-1)\alpha}{4\rho^2 P^{*2}_{i0}(1+Q^*_{i1})} - \frac{\alpha^2 M_i Q^*_{i1}}{2\rho^2 P^*_{i0}P^*_{i1}(1+Q^*_{i1})^2} - \frac{\alpha M_i Q^*_{i1}(1+2\alpha-(2\alpha-1)Q^*_{i1})}{4\rho^2 P^{*2}_{i1}(1+Q^*_{i1})^3} \\
&= \frac{1}{\rho}\frac{\partial^2 \pi}{\partial P^2_{i0}}. \qquad (16)
\end{aligned}$$

The last equality follows from symmetry and the fact that $N^*_{it} \to \rho P^*_{it}$, as $\beta \to 0$. Similarly,

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial N_{j0}N_{i0}} &= (M_i Pr_{i1} - Pr_{j1})\frac{\partial^2 Pr_{i0}}{\partial N_{j0}\partial N_{i0}} - \frac{\partial Pr_{i0}}{\partial N_{i0}}\frac{\partial Pr_{j1}}{\partial N_{j0}} + M_i\frac{\partial Pr_{i0}}{\partial N_{j0}}\frac{\partial Pr_{i1}}{\partial N_{i0}} \\
&= \left(\frac{M_i}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}}\right)\frac{\alpha^2 Q^*_{i0}(1-Q^*_{i0})}{\rho^2 P^*_{i0}P^*_{j0}(1+Q_{i0})^3} + \frac{\alpha^2 Q^*_{i0}Q^*_{j1}}{\rho^2 P^*_{i0}P^*_{j1}(1+Q^*_{i0})^2(1+Q^*_{j1})^2} \\
&\qquad + \frac{\alpha^2 Q^*_{i0}Q^*_{i1}M_i}{\rho^2 P^*_{j0}P^*_{i1}(1+Q^*_{i0})^2(1+Q^*_{i1})^2} \qquad (17) \\
&= \frac{\alpha^2 M_i Q^*_{i1}}{\rho^2 P^*_{i1}P^*_{j0}(1+Q_{i0})^2(1+Q^*_{i1})^2} + \frac{\alpha^2 Q^*_{i0}Q^*_{j1}}{\rho^2 P^*_{i0}P^*_{j1}(1+Q^*_{i0})^2(1+Q^*_{j1})^2} \\
&= \frac{(M_i+1)\alpha^2 Q^*_{i1}}{4\rho^2 P^*_{i1}P^*_{i0}(1+Q_{i1})^2} = \frac{1}{\rho}\frac{\partial^2 \pi}{\partial P_{j0}N_{i0}}.
\end{aligned}$$

Therefore, from equations (15)–(17), as $\beta \to 0$,

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2 \pi}{\partial N^2_{i0}} &= -\frac{\alpha(M_i-1)}{4\rho^2 P^{*2}_{i0}(1+Q^*_{i1})^2} - \left(\frac{(M_i-1)\alpha^2 Q^*_{i1}}{4\rho^2 P^*_{i0}P^*_{i1}(1+Q^*_{i1})^2} + \frac{\alpha M_i Q^*_{i1}(1+2\alpha-(2\alpha-1)Q^*_{i1})}{4\rho^2 P^{*2}_{i1}(1+Q^*_{i1})^3}\right) \\
&= -\frac{\alpha(M_i-1)}{4\rho^2 P^{*2}_{i0}(1+Q^*_{i1})^2} + \frac{1}{\rho}\left(\frac{\partial^2 \pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0}\partial N_{i0}}\right) < 0. \qquad (18)
\end{aligned}$$

Further, one can show that

$$\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2} = \frac{1}{\rho^2}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right) < 0. \tag{19}$$

Step 3. We show that the denominator of the expression in equation (9),

$$\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) - \left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right)\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right),$$

is negative. From equations (12), (13) and (19), this reduces to showing that

$$\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)^2 - \rho^2\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right)^2 < 0.$$

This holds, as equations (15) and (18) imply that

$$0 > \left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) > \rho\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right).$$

Step 4. We show that the numerator of the expression in equation (9),

$$\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right) - \frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right),$$

is positive in equilibrium. From equations (12), (13) and (19), the numerator reduces to

$$\rho^2\underbrace{\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}}_{<0}\underbrace{\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right)}_{<0} - \frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\underbrace{\left(\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}}\right)}_{<0}.$$

Now,

$$\begin{aligned}
\frac{\partial^2\pi}{\partial\beta\partial P_{i0}} &= \frac{\partial Pr_{i0}}{\partial P_{i0}}\left(M_i\frac{\partial Pr_{i1}}{\partial\beta} - \frac{\partial Pr_{j1}}{\partial\beta}\right) + M_i Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial\beta\partial P_{i0}} \\
&= -\frac{\alpha^2(M_i-1)(N_{i0}^*-P_{i0}^*)Q_{i1}^*}{4P_{i0}(1+Q_{i1}^*)^2} - \frac{\alpha M_i Q_{i1}^*(\alpha\rho(N_{i0}^*-P_{i0}^*)(1-Q_{i1}^*) - N_{i1}^*(1+Q_{i1}^*))}{2N_{i1}^*(1+Q_{i1}^*)^3} \\
&= \rho\left(-\frac{\alpha^2(M_i-1)(N_{i0}^*-P_{i0}^*)Q_{i1}^*}{4N_{i0}(1+Q_{i1}^*)^2} - \frac{\alpha^2 M_i Q_{i1}^*(N_{i0}^*-P_{i0}^*)(1-Q_{i1}^*)}{2N_{i1}^*(1+Q_{i1}^*)^3} - \frac{\alpha Q_{i1}^*}{2(1+Q_{i1}^*)^2}\right) \\
&\qquad + \frac{\alpha Q_{i1}^*(M_i+\rho)}{2(1+Q_{i1}^*)^2} \\
&= \rho\frac{\partial^2\pi}{\partial\beta\partial N_{i0}} + \frac{\alpha Q_{i1}^*(M_i+\rho)}{2(1+Q_{i1}^*)^2}.
\end{aligned} \tag{20}$$

To see that the numerator is always positive, first note that if $\frac{\partial^2 \pi}{\partial \beta \partial P_{i0}} > 0$, then the numerator is positive. If, instead $\frac{\partial^2 \pi}{\partial \beta \partial P_{i0}} < 0$, then from equations (15), (18) and (20),

$$0 > \frac{\partial^2 \pi}{\partial \beta \partial P_{i0}} > \rho \frac{\partial^2 \pi}{\partial \beta \partial N_{i0}}, \quad 0 > \left(\frac{\partial^2 \pi}{\partial N_{j0} \partial P_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}} \right) > \rho \left(\frac{\partial^2 \pi}{\partial N_{j0} \partial N_{i0}} + \frac{\partial^2 \pi}{\partial N_{i0}^2} \right)$$

implying that the numerator in equation (9) is positive. Since the denominator in equation (9) is negative (step 3), it follows that N_{i0}^* (and hence n_{i0}^*) decrease in β.

To determine the impact of increasing β on p_{i0}^*, we rewrite equation (8) below with the signs of some of the expressions contained in it.

$$\frac{\partial^2 \pi}{\partial \beta \partial P_{i0}} + \underbrace{\left(\frac{\partial^2 \pi}{\partial N_{i0} \partial P_{i0}} + \frac{\partial^2 \pi}{\partial N_{j0} \partial P_{i0}} \right)}_{<0} \underbrace{\frac{dN_{i0}}{d\beta}}_{>0} + \underbrace{\left(\frac{\partial^2 \pi}{\partial P_{i0}^2} + \frac{\partial^2 \pi}{\partial P_{j0} \partial P_{i0}} \right)}_{<0} \frac{dP_{i0}}{d\beta} = 0.$$

Then, it follows that $\frac{dP_{i0}}{d\beta} > 0$ when $\frac{\partial^2 \pi}{\partial \beta \partial P_{i0}} > 0$. Below, we show that $\frac{\partial^2 \pi^i}{\partial \beta \partial P_{i0}} > 0$ as long the challenger resources are not too great.

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial \beta \partial P_{i0}} &= \frac{\partial Pr_{i0}}{\partial P_{i0}} \left(M_i \frac{\partial Pr_{i1}}{\partial \beta} - \frac{\partial Pr_{j1}}{\partial \beta} \right) + M_i Pr_{i0} \frac{\partial^2 Pr_{i1}}{\partial \beta \partial P_{i0}} \\
&= -\frac{\alpha^2 (M_i - 1)(N_{i0}^* - P_{i0}^*) Q_{i1}^*}{4 P_{i0} (1 + Q_{i1}^*)^2} - \frac{\alpha M_i Q_{i1}^* (\alpha \rho (N_{i0}^* - P_{i0}^*)(1 - Q_{i1}^*) - N_{i1}^* (1 + Q_{i1}^*))}{2 N_{i1}^* (1 + Q_{i1}^*)^3} > 0 \\
&\iff -\frac{\alpha (M_i - 1)(\rho - 1)}{2} - \frac{\alpha (M_i - 1)(\rho - 1)(1 - Q_{i1}^*)}{2 Q_{i1}^*} + M_i > 0 \\
&\iff Q_{i1}^* > \frac{\alpha (M_i - 1)(\rho - 1)}{2 M_i},
\end{aligned}$$

i.e., if the challenger resources are not too high. The equivalences follow from $N_{i0}^* \to \rho P_{i0}^*$ as $\beta \to 0$, rearranging terms and substituting $\frac{P_{i0}^*}{P_{i1}^*} = \frac{(M_i - 1)(1 + Q_{i1}^*)}{2 M_i Q_{i1}^*}$ from equation (5). Following the same steps as above one can also establish the result when challengers only campaign negatively in the general election.

Finally, we show that the increase in positive campaigning exceeds the decrease in negative campaigning so that total campaigning expenditures in the primary election increase.

To see this, note that from equations (9) and (10) and the result in step 3, it follows that

$$\frac{dP_{i0}}{d\beta} > \left|\frac{dN_{i0}}{d\beta}\right| \Longleftrightarrow \frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right) - \frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)$$
$$< \frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) - \frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left(\frac{\partial^2\pi}{\partial {N_{i0}}^2} + \frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right)$$

$$\begin{aligned}\Leftrightarrow \frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\left[\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right) - \left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right)\right] \\ < \frac{\partial^2\pi}{\partial\beta\partial P_{i0}}\left[\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) - \left(\frac{\partial^2\pi}{\partial {N_{i0}}^2} + \frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right)\right].\end{aligned} \tag{21}$$

We now show that this inequality holds.To see this, note that from equation (11), as $\rho \to 1$,

$$\frac{\partial^2\pi}{\partial\beta\partial N_{i0}} \to -\frac{\alpha Q_{i1}^*}{2(1+Q_{i1}^{*2})}. \tag{22}$$

Therefore, from equations (20) and (22), as $\rho \to 1$,

$$\frac{\partial^2\pi}{\partial\beta\partial P_{i0}} \to \frac{\alpha M_i Q_{i1}^*}{2(1+Q_{i1}^{*2})} > \left|\frac{\partial^2\pi}{\partial\beta\partial N_{i0}}\right| \to \frac{\alpha Q_{i1}^*}{2(1+Q_{i1}^{*2})}. \tag{23}$$

However, from equations (12), (13) and (18),

$$\begin{aligned}\left(\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) &- \left(\frac{\partial^2\pi}{\partial {N_{i0}}^2} + \frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right) \\ &= \left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) - \left(\frac{\partial^2\pi}{\partial {N_{i0}}^2} + \frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}}\right) \\ &= \rho\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right) + \frac{\alpha(M_i-1)}{4\rho P_{i0}^{*2}(1+Q_{i1}^*)^2} - \left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right) \\ &\Rightarrow \frac{\alpha(M_i-1)}{4\rho P_{i0}^{*2}(1+Q_{i1}^*)^2} > 0 \quad (\text{when } \rho \to 1).\end{aligned} \tag{24}$$

Similarly, from equations (14), (15), (17) and (18),

$$\begin{aligned}&\left(\frac{\partial^2\pi}{\partial P_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}^2}\right) - \left(\frac{\partial^2\pi}{\partial N_{j0}\partial P_{i0}} + \frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}}\right) \\ &= \rho^2\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right) - \left[\rho\left(\frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{i0}^2}\right) + \frac{\alpha(M_i-1)}{4\rho P_{i0}^{*2}(1+Q_{i1}^*)^2}\right] \\ &\Rightarrow -\frac{\alpha(M_i-1)}{4\rho P_{i0}^{*2}(1+Q_{i1}^*)^2} < 0 \quad (\text{when } \rho \to 1).\end{aligned} \tag{25}$$

Thus, from equations (23), (24) and (25),

$$\frac{\partial^2 \pi}{\partial \beta \partial P_{i0}} > \left| \frac{\partial^2 \pi}{\partial \beta \partial N_{i0}} \right| \quad \text{and}$$

$$\left(\frac{\partial^2 \pi}{\partial P_{j0} \partial N_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}} \right) - \left(\frac{\partial^2 \pi}{\partial {N_{i0}}^2} + \frac{\partial^2 \pi}{\partial N_{j0} \partial N_{i0}} \right) = \left| \left(\frac{\partial^2 \pi}{\partial P_{j0} \partial P_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0}^2} \right) - \left(\frac{\partial^2 \pi}{\partial N_{j0} \partial P_{i0}} + \frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}} \right) \right|.$$

Thus, the inequality in (21) holds. ■

Proof of Proposition 4. With symmetry, $Q^*_{i0} = 1, Q^*_{i1} = Q^*_{j1}$. Substituting these expressions and $\beta = 1$ into equation (3) yields:

$$\frac{M_i Q^*_{i1}}{(1 + Q^*_{i1})^2} \left(\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} + 1 \right) + \frac{Q^*_{i1}}{(1 + Q^*_{i1})^2} = 0.$$

Canceling terms on both sides, we get

$$M_i \left(\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} + 1 \right) + 1 = 0.$$

Next, we re-arrange this equality to obtain

$$\left(\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} \right) = -\frac{M_i + 1}{M_i} = -1 - \frac{1}{M_i}.$$

Since $M_i > 1$, it follows that

$$\left(\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} \right) \in (-2, -1),$$

and hence

$$\left(\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} \right) < -1. \tag{26}$$

Now, recall from Proposition 1 that $p^*_{i1} = \frac{B_{i1}}{2} - \frac{\rho - 1}{2\rho}$ and $n^*_{i1} = \frac{B_{i1}}{2} + \frac{\rho - 1}{2\rho}$. Therefore,

$$P^*_{i1} = 1 + p^*_{i1} = \frac{\rho + 1}{2\rho} + \frac{B_{i1}}{2} \quad \text{and} \quad N^*_{i1} = 1 + \rho n^*_{i1} = \frac{\rho + 1}{2} + \frac{\rho B_{i1}}{2}, \quad \text{i.e.} \quad N^*_{i1} = \rho P^*_{i1}.$$

Substituting $N^*_{i1} = \rho P^*_{i1}$, the inequality in (26) simplifies to:

$$\left(\frac{N^*_{i0}}{\rho} - P^*_{i0} \right) < -P^*_{i1}. \tag{27}$$

Further, since $P^*_{i0} = 1 + p^*_{i0}$ and $N^*_{i0} = 1 + \rho n^*_{i0}$, it follows that $B_{i1} = \bar{B}_i - (p^*_{i0} + n^*_{i0}) =$

$\bar{B}_i - (P^*_{i0} - 1) - \frac{N^*_{i0}-1}{\rho}$, so that

$$P^*_{i1} = \frac{\rho+1}{2\rho} + \frac{B_{i1}}{2} = \frac{1}{\rho}\left(1 + \rho + \frac{\rho\bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2}\right).$$

Hence, the inequality in (27) may be rewritten as

$$\left(\frac{N^*_{i0}}{\rho} - P^*_{i0}\right) < -\frac{1}{\rho}\left(1 + \rho + \frac{\rho\bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2}\right).$$

Multiplying both sides by ρ and rearranging the right-hand side slightly yields:

$$N^*_{i0} - \rho P^*_{i0} < 1 - \rho - \left(2 + \frac{\rho\bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2}\right). \tag{28}$$

Now, recall from Proposition 1 that challengers devote resources to both negative and positive campaigning in the general election when $B_{i1} = p^*_{i1} + n^*_{i1} = \bar{B}_i - (p^*_{i0} + n^*_{i0}) \geq \frac{\rho-1}{\rho}$, i.e.

$$\bar{B}_i - (P^*_{i0} - 1) - \left(\frac{N^*_{i0} - 1}{\rho}\right) \geq \frac{\rho - 1}{\rho}. \tag{29}$$

Rearranging terms, we rewrite (29) as

$$1 + \frac{\rho\bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2} \geq 0. \tag{30}$$

Hence, from (28) and (30),

$$N^*_{i0} - \rho P^*_{i0} < 1 - \rho - \underbrace{\left(2 + \frac{\rho\bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2}\right)}_{> 0 \text{ from } (30)} < 1 - \rho, \quad \text{i.e.,} \quad N^*_{i0} - \rho P^*_{i0} < 1 - \rho.$$

Substituting $N^*_{i0} = 1 + \rho n^*_{i0}$ and $P^*_{i0} = 1 + p^*_{i0}$ to this inequality yields

$$1 + \rho n^*_{i0} - (\rho(1 + p^*_{i0}) < 1 - \rho \Leftrightarrow \rho(n^*_{i0} - p^*_{i0}) < 0.$$

Continuity of n^*_{i0}, p^*_{i0} in β then implies that the result holds for all β sufficiently small. ■

Proof of Lemma 1. At an interior optimum, $N_{i0}^*, P_{i0}^*, N_{j0}^*, P_{j0}^*$ satisfy

$$\underbrace{\frac{\partial^2 \pi}{\partial \theta \partial N_{i0}}}_{\text{direct effect of a change in } \theta} + \overbrace{\frac{\partial^2 \pi}{\partial {N_{i0}}^2}\frac{dN_{i0}}{d\theta} + \frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}}\frac{dP_{i0}}{d\theta}}^{\text{indirect effect via change in } i\text{'s actions}} + \underbrace{\frac{\partial^2 \pi}{\partial P_{j0} \partial N_{i0}}\frac{dP_{j0}}{d\theta} + \frac{\partial^2 \pi}{\partial N_{j0} \partial N_{i0}}\frac{dN_{j0}}{d\theta}}_{\text{indirect effect via change in } j\text{'s actions}} = 0 \tag{31}$$

for $\theta \in \bar{X}_I, \bar{B}_I, \alpha$.

From equation (6) it follows that when $\beta = 0$, under assumptions of symmetry, $\frac{dN_{i0}}{d\theta} = \rho \frac{dP_{i0}}{d\theta} = \rho \frac{dP_{j0}}{d\theta} = \frac{dN_{j0}}{d\theta}$, at the equilibrium. Therefore, equation (31) may be rewritten as

$$\frac{\partial^2 \pi}{\partial \theta \partial N_{i0}} + \frac{dN_{i0}}{d\theta}\left(\frac{\partial^2 \pi}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2 \pi}{\partial P_{j0} \partial N_{i0}} + \frac{\partial^2 \pi}{\partial N_{j0} \partial N_{i0}}\right) = 0. \tag{32}$$

Then, if

$$\frac{\partial^2 \pi}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2 \pi}{\partial P_{i0} \partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2 \pi}{\partial P_{j0} \partial N_{i0}} + \frac{\partial^2 \pi}{\partial N_{j0} \partial N_{i0}} < 0 \tag{33}$$

$\frac{\partial^2 \pi}{\partial \theta \partial N_{i0}}$ and $\frac{dN_{i0}}{d\theta}$ have the same sign. Similarly, for changes in $\bar{B}_i, \bar{B}_j$, or $\bar{X}_i, \bar{X}_j$ or M_i, M_j, $\frac{dn_{i0}}{d\theta_i} + \frac{dn_{i0}}{d\theta_j}, \frac{\partial^2 \pi}{\partial \theta_i \partial n_{i0}} + \frac{\partial^2 \pi}{\partial \theta_j \partial n_{i0}}$ have the same sign. Thus, the direct effect is reinforced by, or if in the opposite direction, outweighs the indirect effect. We now show that the inequality above indeed holds at equilibrium under symmetry. To see this, note that from equations (13), (14), (16) and (17), the left-hand side of (33) reduces to

$$\begin{aligned} &\frac{-(M_i - 1)\alpha}{4\rho^2 P_{i0}^{*2}(1 + Q_{i1}^*)} - \frac{\alpha M_i Q_{i1}^*(1 + 2\alpha - (2\alpha - 1)Q_{i1}^*)}{2\rho^2 P_{i1}^{*2}(1 + Q_{i1}^*)^3} - \frac{(M_i - 1)\alpha^2 Q_{i1}^*}{2\rho^2 P_{i0}^* P_{i1}^*(1 + Q_{i1}^*)^2} \\ = &\frac{\alpha}{4\rho^2(1 + Q_{i1}^*)}\left(-\frac{(M_i - 1)}{P_{i0}^{*2}} - \frac{2M_i Q_{i1}^*(1 + 2\alpha - (2\alpha - 1)Q_{i1}^*)}{P_{i1}^{*2}(1 + Q_{i1}^*)^2} - \frac{2(M_i - 1)\alpha Q_{i1}^*}{P_{i0}^* P_{i1}^*(1 + Q_{i1}^*)}\right). \end{aligned} \tag{34}$$

Multiplying both numerator and denominator by P_{i0}^{*2}, and substituting $\frac{P_{i0}^*}{P_{i1}^*} = \frac{(M_i-1)(1+Q_{i1}^*)}{2M_i Q_{i1}^*}$, from equation (5) under symmetry yields

$$\frac{\alpha(M_i - 1)}{4\rho^2 P_{i0}^{*2}(1 + Q_{i1}^*)}\left[-1 - \left(\frac{1 + 2\alpha - (2\alpha - 1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i - 1}{2M_i} - \frac{\alpha(M_i - 1)}{M_i}\right]. \tag{35}$$

Therefore, (33) holds if

$$-1 - \left(\frac{1 + 2\alpha - (2\alpha - 1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i - 1}{2M_i} - \frac{\alpha(M_i - 1)}{M_i} < 0. \tag{36}$$

Simplifying the inequality in (36) yields

$$Q_{i1}^* > -\frac{(1+2\alpha)(M_i-1)}{3M_i-1},$$

which always holds as $M_i > 1$.

Similarly, one can show that when the resources in the general election are low enough that a challenger only campaigns negatively, the left-hand side of (33) reduces to

$$\frac{-(M_i-1)\alpha}{4N_{i0}^{*2}(1+Q_{i1}^*)} - \frac{\alpha M_i Q_{i1}^*(1+\alpha-(\alpha-1)Q_{i1}^*)}{N_{i1}^{*2}(1+Q_{i1}^*)^3} - \frac{(M_i-1)\alpha^2 Q_{i1}^*}{2N_{i0}^* N_{i1}^*(1+Q_{i1}^*)^2}.$$

Multiply by N_{i0}^2, substitute the first-order conditions and rearrange to show that (33) holds if

$$1+\left(\frac{1+\alpha-(\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{M_i}+\frac{\alpha(M_i-1)}{M_i} > 0,$$

or equivalently if

$$Q_{i1}^* > -\frac{(1+\alpha)(M_i-1)}{2M_i-1},$$

which always holds. ■

Proof of Proposition 5. From Lemma 1, we only need to focus on the partial derivative capturing the direct effect of a change in a parameter. Thus, N_{i0}^* increases in the challengers' reputations $\bar{X}_i, \bar{X}_j$ if and only if $\frac{\partial^2\pi}{\partial\bar{X}_i\partial N_{i0}}+\frac{\partial^2\pi}{\partial\bar{X}_j\partial N_{i0}} > 0$. We show that this holds in equilibrium:

$$\begin{aligned}\frac{\partial^2\pi}{\partial\bar{X}_i\partial N_{i0}}+\frac{\partial^2\pi}{\partial\bar{X}_j\partial N_{i0}} &= (M_iPr_{i1}-Pr_{j1})\left(\frac{\partial^2 Pr_{i0}}{\partial\bar{X}_i\partial N_{i0}}+\frac{\partial^2 Pr_{i0}}{\partial\bar{X}_j\partial N_{i0}}\right)+\\ &\frac{\partial Pr_{i0}}{\partial N_{i0}}\left(M_i\frac{\partial Pr_{i1}}{\partial\bar{X}_i}-\frac{\partial Pr_{j1}}{\partial\bar{X}_j}\right)+M_iPr_{i0}\frac{\partial^2 Pr_{i1}}{\partial\bar{X}_i\partial N_{i0}}+M_i\frac{\partial Pr_{i1}}{\partial N_{i0}}\left(\frac{\partial Pr_{i0}}{\partial\bar{X}_i}+\frac{\partial Pr_{i0}}{\partial\bar{X}_j}\right).\end{aligned}$$

Now, under symmetry, $\frac{\partial Pr_{i0}}{\partial\bar{X}_i}+\frac{\partial Pr_{i0}}{\partial\bar{X}_j} = \frac{Q_{i0}^*}{\bar{X}_i(1+Q_{i0}^*)^2}-\frac{Q_{i0}^*}{\bar{X}_j(1+Q_{i0}^*)^2} = 0$ and since $Q_{i0}^* = 1$, it follows that $\frac{\partial^2 Pr_{i0}}{\partial\bar{X}_i\partial N_{i0}} = -\frac{\alpha Q_{i0}^*(1-Q_{i0}^*)}{\bar{X}_iN_{i0}^*(1+Q_{i0}^*)^3} = 0$. Therefore,

$$\begin{aligned}\frac{\partial^2\pi}{\partial\bar{X}_i\partial N_{i0}}+\frac{\partial^2\pi}{\partial\bar{X}_j\partial N_{i0}} &= \frac{\partial Pr_{i0}}{\partial N_{i0}}\left(M_i\frac{\partial Pr_{i1}}{\partial\bar{X}_i}-\frac{\partial Pr_{j1}}{\partial\bar{X}_j}\right)+M_iPr_{i0}\frac{\partial^2 Pr_{i1}}{\partial\bar{X}_i\partial N_{i0}}\\ &= \frac{\alpha Q_{i1}^*}{2(1+Q_{i1}^*)^2\bar{X}_i}\left(\frac{M_i-1}{2N_{i0}^*}+\frac{M_i(1-Q_{i1}^*)}{N_{i1}^*(1+Q_{i1}^*)}\right).\end{aligned}$$

Multiplying the numerator and denominator by N_{i0}^* and substituting $\frac{N_{i0}^*}{N_{i1}^*} = \frac{(M_i-1)(1+Q_{i1}^*)}{2M_iQ_{i1}^*}$

from the first-order conditions yields

$$\frac{\partial^2 \pi}{\partial \bar{X}_i \partial N_{i0}} + \frac{\partial^2 \pi}{\partial \bar{X}_j \partial N_{i0}} = \frac{\alpha(M_i - 1)}{4N_{i0}^*(1 + Q_{i1}^*)^2 \bar{X}_i} > 0. \tag{37}$$

Thus, N_{i0}^*, P_{i0}^* and hence, total primary expenditures rise with the challenger's reputation.

The probability of winning the primary is one-half in a symmetric equilibrium. Thus, the probability candidate i wins office only varies with the change in his chances against the incumbent. Since $Pr_{i1}^* = \frac{1}{1+Q_{i1}^*}$, it suffices to derive the impact on Q_{i1}^*. Increasing $\bar{X}_i$ reduces N_{i1}^*, P_{i1}^*, so the net impact on the probability that i wins the general election depends on the relative change in $\bar{X}_i$ and N_{i1}^*, P_{i1}^*. We now show that the direct effect (improved reputation) dominates the indirect effect (reduced general election campaigning), implying that the probability of ousting the incumbent rises. To see this, recall that $Q_{i1}^* = \frac{\bar{X}_I (P_I^* N_I^*)^\alpha}{\bar{X}_i (P_{i1}^* N_{i1}^*)^\alpha} = \frac{\bar{X}_I (N_I^*)^{2\alpha}}{\bar{X}_i (N_{i1}^*)^{2\alpha}}$, so

$$\left(\frac{dQ_{i1}}{d\bar{X}_i} + \frac{dQ_{i1}}{d\bar{X}_j}\right) < 0 \Longleftrightarrow 1 + \frac{2\alpha \bar{X}_i}{N_{i1}} \left(\frac{dN_{i1}}{d\bar{X}_i} + \frac{dN_{i1}}{d\bar{X}_j}\right) > 0 \Longleftrightarrow 1 - \frac{2\alpha \bar{X}_i}{N_{i1}} \left(\frac{dN_{i0}}{d\bar{X}_i} + \frac{dN_{i0}}{d\bar{X}_j}\right) > 0. \tag{38}$$

The last equivalence in (38) follows from

$$N_{i1}^* = 1 + \rho n_{i1}^* = \frac{\rho + 1}{2} + \frac{\rho B_{i1}}{2} = 1 + \rho + \frac{\rho \bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2} = 1 + \rho + \frac{\rho \bar{B}_i}{2} - N_{i0} \quad (\text{since } N_{i0}^* = \rho P_{i0}^*).$$

Now, from equation (32),

$$\frac{dN_{i0}}{d\bar{X}_i} + \frac{dN_{i0}}{d\bar{X}_j} = -\frac{\frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}} + \frac{\partial^2 \pi^i}{\partial \bar{X}_j \partial N_{i0}}}{\frac{\partial^2 \pi^i}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{i0} \partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{j0} \partial N_{i0}} + \frac{\partial^2 \pi^i}{\partial N_{j0} \partial N_{i0}}}.$$

From equations (35) and (37), the right-hand side may be written as

$$\frac{\frac{\alpha(M_i-1)}{4N_{i0}^* \bar{X}_i (1+Q_{i1}^*)^2}}{\frac{\alpha(M_i-1)}{4\rho^2 P_{i0}^{*2}(1+Q_{i1}^*)}\left[1 + \left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} + \frac{\alpha(M_i-1)}{M_i}\right]}$$

$$= \frac{\frac{N_{i0}^*}{\bar{X}_i(1+Q_{i1}^*)}}{\left[1 + \left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} + \frac{\alpha(M_i-1)}{M_i}\right]}, \text{ since } N_{i0}^* = \rho P_{i0}^*.$$

One can show that the last inequality in (38) holds by rearranging terms and substituting for $\frac{N_{i0}^*}{N_{i1}^*} = \frac{(M_i-1)(1+Q_{i1}^*)}{2M_i Q_{i1}^*}$. These results also extend to when a challenger only campaigns negatively in the general election:

$$\frac{dQ_{i1}}{d\bar{X}_i}+\frac{dQ_{i1}}{d\bar{X}_j} = \left(\frac{d}{d\bar{X}_i}+\frac{d}{d\bar{X}_j}\right)\left(\frac{\bar{X}_I N_I 2\alpha}{\bar{X}_i(\rho N_{i1})^{\alpha}}\right) < 0 \Longleftrightarrow \frac{dN_{i1}}{d\bar{X}_i}+\frac{dN_{i1}}{d\bar{X}_j} = -2\left(\frac{dN_{i0}}{d\bar{X}_i}+\frac{dN_{i0}}{d\bar{X}_j}\right) > -\frac{N_{i1}}{\alpha\bar{X}_i},$$

since

$$N_{i1}^* = 1+\rho B_{i1} = 2+\rho+\rho\bar{B}_i-\rho P_{i0}^*-N_{i0}^* = 2+\rho+\rho\bar{B}_i-2N_{i0}^*.$$

Now,

$$\begin{aligned}\left(\frac{dN_{i0}}{d\bar{X}_i}+\frac{dN_{i0}}{d\bar{X}_j}\right) &= \frac{\frac{\alpha(M_i-1)}{4N_{i0}^*(1+Q_{i1}^*)^2\bar{X}_i}}{\frac{\alpha(M_i-1)}{4\rho^2 P_{i0}^{*2}(1+Q_{i1}^*)}\left[1+\left(\frac{1+\alpha-(\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{M_i}+\frac{\alpha(M_i-1)}{M_i}\right]} \\ &= \frac{\frac{N_{i0}^*}{\bar{X}_I(1+Q_{i1}^*)}}{\left[1+\left(\frac{1+\alpha-(\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{M_i}+\frac{\alpha(M_i-1)}{M_i}\right]}.\end{aligned}$$

Rearranging terms and substituting for $\frac{N_{i0}^*}{N_{i1}^*}$ from the first order conditions yields the result.

Similarly, consider the impact of increasing the challenger resources. Under symmetry,

$$\begin{aligned}\frac{\partial^2\pi}{\partial\bar{B}_i\partial N_{i0}}+\frac{\partial^2\pi}{\partial\bar{B}_j\partial N_{i0}} &= \frac{\partial Pr_{i0}}{\partial N_{i0}}\left(M_i\frac{\partial Pr_{i1}}{\partial\bar{B}_i}-\frac{\partial Pr_{j1}}{\partial\bar{B}_j}\right)+M_i Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial\bar{B}_i\partial N_{i0}} \\ &= \frac{\alpha Q_{i1}^*}{4P_{i1}^*(1+Q_{i1}^*)^2}\left[\frac{\alpha(M_i-1)}{N_{i0}^*}+\frac{M_i}{N_{i1}^*(1+Q_{i1}^*)}(1+2\alpha-(2\alpha-1)Q_{i1}^*)\right] \\ &= \frac{\alpha(M_i-1)(1+2\alpha+Q_{i1}^*)}{8N_{i0}^*P_{i1}^*(1+Q_{i1}^*)^2} > 0 \qquad (39)\end{aligned}$$

at equilibrium.

Again, since $P_{i0}^* = \frac{N_{i0}^*}{\rho}$, increases i challenger resources raise P_{i0}^*. Thus, N_{i0}^*, P_{i0}^* and total primary campaigning expenditure rise with the challenger resources and reputations.

We next argue that increasing challenger resources increases P_{i1}^*, N_{i1}^* (and hence the probability of winning the general election), i.e., the increase in primary campaigning expenditures does not entirely exhaust the increase in resources. This follows if and only if $\frac{dN_{i0}}{dB_i}+\frac{dN_{i0}}{dB_j} < \frac{\rho}{2}$ as $N_{i1}^* = 1+\rho+\frac{\rho\bar{B}_i}{2}-N_{i0}$. As before, one can show that given equations (32), (35) and (39), this always holds in equilibrium. This result also carries over to when a challenger only campaigns negatively in the general election.

Lastly, consider the effect of an increase in M_i:

$$\frac{\partial^2 \pi^i}{\partial M_i \partial N_{i0}} = Pr_{i0}\frac{\partial Pr_{i1}}{\partial N_{i0}} + Pr_{i1}\frac{\partial Pr_{i0}}{\partial N_{i0}}$$
$$= \frac{\alpha}{(1+Q^*_{i0})(1+Q^*_{i1})}\left(\frac{Q^*_{i0}}{N^*_{i0}(1+Q^*_{i0})} - \frac{Q^*_{i1}}{N^*_{i1}(1+Q^*_{i1})}\right)$$
$$= \frac{\alpha Q^*_{i0}}{M_i N^*_{i0}(1+Q^*_{i0})^2(1+Q^*_{j1})} > 0 \quad \text{(from equation (4))}.$$

Thus, N^*_{i0} is increasing in M_i. Since $M_i = \frac{U_i}{V_i} > 1$, it follows that an increasing U_i increases negative campaigning in the primary election, while increasing V_i, leads to a decrease in N^*_{i0}. It also follows that equal changes in U_i, V_i lower M_i and hence N^*_{i0}. Since, $P^*_{i0} = \frac{N^*_{i0}}{\rho}$, the same results apply to P^*_{i0}, too. Again, via the budget constraint, P^*_{i1}, N^*_{i1} are decreasing in M_i. Consequently, the chances of winning the general election decrease in M_i. As above, this also holds when challengers only campaign negatively in the general election.

Note that changes in n^*_{i0}, p^*_{i0} mirror changes in N^*_{i0}, P^*_{i0}. Finally, the continuity of the derivatives in β, implies that there exists a $\bar{\beta}$ such that the results hold for all $\beta \in [0, \bar{\beta})$. ■

Proof of Proposition 6. From the continuity of the derivatives in $\bar{X}_i, \bar{B}_i, M_i$, it suffices to prove the result when challengers are symmetric. Further, from Lemma 1, we only need to focus on the partial derivative capturing the direct effect of a change in a parameter on N^*_{i0}. Consider the impact of increasing α.

$$\frac{\partial^2 \pi^i}{\partial \alpha \partial N_{i0}} = (M_i Pr_{i1} - Pr_{j1})\frac{\partial^2 Pr_{i0}}{\partial \alpha \partial N_{i0}} + \frac{\partial Pr_{i0}}{\partial N_{i0}}\left(M_i \frac{\partial Pr_{i1}}{\partial \alpha} - \frac{\partial Pr_{j1}}{\partial \alpha}\right) + M_i Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial \alpha \partial N_{i0}} + M_i \frac{\partial Pr_{i0}}{\partial \alpha}\frac{\partial Pr_{i1}}{\partial N_{i0}}$$
$$= (M_i Pr_{i1} - Pr_{j1})\left[\frac{Q^*_{i0}\left(1 - \ln\left(\frac{\bar{X}_j}{\bar{X}_i Q^*_{i0}}\right) + \left(1 + \ln\left(\frac{\bar{X}_j}{\bar{X}_i Q^*_{i0}}\right)\right)Q^*_{i0}\right)}{N^*_{i0}(1+Q_{i0})^3}\right]$$
$$+ \frac{Q^*_{i0}}{N^*_{i0}(1+Q^*_{i0})^2}\left(\frac{M_i Q^*_{i1}\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q^*_{i1}}\right)}{(1+Q^*_{i1})^2} - \frac{Q^*_{j1}\ln\left(\frac{\bar{X}_I}{\bar{X}_j Q^*_{j1}}\right)}{(1+Q^*_{j1})^2}\right) - \frac{M_i Q^*_{i0} Q^*_{i1}\ln\left(\frac{\bar{X}_j}{\bar{X}_i Q^*_{i0}}\right)}{N^*_{i1}(1+Q^*_{i0})^2(1+Q^*_{i1})^2}$$
$$+ \frac{M_i Q^*_{i1}}{N^*_{i1}(1+Q^*_{i1})^3(1+Q^*_{i0})}\left[-1 + \ln\left(\frac{\bar{X}_I}{\bar{X}_i Q^*_{i1}}\right) - \left(1 + \ln\left(\frac{\bar{X}_I}{\bar{X}_i Q^*_{i1}}\right)\right)Q^*_{i1}\right].$$

Thus,

$$\begin{aligned}
\frac{\partial^2 \pi^i}{\partial\alpha\partial N_{i0}} &= \frac{M_i Q_{i1}^*}{N_{i1}^*(1+Q_{i0}^*)^2(1+Q_{i1}^*)^2}\left(1-\ln\left(\frac{\bar{X}_j}{\bar{X}_i Q_{i0}^*}\right)+Q_{i0}^*\right) \\
&\quad + \frac{Q_{i0}^*}{N_{i0}^*(1+Q_{i0}^*)^2}\left(\frac{M_i Q_{i1}^*\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right)}{(1+Q_{i1}^*)^2}-\frac{Q_{j1}^*\ln\left(\frac{\bar{X}_I}{\bar{X}_j Q_{j1}^*}\right)}{(1+Q_{j1}^*)^2}\right) \\
&\quad + \frac{M_i Q_{i1}^*}{N_{i1}^*(1+Q_{i1}^*)^3(1+Q_{i0}^*)}\left[-1+\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right)-\left(1+\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right)\right)Q_{i1}^*\right] \\
&= \frac{M_i Q_{i1}^*\ln\left(\frac{\bar{X}_i Q_{i0}^*}{\bar{X}_j}\right)}{N_{i1}^*(1+Q_{i0}^*)^2(1+Q_{i1}^*)^2}+\frac{\left(M_i-\frac{(1-Q_{i1}^{*2})}{1+Q_{j1}}\right)Q_{i0}^*}{N_{i0}^*(1+Q_{i0}^*)^2(1+Q_{i1}^*)^2}\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right) \\
&\quad + \frac{Q_{i0}^* Q_{j1}^*}{N_{i0}^*(1+Q_{i0}^*)^2(1+Q_{j1}^*)^2}\ln\left(\frac{\bar{X}_j Q_{j1}^*}{\bar{X}_I}\right) \\
&= \frac{M_i-(1-Q_{i1}^*)}{4N_{i0}^*(1+Q_{i1}^*)^2}\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right)-\frac{Q_{i1}^*}{4N_{i0}^*(1+Q_{i1}^*)^2}\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right)\ (\text{when } Q_{i0}^*=1, Q_{i1}^*=Q_{j1}^*) \\
&= \frac{M_i-1}{4N_{i0}^*(1+Q_{i1}^*)^2}\ln\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right) \gtreqless 0,
\end{aligned} \tag{40}$$

according to whether $\left(\frac{\bar{X}_I}{\bar{X}_i Q_{i1}^*}\right) = \left(\frac{N_{i1}^*}{N_I}\right)^{2\alpha} \gtreqless 1$, i.e., according to whether $\bar{B}_i \gtreqless \bar{B}_i^*$. The impact of raising the sensitivity of reputations to campaigning on the probability of winning the general election similarly depends on challenger resources. To see this, note that

$$\frac{dQ_{i1}}{d\alpha} \gtreqless 0 \Longleftrightarrow -\frac{dN_{i0}}{d\alpha} = \frac{dN_{i1}}{d\alpha} \lesseqgtr \frac{N_{i1}^*}{\alpha}\ln\left(\frac{N_I^*}{N_{i1}^*}\right).$$

From equations (32), (35) and (40),

$$\begin{aligned}
-\frac{dN_{i0}}{d\alpha} &= \frac{\frac{2\alpha(M_i-1)}{4N_{i0}^*(1+Q_{i1}^*)^2}\ln\left(\frac{N_I^*}{N_{i1}^*}\right)}{\frac{\alpha(M_i-1)}{4\rho^2 P_{i0}^{*2}(1+Q_{i1}^*)}\left[1+\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i}+\frac{\alpha(M_i-1)}{M_i}\right]} \\
&= \frac{\frac{2N_{i0}^*}{(1+Q_{i1}^*)}\ln\left(\frac{N_I^*}{N_{i1}^*}\right)}{1+\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i}+\frac{\alpha(M_i-1)}{M_i}}.
\end{aligned}$$

As before, by rearranging terms and substituting for $\frac{N_{i0}^*}{N_{i1}^*} = \frac{(M_i-1)(1+Q_{i1}^*)}{2M_i Q_{i1}^*}$, one can show that if $\ln\left(\frac{N_I^*}{N_{i1}^*}\right) \gtreqless 0$, i.e., if $\left(\frac{N_I^*}{N_{i1}^*}\right) \gtreqless 1$, so that N_{i0}^* is decreasing, invariant or increasing in α respectively, then $\frac{dQ_{i1}}{d\alpha} \gtreqless 0$; that is, the chances of winning the general election are decreasing, invariant or increasing in α, respectively. Thus, when $\bar{B}_i < \bar{B}_i^*$, their campaigning expenditures in the general election are relatively low, so that increasing α causes them to reduce N_{i0}^* and increase N_{i1}^*, but this is insufficient to offset the incumbent's increased advantage from his greater resources due to the heightened sensitivity of reputations to campaigning—so that the challenger is less likely to win the general election. The reverse holds when $\bar{B}_i > \bar{B}_i^*$. When $\bar{B}_i = \bar{B}_i^*$, an increase in α has no impact on campaign levels or the probability of a win for the challengers. One can show that these results carry over to when the challenger's campaign in the general election is exclusively negative.

Next, consider the impact of improving the incumbent's reputation. Note that N_{i0}^* decreases in $\bar{X}_I$ if and only if $\frac{\partial^2 \pi}{\partial \bar{X}_I \partial N_{i0}} < 0$, which holds in equilibrium:

$$\begin{aligned}
\frac{\partial^2 \pi^i}{\partial \bar{X}_I \partial N_{i0}} &= \frac{\partial Pr_{i0}}{\partial N_{i0}} \frac{\partial}{\partial \bar{X}_I} \left(M_i Pr_{i1} - Pr_{j1}\right) + M_i Pr_{i0} \frac{\partial^2 Pr_{i1}}{\partial \bar{X}_I \partial N_{i0}} \\
&= \frac{\alpha Q_{i0}^*}{N_{i0}(1+Q_{i0}^*)^2} \left(\frac{Q_{j1}^*(1+Q_{i1}^*)^2 - Q_{i1}^*(1+Q_{j1}^*)^2 M_i}{\bar{X}_I (1+Q_{i1}^*)^2 (1+Q_{j1}^*)^2} \right) - \frac{\alpha Q_{i1}^*(1-Q_{i1}^*) M_i}{\bar{X}_I N_{i1}^* (1+Q_{i0}^*)(1+Q_{i1}^*)^3} \\
&= \frac{\alpha Q_{i0}^*}{\bar{X}_I (1+Q_{i0}^*)^2 N_{i0}^*} \left(\frac{Q_{j1}^*}{(1+Q_{j1}^*)^2} - \frac{M_i}{(1+Q_{i1}^*)^2} + \frac{1-Q_{i1}^*}{(1+Q_{i1}^*)(1+Q_{j1}^*)} \right) \\
&= \frac{\alpha Q_{i0}^*}{\bar{X}_I (1+Q_{i0}^*)^2 N_{i0}^*} \left(\frac{-(M_i-1)(2Q_{j1}^*+1) + 2Q_{i1}^* Q_{j1}^* - M_i Q_{j1}^{*2} - Q_{i1}^{*2}}{(1+Q_{i1}^*)^2 (1+Q_{j1}^*)^2} \right) < 0.
\end{aligned} \tag{41}$$

Similarly, it is easy to show that negative campaigning in the primary falls with the incumbent's resources, $\bar{B}_I$:

$$\begin{aligned}
\frac{\partial^2 \pi^i}{\partial \bar{B}_I \partial N_{i0}} &= \frac{\partial Pr_{i0}}{\partial N_{i0}} \left(M_i \frac{\partial Pr_{i1}}{\partial \bar{B}_I} - \frac{\partial Pr_{j1}}{\partial \bar{B}_I} \right) + M_i Pr_{i0} \frac{\partial^2 Pr_{i1}}{\partial \bar{B}_I \partial N_{i0}} \\
&= \frac{\alpha Q_{i0}^*}{N_{i0}^*(1+Q_{i0}^*)^2} \left(\frac{-\alpha M_i Q_{i1}^*}{P_I^*(1+Q_{i1}^*)^2} + \frac{\alpha Q_{j1}^*}{P_I^*(1+Q_{j1}^*)^2} \right) - \frac{\alpha^2 M_i Q_{i1}^*(1-Q_{i1}^*)}{P_{i1}^*(1+Q_{i0}^*) N_I^* (1+Q_{i1}^*)^3} \\
&= \frac{\alpha^2}{\rho P_I^*(1+Q_{i0}^*)} \left[\frac{Q_{i0}^*}{P_{i0}^*(1+Q_{i0}^*)} \left(\frac{-M_i Q_{i1}^*}{(1+Q_{i1}^*)^2} + \frac{Q_{j1}^*}{(1+Q_{j1}^*)^2} \right) - \frac{M_i Q_{i1}^*(1-Q_{i1}^*)}{P_{i1}^*(1+Q_{i1}^*)^3} \right] \\
&= \frac{\alpha^2 Q_{i0}^*}{\rho P_I^* P_{i0}^*(1+Q_{i0}^*)^2} \left(\frac{-M_i Q_{i1}^*}{(1+Q_{i1}^*)^2} + \frac{Q_{j1}^*}{(1+Q_{j1}^*)^2} - \frac{(M_i Pr_{i1}^* - Pr_{j1}^*)(1-Q_{i1}^*)}{(1+Q_{i1}^*)} \right) \\
&= -\frac{\alpha^2 Q_{i0}^* \Big((M_i-1)(1+Q_{j1}^*)^2 + (Q_{i1}^* - Q_{j1}^*)^2 \Big)}{\rho P_I^* P_{i0}^* (1+Q_{i0}^*)^2 (1+Q_{i1}^*)^2 (1+Q_{j1}^*)^2} < 0.
\end{aligned} \tag{42}$$

The fourth equality follows from equation (5). Since $P^*_{i0} = \frac{N^*_{i0}}{\rho}$, it follows that$P^*_{i0}$ decreases in $\bar{X}_I, \bar{B}_I$ too. Then, total primary campaigning expenditures fall with the incumbent's resources and reputation, too. Further, since P^*_{i1}, N^*_{i1} decrease in N^*_{i0}, P^*_{i0}, they rise with $\bar{X}_I, \bar{B}_I$, implying that challengers now spend more on campaigning in the general election than before.

Thus, improving the incumbent's reputation induces challengers to campaign more extensively in the general election. However, the probability that a challenger wins the general election still falls with $\bar{X}_I$, i.e., the direct effect dominates the indirect strategic effect on challengers campaigning in the primary. The incumbent's campaigning levels depend on his budget alone, so

$$\frac{dQ_{i1}}{d\bar{X}_I} = \frac{d}{d\bar{X}_I}\left(\frac{\bar{X}_I N_I^{2\alpha}}{\bar{X}_i N_{i1}^{*2\alpha}}\right) > 0 \Longleftrightarrow \frac{dN_{i1}}{d\bar{X}_I} = -\frac{dN_{i0}}{d\bar{X}_I} < \frac{N_{i1}}{2\alpha\bar{X}_I} \Longleftrightarrow \frac{\frac{dN_{i1}}{N_{i1}}}{\frac{d\bar{X}_I}{\bar{X}_I}} < \frac{1}{2\alpha}. \tag{43}$$

Intuitively, increasing both $\bar{X}_I$ and N^*_{i1} have opposing effects on Q^*_{i1}, so that the relative strength of the two effects decides the net impact on the probability that the challenger wins the general election. Now, from equation (41), substituting the symmetry conditions, $Q^*_{i0} = 1, Q^*_{i1} = Q^*_{j1}$, yields

$$\frac{dN_{i0}}{d\bar{X}_I} = \frac{\alpha(M_i - 1)}{4\rho P^*_{i0}\bar{X}_I(1 + Q^*_{i1})^2} \tag{44}$$

Therefore, from equations (32), (35) and (44),

$$\begin{aligned}
-\frac{dN_{i0}}{d\bar{X}_I} &= \frac{\frac{\partial^2\pi^i}{\partial\bar{X}_I\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial N_{i0}{}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}}} \\
&= \frac{\frac{\alpha(M_i-1)}{4\rho P^*_{i0}\bar{X}_I(1+Q^*_{i1})^2}}{\frac{\alpha(M_i-1)}{4\rho^2 P^{*2}_{i0}(1+Q^*_{i1})}\left[1 + \left(\frac{1+2\alpha-(2\alpha-1)Q^*_{i1}}{Q^*_{i1}}\right)\frac{M_i-1}{2M_i} + \frac{\alpha(M_i-1)}{M_i}\right]} \\
&= \frac{\frac{N^*_{i0}}{\bar{X}_I(1+Q^*_{i1})}}{\left[1 + \left(\frac{1+2\alpha-(2\alpha-1)Q^*_{i1}}{Q^*_{i1}}\right)\frac{M_i-1}{2M_i} + \frac{\alpha(M_i-1)}{M_i}\right]}.
\end{aligned}$$

One can show that the inequality in (43) holds by rearranging terms and substituting $\frac{N^*_{i0}}{N^*_{i1}} = \frac{(M_i-1)(1+Q^*_{i1})}{2M_iQ^*_{i1}}$.

Similarly, one can show that increasing the incumbent's resources reduces a challenger's chances of winning the general election. This holds if -$\frac{dN_{i0}}{d\bar{B}_I} < \frac{\rho N_{i1}}{2N_{I1}}$ and one can show that this holds the equilibrium, given equations (32), (35) and (42). These results extend to when a challenger only campaigns negatively in the general election.

Finally, note that for a given ρ, an increase (decrease) in N_{it}^*, P_{it}^*, $t \in \{0,1\}$, implies an increase (decrease) in n_{it}^*, p_{it}^*. Again, by the continuity of the derivatives in β and other parameters, there exists a $\bar{\beta}$ such that these results hold for all $\beta \in [0, \bar{\beta})$ in the neighborhood of symmetry. ■

Proof of Proposition 8.

At the equilibrium $(N_{i0}^*, P_{i0}^*, N_{j0}^*, P_{j0}^*)$,

$$\underbrace{\frac{\partial^2 \pi^i}{\partial\theta\partial N_{i0}}}_{\text{direct effect of a change in } \theta} + \overbrace{\frac{\partial^2 \pi^i}{\partial {N_{i0}}^2}\frac{dN_{i0}}{d\theta} + \frac{\partial^2 \pi^i}{\partial P_{i0}\partial N_{i0}}\frac{dP_{i0}}{d\theta}}^{\text{indirect effect via change in } i\text{'s actions}} + \underbrace{\frac{\partial^2 \pi^i}{\partial P_{j0}\partial N_{i0}}\frac{dP_{j0}}{d\theta} + \frac{\partial^2 \pi^i}{\partial N_{j0}\partial N_{i0}}\frac{dN_{j0}}{d\theta}}_{\text{indirect effect via change in } j\text{'s actions}} = 0$$

and

$$\underbrace{\frac{\partial^2 \pi^j}{\partial\theta\partial N_{j0}}}_{\text{direct effect of a change in } \theta} + \overbrace{\frac{\partial^2 \pi^j}{\partial N_{i0}\partial N_{j0}}\frac{dN_{i0}}{d\theta} + \frac{\partial^2 \pi^j}{\partial P_{i0}\partial N_{j0}}\frac{dP_{i0}}{d\theta}}^{\text{indirect effect via change in } i\text{'s actions}} + \underbrace{\frac{\partial^2 \pi^j}{\partial P_{j0}\partial N_{j0}}\frac{dP_{j0}}{d\theta} + \frac{\partial^2 \pi^j}{\partial N_{j0}^2}\frac{dN_{j0}}{d\theta}}_{\text{indirect effect via change in } j\text{'s actions}} = 0.$$

Since $\rho P_{i0}^* = N_{i0}^*$ and $\rho P_{j0}^* = N_{j0}^*$ at an interior optimum when $\beta = 0$ (from equation (6)), the equations reduce to,

$$\frac{\partial^2 \pi^i}{\partial\theta\partial N_{i0}} + \left(\frac{\partial^2 \pi^i}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{i0}\partial N_{i0}}\right)\frac{dN_{i0}}{d\theta} + \left(\frac{\partial^2 \pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{j0}\partial N_{i0}}\right)\frac{dN_{j0}}{d\theta} = 0$$

and

$$\frac{\partial^2 \pi^j}{\partial\theta\partial N_{j0}} + \left(\frac{\partial^2 \pi^j}{\partial N_{i0}\partial N_{j0}} + \frac{1}{\rho}\frac{\partial^2 \pi^j}{\partial P_{i0}\partial N_{j0}}\right)\frac{dN_{i0}}{d\theta} + \left(\frac{\partial^2 \pi^j}{\partial N_{j0}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^j}{\partial P_{j0}\partial N_{j0}}\right)\frac{dN_{j0}}{d\theta} = 0.$$

Next, solve these equations for

$$\frac{dN_{i0}}{d\theta} = \frac{\frac{\partial^2 \pi^i}{\partial\theta\partial N_{i0}}\left(\frac{\partial^2 \pi^j}{\partial N_{j0}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^j}{\partial P_{j0}\partial N_{j0}}\right) - \frac{\partial^2 \pi^j}{\partial\theta\partial N_{j0}}\left(\frac{\partial^2 \pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{j0}\partial N_{i0}}\right)}{\left(\frac{\partial^2 \pi^j}{\partial N_{i0}\partial N_{j0}} + \frac{1}{\rho}\frac{\partial^2 \pi^j}{\partial P_{i0}\partial N_{j0}}\right)\left(\frac{\partial^2 \pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{j0}\partial N_{i0}}\right) - \left(\frac{\partial^2 \pi^i}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^i}{\partial P_{i0}\partial N_{i0}}\right)\left(\frac{\partial^2 \pi^j}{\partial N_{j0}^2} + \frac{1}{\rho}\frac{\partial^2 \pi^j}{\partial P_{j0}\partial N_{j0}}\right)} \tag{45}$$

and similarly for $\frac{dN_{j0}}{d\theta}$. It follows from the second-order conditions and equation (16) (which holds in the neighborhood of symmetry from the continuity of the derivatives in $\bar{X}_i, \bar{B}_i, M_i$, etc.) that

$$\frac{\partial^2\pi^i}{\partial N_{i0}^2}\frac{\partial^2\pi^i}{\partial P_{i0}^2} - \left(\frac{\partial^2\pi^i}{\partial N_{i0}\partial P_{i0}}\right)^2 \to \left(\rho\frac{\partial^2\pi^i}{\partial N_{i0}^2}\right)^2 - \left(\frac{\partial^2\pi^i}{\partial N_{i0}\partial P_{i0}}\right)^2 > 0 \Rightarrow \frac{\partial^2\pi^i}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}} < 0. \tag{46}$$

Also, recall from equation (17) that

$$\frac{\partial^2\pi^i}{\partial N_{j0}N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}N_{i0}} > 0. \tag{47}$$

Further, given the continuity of derivatives in the parameters, it follows from Lemma 1 that in the neighborhood of symmetry,

$$\frac{\partial^2\pi}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2\pi}{\partial P_{i0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi}{\partial N_{j0}\partial N_{i0}} < 0. \tag{48}$$

Then, from equations (47) and (48) it follows that

$$0 < \frac{\partial^2\pi^i}{\partial N_{j0}N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}N_{i0}} < \left|\frac{\partial^2\pi^i}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}\right|$$

Analogous inequalities hold for challenger j, implying that the denominator of the expression on the right-hand side in equation (45) is negative. Below, we reproduce equation (45) along with the signs of expressions contained in it:

$$\frac{dN_{i0}}{d\theta} = \frac{\frac{\partial^2\pi^i}{\partial\theta\partial N_{i0}}\overbrace{\left(\frac{\partial^2\pi^j}{\partial N_{j0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{j0}\partial N_{j0}}\right)}^{<0} - \frac{\partial^2\pi^j}{\partial\theta\partial N_{j0}}\overbrace{\left(\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}\right)}^{>0}}{\underbrace{\left(\frac{\partial^2\pi^j}{\partial N_{i0}\partial N_{j0}} + \frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{i0}\partial N_{j0}}\right)\left(\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}\right) - \left(\frac{\partial^2\pi^i}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}\right)\left(\frac{\partial^2\pi^j}{\partial N_{j0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{j0}\partial N_{j0}}\right)}_{<0}}. \tag{49}$$

Similarly,

$$\frac{dN_{j0}}{d\theta} = \frac{\frac{\partial^2\pi^j}{\partial\theta\partial N_{j0}}\overbrace{\left(\frac{\partial^2\pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}\right)}^{<0} - \frac{\partial^2\pi^i}{\partial\theta\partial N_{i0}}\overbrace{\left(\frac{\partial^2\pi^j}{\partial N_{i0}\partial N_{j0}} + \frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{i0}\partial N_{j0}}\right)}^{>0}}{\underbrace{\left(\frac{\partial^2\pi^j}{\partial N_{i0}\partial N_{j0}} + \frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{i0}\partial N_{j0}}\right)\left(\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}\right) - \left(\frac{\partial^2\pi^i}{\partial {N_{i0}}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}\right)\left(\frac{\partial^2\pi^j}{\partial N_{j0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^j}{\partial P_{j0}\partial N_{j0}}\right)}_{<0}}. \tag{50}$$

Now, consider the impact of improving *only* challenger i's reputation. We first show that as long as a challenger's payoff from winning office significantly exceeds his payoff if his primary rival wins, so that $M_i > 3$, then $\frac{\partial^2 \pi^j}{\partial \bar{X}_i \partial N_{j0}} > 0$. To show this, we first note that,

$$\pi^j = -(M_j Pr_{j1} - Pr_{i1}) Pr_{i0} + M_j Pr_{j1}$$

so that,

$$\begin{aligned}
\frac{\partial^2 \pi^j}{\partial \bar{X}_i \partial N_{j0}} &= -\frac{\partial^2 Pr_{i0}}{\partial \bar{X}_i \partial N_{j0}}(M_j Pr_{j1} - Pr_{i1}) - M_j \frac{\partial Pr_{j1}}{\partial N_{j0}} \frac{\partial Pr_{i0}}{\partial \bar{X}_i} + \frac{\partial Pr_{i0}}{\partial N_{j0}} \frac{\partial Pr_{i1}}{\partial \bar{X}_i} \\
&= (M_j Pr_{j1} - Pr_{i1}) \frac{\alpha Q_{i0}^*(1 - Q_{i0}^*)}{\bar{X}_i N_{j0}^*(1 + Q_{i0}^*)^3} - \frac{\alpha Q_{i0}^* Q_{i1}^*}{\bar{X}_i N_{j0}^*(1 + Q_{i0}^*)^2(1 + Q_{i1}^*)^2} \\
&\qquad + \frac{\alpha M_j Q_{i0}^* Q_{j1}^*}{\bar{X}_i N_{j1}^*(1 + Q_{i0}^*)^2(1 + Q_{j1}^*)^2} \qquad (51) \\
&\to \frac{\alpha Q_{i1}^*(M_i N_{i0}^* - N_{i1}^*)}{4\bar{X}_i N_{i0}^* N_{i1}^*(1 + Q_{i1}^*)^2} \quad (\text{as } Q_{i0}^* = 1, Q_{i1}^* = Q_{j1}^*) \\
&= \frac{\alpha(M_i - 1 + (M_i - 3)Q_{i1}^*)}{8\bar{X}_i N_{i0}^*(1 + Q_{i1}^*)^2} > 0,
\end{aligned}$$

when $M_i \geq 3$. Recall that N_{i0}^* increases in M_i. Therefore, N_{i1}^* decreases in M_i, so that $M_i N_{i0}^* - N_{i1}^* > 0$ when M_i is high. In fact, the last equality in (51), which is obtained by dividing both numerator and denominator by N_{i1}^* and substituting $\frac{N_{i0}^*}{N_{i1}^*} = \frac{(M_i-1)(1+Q_{i1}^*)}{2M_i Q_{i1}^*}$, indicates that $M_i > 3$ suffices, and in what follows, we assume that this holds. Similarly,

$$\begin{aligned}
\frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}} &= \frac{\partial^2 Pr_{i0}}{\partial \bar{X}_i \partial N_{i0}}(M_i Pr_{i1} - Pr_{j1}) + M_i \frac{\partial Pr_{i1}}{\partial N_{i0}} \frac{\partial Pr_{i0}}{\partial \bar{X}_i} + M_i \frac{\partial Pr_{i0}}{\partial N_{i0}} \frac{\partial Pr_{i1}}{\partial \bar{X}_i} + M_i Pr_{i0} \frac{\partial^2 Pr_{i1}}{\partial \bar{X}_i \partial N_{i0}} \\
&= (M_i Pr_{i1} - Pr_{j1}) \frac{\alpha Q_{i0}^*(1 - Q_{i0}^*)}{\bar{X}_i N_{i0}^*(1 + Q_{i0}^*)^3} + \frac{\alpha M_i Q_{i0}^* Q_{i1}^*}{\bar{X}_i N_{i0}^*(1 + Q_{i0}^*)^2(1 + Q_{i1}^*)^2} + \frac{\alpha M_i Q_{i1}^*(1 - Q_{i1}^*)}{\bar{X}_i N_{i1}^*(1 + Q_{i0}^*)(1 + Q_{i1}^*)^3} \\
&\qquad - \frac{\alpha M_i Q_{i1}^* Q_{i0}^*}{\bar{X}_i N_{i1}^*(1 + Q_{i0}^*)^2(1 + Q_{i1}^*)^2} \\
&\to \frac{\alpha M_i Q_{i1}^*}{4\bar{X}_i N_{i0}^*(1 + Q_{i1}^*)^2} + \frac{\alpha M_i Q_{i1}^*(1 - Q_{i1}^*)}{2\bar{X}_i N_{i1}^*(1 + Q_{i1}^*)^3} - \frac{\alpha M_i Q_{i1}^*}{4\bar{X}_i N_{i1}^*(1 + Q_{i1}^*)^2} = \frac{\alpha(M_i - 3)(\frac{M_i - 1}{M_i - 3} - Q_{i1}^*)}{8\bar{X}_i N_{i0}^*(1 + Q_{i1}^*)^2}.
\end{aligned} \qquad (52)$$

Now, note that $\frac{M_i-1}{M_i-3}$ decreases in M_i, while Q_{i1}^* increases in M_i (when M_i increases, N_{i0}^*, P_{i0}^* increase and N_{i1}^*, P_{i1}^* decrease implying Q_{i1}^* increases). Therefore, when M_i is high, $\frac{M_i-1}{M_i-3} - Q_{i1}^*$ is low. If it is positive, i.e., if $Q_{i1}^* < \frac{M_i-1}{M_i-3}$, then so is $\frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}}$, and from equations (49) and (50), $\frac{dN_{i0}}{d\bar{X}_i} > 0, \frac{dN_{j0}}{d\bar{X}_i} > 0$. If it is negative, i.e., if $Q_{i1}^* > \frac{M_i-1}{M_i-3}$, so that $\frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}} < 0$, then

$\frac{dN_{j0}}{d\bar{X}_i} > 0$, since from equations (51) and (52),

$$\frac{\partial^2 \pi^j}{\partial \bar{X}_i \partial N_{j0}} > \left| \frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}} \right|$$

while, as $\beta \to 0$,

$$\left(\frac{\partial^2 \pi^j}{\partial N_{i0} \partial N_{j0}} + \frac{1}{\rho} \frac{\partial^2 \pi^j}{\partial P_{i0} \partial N_{j0}} \right) = \left(\frac{\partial^2 \pi^i}{\partial N_{i0} \partial N_{j0}} + \frac{1}{\rho} \frac{\partial^2 \pi^i}{\partial P_{j0} \partial N_{i0}} \right) < \left| \frac{\partial^2 \pi^i}{\partial N_{i0}^2} + \frac{1}{\rho} \frac{\partial^2 \pi^i}{\partial P_{i0} \partial N_{i0}} \right|$$

from Lemma 1. Hence, the numerator of expression (50) is still negative. Thus, challenger j always raises his campaigning levels in response to an improvement in the reputation of the other challenger. However, the analysis of the direction of change in N_{i0}^* is not so straightforward anymore. From equation (49),

$$\frac{dN_{i0}}{d\bar{X}_i} \gtrless 0 \iff \frac{\frac{\partial^2 \pi^j}{\partial N_{j0}^2} + \frac{1}{\rho} \frac{\partial^2 \pi^j}{\partial P_{j0} \partial N_{j0}}}{\frac{\partial^2 \pi^i}{\partial N_{j0} \partial N_{i0}} + \frac{1}{\rho} \frac{\partial^2 \pi^i}{\partial P_{j0} \partial N_{i0}}} \gtrless \frac{\frac{\partial^2 \pi^j}{\partial \bar{X}_i \partial N_{j0}}}{\frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}}}.$$

Adding one to both sides and substituting

$$\frac{\partial^2 \pi^j}{\partial N_{j0}^2} + \frac{1}{\rho} \frac{\partial^2 \pi^j}{\partial P_{j0} \partial N_{j0}} = \frac{\partial^2 \pi^i}{\partial N_{i0}^2} + \frac{1}{\rho} \frac{\partial^2 \pi^i}{\partial P_{i0} \partial N_{i0}},$$

which holds as $\beta \to 0$, yields

$$\frac{\frac{\partial^2 \pi^i}{\partial N_{i0}^2} + \frac{1}{\rho} \frac{\partial^2 \pi^i}{\partial P_{i0} \partial N_{i0}} + \frac{\partial^2 \pi^i}{\partial N_{j0} \partial N_{i0}} + \frac{1}{\rho} \frac{\partial^2 \pi^i}{\partial P_{j0} \partial N_{i0}}}{\frac{\partial^2 \pi^i}{\partial N_{j0} \partial N_{i0}} + \frac{1}{\rho} \frac{\partial^2 \pi^i}{\partial P_{j0} \partial N_{i0}}} \gtrless \frac{\frac{\partial^2 \pi^j}{\partial \bar{X}_i \partial N_{j0}} + \frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}}}{\frac{\partial^2 \pi^i}{\partial \bar{X}_i \partial N_{i0}}} \tag{53}$$

From Lemma 1, as $\beta \to 0$, the expression on the left-hand side of (53) goes to

$$\frac{\frac{\alpha(M_i-1)}{4\rho^2 P_{i0}^{*2}(1+Q_{i1}^*)} \left[1 + \left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*} \right) \frac{M_i-1}{2M_i} + \frac{\alpha(M_i-1)}{M_i} \right]}{\frac{(M_i+1)\alpha^2 Q_{i1}^*}{2\rho^2 P_{i1}^* P_{i0}^*(1+Q_{i1})^2}}.$$

Rearranging terms and substituting $\frac{P_{i0}^*}{P_{i1}^*} \to \frac{(M_i-1)(1+Q_{i1}^*)}{2M_i Q_{i1}^*}$, this reduces to

$$\frac{M_i}{\alpha(M_i+1)} \left[-1 - \left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*} \right) \frac{M_i-1}{2M_i} - \frac{\alpha(M_i-1)}{M_i} \right]. \tag{54}$$

From equations (51) and (52), after canceling terms, the right-hand side of (53) is $\frac{\frac{2(M_i-1)}{M_i-3}}{\frac{M_i-1}{M_i-3} - Q_{i1}^*}$.

Thus, when $Q_{i1}^* > \frac{M_i-1}{M_i-3}$,

$$\frac{dN_{i0}}{d\bar{X}_i} \gtrless 0 \Longleftrightarrow \frac{M_i}{\alpha(M_i+1)}\left[-1-\left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i}-\frac{\alpha(M_i-1)}{M_i}\right] \gtrless \frac{\frac{2(M_i-1)}{M_i-3}}{\frac{M_i-1}{M_i-3}-Q_{i1}^*}$$

or,

$$\frac{dN_{i0}}{d\bar{X}_i} \gtrless 0 \Longleftrightarrow \frac{(1+2\alpha)(M_i-1)+Q_{i1}^*(3M_i-1)}{2\alpha Q_{i1}^*(M_i+1)} + \frac{2(M_i-1)}{M_i-1-(M_i-3)Q_{i1}^*} \lessgtr 0$$
$$\Longleftrightarrow \left((1+2\alpha)(M_i-1)+Q_{i1}^*(3M_i-1)\right)\left(M_i-1-(M_i-3)Q_{i1}^*\right)+4(M_i^2-1)\alpha Q_{i1}^* \gtrless 0. \tag{55}$$

Clearly, there exists a $\bar{Q}$, such that for $Q_{i1}^* \in (\frac{M_i-1}{M_i-3}, \bar{Q})$, the above expression is positive, implying that negative primary campaigning by i increases in $\bar{X}_i$, and from the analysis above, it is increasing for $Q_{i1}^* < \frac{M_i-1}{M_i-3}$ as well. Thus, if the incumbent is weak (e.g., due to a poor reputation or limited resources), so that he is likely to be ousted by a challenger, i.e., $Q_{i1}^* \in (0, \bar{Q})$, then the challenger's primary campaigning increases as his reputation improves, while the opposite holds if the incumbent is strong.

Next, consider the impact of increasing challenger i's resources. We first show that it results in increased negative primary campaigning by both challengers when challengers have nearly identical endowments, preferences and reputations. To see this, note that when $\beta = 0$,

$$\frac{\partial^2 \pi^j}{\partial \bar{B}_i \partial N_{j0}} = \frac{\partial Pr_{i0}}{\partial N_{j0}}\frac{\partial Pr_{i1}}{\partial \bar{B}_i} = \frac{-\alpha Q_{i0}^*}{N_{j0}^*(1+Q_{i0}^*)^2}\frac{\alpha Q_{i1}^*}{P_{i1}^*(1+Q_{i1}^*)^2} \to \frac{-\alpha^2 Q_{i1}^*}{4N_{i0}^* P_{i1}^*(1+Q_{i1}^*)^2} < 0, \tag{56}$$

and

$$\begin{aligned}\frac{\partial^2 \pi^i}{\partial \bar{B}_i \partial N_{i0}} &= M_i \frac{\partial Pr_{i0}}{\partial N_{i0}}\frac{\partial Pr_{i1}}{\partial \bar{B}_i} + M_i Pr_{i0}\frac{\partial^2 Pr_{i1}}{\partial \bar{B}_i \partial N_{i0}} \\ &= \frac{\alpha^2 M_i Q_{i0}^* Q_{i1}^*}{N_{i0}^* P_{i1}^*(1+Q_{i0}^*)^2(1+Q_{i1}^*)^2} + \frac{\alpha M_i Q_{i1}^*(1+2\alpha-(2\alpha-1)Q_{i1}^*)}{2N_{i1}^* P_{i1}^*(1+Q_{i0}^*)(1+Q_{i1}^*)^3} \\ &= \frac{\alpha^2 \rho M_i Q_{i1}^*}{4N_{i0}^* N_{i1}^*(1+Q_{i1}^*)^2} + \frac{\alpha\rho M_i Q_{i1}^*(1+2\alpha-(2\alpha-1)Q_{i1}^*)}{4N_{i1}^{*2}(1+Q_{i1}^*)^3} \\ &= \frac{\alpha\rho(M_i-1)}{8N_{i0}^{*2}(1+Q_{i1}^*)}\left(\alpha + \frac{(M_i-1)(1+2\alpha-(2\alpha-1)Q_{i1}^*)}{2M_i Q_{i1}^*}\right) \\ &= \frac{\alpha\rho(M_i-1)((M_i-1)(1+2\alpha+Q_{i1}^*)+2\alpha Q_{i1}^*)}{16N_{i0}^{*2}M_i Q_{i1}^*(1+Q_{i1}^*)} > 0.\end{aligned} \tag{57}$$

Then, it follows from equation (49) that as $\beta \to 0$,

$$\frac{dN_{i0}}{d\bar{B}_i} > 0 \iff \frac{\frac{\partial^2\pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}} < \frac{\frac{\partial^2\pi^j}{\partial \bar{B}_i\partial N_{j0}}}{\frac{\partial^2\pi^i}{\partial \bar{B}_i\partial N_{i0}}}$$
$$\iff \frac{\frac{\partial^2\pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}} + \frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}}} < \frac{\frac{\partial^2\pi^j}{\partial \bar{B}_i\partial N_{j0}} + \frac{\partial^2\pi^i}{\partial \bar{B}_i\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial \bar{B}_i\partial N_{i0}}}.$$

From equations (54), (56) and (57), and substituting for $\frac{N_{i0}^*}{N_{i1}^*} \to \frac{(M_i-1)(1+Q_{i1}^*)}{2M_iQ_{i1}^*}$, rewrite this as

$$\frac{M_i}{\alpha(M_i+1)}\left[-1 - \left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} - \frac{\alpha(M_i-1)}{M_i}\right] < \frac{(M_i-1)(1+2\alpha+Q_{i1}^*)}{(M_i-1)(1+2\alpha+Q_{i1}^*)+2\alpha Q_{i1}^*}$$
$$\text{or, } -\frac{(1+2\alpha)(M_i-1)+Q_{i1}^*(3M_i-1)}{2\alpha Q_{i1}^*(M_i+1)} - \frac{(M_i-1)(1+2\alpha+Q_{i1}^*)}{(M_i-1)(1+2\alpha+Q_{i1}^*)+2\alpha Q_{i1}^*} < 0,$$

which always holds as $M_i > 1$. Similarly,

$$\frac{dN_{j0}}{d\bar{B}_i} > 0 \iff \frac{\frac{\partial^2\pi^i}{\partial N_{j0}\partial N_{i0}} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{j0}\partial N_{i0}} + \frac{\partial^2\pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial N_{i0}^2} + \frac{1}{\rho}\frac{\partial^2\pi^i}{\partial P_{i0}\partial N_{i0}}} < \frac{\frac{\partial^2\pi^j}{\partial \bar{B}_i\partial N_{j0}} + \frac{\partial^2\pi^i}{\partial \bar{B}_i\partial N_{i0}}}{\frac{\partial^2\pi^i}{\partial \bar{B}_i\partial N_{i0}}}$$
$$\text{or, } \frac{1 + \left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} + \frac{\alpha(M_i-1)}{M_i}}{1 + \left(\frac{1+2\alpha-(2\alpha-1)Q_{i1}^*}{Q_{i1}^*}\right)\frac{M_i-1}{2M_i} + 2\alpha} < \frac{(M_i-1)(1+2\alpha+Q_{i1}^*)}{(M_i-1)(1+2\alpha+Q_{i1}^*)+2\alpha Q_{i1}^*}$$
$$\iff M_i(M_i-1)(1+2\alpha) + M_iQ_{i1}^*(M_i-3) > 0$$

which always holds. Thus, negative primary campaigning by challengers initially increases with challenger i's resources in the neighborhood of symmetry.

Finally, consider the impact of an increase in the relative payoff, M_i, of challenger i. Note that $\frac{\partial^2\pi^j}{\partial M_i\partial N_{j0}} = 0$, while $\frac{\partial^2\pi^i}{\partial M_i\partial N_{i0}} > 0$, as shown earlier. Then, from equation (49) and (50) it follows that $\frac{\partial N_{i0}}{\partial M_i} > 0, \frac{\partial N_{j0}}{\partial M_i} > 0$. Since $\rho P_{i0}^* = N_{i0}^*, \rho P_{j0}^* = N_{j0}^*$, positive campaigning levels increase as well. Thus, the challengers' campaigning levels and expenditure in the primary increase. Note that the probability of a win for either challenger in the general election declines.

By the continuity of derivatives in β, there exists a $\tilde{\beta}$ such that for all $\beta \leq \tilde{\beta}$ the result holds. Lastly, it is easy to show that these results carry over to the case where the challenger campaigns only negatively in the general election. ■