Vanguards in Revolution

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Abstract

We analyze strategic interactions between revolutionary vanguards and citizens who have private information about payoffs from successful revolution. Vanguards sacrifice by initiating revolts that may not win support, mitigating a follower’s risk of revolting alone. We show regimes may prefer radical vanguards to moderately-conservative ones because even though radical vanguards revolt more, they generate less following. If follower status quo payoffs fall below a threshold, followers prefer extremely radical vanguards who always initiate revolt; but even when vanguards can punish non-compliant followers, incentives to exploit their information limit their use of coercion. We then study the emergence of organic leaders, showing that once a revolution begins, it is more likely to succeed if it is initiated organically—i.e., without established vanguards than with established vanguards.
1 Introduction

We study the strategic interactions between revolutionary vanguards\(^1\) and citizens in a setting where the merits of changing the status quo are uncertain, and both vanguards and citizens have private information about those merits. Censorship and restrictions on assembly together with the fear of punishment by the regime sharply limit direct communication between vanguards and citizens, resulting in both parties having substantial private, uncommunicated information about the merits of revolution.\(^2\) We characterize the coordination and information aggregation considerations that arise. We study how a vanguard’s radicalism affects the likelihood of successful revolution, and the optimal level of radicalism in a vanguard from a follower’s perspective. We derive the optimal use of coercion with which vanguards threaten citizens for disloyalty. We describe the subtleties that enter a state’s choice of how to combine rewards and punishments to maintain the status quo. Finally, we study the endogenous emergence of revolutionary leaders and its effects on the likelihood of revolution.

Our base model features a representative vanguard and a representative citizen “follower”. The vanguard first decides whether to revolt. The follower observes the vanguard’s action and decides whether to revolt. For the revolution to succeed, both the vanguard and follower must revolt, or else the status quo prevails. Moreover, a solo revolter is punished by the regime: by moving first, the vanguard incurs the risk of being the sole revolter, but this sacrifice mitigates that risk for the follower.

The vanguard’s strategic considerations are subtle: with a more compliant follower, the vanguard faces less risk of a failed revolt; but a more compliant follower

\(^1\)The word vanguard is from the Anglo-French word *avantgarde*, where *avant-* means “front” and *garde* means “guard”. By vanguards we mean activists with skills and experience in organizing anti-regime activities. This notion of vanguard resonates with the “professional revolutionaries” advocated in Lenin’s “What is to be Done?”, as well as the notions of “revolutionary entrepreneurs” in formal models of revolution (Bueno de Mesquita 2010; Roemer 1985), and “social entrepreneurs” and “entrepreneurs of violence” in the social movements literature (Della Porta 1995, 195-201; Diani 2003; Tilly and Tarrow 2007, 29-31). However, our vanguards need not be committed revolutionaries. What distinguishes them from other citizens is that they have the know-hows of “protest technology”; that is, they have access to organizational resources (e.g., religious funds or networks) or are endowed with skills that enable them to initiate anti-regime activities such as protests or armed attacks.

\(^2\)An alternative interpretation is that only some citizens can communicate with vanguards, and we consider the interaction between the vanguards and the remaining citizens.
also supports change even when his private information indicates that the vanguard is “mistaken”, i.e., that a successful protest would be worse than the status quo. This reduces the vanguard’s ability to use the citizen’s information to protect against outcomes that could be far worse than the costs of revolting alone, for example, protecting the vanguard against a successful revolution that “devours its own”, as happened to the Jacobins in the French Revolution and the Marxists in the Iranian Revolution (Abrahamian 1982, 1999). We establish the strategic substitute/complement structure of strategies and show that a unique equilibrium exists.

We then investigate what happens when the vanguard is more (less) radical than the follower. By definition, a radical vanguard derives a higher payoff from a successful revolution than the follower, so that a more radical vanguard revolts after worse information about the successful revolution payoff. In turn, this makes the follower less willing to support the vanguard: for the follower to revolt, his information about the successful revolution payoff must be better. Thus, a more radical vanguard revolts more, but is less likely to generate a following.

This simple result has sharp implications for a state that seeks to minimize the probability of a successful revolution. Obviously, the state would like to have a very conservative vanguard, one that is reluctant to overturn the status quo. So, too, the state would like to be able to catch and harshly punish any vanguard in a failed revolt—as then a vanguard is very unlikely to initiate a revolt. But what should a state do if it can neither anoint a puppet as a leader (e.g., a faux union leader), nor punish so harshly? We show that a state may be better off with an extremely radical vanguard than with a moderate vanguard; and it may be better off not punishing leaders of a failed revolt than punishing moderately. While more radical and less harshly-punished vanguards revolt more, they are also less likely to be followed; and what matters to the state is the probability of joint action, and not unilateral action. We find that the probability of a successful revolution may be maximized by a slightly conservative vanguard, or by moderate punishments for a failed revolt. This provides a rationale for why Assad “has funded and co-operated with al-Qaeda in a complex double game even as the terrorists fight Damascus (Telegraph, 20 Jan 2014).” Or
for why a regime might implement surprisingly light punishments for leaders of a failed revolt. For example, in the early 1960s when Khomeini first openly criticized the Shah, the Pahlavi regime kept him under arrest for a short period of time although the regime responded harshly to protesters on the streets. When he publicly denounced the Shah again after his release, he was only forced into exile. Relatedly, a state must be careful when deciding how to combine a reward that it gives a potential revolutionary follower for defying a vanguard together with anointing a more conservative vanguard. We illustrate how, when the reward to a disloyal follower is raised, the probability of successful revolt is maximized by a more conservative vanguard, i.e., by a vanguard who is more reluctant to overturn the status quo.

We then highlight a distinction between a vanguard that is truly radical and one that only appears to be. A vanguard appears to be more radical than her follower when she would revolt following signals that would not cause her follower to do so. When the punishment for leading a failed revolt is high enough, the follower always appears to be more radical than the vanguard. However, as long as this punishment is not too high, a vanguard’s appearance is non-monotone in the follower’s payoff for non-support. The vanguard appears more conservative than the follower both when the state indiscriminately punishes everyone following a failed revolt—so that the follower’s payoff from non-support is very low—or when the state generously rewards a disloyal follower—so that this payoff is very high. However, when the follower’s payoff for non-support is intermediate, the vanguard appears more radical.

The intuition for why a vanguard appears more conservative when a follower is harshly punished for failing to revolt is information-based: since the follower fears the consequences of not revolting, he revolts even after very bad signals, causing the vanguard to revolt more selectively in order to avoid “successful” revolutions that turn out badly. The intuition for why a vanguard appears more conservative when a follower is generously rewarded by the regime for disloyalty reflects the risks of miscoordination: the vanguard’s fear of punishment is so high that she revolts less than the follower.

The extent of a vanguard’s radicalism directly affects her follower’s payoffs; and the follower’s payoff from not providing support directly affects the vanguard. These
observations give rise to the natural questions: If a follower could choose the vanguard, would he select a radical vanguard that is likely to initiate revolt or a conservative one that seldom does? And, if a vanguard can employ coercion to reduce a follower’s payoffs if he does not provide support, how much coercion should she employ?

We first show that a follower’s preferred level of radicalism in a vanguard induces the optimal choice by the vanguard from the follower’s perspective of when to revolt. Thus, if their status quo payoffs are equal, a follower prefers a radical vanguard if and only if his reward for not supporting the vanguard exceeds the vanguard’s payoff from a failed revolt. Further, the higher is a vanguard’s status quo payoff or the harsher are punishments for failed revolt, the more reluctant she is to revolt—to unwind this, a follower prefers a more radical vanguard. A follower also prefers a more radical vanguard when his status quo payoffs are lower. In fact, if the follower’s status quo payoffs are below a threshold, he prefers an extremely radical vanguard who always revolts.

A vanguard’s design of the coercion to employ on a follower reflects different considerations. In settings such as civil wars or guerrilla movements, a vanguard can punish a follower who does not follow her lead. For example, a clandestine armed organization (the vanguard) contemplating an attack on government forces can punish villagers (the follower) who have information about the strength and location of those forces if they do not cooperate (burning houses, etc.). When choosing coercion, a vanguard internalizes that a follower’s information may suggest that the outcome of joint action is likely to be so bad that the vanguard would be better off if the follower withheld support, causing the joint action to fail. This means that a vanguard never wants to coerce a follower too harshly—in contrast to a follower who may want an unboundedly radical vanguard. We show that if a vanguard can freely choose the extent of coercion, then she punishes a “disloyal” follower by just enough that he faces the same payoffs as the vanguard. The logic driving the limited use of coercion is strategic in nature: a vanguard wants a follower to take whatever action the vanguard would were she in his shoes, i.e., were the vanguard taking the follower’s decision given his information, which means that the vanguard wants the follower to face the same payoffs. Thus, the more a vanguard sacrifices, the more coercion she
wants to use. Paradoxically, a follower can benefit from a vanguard’s use of coercion, as it makes the vanguard more willing to initiate revolt.

Extending this reasoning, we show that if, rather than having common beliefs about the informational structure, a vanguard has more confidence in her knowledge than does her follower, then she wants to use harsher coercion. A follower who has less faith than the vanguard in the vanguard’s information weighs her information by less. The vanguard then wants to punish more harshly in order to adjust for her follower’s “ignorance”. These findings highlight that the source of extreme coercive measures by vanguards is not that vanguards believe they know far more than their followers, but rather that they believe that their followers do not understand how much they know. This also suggests why ideological vanguards (e.g., Bolsheviks), who tend to be overconfident about their knowledge, tend to use more coercive measures.

Our primary analysis characterizes settings where the identity of the potential leader of a revolution is well-established—it is a revolutionary vanguard, an activist with skills at organizing anti-regime activities. However, in other settings, the identity of who can take the first steps to initiate a protest or revolution is less clear-cut—and citizens can choose whether to lead or follow. Our concluding analysis studies the emergence of organic leaders in such a setting. Two ex-ante identical citizens receive private signals about the payoffs from successful action. Now, however, each citizen has two chances to act. By initiating revolt at date 1, a citizen can endogenously assume the mantle of a leader, revealing that he/she has good news about the revolution. Alternatively, a citizen can wait to see what the other citizen did, and then base date-2 decisions on the information revealed. Now, citizens have incentives both to lead, risking punishment to convey a positive signal about revolution payoffs and deliver a successful revolution; and to defer, in order to free ride on the information conveyed by the other’s actions, and avoid punishment for leading a failed revolt.

One might conjecture that these free-riding incentives reduce the likelihood of revolt and citizen welfare relative to settings with an established vanguard. Indeed, we prove that an established vanguard is always more likely to initiate revolt because the revolution cannot succeed unless she acts immediately, regardless of her follower’s
potential information. However, the relative reluctance of an organic leader to act raises the good news conveyed by acting, making it more attractive to support an organic leader. Thus, once a revolution begins, it is more likely to succeed if it was initiated organically. We describe conditions under which the likelihood of successful revolution is higher in one setting than the other, and provide a numerical analysis suggesting that citizen welfare is higher when leaders emerge organically.

**Literature Review**

Bueno de Mesquita (2010) studies the signaling role of vanguards. There is a vanguard and a continuum of citizens distinguished by their private levels of “anti-regime sentiments”. Vanguards exert costly efforts to foment violence, which is publicly observed. Then, citizens decide whether to revolt. By assumption, the intensity of violence is a noisy public signal of anti-regime sentiments in the population—for any effort level of the vanguard, violence rises with those sentiments. Thus, a vanguard’s effort reduces the strategic risk of revolting, and hence can enhance coordination among citizens.

Bueno de Mesquita (2013) explores rebel tactics: after observing the level of mobilization in the population, a rebel leader must decide among conventional war, irregular war, or withdrawal. Conventional war has the strongest complementary link with mobilization. Thus, irregular war occurs when the outside option is intermediate: when it is low, mobilization is high, and the leader chooses conventional war; when it is high, the leader completely withdraws from conflict. Dynamically, a leader may continue conflict despite low mobilization levels, hoping that future exogenous shocks raise mobilization, i.e., conflict has an option value. We abstract from a vanguard’s choice of tactics, focusing on coordination and information aggregation issues and their consequences for the state’s response to dissent, a vanguard’s use of coercion, and radicalism. Our analysis of coercion is loosely related to rebel tactics—but its driving force is information aggregation rather than complementarities in fighting technology.

Majumdar and Mukand (2008) focus on a leader’s information acquisition. The leader and citizens must decide whether to participate in a costly action to change the status quo. Greater participation raises the likelihood of changing the status quo. Cit-
izens first decide whether to “invest in activism” to reduce future costs of action. Next, the leader exerts effort to learn about the payoffs of change, which he reveals to the population. Then, costs are realized and individuals decide whether to act. They analyze the complimentary links between citizens’ investments, a leader’s information acquisition efforts, and participation levels. These complementarities can result in multiple equilibria, including one with no action, and hence give rise to “threshold effects”. For example, when a leader’s ability to acquire information is below a threshold, then in the unique equilibrium, no one acts so that the status quo prevails. They also identify tradeoffs that arise when a leader’s preferences are not aligned with citizens: Such a leader has low credibility, but may exert more effort to learn the payoffs from change because, even when change does not benefit citizens, the leader can still gain.

Much of the leadership literature focuses on the role of “leader as communicator”. In Hermalin (1998), the leader is a team member who has more information about the returns to effort, and tries to credibly signal this information to other members to induce them to work harder to produce output that they will share. Komai et al. (2007) build on Hermalin (1998) to show that giving a member exclusive information can improve efficiency by reducing the incentives of others to shirk. Our focus is different: although a vanguard’s action is informative about its private information, we do not focus on this signaling role. Instead, our focus is on information aggregation and coordination considerations that arise in equilibrium, and their interactions with a vanguard’s radicalism and use of coercion, and the state’s decisions to prevent revolution.

A more distant literature studies the tradeoffs between coordination and adaptation. In Dewan and Myatt (2008), the leaders are party activists with noisy private signals about the unknown best action for the party, θ. Each leader sends a public signal to other members who simultaneously choose actions. A member cares about conformity and taking the best action θ: his payoff from taking action a is $-\pi (a - \theta)^2 - (1 - \pi)(a - \bar{a})^2$, where $\bar{a}$ is the average of members’ actions and $\pi$ is a weight. See also Wilson and Rhodes (1997). Alonso et al. (2008) and Rantakari (2008) also study the tradeoffs between coordination and adaptation in the context of organizational design, in which, unlike Dewan and Myatt, members have conflicting interests. A firm
has two local managers and a headquarter manager. The best action for the local office \( i \in \{1, 2\} \) is \( \theta_i \), which is the private information of its local manager; but profits also depend on how coordinated local offices are: office \( i \)'s profit from action \( a_i \) is \( K_i - (a_i - \theta_i)^2 - \delta(a_i - a_j)^2 \), where \( \delta \) is a weight and \( K_i \) is a constant. Under centralization, each local manager sends a (cheap talk) message to the headquarters, which then decides which actions to be taken in local offices. With decentralization, each local manager sends a message to the other, and then each decides the action for her local office.

Bolton et al. (2013) analyze the role of a leader’s overconfidence and corporate culture in an organization. The leader sends a public signal (“mission statement”) revealing her original private signal about the unknown state of the world \( \theta \) to employees. Next, each employee \( i \in [0, 1] \) receives his own noisy private signal about \( \theta \), and takes an action \( a_i \) that the leader does not see. Then, the leader receives an additional signal about \( \theta \), and decides on an action \( a_L \) (“organization’s strategy”). An employee \( i \)'s payoff from taking action \( a_i \) is \( \Pi_i = -\int (a_j - \bar{a})^2 \, dj - (a_i - a_L)^2 - (a_L - \theta)^2 \), where \( \bar{a} \) is the average employees’ actions; the leader’s payoff is the sum of the employees’ payoffs: \( \Pi = \int \Pi_j \, dj \). To allow for “bottom-up information flow”, they extend the game to allow the leader to observe a signal of the average action \( \bar{a} \) of employees, yielding multiple equilibria, and hence the role of corporate culture in equilibrium selection.

The tradeoff between coordination and adaptation takes a different form in our model: Coordination concerns arise because acting alone is costly, and “adaptation” concerns arise because the best course of action (whether to revolt) depends on the unknown revolution payoffs. However, in that literature, there is no information aggregation in equilibrium—i.e., no learning about others’ information via equilibrium actions, e.g., winner’s curse (Wilson 1977), swing voter’s curse (Feddersen and Pesendorfer 1996) or jury decisions. For example, in Dewan and Myatt, a party activist’s strategy given his signal \( s_i \) is \( A(s_i) = \pi E[\theta|s_i] + (1 - \pi)E[A(s_j)|s_i] \), implying that an activist only uses his own signal to estimate the common value \( \theta \). Further, unlike in the firms or party conferences settings studied in the literature, in our revolution, civil war or protest settings, contracting is implausible and communication is limited.
2 Model

A representative vanguard and a representative citizen (follower), sequentially decide whether or not to revolt. The vanguard moves first and the follower moves second. Figure 1 shows the sequence of moves and payoffs. The payoff $\theta$ from a successful revolution is uncertain, and citizens receive private signals $s_1$ and $s_2$ about $\theta$. The other expected payoffs are common knowledge, with $h_i > l_i$. Thus, when only one citizen revolts, the revolution fails, and the sole revolter incurs a punishment cost.$^3$

Figure 1: vanguard-follower Game. $R$ indicates revolt and $\neg R$ indicates no revolt. There are miscoordination costs: $l_i < h_i$; and citizens have private information about the successful revolution payoff $\theta$.

The signals and $\theta$ are jointly distributed with a strictly positive, continuously differentiable density $f(s_1, s_2, \theta)$ on $\mathbb{R}^3$. We assume that $s_1$, $s_2$ and $\theta$ are strictly affiliated, so that, for example, when the vanguard receives a higher signal $s_1$, the follower is more likely to receive higher signals $s_2$, and higher payoffs $\theta$ from successful revolution are more likely.$^4$ We impose minimal structure on the tail properties of $f(s_1, s_2, \theta)$: we require that expected revolution payoffs be very high following a very good signal, and very low following a very bad signal. Further, when an agent receives a very good or very bad signal, the other agent is very likely to receive a qualitatively similar very good or bad signal; and the likelihood that the follower’s signal exceeds

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$^3$Because only the net expected payoffs of revolt or not enter action choices, if revolt has an expected cost of $c$, the payoffs in Figure 1 capture it via normalization of $w_i$, $h_i$, and $l_i$. If expected costs differ, i.e., $c_1 \neq c_2$, then the payoffs in Figure 3 capture them via normalization of payoffs.

$^4$s_1, s_2 and $\theta$ are strictly affiliated if, for all $z$ and $z' \in \mathbb{R}^3$, $f(\min\{z, z'\})f(\max\{z, z'\}) > f(z)f(z')$, where min and max are defined component-wise (Milgrom and Weber 1982; see also de Castro 2010).
the vanguard’s vanishes sufficiently fast as the vanguard’s signal becomes very good:

**Assumption A1.** For every $k$, for $i, j = 1, 2$, with $j \neq i$,

\[ \begin{align*}
(a) & \lim_{s_i \to \infty} E[\theta | s_j < k, s_i] = \infty, \quad \lim_{s_i \to -\infty} E[\theta | s_j > k, s_i] = -\infty \\
(b) & \lim_{s_i \to \infty} \Pr(s_j > k | s_i) = 1, \quad \lim_{s_i \to -\infty} \Pr(s_j > k | s_i) = 0 \\
(c) & \lim_{k \to \infty} \Pr(s_j > k | k) E[\theta | s_j > k, k] = 0 \\
(d) & \lim_{s_i \to -\infty} \Pr(s_2 > k | s_1) E[\theta | k, s_1] > -\infty.
\end{align*} \]

A1 obviously holds in an additive, normal noise signal setting where $s_i = \theta + \nu_i$, $i \in \{1, 2\}$, and $\theta$ and $\nu_i$s are independently distributed normal random variables.\(^5\)

Our modeling assumption that the vanguard must move first for the revolution to succeeds reflects the literature on social movements (Gamson 1975; McAdam 1999; McAdam et al. 2004; Tarrow 1998; Tilly 1978, 1996, 2004) that without the vanguard’s tactical and organizational skills, a “spontaneous” protest cannot sustain to change the status quo; and that the general population typically plays a more passive role—they may join a protest, but they do not initiate it. To captures settings where non-activist citizens may initiate revolution, section 7 analyzes the endogenous emergence of revolutionary leaders when it is not the heterogeneous attributes of citizens—the skills, experience or radical preferences—that determines who initiates a protest, but rather their private information about the payoffs from successful revolution.\(^6\)

**Strategies.** A pure strategy for the vanguard is a function $\rho_1$ mapping her private signal $s_1$ into an action choice, $a_1 \in \{-R, R\}$, where $-R$ indicates no revolt and $R$ indicates revolt. A pure strategy for the follower is a function $\rho_2$ mapping his private signal $s_2$ and the vanguard’s action $a_1$ into an action choice, $a_2 \in \{-R, R\}$. That is, $\rho_2 : S \times \{-R, R\} \to \{-R, R\}$. The equilibrium concept is Perfect Bayesian Equilibrium.

\(^5\)In most revolution settings, the possibilities of pre-play communication are slim as regimes monitor such activities, limiting the channels of communication, creating a risk of premature revelation of intent to the regime. Hence, we do not allow for pre-play communication.

\(^6\)Such spontaneous initiation of revolt by non-activist citizens seems to have occurred in the Arab Spring. However, Khatib and Lust (2014) argue that “the uprisings did not emerge out of nowhere but were part of a longer revolution in the landscape of activism persistently challenging the regime.... Nor were they entirely spontaneous; activists simultaneously capitalized on a changing set of skills and tactics they had developed in the previous decades and continued to adapt to new realities as the uprisings continued” (p. 15; see Ch. 2 for an analysis of the 2011 Egyptian Revolution).
3 Equilibrium

We first identify the forces that shape the incentives of the vanguard and follower to revolt. When the vanguard does not revolt, the positive punishment cost $\mu_2 \equiv h_2 - l_2 > 0$ means that the follower has a dominant strategy to do the same. Lemma 4 in the Appendix shows that when the vanguard revolts, the follower’s best response takes a cutoff form in which he revolts whenever his signal $s_2$ exceeds a threshold $k_2$ that depends on the vanguard’s strategy. Lemma 5 in the Appendix shows that if the follower’s strategy takes a cutoff form, then so does the vanguard’s. That is, the vanguard revolts whenever $s_1 \geq k_1$ for some $k_1$. These lemmas imply that in any equilibrium in which the vanguard sometimes revolts, both agents adopt cutoff strategies. Lemma 1 identifies the strategic forces that shape a follower’s incentive to revolt.

**Lemma 1** As the vanguard becomes more willing to revolt, the follower revolts less. That is, the follower’s best response always features strategic substitutes: $\frac{\partial h_2(k_1)}{\partial k_1} < 0$.

A vanguard who is more willing to revolt does so following worse signals about the payoff from revolution—her cutoff $k_1$ is lower. This lowers a follower’s forecast of the payoff from successful revolution—$E[\theta|s_1 \geq k_1, s_2]$ falls—reducing his incentive to revolt.

The vanguard’s strategic calculation is more complex, as it involves both coordination and information aggregation elements. Unlike the follower, the vanguard must decide whether to revolt without knowing whether she will be joined; and if the vanguard is the sole challenger to the regime, she expects to be punished. Thus, the vanguard faces a type of cost that her follower does not: miscoordination costs, $\mu_1 \equiv h_1 - l_1 > 0$, that she pays when she revolts, but her follower does not.\footnote{While the follower faces miscoordination costs $\mu_2 = h_2 - l_2$ if he revolts alone, he never does so in equilibrium because he observes the vanguard’s action before deciding whether to revolt. Thus, he has a dominant strategy not to revolt (to avoid $\mu_2$) when the vanguard does not. Our results extend directly if there is a chance that revolution fails even when both citizens revolt: this amounts to a renormalization of payoffs. Also, our results extend qualitatively if there is a small probability $p$ that a revolution succeeds even if only one party revolts. When $p$ is positive and the vanguard does not revolt, then the follower will revolt whenever his signal about the payoff from successful revolution is so high that it offsets the low probability of success and the negative news conveyed by the vanguard’s decision not to revolt. As long as $w_2 - h_2$ is not excessively large, the follower sets a higher cutoff for revolt when the vanguard does not act, reflecting that the vanguard’s decision...}

With a more compliant follower (someone who is more likely to follow the vanguard), the vanguard is less
likely to incur the costs of miscoordination, and hence has more incentive to revolt. This provides a force for strategic complements in the vanguard’s calculations.

However, as a follower grows more compliant it also means that he follows the vanguard after receiving worse signals. This reduces the vanguard’s expectation of the payoff from successful revolution, i.e., \( E[\theta|s_2 \geq k_2, s_1] \) falls, reducing her incentive to revolt. That is, excessive compliance by a follower deprives the vanguard of effectively aggregating the follower’s information, information that can protect against a successful revolution that results in much worse outcomes than the status quo. This constitutes the force for strategic substitutes in the vanguard’s calculations.

Lemma 2 shows that with a barely-compliant follower who rarely follows the vanguard, the force for strategic complements dominates: as the follower becomes more likely to follow, the vanguard revolts more. However, as the follower grows more compliant, the force for strategic substitutes rises relative to that for strategic complements. In fact, there is a unique threshold on the follower’s level of compliance after which the force for strategic substitutes dominates: thereafter, as her follower grows more compliant, the vanguard revolts less.

**Lemma 2** Suppose \( A_1 \) holds. Then there is a critical level \( k^* \) of the follower’s cutoff that determines whether a vanguard’s best response features strategic complements or substitutes: if a follower is unlikely to revolt, so \( k_2 > k^* \), the vanguard’s best response features strategic complements; if, instead, \( k_2 < k^* \), it features strategic substitutes.

If the vanguard’s best response featured global strategic complements, equilibrium would necessarily be unique—\( k_1(k_2) \) would be strictly increasing, and \( k_2(k_1) \) is strictly decreasing. However, because a vanguard’s best response exhibits strategic substitutes when \( k_2 \) is low, multiple equilibria might exist. When \( w_2 \) is sufficiently large, the crossing can only occur on the strategic complement part of the vanguard’s best response, so equilibrium is unique. To prove uniqueness more generally, we impose Assumption \( A_2 \), which ensures that best responses cross at most once.

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not to revolt conveys negative information about the expected payoffs from successful revolt, the low likelihood of success, and the high likelihood of being punished.
Assumption A2. For every $x$ and $y$,

$$\frac{\partial E[\theta|x, s_2 \geq y]}{\partial x} \frac{\partial E[\theta|s_1 \geq x, y]}{\partial y} > \frac{\partial E[\theta|x, s_2 \geq y]}{\partial y} \frac{\partial E[\theta|s_1 \geq x, y]}{\partial x}.$$ 

Assumption A2 states that the conditional expectation of $\theta$ is more sensitive to changes in a signal $s_i = x$ than to changes in the cutoff $x \leq s_i$. Lemma 3 shows this assumption holds for the standard signal structure used in the literature.

Lemma 3 Assumption A2 holds with an additive, normal noise signal structure, where $s_i = \theta + \nu_i$ and $\theta$, $\nu_1$ and $\nu_2$ are independently distributed as $\theta \sim N(0, \sigma_0^2)$ and $\nu_i \sim N(0, \sigma_i^2)$.

This leaves the possibility that, regardless of her signal, a vanguard never revolts. Then, a follower’s beliefs cannot be determined via Bayes rule on an off-equilibrium path where the vanguard revolts. We impose a minimal plausibility condition on a follower’s off-equilibrium beliefs: if the vanguard revolts, then $E[\theta|a_1 = R, s_2]$ exceeds $w_2$ for all sufficiently high values of the follower’s signal $s_2$, reflecting affiliation. With this condition, an equilibrium in which there is never revolution does not exist.

Proposition 1 Suppose A1 and A2 hold. Then a unique equilibrium exists. In equilibrium, both agents adopt finite cutoff strategies, revolting if and only if their signals about $\theta$ are sufficiently high.

Sacrifice and Coordination. This result contrasts with what happens when there is no vanguard—i.e., when citizens move simultaneously. Shadmehr and Bernhardt (2013) show that without a vanguard, there is always an equilibrium in which citizens never revolt, and this is the sole equilibrium if miscoordination costs are high. Absent a vanguard, coordination breaks down because if each citizen believes that the other does not revolt, then s/he does not revolt to avoid paying the miscoordination costs $h_i - l_i$. Proposition 1 shows that a vanguard always facilitates some coordination on revolution. By moving first, the vanguard mitigates miscoordination costs for the follower, eliminating the force for strategic complements in his considerations that underlies the multiple equilibria, including the one in which all coordination breaks down.
The revolutionary vanguard restores coordination by making a sacrifice: by being the first to challenge a regime, she takes on the risk of being the sole revolter, in which case she is punished, incurring miscoordination costs $\mu_1 = h_1 - l_1$. The level of $\mu_1$ captures the magnitude of the vanguard’s sacrifice. By publicly risking this sacrifice, the vanguard eliminates this risk for a follower by ensuring that if he revolts, he will not be alone. This coordination-enhancing effect of a vanguard resonates with the “tipping point” mechanism through which vanguards, by moving first, start a “snowballing process” that transforms “sparks to prairie fire” (Granovetter 1978; Kuran 1991; Lohmann 1994; Schelling 1969, 1971; see Bueno de Mesquita (2010) for a discussion). It contrasts with other coordination-enhancing mechanisms in which vanguards send public signals that reduce strategic risks precisely because they are publicly observed (Bolton et al. 2013; Dewan and Myatt 2007, 2008) and convey information about levels of anti-regime sentiments (Bueno de Mesquita 2010; Lohmann 1994) or the returns to effort (Hermalin 1998; see also Majumdar and Mukand 2008).

Such sacrifices play a critical role at facilitating revolt even when there is no vanguard to move first: When the agents move simultaneously, it facilitates coordination if miscoordination costs are divided asymmetrically, so that one citizen expects to bear a greater share of the sacrifice costs. To see why, consider the simultaneous move game analyzed in Shadmehr and Bernhardt (2013). For numerical investigation, we assume that citizens receive noisy, independently normally distributed signals about $\theta$. The following result describes the equilibria of the game in cutoff strategies.

**Result.** *(Shadmehr and Bernhardt 2013)* In the simultaneous move game, an equilibrium always exists in which citizens never revolt. There is a threshold $\mu^*$ for the punishment cost $\mu$ such that if $\mu > \mu^*$ then only this equilibrium exists. If $\mu = \mu^*$, there is an equilibrium in which citizens revolt with positive probability; and if $\mu < \mu^*$, there are two equilibria in which citizens revolt with positive probability.

Figure 2 shows that when the punishment cost $\mu$ is close to $\mu^*$, then asymmetric divisions of punishment costs, $(\mu_1, \mu_2) = (\mu - \epsilon, \mu + \epsilon)$ facilitate greater coordination. Quite generally, asymmetric divisions of costs retrieve coordination on revolution when no coordination was originally possible (because $\mu > \mu^*$), and they raise coor-
Figure 2: Best response functions when there is no vanguard (simultaneous move game). Left: symmetric miscoordination costs, $\mu_1 = \mu_2 = \mu^* \approx 0.16$. Right: asymmetric miscoordination costs, $\mu_1 = 0.23$, $\mu_2 = 0.09$.

dination on revolution in cases where there was originally some coordination. That is, with a revolutionary vanguard that takes on a greater share of the risk of punishment, a situation can tip from one in which revolution does not occur to one in which it does.

The reason is that at $\mu^*$, a slight $\epsilon$ reduction in $\mu_1$ to $\mu_1 = \mu^* - \epsilon$ causes agent 1 to reduce his cutoff for revolting slightly, reducing the miscoordination risk for agent 2; and a slight $\epsilon$ increase in $\mu_2$ to $\mu_2 = \mu^* + \epsilon$ causes agent 2 to raise his cutoff slightly, raising the miscoordination risk for agent 1. However, the asymmetry in miscoordination costs means that the high miscoordination cost agent 2 benefits more from the reduced miscoordination risk (giving rise to a relatively greater increased willingness to revolt), and the low miscoordination cost citizen 1 is hurt less by the increased miscoordination risk (so its reduction in willingness to revolt is less). As a result, greater coordination on revolution is sustained. More generally, more asymmetric (higher $\epsilon$) divisions of miscoordination costs facilitate ever greater coordination on revolution.
4 Radicalism

To study the effects of a vanguard’s relative preference for revolution compared to her follower, we modify her payoffs from successful revolution, so that she receives \( \theta + z \) when revolution succeeds—see Figure 3. We say that a vanguard is more radical than her follower if \( z > 0 \), and she is more conservative if \( z < 0 \). A more radical vanguard revolts more often, which means that she revolts after receiving worse signals about \( \theta \), i.e., \( k_1 \) falls. This reduces her follower’s incentive to revolt because she lowers the follower’s estimate of \( \theta \), \( E[\theta|s_1 \geq k_1, s_2] \). Formally,

Proposition 2 Suppose that A1 and A2 hold. Then, the more radical is the revolutionary vanguard, the more likely she is to revolt, and the less likely is her follower:

\[
\frac{\partial \bar{k}_1(z)}{\partial z} < 0 < \frac{\partial \bar{k}_2(z)}{\partial z},
\]

where \( \bar{k}_1 \) and \( \bar{k}_2 \) are the equilibrium cutoffs.

Figure 3: Radicalism. \( z \) captures a vanguard’s radicalism: \( z > 0 \) (\( z < 0 \)) indicates that the vanguard is more radical (conservative) than the follower.

Proposition 2 highlights key tradeoffs for a regime. One might think that greater rewards to a follower for defying a revolutionary vanguard, harsher punishments for failed revolt and more conservative vanguards are complementary tools for the state, and that increasing any of these measures always has value. Indeed, if a regime can punish failed revolters extremely harshly at minimal expense, or radically raise the re-
ward $w_2$ to a follower for defying a vanguard, or “deradicalize” a vanguard by decreasing $z$ sufficiently then it can reduce the probability of successful revolt almost to zero.

However, when this is not possible, the state must be more careful. The state does not care about the probability of revolt, per se, but rather the probability that a revolt succeeds, and successful revolt requires both the vanguard and follower to act. As a result, to reduce the probability of successful revolt, rather than anoint a modestly conservative citizen as a puppet vanguard of the opposition (e.g., a union vanguard), the regime may do better to radicalize the vanguard. A vanguard with a larger $z$ is more eager to revolt, delegitimizing her in the eyes of a potential follower.

Increasing $z$ has conflicting effects: (1) the direct, non-strategic effect is to increase the vanguard’s incentive to revolt; (2) but the indirect, strategic effect is to decrease the follower’s incentive to follow the vanguard. This strategic effect further feeds back into the revolutionary vanguard’s strategic considerations, which can mitigate (amplify) the direct effect when the vanguard’s best response features strategic complements (substitutes). When the strategic effect dominates, the likelihood of successful revolution falls as the vanguard become more radical—see Figure 4. Let $P$ be the probability of successful revolution, i.e., $P = Pr(s_1 \geq k_1, s_2 \geq k_2)$. Then,

$$\frac{dP}{dz} = \left( \frac{\partial P}{\partial k_1} + \frac{\partial P}{\partial k_2} \frac{\partial k_2}{\partial k_1} \right) \frac{dk_1}{dz}.$$ 

At the interior maximum where the probability of successful revolution is highest,

$$-\frac{\partial P}{\partial k_1} \bigg/ \frac{\partial P}{\partial k_2} = \frac{\partial k_2}{\partial k_1}. \quad (1)$$

The left-hand side is the marginal rate of substitution between a vanguard’s willingness to revolt and her follower’s willingness, while the right-hand side is the slope of the follower’s best response. Equation (1) shows that as a vanguard’s cutoff $k_1$ is varied, the likelihood of successful revolt is highest when the marginal rate of substitution in the probability of successful revolt equals the slope of the follower’s best response.

As Figure 4 shows, when a follower’s predatory payoff $w_2$ is high, the probability of a successful revolution may be highest with a slightly conservative vanguard, whose payoff from successful revolt is less than her follower’s. In turn, facing such
Figure 4: Probability of successful revolution, $Pr(s_1 \geq \overline{r}_1(z), s_2 \geq \overline{r}_2(z))$, as a function of the revolutionary vanguard’s radicalism $z$. $s_i = \theta + \epsilon_i$, $\theta \sim N(0,1)$, $\epsilon_i \sim N(0,1)$, $h_1 = 0.1$, $l_1 = 0$, $w_2 = 1.2$.

Figure 5: The revolutionary vanguard’s level of radicalism $z^*(w_2)$ that maximizes the probability of successful revolution as a function of the follower’s disloyalty rewards $w_2$. $s_i = \theta + \epsilon_i$, $\theta \sim N(0,1)$, $\epsilon_i \sim N(0,1)$, $h_1 = 0.1$, $l_1 = 0$.

For a conservative vanguard, the state may want to reduce its punishment of a failed revolt, in order to make the vanguard more willing to revolt. Paradoxically, the vanguard’s increased willingness to revolt makes the follower sufficiently less willing to follow that it reduces the likelihood that the regime is overthrown. In fact, Figure 5 numerically illustrates how with normally distributed uncertainty, when the state
raises the follower’s predatory payoff $w_2$, the level of the vanguard’s radicalism that maximizes the likelihood of successful revolution falls. Thus, a regime must be wary about combining the twin tools of more generously rewarding a follower who turns on his vanguard, and of also punishing the vanguard of a failed revolt somewhat more harshly or anointing a more conservative puppet vanguard. Doing so can backfire and increase the probability of successful revolt.

5 Who is more willing to revolt?

When $w_2 < l_1$, the follower’s payoff from preventing revolt is even worse than the vanguard’s “punishment” when this happens. This might lead one to conjecture that when the quality of a vanguard’s information is the same as her follower’s, and $w_2 < l_1$, then a non-radical vanguard may be more willing to revolt than her follower, i.e., $w_2 < l_1$ could imply $k_2(z = 0) > k_1(z = 0)$. In fact, the opposite is true. Proposition 3 shows that $w_2 > h_1$ is a necessary condition for a non-radical vanguard to revolt more than her follower. More generally, we prove that a non-radical vanguard revolts more than her follower if and only if the follower’s predatory payoff $w_2$ is high, but not too high. To ease analysis, we impose structure on the vanguard’s best response function, structure that holds in the standard additive noise, normal signal setting.

Assumption A3. The strategic complements segment of the vanguard’s best response is convex, i.e., if $\frac{\partial k_1(k_2)}{\partial k_2} > 0$, then $\frac{\partial^2 k_1(k_2)}{\partial (k_2)^2} > 0$.

Assumption A3 ensures that once the force for strategic complements dominates, then when a follower revolts even less, the degree of strategic complementarities in the vanguard’s best response rises.

Proposition 3 Suppose that A1, A2 and A3 hold. Then

- There exist a threshold $h_1^*$ on a vanguard’s status quo payoff and thresholds

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8A weaker sufficient condition (implied by A3) for the results in Proposition 3 can be imposed on the vanguard’s net expected payoff from revolting when she receives a signal that just equals the cutoff $k_2$ set by her follower. The vanguard is more willing to revolt than her follower if and only if at an equilibrium cutoff $k_2$, her net expected payoff from revolting when $s_1 = k_2$ is positive; and a sufficient condition for the results is that this net expected payoff be a single-peaked function of $k_2$. 

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$\overline{w}(h_1) > \underline{w}(h_1)$ on the follower’s predatory payoff $w_2$ such that the vanguard sets a lower cutoff for revolt than her follower if and only if $h_1 < h_1^*$ and $\underline{w}(h_1) < w_2 < \overline{w}(h_1)$.

- $\underline{w}(h_1)$ is strictly increasing in $h_1$, while $\overline{w}(h_1)$ is strictly decreasing in $h_1$.

- If the signals of the vanguard and the follower have identical distributions, i.e., if $s_1 \sim s_2$, then $\underline{w}(h_1) > h_1$.

When the follower’s predatory payoff $w_2$ is very low, the follower almost always supports the vanguard, but the vanguard revolts only when her signal is sufficiently high: $E[\theta|s_1] > h_1$. As $w_2$ increases, the follower tends to revolt less. Then, the strategic complements force acts to decrease the vanguard’s incentive to revolt, while the strategic substitutes force acts to raise it. When a vanguard’s status quo payoff is high, $h_1 \geq h_1^*$, the vanguard has a lot to lose from revolting alone. As a result, the strategic complements force is so strong that a vanguard never revolts more than her follower. When, instead, $h_1 < h_1^*$, the strategic complements force is less strong. As the follower’s predatory payoff $w_2$ increases, he revolts less, and just as $w_2$ passes a threshold $\underline{w}(\mu_1)$, the vanguard’s incentive to revolt exceeds her follower’s. But as $w_2$ further increases, the follower revolts even less, and eventually, the strategic complements force dominates (Lemma 2), so that $\frac{\partial k_1(k_2)}{\partial k_2} > 0$. In addition, if the degree of strategic complementarities rises as the follower revolts less, it eventually reaches a level that the vanguard’s incentive to revolt falls even more than her follower’s with increases in $w_2$. Consequently, once $w_2$ passes a threshold $\overline{w}(\mu_1) > \underline{w}(\mu_1)$, the vanguard’s incentive to revolt again becomes less than her follower’s.

When $w_2 > h_1$, the follower faces a higher opportunity cost of revolting. This makes the vanguard more likely to revolt than the follower as long as the likelihood of miscoordination, and its associated costs, which the vanguard alone incurs, are not too great. With symmetric signal structures, as $h_1 \rightarrow l_1$, we have $\overline{w}(h_1) \rightarrow \infty$ and $\underline{w}(h_1) \rightarrow h_1$: whether the vanguard is more eager to revolt than her follower boils down to a comparison of what they receive when they do not revolt.
6 What do vanguards and followers want in each other?

If a follower could select the vanguard, would he choose a radical vanguard that often initiates revolt or a conservative one that seldom does? Conversely, if a vanguard could punish her follower when he does not follow her lead, would she punish him harshly or mildly? In this section, we characterize a follower’s preferred level of radicalism in a vanguard and the vanguard’s choice of the coercion with which to threaten a follower.

6.1 The follower’s preferred level of radicalism

The follower’s preferred level of radicalism $z$ induces the optimal choice by the vanguard from his perspective of when to revolt—i.e., it induces the vanguard to select the optimal cutoff from the follower’s perspective.\footnote{We thank Ethan Bueno de Mesquita for encouraging us to do this analysis.}

**Proposition 4** Fixing the other parameters, there exists a $\bar{h} < w_2$ such that if $h_2 > \bar{h}$ then the follower’s preferred level of radicalism in a vanguard is given by

$$z^* = \frac{Pr(s_2 < k_2|k_1)(w_2 - l_1) - (h_2 - h_1)}{Pr(s_2 \geq k_2|k_1)},$$

where $k_1$ and $k_2$ are endogenous equilibrium cutoffs. If, instead $h_2 < \bar{h}$ then the follower always wants the vanguard to revolt, i.e., $z^* = \infty$.

When a follower’s status quo payoff $h_2$ is not too low, he prefers a vanguard who revolts whenever her signal indicates that revolution payoffs are high—$z^*$, and hence $k_1$, are finite. However, once $h_2$ is reduced below a threshold $\bar{h}$, then because the value of information revealed about $\theta$ when a vanguard revolts more selectively is bounded, a follower prefers a truly radical vanguard who always revolts, i.e., $z^* = \infty$. The follower then supports the vanguard if $E[\theta|s_2] > w_2$, but turns on it if the inequality is reversed.

This discontinuous change may shed light on abrupt surges of support for vanguards with extreme interests in revolution. For example, in the late Pahlavi Regime, many politically-active Iranians supported the Liberation Movement of Iran (LMI)
and the Nationalists who organized protests, but were less eager than Khomeini’s “faction” to mount a revolution. However, in the years just preceding the Iranian Revolution, support for Khomeini grew rapidly to the extent that even Bazargan, a founding member of the LMI, was marginalized for being too conservative in advocating revolution. Some researchers have attributed this abrupt shift in support to psychological factors such as an “identity crisis” in a rapidly changing society. Our model suggests that the Shah’s increased authoritarian approach following the oil boom of the mid-1970s (e.g., he dismantled the existing political parties and increased repression) together with the economic crisis and high inflation that followed reduced status quo payoffs below the threshold that justified following a “cautious vanguard” who would initiate revolt selectively, causing many Iranians to switch support to Khomeini.

When status quo payoffs are not so low, the follower wants a radical vanguard \((z^* > 0)\) if and only if \(Pr(s_2 < k_2|k_1) (w_2 - l_1) - (h_2 - h_1) > 0\). For example, if the follower and vanguard have the same status quo payoffs, i.e., \(h_1 = h_2\), then the follower prefers a radical vanguard if and only if \(w_2 > l_1\). In most settings, \(w_2 >> l_1\), i.e., the follower’s payoff when he does not support his vanguard in a revolution exceeds the vanguard’s payoff from leading a failed revolt. But, it may well be that \(h_2 > h_1\), i.e., the vanguard also dislikes the status quo by more than its follower.

To understand how a follower’s preferred level of radicalism in a vanguard varies with their other characteristics, recognize that a choice of \(z\) amounts to choosing the equilibrium level of the vanguard’s cutoff \(k_1\). The optimal cutoff \(k_1\) from the perspective of the follower only depends on his payoffs \(w_2\) and \(h_2\). However, the vanguard’s payoffs \(h_1\) and \(l_1\) influence her willingness to revolt, and hence the level of radicalism that the follower seeks in a vanguard.

When the vanguard’s status quo payoff, \(h_1\), is higher, the vanguard is more reluctant to revolt. To reduce \(k_1\) back to the follower’s preferred level, the follower wants to increase \(z\). Thus, \(z^*\) increases in \(h_1\). So, too, when the vanguard’s punishment for

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10 This was not an issue of resorting to violent tactics because (1) Khomeini, too, had never endorsed violent revolutionary actions before the revolution, and (2) LMI had contacts with Mojahedin and other armed organizations fighting against the Pahlavi regime.

11 In 1974, the secretary general of Amnesty International stated that “no country in the world has a worse record in human rights than Iran” (Bill 1988, p. 187).
failed revolt falls, i.e., when $l_1$ is higher, the vanguard is more willing to revolt. To raise $k_1$ back to its optimal level, the follower wants to reduce $z$, i.e., $z^*$ decreases in $l_1$. The follower also prefers a less radical vanguard when the follower’s status quo payoff, $h_2$ is higher. Increasing the follower’s status quo payoff $h_2$ does not affect the follower’s equilibrium cutoff $k_2$ directly, and hence does not affect the vanguard’s cutoff $k_1$ directly. However, with a higher status quo payoff, the follower wants the vanguard to revolt less. Hence, the follower wants a less radical vanguard, i.e., $z^*$ decreases in $h_2$.

The effect of raising a follower’s predatory payoff $w_2$ on the optimal level of the vanguard’s radicalism is more complicated because the direct effect of raising $w_2$ is to cause the follower to revolt less, i.e., to increase $k_2$; and the impact of raising $k_2$ on $k_1$ depends on whether the vanguard’s best response exhibits strategic complements ($k_1$ rises) or strategic substitutes ($k_1$ falls).

**Corollary 1** Suppose $h_2 = h_1$, $w_2 > l_1$, and the vanguard’s best response features strategic substitutes. Then, $z^*$ is increasing in $w_2$.

### 6.2 Coercion

In settings such as civil wars or guerrilla movements, a vanguard can punish a follower who does not follow her lead. For example, consider a guerrilla organization (the vanguard) contemplating an attack on a government military post near a village. The attack succeeds if the villagers (the follower) cooperate, e.g., if they provide accurate information about government force activity (Kalyvas 2006; Wood 2003). In this context, if the guerrillas do not attack, the status quo prevails as the villagers are not equipped to fight. If the guerrillas attack, but the villagers do not cooperate, the guerrillas incur costs, e.g., some are killed, but the guerrilla organization can punish the villagers, e.g., by setting houses on fire or kidnapping some villagers.\(^\text{12}\) We now focus on a vanguard’s use of punishment for non-compliant followers. When we say that a vanguard uses coercion, we mean that she imposes sanctions on those who do not

\(^{12}\)More generally, as Ahlquist and Levi (2011) discuss in their review of the literature on leadership, in many settings, vanguards can use coercion to make others follow their lead, and yet “no model so far encapsulates noninformational tools available to vanguards, such as coercion (p. 14).”
follow her: by using coercion, a vanguard decreases her follower’s payoff when the vanguard revolts, but the follower does not, reducing it from $w_2$ to $w_2 - c$. By threatening a very large $c$, a vanguard can coerce her follower to almost always follow her lead.

How much coercion should a vanguard employ? One might posit that, if possible, a vanguard should coerce her follower into almost always following her lead, in order to eliminate her risk of being punished by the regime. We now show that such high levels of coercion are not in the vanguard’s interest. By punishing her follower severely whenever he does not follow, a vanguard induces her follower to revolt even when his information suggests that the outcome of successful revolution would be far worse than the status quo, i.e., when $E[\theta|s_2]$ is very low. This, in turn, hurts the vanguard by preventing her from effectively using her follower’s information. In other words, a vanguard should allow some level of “dissent” in order to make more effective use of a follower’s information, thereby improving the vanguard’s payoffs.

One might also expect that as the precision of a vanguard’s information grows so that she becomes more confident in her information, she feels less need to rely on her follower’s information, and hence finds harsher coercive measures optimal. In particular, one might think that if a vanguard’s signal is very precise while her follower’s is very imprecise, then the vanguard would like to coerce the follower so harshly that he almost always follows her. This reasoning is wrong: the follower already accounts for the precision of his vanguard’s information in his strategic calculations. Thus, if he believes that the vanguard’s information is far more precise than his, then he follows the vanguard as long as his own information suggests that the outcome will not be absolutely disastrous. In fact, the optimal level of coercion is unrelated to the quality of the follower’s and vanguard’s information:

**Proposition 5** Suppose A1 and A2 hold. If, prior to seeing $s_1$, a vanguard can choose how hard to punish a non-compliant follower, then she sets $c = \max\{w_2 - l_1, 0\}$.

If $w_2 - l_1 < 0$, so that a follower gets hurt more than the vanguard when he does not follow her lead, then the vanguard would like to compensate her follower—if she could. This situation may arise in a civil war in which the government uses violence indiscriminately in a region with guerrilla activities. However, in such scenarios it is unlikely
that guerrillas can protect the natives who do not cooperate with them. Thus, we focus on the case with $w_2 > l_1$ and hence $c > 0$. The follower’s problem is simple—he revolts if $E[\theta|s_1 \geq k_1, s_2] \geq w_2$, and he does not revolt if the inequality is reversed. By having the follower internalize her payoff $l_1$ when he does not support it, the vanguard induces the follower to make the decision that is optimal from the vanguard’s perspective based solely on the follower’s information. As a result, sometimes the follower will not follow the vanguard, and the vanguard is punished, receiving $l_1$; however, when this happens the vanguard is protecting herself from leading a successful revolution that the follower’s information suggests has an even worse expected payoff.\footnote{This result has a surface similarity to one in Landier et al. (2009). They show that some “dissent” in organizations can be optimal. They define “dissent” as divergent preferences between a decision maker, who chooses a project first, and an implementer, who exerts effort to implement the project chosen. A “dissenting” implementer fosters the use of objective information by the decision maker, which raises the likelihood that the project succeeds, and hence increases the implementer’s effort. In our paper, the vanguard does not choose extreme punishment to foster information aggregation, while in Landier et al., “dissent” is optimal because it constrains selfish decisions by the decision maker. See also Che and Kartik (2009).}

The result that a vanguard wants to align her follower’s preferences with hers contrasts sharply with what a follower wants in the vanguard. Most obviously, when the follower’s status quo payoffs $h_2$ are sufficiently below his predatory payoffs $w_2$, the follower wants an extremely radical vanguard who always revolts. This allows the follower to avoid the low status quo payoffs and choose between the predatory and revolution payoffs, albeit at the expense of learning less about those revolution payoffs.

The magnitude of a vanguard’s sacrifice is captured by the cost $\mu_1 = h_1 - l_1$ she pays when her follower does not support her revolt. Thus, a lower $l_1$ means that a vanguard sacrifices more. By reducing her follower’s payoff for non-support to equal to her own—by setting $c = w_2 - l_1$—a vanguard ensures that if she has to make a greater sacrifice, then so does her follower.

**Corollary 2** Suppose that A1 and A2 hold. Then, the more a vanguard has to sacrifice, the more coercion she wants to use.

One might naively conjecture that a follower must necessarily be hurt, at least in expectation, when his vanguard coercively threatens a significant punishment for
defiance. However, this conjecture is off-base. It ignores the fact that the coercive measure raises the vanguard’s confidence that her decision to revolt will be supported, making her more willing to act. As a result, coercion raises the likelihood that the follower receives the revolutionary payoff \( \theta \), rather than the status quo payoff \( h_2 \). To highlight most transparently that the follower can gain from coercion, suppose that \( h_2 = h_1 = h \) so that with optimal coercion, the vanguard and follower’s payoffs are the same. Then, when \( E[\theta] \gg h \) and \( w_2 \) is sufficiently large, not only the vanguard, but also the follower, benefit from the vanguard’s coercion. When \( w_2 \) is very large, absent coercion, the vanguard almost never revolts, so the follower and vanguard almost always receive \( h \), and the follower’s expected payoff is only marginally above \( h \) (under A1). With coercion, the vanguard routinely revolts whenever her signal is high enough that she expects to gain over the status quo, and since the follower’s payoff with coercion equals his vanguard’s, the follower must also benefit.

The result that the vanguard wants to set coercion equal to \( c = w_2 - l_1 \) hinges on the assumption that a vanguard and her follower share common beliefs about the quality of each others’ signals. If, instead, the vanguard has more faith in her information than does her follower, i.e., if she thinks that the follower believes that the vanguard’s information is less precise than the vanguard believes her own information to be, then she wants to use more coercion to adjust for her follower’s lesser incentive to revolt. Proposition 6 formalizes this finding. It uses Assumption A4, which is satisfied by the additive normal noise signal structure (see equation (9) in the Appendix) as well as most empirically-relevant distributions of noise.

**Assumption A4.** \( E[\theta|s_1 \geq k_1, s_2] \) is strictly decreasing in the variance of \( \theta|s_1 \).

**Proposition 6** Suppose that A1, A2 and A4 hold. If the follower believes that the vanguard’s information is less precise than the vanguard, herself, does, then the vanguard chooses harsher coercive measures, setting \( c > w_2 - l_1 \). That is, if the follower believes that \( \text{var}(\theta|s_1) \) is higher than what the vanguard believes, then \( c > w_2 - l_1 \).

For example, if the follower believes that the normally-distributed noise \( \nu_1 \) in the vanguard’s signal, \( s_1 = \theta + \nu_1 \), has variance \( \tilde{\sigma}_\nu^2 \), but the vanguard believes that the
noise only has variance $\sigma^2_\nu < \bar{\sigma}^2_\nu$, then the vanguard chooses $c > w_2 - l_1$. In effect, when a follower believes that the vanguard’s information is less precise, then from the vanguard’s perspective, the follower underweights the positive information about the revolution payoffs conveyed by the vanguard’s decision to revolt. This makes the follower less willing to revolt. To correct for this, the vanguard reduces the follower’s payoff when he fails to support her. Many vanguards have strong ideological convictions that cause them to be overconfident in their knowledge, others live underground with limited contacts where “correlation neglect” (Levy and Razin 2013) may cause them to put too much weight on the confirmations they receive from their like-minded comrades. Therefore, we believe that in most real world cases $c > w_2 - l_1$.

7 Who Leads?

A central premise of our analysis is that the identity of the potential leader of a revolution is well-established. In many settings, this identity is clear—it is the revolutionary vanguard, i.e., an activist with skills in organizing anti-regime activities, who must lead for a revolution to occur. However, in other settings, the identity of who can take the first steps to initiate a protest is less clear-cut—and citizens can choose whether to lead or follow. For example, the successful revolutions in the so-called Arab Spring were not initiated by experienced activists.

This raises the question of how outcomes are affected when it is not the heterogeneous attributes of the citizens—the skills, experience and possibly radical preferences of a vanguard—that determines who initiates a protest, but rather their information. That is, what happens when potential leaders of a revolution emerge organically based on their private information about the payoffs from successful revolution? And how does this endogenous flexibility affect revolution outcomes?

To address such questions, we suppose that two ex-ante identical citizens first receive private signals about the payoffs from successful revolution. Now, however, citizens have two opportunities to act—by initiating revolt in the first period, a cit-

\footnote{We thank Gilat Levy, our discussant at the Princeton-Warwick Political Economy Conference, for encouraging us to do this analysis.}
izen can endogenously assume the mantle of a leader, revealing to the other citizen that he/she possesses information indicating that revolution is worthwhile. In the second period, citizens can also act, and a citizen can base his or her decision on the information revealed by the first-period choice of the other citizen to act or not. Now citizens have incentives both to lead in the first period, in order to signal good news about revolution payoffs and deliver a successful revolution; and to defer in the first period, in order to free ride on the information conveyed by the other citizen’s actions, thereby reducing the risk of being punished for leading a failed revolt.

We focus on symmetric equilibria in which citizens set cutoff $\alpha$ for revolt in period 1; cutoff $\beta$ for revolt in period 2 if the other citizen revolted in period 1; and cutoff $\gamma$ in period 2 if no one revolted in period 1. We further assume $w \geq h$ to ease presentation.

**Proposition 7** A symmetric equilibrium with finite cutoffs $\alpha$ and $\beta$ always exists. In equilibrium, $\alpha > \beta$: a citizen is strictly more willing to revolt when the other citizen takes on the mantle of leadership. Further, if $\gamma$ is finite (i.e., if a failure of a leader to emerge does not preclude revolution), then $\alpha > \gamma > \beta$.

To understand the tradeoffs, first consider the case in which if no one revolts in period 1, then no one revolts in period 2—supported by beliefs that if no one revolted in period 1 then no one revolts in period 2. Initiating a revolt in period 1 causes the other citizen to support the revolution in period 2 when she has signal $s \in [\beta, \alpha)$, but risks punishment when her signal is $s < \beta$. Deferring revolt forgoes successful revolution outcomes when the other citizen sees signal $s \in [\beta, \alpha)$, but does not risk punishment for leading a failed revolt when she receives signal $s < \beta$. In equilibrium, a citizen with threshold signal $\alpha$ must be indifferent between initiating revolt to signal her information and deferring to free-ride:

$$Pr(\beta \leq s_2 < \alpha|s_1 = \alpha) \ (E[\theta|s_1 = \alpha, \beta \leq s_2 < \alpha] - h) = Pr(s_2 < \beta|s_1 = \alpha) \ (h - l).$$

Now, consider the tradeoffs given that revolt is sometimes initiated in period 2: if no one revolts in period 1, then a citizen revolts in period 2 whenever his signal exceeds $\gamma < \alpha$.$^{15}$ Consider citizen 1 with signal $s_1 = \alpha$ in period 1. If citizen 2 revolts,

$^{15}$We have not been able to establish that an equilibrium in which revolt is initiated in period 2
then revolting and not revolting in period 1 does not make a difference because citizen 1 will revolt in period 2 as \( s_1 = \alpha > \beta \). However, if citizen 2 does not revolt, then citizen 1 faces a tradeoff. Revolting (versus not revolting) induces 2 to revolt for signals \( s_2 \in [\beta, \gamma) \), but risks extra punishment when \( s_2 \in [\beta, \gamma) \).\(^{16}\) Equilibrium requires that

\[
Pr(s_2 \in [\beta, \gamma) | \alpha)(h - E[\theta | s_1 = \alpha, s_2 \in [\beta, \gamma]) = Pr(s_2 \in [\beta, \gamma) | \alpha)(h - l).
\] (3)

Put differently, a citizen seeing \( \alpha \) must gain from delaying by discouraging citizens with signals \( s \) slightly above \( \beta \) from revolting. The costs are a heightened probability of punishment—the citizen is now ‘unnecessarily’ punished when the other citizen receives signal \( s \in [\beta, \gamma) \). In the proof of Proposition 7, we show that necessary demanding equilibrium conditions for the existence of an equilibrium cutoff \( \gamma < \alpha \) are:

\[
E[\theta | s_1 = \alpha, s_2 \in [\beta, \gamma)] = l, \quad \text{and} \quad E[\theta | s_1 = \gamma, s_2 \in [\gamma, \alpha)] > E[\theta | s_1 \geq \alpha, s_2 = \beta] = w.
\]

The first equality is the indifference condition for cutoff \( \alpha \)—equation (3) above. The last equality is the indifference condition for \( \beta \). The inequality is implied by the indifference condition for \( \gamma \)—see equation (27) in the Appendix—which also implies \( \gamma > \beta \).

We next contrast the extent of revolt in our initial setting where there is an established vanguard, with what happens when revolutionary leaders emerge endogenously.

**Proposition 8** An established vanguard leader is more likely to initiate revolt than is an organic leader, but his revolt is less likely to be followed: \( \beta < k_2 < k_1 < \alpha \).

This proposition holds regardless of whether or not citizens initiate revolution in the second period. To see the intuition, note that for an established vanguard, “waiting” is very costly: if the vanguard doesn’t move, then revolution will not happen. This opportunity cost of “not revolting” is significantly lower in the endogenous setting because when one citizen does not revolt in the first period, a revolution can still happen: the other citizen may have moved (when her signal is high enough), which can exist; our characterization is a necessary description if such an equilibrium exists.

\(^{16}\)If citizen 1 (with signal \( s_1 = \alpha \)) revolts in period one, he is punished when \( s_2 < \beta \); and if he does not revolt, then he will revolt in period 2 (because \( \gamma < s_1 = \alpha \)) and is punished when \( s_2 < \gamma \). Thus, the change in the expected punishment from revolting versus not revolting is

\[
[Pr(s_2 < \beta | s_1 = \alpha) - Pr(s_2 < \gamma | s_1 = \alpha)](h - l) = Pr(\beta \leq s_2 < \gamma | s_1 = \alpha)(h - l).
\]
would allow the first citizen to choose to follow in the second period. Therefore, in the endogenous setting, citizens set a higher cutoff in the first period than does an established vanguard. In turn, because citizens set a higher cutoff for acting in the first period, more good news is revealed when someone revolts, causing citizens to set a lower cutoff in the second period for supporting a revolt in the endogenous setting.

Proposition 8 says that an established vanguard leader is more likely to revolt than an organic leader. This result is driven by the incentives to free-ride on the other citizen, to reduce the possibility of being punished. One might conjecture that these free-riding incentives must reduce the probabilities of revolution and successful revolution below their levels with an established vanguard, impairing citizen welfare. In fact, this reasoning is flawed—it is not the case that revolution is necessarily more likely with an established vanguard-follower structure: either citizen can lead a revolution when the leadership mantle is endogenous, whereas with an established vanguard, no matter how promising the follower believes revolution payoffs to be, the vanguard must initiate for a revolution to succeed. Moreover, conditional on a revolution being initiated, a revolution is more likely to succeed when leaders emerge endogenously, as the endogenous follower is always more willing to revolt—\( k_2 > \beta \), i.e., followers are more confident that an organic leader has good cause to initiate revolt. In turn, the implications for welfare are subtle.

To glean insights into these comparisons, we numerically investigate how the leadership structure affects the probability of successful revolution in settings with the additive, normal noise signal structure when \( w = h \). An extensive analysis suggests that, when (a) the expected status quo payoff is better than the ex-ante expected revolution payoff, and (b) the punishment from leading a failed revolt is not too small (\( h > E[\theta] = 0 \) and \( h >> l \)), then the probabilities of successful revolution are highest when either citizen’s information can induce him to lead, intermediate with the established vanguard-follower structure, and lowest when citizens must simultaneously decide whether or not to revolt. Figure 6 illustrates. Obviously, when the costs of leading a failed revolution are high enough, the probability of a successful revolution is always lowest when citizens act simultaneously—the risk of miscoordination...
Figure 6: Probability of successful revolution. Left: The solid (dashed) curve is the probability of successful revolution in the endogenous (established vanguard) setting. Right: The solid curve minus the dashed curve. Parameters: $\sigma^2 = 1$, $\sigma^2_\nu = 0.85$, $l = -2$, and $h$ varies from $-1.95$ to $1.95$ in intervals of $0.05$.

completely deters citizens from revolting because they have no way to signal their information to each other. More generally, when leading a failed revolution has sufficient costs, revolution is most likely to be initiated and succeed with organic leaders. However, this ordering on the likelihoods of revolution ceases to hold when (a) leading a failed revolt has modest costs ($\mu = h - l$ is small), and (b) the status quo is bad ($h < 0$) so that the likelihood of revolution is high. In such settings, the likelihood of successful revolution can be highest with an established (exogenous) vanguard-follower structure, intermediate when citizens simultaneously decide whether to act, and lowest in the endogenous leader setting.

The intuition reflects the general result that when $\mu$ is small, the fact that the vanguard is less likely to be followed than an endogenous leader matters less, meaning it has less influence on his decision to act. In particular, when the punishment for leading a failed revolt is negligible, i.e., $h \approx l$, then with an established vanguard (and the simultaneous move game), (1) both the vanguard and the follower set (approximately) the same cutoffs, and (2) these cutoffs are set so low that if both citizens receive the cutoff signal, they both become worse off than the status quo (experiencing ex-post regret): $E[\theta|s_2 \geq k, s_1 = k] \approx h \Rightarrow E[\theta|s_2 = k, s_1 = k] < h$. That is, a citizen may initiate revolt even though, based solely on his own information, the
status quo is preferred to a successful revolution, i.e., $E[\theta|s_1 = k] < h$. A citizen does this to let the other citizen’s information determine whether the revolution succeeds, reflecting that with a single opportunity to act, a citizen must revolt to let the other citizen’s information be pivotal in determining outcomes. When the punishment for failed revolt is small, the value of revolting to allow the other citizen’s information determine what happens swamps the cost (in terms of $\mu$ and the ex post regret following successful revolution when the other citizen’s signal only modestly exceeds the cutoff). This incentive to revolt when $\mu$ is small exists both with an established vanguard and when citizens act simultaneously, but it is absent in the endogenous setting. Qualitatively, however, once the punishment $\mu$ for leading a failed revolt takes on more than modest values, or the status quo is “good enough”, this incentive is swamped, and the probability of successful revolution is highest in the endogenous setting, and smallest when citizens act simultaneously.

**Welfare.** We begin by establishing that when $w = h$, so that the payoff of a citizen who does not revolt is unaffected by the other citizen’s action, then citizens are overly reluctant to revolt from a welfare perspective. As the logic is identical, we prove the result in the setting where leaders emerge endogenously. We then compare welfare in the endogenous leader and exogenous vanguard-follower frameworks.

Consider the endogenous setting in which revolution is not initiated in the second period. We show that, from one citizen’s perspective, the other citizen does not revolt enough, i.e., his cutoffs $\alpha$ and $\beta$ for revolution are too high. To establish this result, first suppose that citizen 1 has revolted, and citizen 2 must decide whether to follow. Citizen 2 compares the payoff $w$ that he receives when he does not revolt, with his expected payoff from revolt, whereas citizen 1 would have citizen 2 weigh the payoff $l < w$ that citizen 1 receives when 2 does not revolt against the expected payoff from revolt. It follows that citizen 1 would prefer citizen 2 lower his cutoff below $\beta$.

Now consider the choice of $\alpha$. Citizen 1’s payoffs are only affected by a marginal reduction in $\alpha$ set by citizen 2 when citizen 1 receives a signal $s_1 \in [\beta, \alpha]$: with the reduction, citizen 1 would receive $E[\theta|\alpha, s_1]$ rather than $h$. Citizen 2’s indifference condition for initiating revolt given signal $\alpha$ implies: $E[\theta|s_2 = \alpha, \beta \leq s_1 < \alpha] - h > 0$. 

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Figure 7: Ex-ante social welfare for the endogenous setting. Parameters: \( \sigma^2 = 1, \ \sigma^2_\nu = 0.85, \ h = 2, \) and \( l \) varies from 1.95 to −3.

It follows that citizens are too reluctant from a social welfare perspective to initiate revolt. In particular, the socially optimal symmetric cutoffs \( \alpha_s \) and \( \beta_s \) solve:

\[
Pr(\beta_s \leq s_2 < \alpha_s | s_1 = \alpha_s, \beta_s \leq s_2 < \alpha_s) - h = Pr(s_2 < \beta_s | s_1 = \alpha_s) \frac{h - l}{2},
\]

and

\[
E[\theta | s_1 \geq \alpha_s, s_2 = \beta_s] = \frac{h + l}{2}.
\]

Equilibrium indifference conditions demand that the term \( \frac{h - l}{2} \) in the first equality be \((h - l)\) and the term \( \frac{h + l}{2} \) in the second equality be \( h \). Thus, citizens would gain from an ex-ante perspective if they could commit to reducing cutoffs below equilibrium levels.

We next illustrate how primitive parameters affect information aggregation from equilibrium actions, and the possibly perverse welfare effects that can result. Most provocatively, consider the effects of harshening the consequences for leading a failed revolution. Obviously, the direct effect of reducing \( l \) is to reduce citizen welfare. Moreover, the first order indirect strategic effect reduces welfare further: reducing \( l \) makes citizens more reluctant to revolt, i.e., they raise \( \alpha \). But, the second order indirect strategic effect raises citizen welfare: increases in \( \alpha \) cause citizens who see that a revolt was initiated to update more positively about revolution payoffs, causing them to reduce the cutoff \( \beta \) for providing support. Figure 7 presents a setting where the welfare-increasing effect of the reduction in \( \beta \) can dominate: Citizen welfare is a single-peaked function of \( l \). Similar conflicting welfare forces can arise with an established vanguard.

Analytical comparisons of welfare with the two leadership structures are difficult:
even absent punishment considerations, one must calculate expected utilities in areas of the signal space that are not easily ordered because $\alpha > k_1 > k_2 > \beta$. Thus, even if the likelihood of successful revolution is higher in one setting, it may yield lower citizen welfare. However, an extensive numerical investigation of the standard normal signal structure suggests that welfare is higher when leaders emerge organically.

8 Conclusion

Our paper considers the problem of a representative revolutionary vanguard who is unhappy with the status quo. The vanguard also knows that she does not know everything, and that her followers have valuable information that may bear on whether or not a revolution would be a good idea. The vanguard knows that if she initiates a revolution, but her follower is less sanguine about the revolution payoffs and hence fails to provide support, then the attempted revolution will fail, and the regime will punish her. But, if the vanguard does not act, the revolution has no chance, and a follower’s information cannot influence the revolution outcome. What is the vanguard to do?

We characterize the strategic interactions between a vanguard and its follower. We show that under mild conditions a unique equilibrium exists. In equilibrium, the vanguard takes on the risk of being the only one to act: by observably moving, it eliminates that risk for a follower. Via its willingness to sacrifice and bear alone the risk of punishment, the vanguard precludes the possibility of failing to act even when each has very positive information about the benefits of revolution.

We establish how a vanguard would like to use coercion on her follower, not as a blunt tool that induces blind obedience, but rather as a delicate instrument that aligns the follower’s incentives so that he sacrifices similarly to the vanguard when he does not follow her lead. An excessively-coerced follower compliantly follows his vanguard even when he believes a “successful” revolution will turn out disastrously. In contrast, whether a follower values learning from a vanguard’s actions depends on his status quo and predatory payoffs. If his status quo payoff is high, a follower seeks limited radicalism in a vanguard, as he trades off between learning more about the vanguard’s signal and the option value of being able to determine whether he receives
the successful revolution or predatory payoff. However, if his status quo payoff is lower, the follower foresakes learning from the vanguard’s action, preferring a truly radical vanguard who always revolts, to ensure that the status quo does not prevail.

We show how a state that wants to minimizes the likelihood that it is overthrown must be careful in how it deploys the tools at its disposal. When the state cannot punish the leader of an unsuccessful revolt too harshly, if it raises the reward to a follower who turns on his vanguard, it may also want to reduce the punishment for failed revolt. While a less reliable follower or harsher punishment both reduce the attraction to the vanguard of revolting, paradoxically, they may raise the probability of successful revolt, because when the vanguard acts it conveys better news to her follower about the payoffs from successful joint action.

We show that even though a vanguard alone risks punishment, she may still be more willing than a follower to act given the same signal about revolution payoffs. This is because a vanguard also weighs the possible gains of acting in order to let her follower’s information determine whether revolution succeeds. In fact, a vanguard is more willing to act than her follower when the follower’s payoff from not supporting the vanguard is high (so the follower is not too willing to act), but not too high (so the vanguard does not face an excessive risk of miscoordination, and hence punishment).

Our primary analysis takes the leadership structure as given. Our concluding analysis considers a setting in which anyone can lead—a citizen’s information rather than his expertise at organizing anti-regime activities determines whether he acts to lead a revolution. The endogenous emergence of leadership gives rise to a host of subtle considerations. Absent a formal leadership structure, each citizen can initiate revolt and thereby signal positive information about revolution payoffs; but each citizen may also be reluctant to risk leading and punishment, choosing to wait to see what others do. But then if no one acts, citizens update more negatively about the prospects of revolution; and such free-riding can cause citizens to act too little. We show that when a revolution begins organically (without an established structure of leadership), it is more likely to succeed, and our numerical investigation suggests that citizen welfare is higher when the leader emerges endogenously.
9 Appendix

Lemma 4 Suppose the vanguard sometimes revolts, i.e., that $\rho_1(s_1) = R$ for some $s_1$. Then the follower’s best response to $\rho_1(\cdot)$ takes a cutoff form: There exists a finite cutoff $k_2(\rho_1)$ such that the follower revolts if and only if $s_2 \geq k_2(\rho_1)$.

Proof of Lemma 4: The follower’s best response following $a_1$ and $s_2$ is to take action $R$ if and only if $E[\theta|\rho_1(s_1) = R, s_2] \geq w_2$. The limit properties in Part (a) of A1 imply the existence of sufficiently good and bad signals, so that there exists a signal $s_2 = k_2$ such that $E[\theta|\rho_1(s_1) = R, k_2] = w_2$. Strict affiliation of signals implies that $E[\theta|\rho_1(s_1) = R, s_2] > w_2$, $\forall s_2 > k_2$, and $E[\theta|\rho_1(s_1) = R, s_2] < w_2$, $\forall s_2 < k_2$. □

Lemma 5 Suppose that $\rho_2(s_2, R) = R$ if and only if $s_2 \geq k_2$. Then, given A1, there exists a $k_1$ such that the vanguard’s best response is to revolt if and only if $s_1 \geq k_1(k_2)$.

Proof of Lemma 5: Given the follower’s cutoff $k_2$, the vanguard’s expected net pay-off from revolt is $\Delta(s_1; k_2) \equiv Pr(s_2 \geq k_2|s_1) E[\theta|s_2 \geq k_2, s_1] + Pr(s_2 < k_2|s_1)l_1 - h_1$, which simplifies to

$$
\Delta(s_1; k_2) = Pr(s_2 \geq k_2|s_1) (E[\theta|s_2 \geq k_2, s_1] - l_1) + l_1 - h_1.
$$

(4)

Affiliation and the limit properties in A1 imply the vanguard’s best response to a follower’s cutoff strategy is a cutoff strategy with cutoff $k_1(k_2)$, where $\Delta(k_1(k_2); k_2) = 0$.

We have $Pr(s_2 \geq k_2|s_1) > 0$, and $Pr(s_2 \geq k_2|s_1)$ and $E[\theta|s_2 \geq k_2, s_1]$ rise with $s_1$ due to affiliation. Thus, from equation (4), if $\Delta(s_1 = x; k_2) = 0$, then $\Delta(s_1; k_2) > 0$, $\forall s_1 > x$. From A1, $\lim_{s_1 \to -\infty} \Delta(s_1; k_2) < 0 < \lim_{s_1 \to +\infty} \Delta(s_1; k_2)$. Thus, for every $k_2$, there exists a unique $s_1 = k_1$ such that $\Delta(k_1; k_2) = 0$. Further, at $s_1 = k_2$,

$$
\frac{\partial \Delta(s_1; k_2)}{\partial s_1} \bigg|_{s_1=k_1} > 0.
$$

(5)

Proof of Lemma 1: From Lemma 4, $E[\theta|s_1 \geq k_1, k_2] - w_2 = 0$. Thus,

$$
\frac{\partial k_2(k_1)}{\partial k_1} = -\left(\frac{\partial E[\theta|s_1 \geq k_1, k_2]}{\partial k_2}\right)^{-1} \frac{\partial E[\theta|s_1 \geq k_1, k_2]}{\partial k_1}.
$$
By affiliation, both terms are positive, and hence $\frac{\partial k_t(k_1)}{\partial k_1} < 0$. □

Proof of Lemma 2:

\[
\frac{\partial k_1(k_2)}{\partial k_2} = -\left( \frac{\partial \Delta(k_1; k_2)}{\partial k_1} \right)^{-1} \frac{\partial \Delta(k_1; k_2)}{\partial k_2},
\]

(Rewrite equation (4) as

\[
\Delta(k_1; k_2) = \int_{k_2}^{\infty} E[\theta|s_2, k_1] f(s_2|k_1) \, ds_2 + F(k_2|k_1) \, l_1 - h_1.
\]

Let $\delta(k_2, k_1(k_2)) \equiv E[\theta|k_2, k_1(k_2)] - l_1$. Thus,

\[
\frac{\partial \Delta(k_1; k_2)}{\partial k_2} = f(k_2|k_1) \left( -E[\theta|k_2, k_1] + l_1 \right) \equiv -f(k_2|k_1) \, \delta(k_2, k_1),
\]

and hence from equations (5), (6), and (7), $\text{sign} \left( \frac{\partial k_t(k_1)}{\partial k_2} \right) = \text{sign} \left( \delta(k_2, k_1) \right).$ Next, we show $\delta$, establishing its monotonicity properties:

\[
\frac{d\delta(k_2, k_1(k_2))}{dk^2} = \frac{dE[\theta|k_2, k_1(k_2)]}{dk^2} = \frac{\partial E[\theta|k_2, k_1(k_2)]}{\partial k_2} + \frac{\partial E[\theta|k_2, k_1(k_2)]}{\partial k_1} \frac{\partial k_1}{\partial k_2}
\]

\[
= \frac{\partial E[\theta|k_2, k_1(k_2)]}{\partial k_2} - \frac{\partial E[\theta|k_2, k_1(k_2)]}{\partial k_1} \left( \frac{\partial \Delta(k_1; k_2)}{\partial k_1} \right)^{-1} \frac{\partial \Delta(k_1; k_2)}{\partial k_2}
\]

\[
= \frac{\partial E[\theta|k_2, k_1(k_2)]}{\partial k_2} + \frac{\partial E[\theta|k_2, k_1(k_2)]}{\partial k_1} \frac{f(k_2|k_1)}{\frac{\partial \Delta(k_1; k_2)}{\partial k_1}},
\]

where the third equality follows from equation (6) and the fourth from equation (7). Both $\frac{\partial E[\theta|k_2, k_1]}{\partial k_1}$ and $\frac{\partial E[\theta|k_2, k_1]}{\partial k_2}$ are positive because $s_1$, $s_2$, and $\theta$ are affiliated; and $\frac{\partial \Delta(k_1; k_2)}{\partial k_1} > 0$ from equation (5). Thus, $\frac{d\delta}{dk^2} > 0$ for all $\delta \geq 0$, which implies that $\delta(k_2, k_1(k_2))$ has a single-crossing property as a function of $k_2$. Next, we show $\delta$ changes sign from negative (strategic substitutes) to positive (strategic complements). From equation (4) and A1, $\lim_{k_2 \to -\infty} k_1(k_2) < \infty$ and $\lim_{k_2 \to -\infty} k_1(k_2) > -\infty$. To see the latter, note that

\[
\lim_{k_1 \to -\infty} \lim_{k_2 \to -\infty} Pr(s_j > k_j|k_1) \, E[\theta|s_j > k_j, k_1] < \lim_{k \to -\infty} Pr(s_j > k|k) \, E[\theta|s_j > k, k],
\]

and hence part (c) of A1 implies

\[
\lim_{k_1 \to -\infty} \lim_{k_2 \to -\infty} Pr(s_2 > k_2|k_1) \, E[\theta|s_2 > k_2, k_1] < 0.
\]
Thus, from equation (4), \( \lim_{s_1 \to -\infty} \lim_{k_2 \to \infty} \Delta(s_1; k_2) < 0 \), and hence \( \lim_{k_2 \to \infty} k_1(k_2) > -\infty \). Therefore, \( \lim_{k_2 \to \pm \infty} \delta(k_2, k_1(k_2)) = \pm \infty \). \( \square \)

**Proof of Lemma 3:** For \( i \in \{1, 2\} \), let \( b_i = \sigma_0^2 / (\sigma_0^2 + \sigma_i^2) \), \( a_i = \sqrt{(1 + b_i)\sigma_i^2} \), and \( \Sigma = \sigma_0^2 \sigma_1^2 + \sigma_0^2 \sigma_2^2 + \sigma_1^2 \sigma_2^2 \). Then,

\[
E[\theta | k_i, s_j \geq k_j] = b_i k_i + \frac{\sigma_0^2 \sigma_i^2 a_i}{\Sigma} \frac{\phi(x_i)}{1 - \Phi(x_i)},
\]

where \( x_i = (k_j - b_i k_i)/a_i \) and \( \phi(x) \) and \( \Phi(x) \) are pdf and cdf of standard normal distribution, respectively. Let \( A(x) \equiv \frac{\partial}{\partial x} \frac{\phi(x)}{1 - \Phi(x)} \). Moreover, \( A(x) \in (0, 1) \) (Sampford 1953). Thus, Assumption A2 holds if and only if

\[
\left( 1 - \frac{\sigma_0^2 \sigma_1^2}{\Sigma} A(x_1) \right) \left( 1 - \frac{\sigma_0^2 \sigma_2^2}{\Sigma} A(x_2) \right) b_2 b_1 > \left( \frac{\sigma_0^2 \sigma_2^2}{\Sigma} A(x_2) \right) \left( \frac{\sigma_0^2 \sigma_1^2}{\Sigma} A(x_1) \right).
\]

That is,

\[
b_1 b_2 \left( 1 - \frac{\sigma_0^2 \sigma_1^2}{\Sigma} A(x_1) - \frac{\sigma_0^2 \sigma_2^2}{\Sigma} A(x_2) \right) > (1 - b_1 b_2) \left( \frac{\sigma_0^2 \sigma_1^2}{\Sigma} \frac{\sigma_0^2 \sigma_2^2}{\Sigma} A(x_1) A(x_2). \right)
\]

Next, observe that

\[
b_1 b_2 = \frac{\sigma_0^4}{(\sigma_0^2 + \sigma_1^2)(\sigma_0^2 + \sigma_2^2)}, \quad \text{and hence} \quad 1 - b_1 b_2 = \frac{\Sigma}{(\sigma_0^2 + \sigma_1^2)(\sigma_0^2 + \sigma_2^2)}.
\]

Substituting (11) into (10) and rearrangement yields

\[
\Sigma - \sigma_0^2 \sigma_1^2 A(x_1) - \sigma_0^2 \sigma_2^2 A(x_2) > \sigma_1^2 \sigma_2^2 A(x_1) A(x_2),
\]

which is true because \( A(x) \in (0, 1) \), and hence

\[
\Sigma - \sigma_0^2 \sigma_1^2 A(x_1) - \sigma_0^2 \sigma_2^2 A(x_2) > 0 - \sigma_0^2 \sigma_1^2 - \sigma_0^2 \sigma_2^2 = \sigma_1^2 \sigma_2^2 > 0.
\]

**Proof of Proposition 1:** First, we prove an equilibrium exists. The follower’s best response \( k_2(k_1) \) upon observing that the vanguard has revolted solves \( E[\theta | s_1 \geq k_1, k_2(k_1)] = w_2 \). Thus, from part (a) of A1, \( \lim_{k_2 \to +\infty} k_2(k_1) = -\infty \) and \( \lim_{k_1 \to -\infty} k_2(k_1) \) is finite. Moreover, \( \Delta(k_1(k_2); k_2) = 0 \), and hence from A1, \( \lim_{k_2 \to +\infty} k_1(k_2) = -\infty \).

Then, the continuity of \( k_1(k_2) \) and \( k_2(k_1) \) implies that they cross, at least once.

Next, we prove uniqueness for the case where \( l_1 = h_1 \). Then, the vanguard best response satisfies \( E[\theta | k_1, s_2 \geq k_2] = l_1 \), and hence, for the vanguard’s best response,

\[
\frac{\partial k_1(k_2)}{\partial k_2} = - \left( \frac{\partial E[\theta | k_1, s_2 \geq k_2]}{\partial k_1} \right)^{-1} \frac{\partial E[\theta | k_1, s_2 \geq k_2]}{\partial k_2}.
\]

(12)
Next, consider the follower. His best response satisfies $E[\theta | s_1 \geq k_1, k_2] = w_2$. Similar calculations for the follower’s best response yields

$$\frac{\partial k_2(k_1)}{\partial k_1} = -\frac{\partial E[\theta | s_1 \geq k_1, k_2]}{\partial k_1} \left( \frac{\partial E[\theta | s_1 \geq k_1, k_2]}{\partial k_2} \right)^{-1}. \quad (13)$$

To prove uniqueness, it suffices to show that the vanguard’s best response curve in $(k_1, k_2)$-space, for all relevant $k_1$s, always has a sharper negative slope that the follower’s. That is, the inverse of (12) is more a larger negative number than (13), i.e.,

$$-\frac{\partial E[\theta | s_1 \geq k_1, k_2]}{\partial k_1} \left( \frac{\partial E[\theta | s_1 \geq k_1, k_2]}{\partial k_2} \right)^{-1} > -\frac{\partial E[\theta | k_1, s_2 \geq k_2]}{\partial k_1} \left( \frac{\partial E[\theta | k_1, s_2 \geq k_2]}{\partial k_2} \right)^{-1}.$$

Due to affiliation all terms are positive, and hence rearrangement yields

$$\frac{\partial E[\theta | s_1 \geq k_1, k_2]}{\partial k_1} \left( \frac{\partial E[\theta | s_1 \geq k_1, k_2]}{\partial k_2} \right)^{-1} < \frac{\partial E[\theta | k_1, s_2 \geq k_2]}{\partial k_1} \left( \frac{\partial E[\theta | s_1 \geq k_1, k_2]}{\partial k_2} \right),$$

which hold by Assumption A2. If $h_1 > l_1$, then the slope of the strategic substitute segment of the vanguard’s best response becomes even more negative. Thus, if a crossing happens on the strategic substitutes segment of the vanguard’s best response, it is unique. Finally, clearly, crossing can happen only once on the strategic complements segment of the vanguard’s best response.  \(\square\)

**Proof of Proposition 2:** With the new vanguard’s payoff $\theta + z$, the vanguard expected net payoff from taking action $R$, equation (4) becomes

$$\Delta(s_1; k_2, z) = Pr(s_2 \geq k_2 | s_1) \left( E[\theta | s_2 \geq k_2, s_1] + z - l_1 \right) + l_1 - h_1. \quad (14)$$

As in the proof of Lemma 5, one can show that the vanguard’s best response is a finite cutoff strategy with associated cutoff $k_1$ such that $\Delta(k_1; k_2) = 0$, and $\frac{\partial \Delta(s_1; k_2, z)}{\partial s_1} |_{s_1=k_1} > 0$. Moreover, $\frac{\partial \Delta(k_1; k_2, z)}{\partial z} = Pr(s_2 \geq k_2 | k_1) > 0$. Thus, $\frac{\partial k_1(z)}{\partial z} = -\left( \frac{\partial \Delta(k_1; k_2, z)}{\partial k_1} \right)^{-1} \frac{\partial \Delta(k_1; k_2, z)}{\partial z} < 0$ at any best response cutoff $k_1$, including the equilibrium cutoff $k_1(z)$. Hence, since the follower’s best response exhibits strategic substitutes (Lemma 1), 0 < $\frac{\partial k_2(z)}{\partial z}$.  \(\square\)

**Proof of Proposition 3:** First, we prove a lemma.

**Lemma 6** There exists a $h_1^*$ such that $k_1(k_2) > k_2$ if and only if $h_1^* < h_1$. 

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Proof of Lemma 6: $k_1(k_2)$ is continuous in $k_2$, and $\lim_{k_2 \to -\infty} k_1(k_2) > -\infty$. Thus, $k_1(k_2) > k_2$ if and only if $\Delta(k_1(k_2) = k; k_2 = k) \neq 0$, where

$$\Delta(k_1(k_2) = k; k_2 = k) = Pr(s_2 \geq k|k) (E[\theta|s_2 \geq k, k] - l_1) + l_1 - h_1.$$

(15)

From A1, $\lim_{k \to \infty} \Delta(k; k) = l_1 - h_1 < 0$, and $\lim_{k \to -\infty} \Delta(k; k) = -\infty$. Moreover, $\Delta(k; k)$ is continuous, and hence $\Delta(k; k)$ has a global maximum in $\mathbb{R}$, call it $k_{\text{max}}$. Further, $k_{\text{max}}$ is independent of $h_1$, but $\Delta(k; k)$ is uniformly decreasing in $h_1$. Thus, there exists an $h_1^*$ such that $\Delta(k; k) < 0$ if and only if $h_1 > h_1^*$. □

Next, we prove that, for $h_1 < h_1^*$, the vanguard’s best response $k_1(k_2)$ crosses the 45 degree line at exactly two points. Lemma 6 implies that it must cross the 45 degree line at least twice. If $k_1(k^*) < k^*$, so that $k_1(k_2)$ first crosses the 45 degree line on its decreasing (strategic substitutes) segment, it means that the increasing segment of $k_1(k_2)$ starts from under 45 degree line. Then the convexity in Assumption A3 implies that there is at most one crossing. If $k_1(k^*) > k^*$, so that $k_1(k_2)$ first crosses the 45 degree line on its increasing (strategic complements) segment, convexity also implies that $k_1(k_2)$ crosses the 45 degree line at most twice. Call the corresponding crossings $k_2 = \bar{k}$ and $k_2 = \bar{k}$, with $\bar{k} < \bar{k}$. Thus, $k_1(k_2)$ is above the 45 degree line between $k_2 = \bar{k}$ and $k_2 = \bar{k}$. Moreover, from equation (15), $\bar{k}$ rises in $h_1$ while $\bar{k}$ falls.

From the proof of Proposition 1, $k_2(k_1; w_2)$ is strictly decreasing in $k_1$, and uniformly increasing in $w_2$. Thus, there exists a unique $w_2 = \bar{w}$ such that $k_2(k_1 = \bar{k}; \bar{w}) = \bar{k}$, and a unique $w_2 = \bar{w}$ such that $k_2(k_1 = \bar{k}; \bar{w}) = \bar{k}$. That $\bar{w}$ is increasing in $h_1$ and $\bar{w}$ is decreasing follows directly from the relationship between $k$, $\bar{k}$ and $1$.

Finally, we prove that, under symmetry, $w > h_1$. From the proof of Proposition 2, $Pr(s_2 \geq k|s_1 = k) (E[\theta|s_2 \geq k, s_1 = k] - l_1) = h_1 - l_1$. Since $Pr(s_2 \geq k|s_1 = k) < 1$, it follows that $E[\theta|s_2 \geq k, s_1 = k] > h_1$. Further, from the proof of Proposition 1, $E[\theta|s_1 \geq k, s_2 = k] = w$, and from symmetry, $E[\theta|s_2 \geq k, s_1 = k] = \bar{w}$. Thus, $w > h_1$. □

Proof of Proposition 4: Let $E[U_2|k_1, k_2]$ be the follower’s ex ante expected utility.
given equilibrium cutoffs $k_1$ and $k_2$. Then

$$\frac{dE[U_2|k_1, k_2]}{dz} = \left( \frac{\partial E[U_2|k_1, k_2]}{\partial k_1} + \frac{\partial E[U_2|k_1, k_2]}{\partial k_2} \frac{\partial k_2}{\partial k_1} \right) \frac{dk_1}{dz}$$

where we use $\frac{\partial E[U_2|k_1, k_2]}{\partial k_2} = 0$ because $k_2$ is the follower’s equilibrium cutoff. Moreover,

$$E[U_2|k_1, k_2] = Pr(s_1 < k_1)h_2 + Pr(s_1 \geq k_1, s_2 < k_2)w_2$$

$$+ Pr(s_1 \geq k_1, s_2 \geq k_2) E[\theta|s_1 \geq k_1, s_2 \geq k_2]$$

$$= Pr(s_1 < k_1)h_2 + w_2 \int_{s_1=k_1}^{k_2} \int_{s_2=-\infty}^{k_2} f(s_1, s_2) ds_1 ds_2$$

$$+ \int_{s_1=k_1}^{\infty} \int_{s_2=k_2}^{\infty} E[\theta|s_1, s_2] f(s_1, s_2) ds_1 ds_2.$$

Hence, $\frac{\partial E[U_2|k_1, k_2]}{\partial k_1} =$

$$f(k_1) h_2 - w_2 f(k_1) \int_{-\infty}^{k_2} f(s_2|k_1) ds_2 - f(k_1) \int_{k_2}^{\infty} E[\theta|s_1 = k_1, s_2] f(s_2|s_1 = k_1) ds_2$$

$$= f(k_1) \{h_2 - w_2 Pr(s_2 < k_2|s_1 = k_1) - Pr(s_2 \geq k_2|k_1) E[\theta|s_2 \geq k_2, k_1]\}.$$  

(17)

$$= f(k_1) \{h_2 - w_2 + Pr(s_2 \geq k_2|k_1) (E[\theta|s_1 \geq k_1, k_2] - E[\theta|k_1, s_2 \geq k_2])\}.$$  

(18)

First, substituting $\lim_{k_1 \to \infty} k_2(k_1) = -\infty$ into (17) reveals that $\frac{\partial E[U_2|k_1, k_2]}{\partial k_1} < 0$ for $k_1$ sufficiently large. Thus, $\frac{dE[U_2|k_1, k_2]}{dz} > 0$ for sufficiently negative $z$, implying that $z^* > -\infty$.

Second, since $E[\theta|s_1 \geq k_1, k_2] = w_2$ and $\lim_{k_1 \to -\infty} k_2(k_1)$ is finite, $\lim_{k_1 \to -\infty} E[\theta|s_1 \geq k_1, s_2 \geq k_2] = -\infty$ and $\lim_{k_1 \to -\infty} E[\theta|s_1 \geq k_1, k_2] = w_2$. Substituting these limits into (18) reveals that when $h_2 = w_2$, $\frac{\partial E[U_2|k_1, k_2]}{\partial k_1} > 0$ for sufficiently negative $k_1$. Therefore, when $h_2 = w_2$, $\frac{dE[U_2|k_1, k_2]}{dz} < 0$ for all sufficiently large $z$, i.e., $z^* < \infty$ when $h_2 = w_2$.

Third, if $Pr(s_2 \geq k_2|k_1) E[\theta|s_2 \geq k_2, k_1]$ is bounded from below, then fixing $w_2$, substituting sufficiently negative $h_2$ into from (17) reveals that $\frac{\partial E[U_2|k_1, k_2]}{\partial k_1}$ is always negative. Therefore, $\frac{dE[U_2|k_1, k_2]}{dz}$ is always positive, and hence $z^* = \infty$. Moreover, assumption A1 (d) implies $Pr(s_2 \geq k_2|k_1) E[\theta|s_2 \geq k_2, k_1]$ is bounded from below. To see this, note that (i) $\lim_{k_1 \to -\infty} Pr(s_2 \geq k_2|k_1) E[\theta|s_2 \geq k_2, k_1] \geq \lim_{k_1 \to -\infty} Pr(s_2 \geq k_2|k_1) E[\theta|k_2, k_1]$, and (ii) $k_2(k_1)$ is decreasing in $k_1$ with $\lim_{k_1 \to -\infty} k_2(k_1) = \overline{k}_2 \in \mathbb{R}$.
Thus, for any $\epsilon > 0$, $\lim_{k_2 \to -\infty} Pr(s_2 \geq k_2|k_1) E[\theta|k_2, k_1] \geq \lim_{k_2 \to -\infty} Pr(s_2 \geq k_2 - \epsilon|k_1) E[\theta|k_2 - \epsilon, k_1] > -\infty$, where the last inequality follows from A1(d).

Finally, from (17), $\frac{\partial E[U_2|k_1, k_2]}{\partial k_1}$ rises in $h_2$ because $h_2$ does not affect $k_1$ or $k_2$. Hence, $z^*$ falls with $h_2$. Thus, there exists $\bar{h} \in \mathbb{R}$ such that $z^*$ is finite if and only if $h_2 > \bar{h}$.

Next, we derive $z^*$ when it is finite. Because $k_1$ is the vanguard’s equilibrium cutoff,

$$Pr(s_2 \geq k_2|k_1) E[\theta|s_2 \geq k_2, k_1] = h_1 - l_1 Pr(s_2 < k_2|k_1) - Pr(s_2 \geq k_2|k_1)z.$$  

Substituting for $E[\theta|s_2 \geq k_2, k_1]$ using this equilibrium relationship yields

$$f(k_1) \frac{\partial E[U_2|k_1, k_2]}{\partial k_1} = h_2 - w_2 Pr(s_2 < k_2|k_1) - h_1 + l_1 Pr(s_2 < k_2|k_1) + Pr(s_2 \geq k_2|k_1)z$$

$$= (h_2 - h_1) - (w_2 - l_1) Pr(s_2 < k_2|k_1) + Pr(s_2 \geq k_2|k_1)z.$$  

Combining equations (16) and (19) yields

$$\frac{dE[U_2]}{dz} = \frac{dk_1}{dz} f(k_1) [(h_2 - h_1) - (w_2 - l_1) Pr(s_2 < k_2|k_1) + Pr(s_2 \geq k_2|k_1)z].$$  

Solving this first-order condition yields the result in the proposition.  

**Proof of Corollary 1**: By the Implicit Function Theorem, at an interior optimum $z^*$ that maximizes $E[U_2]$, we have $\frac{dz^*}{dw_2} = -\frac{\partial E[U_2]}{\partial w_2} \left/ \frac{\partial^2 E[U_2]}{\partial z^*} \right.$.

Since $\frac{\partial^2 E[U_2]}{\partial z^*} < 0$, $\text{sign}(\frac{dz^*}{dw_2}) = \text{sign}(\frac{\partial E[U_2]}{\partial w_2})$. Using equation (20), one can show that $\text{sign}(\frac{\partial E[U_2]}{\partial w_2})$ is given by the sign of the partial derivative of (2) with respect to $w_2$.

From equation (2), if $h_2 = h_1$ and $w_2 > l_1$, then $z^* > 0$. Substituting $Pr(s_2 \geq k_2|k_1) = 1 - Pr(s_2 < k_2|k_1)$ into (2) and differentiating with respect to $w_2$ yields

$$\frac{\partial}{\partial w_2} \left[ \frac{(w_2 - l_1) - (h_2 - h_1)}{Pr(s_2 \geq k_2|k_1)} - (w_2 - l_1) \right]$$

$$= \frac{1}{Pr(s_2 \geq k_2|k_1)} \left[ \frac{(w_2 - l_1) - (h_2 - h_1)}{[Pr(s_2 \geq k_2|k_1)]^2} \right] \frac{dPr(s_2 \geq k_2|k_1)}{dw_2} - 1$$

$$= \frac{1 - Pr(s_2 \geq k_2|k_1) - (z^* + w_2 - l_1) \frac{dPr(s_2 \geq k_2|k_1)}{dw_2}}{Pr(s_2 \geq k_2|k_1)},$$

where we have substituted for $z^* = \frac{(w_2 - l_1) - (h_2 - h_1)}{Pr(s_2 \geq k_2|k_1)} - (w_2 - l_1)$ in the last equality. When $w_2$ increases, $k_2$ increases, and if the vanguard’s best response features strategic substitutes, $k_1$ falls, which implies $\frac{dPr(s_2 \geq k_2|k_1)}{dw_2} < 0$. Thus, (21) is positive.  

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Proof of Proposition 5: Let \( E[U_1|k_1, k_2] \) be the vanguard’s ex ante expected utility given cutoffs \( k_1 \) and \( k_2 \):

\[
E[U_1|k_1, k_2] = Pr(s_1 \geq k_1, s_2 \geq k_2) E[\theta|s_1 \geq k_1, s_2 \geq k_2] + Pr(s_1 \geq k_1, s_2 < k_2) l_1 + Pr(s_1 < k_1) h_1
\]

\[
= Pr(s_1 \geq k_1) \int_{s_2=k_2}^{\infty} E[\theta|s_1 \geq k_1, s_2] f(s_2|s_1 \geq k_1) ds_2 + Pr(s_1 \geq k_1)Pr(s_2 < k_2|s_1 \geq k_1) l_1 + Pr(s_1 < k_1) h_1. \tag{22}
\]

In equilibrium, \( k_1 \) and \( k_2 \) are best responses, and hence \( \frac{\partial E[U_1|k_1, k_2]}{\partial k_1} = 0 \). Moreover, from Lemmas 4 and 5, only \( k_2 \) explicitly depends on \( w_2 \). Thus,

\[
\frac{\partial E[U_1|k_1(k_2(w_2)), k_2(w_2)]}{\partial w_2} = \frac{\partial E[U_1]}{\partial k_2} \frac{\partial k_2}{\partial w_2} + \frac{\partial E[U_1]}{\partial k_1} \frac{\partial k_1}{\partial k_2} \frac{\partial k_2}{\partial w_2}
\]

\[
= \frac{\partial E[U_1]}{\partial k_2} \frac{\partial k_2}{\partial w_2}.
\]

It is easy to see that \( \frac{\partial k_2}{\partial w_2} > 0 \), and hence the first order condition, \( \frac{\partial E[U_1|k_1(k_2(w_2)), k_2(w_2)]}{\partial w_2} = 0 \), holds if and only if \( \frac{\partial E[U_1]}{\partial k_2} = 0 \). From equation (22),

\[
\frac{\partial E[U_1]}{\partial k_2} = Pr(s_1 \geq k_1) ( - E[\theta|s_1 \geq k_1, k_2] f(k_2|s_1 \geq k_1) + f(k_2|s_1 \geq k_1) Pr(s_1 \geq k_1) l_1
\]

\[
= Pr(s_1 \geq k_1) f(k_2|s_1 \geq k_1) ( l_1 - E[\theta|s_1 \geq k_1, k_2]).
\]

Thus, \( \frac{\partial E[U_1]}{\partial k_2} = 0 \) if and only if \( E[\theta|s_1 \geq k_1(k_2(w_2)), k_2(w_2)] = l_1 \). Moreover, from Lemma 4, \( E[\theta|s_1 \geq k_1(k_2(w_2)), k_2(w_2)] = w_2 \). Thus, the first order condition holds if and only if \( w_2 = l_1 \). It is easy to see that this is a maximum. That is, the vanguard’s expected payoff is maximized when \( w_2 = l_1 \). Thus, it chooses \( c \) such that \( w_2 - c = l_1 \), i.e., \( c = w_2 - l_1 \). Clearly, when we restrict \( c \) to be positive, \( c = \max\{w_2 - l_1, 0\} \). \( \square \)

Proof of Proposition 6: Let \( E_i \) be agent \( i \)'s expectation. From A4, \( E_2[\theta|s_1 \geq k_1(k_2(w_2 - c)), k_2(w_2 - c)] < E_1[\theta|s_1 \geq k_1(k_2(w_2 - c)), k_2(w_2 - c)] \). The equilibrium level of \( k_2 \) is determined by \( E_2[\theta|s_1 \geq k_1(k_2(w_2 - c)), k_2(w_2 - c)] = w_2 - c \), and hence \( w_2 - c < E_1[\theta|s_1 \geq k_1(k_2(w_2 - c)), k_2(w_2 - c)] \) in equilibrium. From Proposition 5, the vanguard’s optimal choice is to pick a \( c \) that yields a \( k_2(w_2 - c) \) that satisfies \( E_1[\theta|s_1 \geq k_1(k_2(w_2 - c)), k_2(w_2 - c)] = l_1 \). This implies that \( w_2 - c < l_1 \), i.e., \( w_2 - l_1 < c \). \( \square \)
Proof of Proposition 7: Citizen 1’s expected net payoff from revolting versus not
revolting at date 1 when he receives signal $s_1$ and citizen 2 sets cutoffs $\alpha, \beta$ and $\gamma$ is:

$$
Pr(s_2 \geq \min\{\alpha, \beta\}|s_1) E[\theta|s_1, s_2 \geq \min\{\alpha, \beta\}] + Pr(s_2 < \min\{\alpha, \beta\}|s_1) l
$$

$$
-Pr(s_2 \geq \alpha|s_1) \max\{w, E[\theta|s_2 \geq \alpha]\} - Pr(s_2 < \alpha|s_1) v,
$$

where $v > l$ is the expected continuation payoff to citizen 1 if neither citizen initiates
revolt at date 1, and we note that $v = h$ is always an equilibrium, supported by beliefs
that there will be no revolt at date 2, if there is none at date 1.

At a symmetric equilibrium, optimization by citizen 1 requires that when
$s_1 = \alpha$, he be indifferent between actions:

$$
Pr(s_2 \geq \min\{\alpha, \beta(\alpha)\}|s_1 = \alpha)[E[\theta|s_1 = \alpha, s_2 \geq \min\{\alpha, \beta(\alpha)\}] - l] + l
$$

$$
= Pr(s_2 \geq \alpha|s_1 = \alpha)[\max\{w, E[\theta|s_2 \geq \alpha]\} - v] + v,
$$

where best-responding when citizen 2 revolts for signals $s_2 \geq \alpha$ means that $\beta(\alpha)$ solves

$$
E[\theta|s_2 \geq \alpha, s_1 = \beta(\alpha)] = w.
$$

The left-hand side of (24) is citizen 1’s expected payoff if he revolts at date 1 follow-
ing signal $s_1 = \alpha$. With probability $Pr(s_2 \geq \min\{\alpha, \beta(\alpha)\}|s_1 = \alpha)$, citizen 2 revolts
too—either at date 1 or date 2—and the revolution succeeds, yielding an expected
payoff of $E[\theta|s_1 = \alpha, s_2 \geq \min\{\alpha, \beta(\alpha)\}]$ from citizen 1’s perspective. With the re-
mainning probability, citizen 2 does not revolt, and citizen 1 gets $l$. The right-hand side
captures citizen 1’s expected payoff if he does not revolt at date 1. With probability
$Pr(s_2 \geq \alpha|s_1)$, citizen 2 initiates revolt at date 1. At date 2, citizen 1 revolt if his
expected payoff from revolting $E[\theta|s_1 = \alpha, s_2 \geq \alpha]$ exceeds that of not revolting $w$.
With the remaining probability, no one revolts at date 1, and both citizens receive $v$.

Because $\min\{\alpha, \beta(\alpha)\} \leq \alpha$, if $\beta(\alpha) \geq \alpha$, then equation (24) becomes

$$
Pr(s_2 \geq \alpha|s_1 = \alpha)[E[\theta|s_1 = \alpha, s_2 \geq \alpha] - l] + l
$$

$$
= Pr(s_2 \geq \alpha|s_1 = \alpha)[\max\{w, E[\theta|s_1 = \alpha, s_2 \geq \alpha]\} - v] + v.
$$
A contradiction immediately obtains since \( v > l \), and \( \max\{w, E[\theta|s_1 = \alpha, s_2 \geq \alpha]\} \geq E[\theta|s_1 = \alpha, s_2 \geq \alpha] \). Hence, \( \alpha > \beta(\alpha) \), and equation (24) becomes

\[
Pr(s_2 \geq \beta(\alpha)|s_1 = \alpha)[E[\theta|s_1 = \alpha, s_2 \geq \beta(\alpha)] - l] + l = Pr(s_2 \geq \alpha|s_1 = \alpha)\max\{w, E[\theta|s_1 = \alpha, s_2 \geq \alpha]\} - v] + v.
\]

Next use \( E[\theta|s_2 \geq \alpha, s_1 = \beta(\alpha)] = w \) together with \( \alpha > \beta(\alpha) \) to see that \( \max\{w, E[\theta|s_1 = \alpha, s_2 \geq \alpha]\} \geq E[\theta|s_1 = \alpha, s_2 \geq \alpha] \). Thus, equation (24) becomes

\[
Pr(s_2 \geq \beta(\alpha)|s_1 = \alpha)[E[\theta|s_1 = \alpha, s_2 \geq \beta(\alpha)] - l] + l = Pr(s_2 \geq \alpha|s_1 = \alpha)\{E[\theta|s_1 = \alpha, s_2 \geq \beta(\alpha)] - l\} - Pr(s_2 < \alpha|s_1 = \alpha)(v - l) = Pr(s_2 \geq \alpha|s_1 = \alpha)\{E[\theta|s_1 = \alpha, s_2 \geq \alpha] - l\}.
\]

(26)

To establish that there exist \( \beta(\alpha^*) \) that satisfy equations (25) and (26), first observe that \( \beta(\alpha) \) is continuous and strictly decreasing in \( \alpha \), with \( \lim_{\alpha \to -\infty} \beta(\alpha) = -\infty \), and \( \lim_{\alpha \to +\infty} \beta(\alpha) = -\infty \). Thus, there exists a unique \( \alpha_\beta \) such that \( \beta(\alpha_\beta) = \alpha_\beta \). Further, so long as \( v > l \), at \( \alpha_\beta \), the left-hand side of (26) is less than the right-hand side; and, as \( \alpha \to \infty \), the left-hand side goes to infinity, while the right-hand side goes to zero. By continuity, there exists \( \beta(\alpha^*) \) that satisfies equations (25) and (26).

When \( \gamma < \alpha \) is a posited cutoff for revolt in the second period given that no one revolted in the first period, equilibrium demands that

\[
Pr(s_2 < \gamma|s_1 = \gamma, s_2 < \alpha)\ l + Pr(s_2 \geq \gamma|s_1 = \gamma, s_2 < \alpha)\ E[\theta|s_1 = \gamma, \gamma \leq s_2 < \alpha] = Pr(s_2 < \gamma|s_1 = \gamma, s_2 < \alpha)\ h + Pr(s_2 \geq \gamma|s_1 = \gamma, s_2 < \alpha)\ w.
\]

Substitute in for \( w = E[\theta|s_1 \geq \alpha, s_2 = \beta(\alpha)] \) and re-arrange:

\[
Pr(s_2 \geq \gamma|s_1 = \gamma, s_2 < \alpha)\ \{E[\theta|s_1 = \gamma, \gamma \leq s_2 < \alpha] - E[\theta|s_1 \geq \alpha, s_2 = \beta(\alpha)]\} = Pr(s_2 < \gamma|s_1 = \gamma, s_2 < \alpha)\ (h - l) > 0.
\]

For the left-hand side to be positive, it must be that

\[
E[\theta|s_1 = \gamma, \gamma \leq s_2 < \alpha] > E[\theta|s_1 \geq \alpha, s_2 = \beta(\alpha)].
\]

(27)
Since $\alpha > \gamma$ is necessary for a revolt at date 2 when there is no revolt at date 1, the symmetry of $f(s_1, s_2)$ then implies that for (27) to hold, we must have $\alpha > \gamma > \beta(\alpha)$.

It remains to verify that at the equilibrium level of $\alpha$

$$Pr(s_2 \geq \beta(\alpha)|s_1) \{E[\theta|s_1, s_2 \geq \beta(\alpha)] - l\} - Pr(s_2 < \alpha|s_1)(v - l)$$

$$- Pr(s_2 \geq \alpha|s_1) \{E[\theta|s_1, s_2 \geq \alpha] - l\}$$

is positive for all $s_1 > \alpha$, and negative for all $s_1 < \alpha$. Equivalently, we must sign

$$Pr(\beta(\alpha) \leq s_2 < \alpha|s_1) \ E[\theta|s_1, \beta(\alpha) \leq s_2 < \alpha]$$

$$+ Pr(s_2 < \beta(\alpha)|s_1) (l - v) - Pr(\beta(\alpha) \leq s_2 < \alpha|s_1) v,$$

which we rewrite as

$$Pr(\beta(\alpha) \leq s_2 < \alpha|s_1) \ (E[\theta|s_1, \beta(\alpha) \leq s_2 < \alpha] - v) - Pr(s_2 < \beta(\alpha)|s_1) \ (v - l),$$

which is equivalent to signing

$$[E[\theta|s_1, \beta(\alpha) \leq s_2 < \alpha] - v] - \frac{Pr(s_2 < \beta(\alpha)|s_1)}{Pr(\beta(\alpha) \leq s_2 < \alpha|s_1)} \ (v - l).$$

Case I: Constant $v$. The bracketed term increases in $s_1$. The last term falls with $s_1$. To see this, let $A \equiv Pr(s_2 < \alpha|s_1)$ and $B \equiv Pr(s_2 < \beta|s_1)$, so that

$$\frac{Pr(s_2 < \beta|s_1)}{Pr(s_2 \leq \alpha|s_1)} = \frac{B}{A-B}.$$ Differentiating with respect to $s_1$ yields

$$\frac{B'(A-B)-(A'B'-B)}{(A-B)^2},$$

which is negative if and only if $\frac{B'}{B} < \frac{A'}{A}$. Thus, a sufficient condition for $\frac{B}{A-B}$ to be decreasing in $s_1$ for all $\alpha$ and $\beta$ with $\beta < \alpha$ is that $\frac{\partial}{\partial s_1} \frac{\partial \ln[Pr(s_2 < x|s_1)]}{\partial s_1} > 0$, i.e.,

$$\frac{\partial}{\partial s_1} \frac{\partial \ln[Pr(s_2 < x|s_1)]}{\partial x} = \frac{\partial}{\partial s_1} \frac{PDF(s_2=x|s_1)}{CDF(s_2=x|s_1)} > 0.$$ But this follows from our assumption that $s_1$ and $s_2$ are affiliated, i.e., they have the monotone likelihood ratio property.

Case II: non-constant $v$. Dropping dependence on $\alpha$

$$v = \begin{cases} h &; s_1 < \gamma \\ Pr(s_2 < \gamma|s_1, s_2 < \alpha) \ l + Pr(\gamma < s_2|s_1, s_2 < \alpha) \ E[\theta|s_1, \gamma < s_2 < \alpha] &; s_1 > \gamma, \end{cases}$$

which one can rewrite as

$$v = \begin{cases} h &; s_1 < \gamma \\ Pr(s_2 < \gamma|s_1) \ l + Pr(\gamma < s_2 < \alpha|s_1) \ E[\theta|s_1, \gamma < s_2 < \alpha] &; s_1 > \gamma, \end{cases}$$

$$Pr(\beta \leq s_2 < \alpha|s_1) \ E[\theta|s_1, \beta \leq s_2 < \alpha] + Pr(s_2 < \beta|s_1) \ l - Pr(s_2 < \alpha|s_1) \ v,$$
and then substitute for \( v \) into this expression to obtain

\[
Pr(\beta \leq s_2 < \alpha|s_1) E[\theta|s_1, \beta \leq s_2 < \alpha] + Pr(s_2 < \beta|s_1) l
- \begin{cases}
Pr(s_2 < \alpha|s_1) h & ; s_1 < \gamma \\
Pr(s_2 < \gamma|s_1) l + Pr(\gamma < s_2 < \alpha|s_1) E[\theta|s_1, \gamma < s_2 < \alpha] & ; s_1 > \gamma,
\end{cases}
\]

Then, if \( s_1 > \gamma \), the above expression becomes

\[
Pr(\beta \leq s_2 < \alpha|s_1) E[\theta|s_1, \beta \leq s_2 < \alpha] + Pr(s_2 < \beta|s_1) l
- Pr(s_2 < \gamma|s_1) l - Pr(\gamma < s_2 < \alpha|s_1) E[\theta|s_1, \gamma < s_2 < \alpha]
= Pr(\beta \leq s_2 < \gamma|s_1) (E[\theta|s_1, \beta \leq s_2 < \gamma] - l).
\]

Hence,

\[
\begin{cases}
Pr(\beta \leq s_2 < \alpha|s_1) (E[\theta|s_1, \beta \leq s_2 < \alpha] - h) - Pr(s_2 < \beta|s_1) (h - l) & ; s_1 < \gamma \\
Pr(\beta \leq s_2 < \gamma|s_1) (E[\theta|s_1, \beta \leq s_2 < \gamma] - l) & ; s_1 > \gamma,
\end{cases}
\]

We have established that the first line for \( s_1 \in \mathbb{R} \) has a single crossing property. The second line, obviously has that too, and hence, if is has a solution, then it is unique. Further, payoffs are continuous at \( s_1 = \gamma \), implying a unique solution in \( s_1 \). That a solution exists follows from the limiting properties of the expectations as \( s_1 \to \pm \infty \). \( \square \)

**Proof of Proposition 8:** Index cutoffs in the exogenous and endogenous frameworks by \( ex \) and \( en \), respectively, so that \( k_1 = \alpha_{ex} \) and \( k_2 = \beta_{ex} \). In both frameworks, the best response function following a revolt at date 1 is the same: \( \beta_{ex}(\cdot) = \beta_{en}(\cdot) \). Thus, \( \beta_{ex}(\alpha_{ex}) < \beta_{en}(\alpha_{en}) \) if and only if \( \alpha_{ex} > \alpha_{en} \). Moreover, in the exogenous setting,

\[
Pr(s_2 \geq \beta_{ex}|s_1 = \alpha_{ex}) (E[\theta|s_1 = \alpha_{ex}, s_2 \geq \beta_{ex}] + Pr(s_2 < \beta_{ex}|s_1 = \alpha_{ex}) l = h, \tag{30}
\]

which has a unique solution by Proposition 1. In the endogenous setting,

\[
Pr(s_2 \geq \beta_{en}|s_1 = \alpha_{en}) E[\theta|s_1 = \alpha_{en}, s_2 \geq \beta_{en}] + Pr(s_2 < \beta_{en}|s_1 = \alpha_{en}) l \tag{31}
= Pr(s_2 \geq \alpha_{en}|s_1 = \alpha_{en}) E[\theta|s_1 = \alpha_{en}, s_2 \geq \alpha_{en}] + Pr(s_2 < \alpha_{en}|s_1 = \alpha_{en}) v > h.
\]

The left-hand sides of (30) and (31) are equal when evaluated at the same \( \alpha \), but the right-hand side of (31) exceeds that of (30) because \( v \geq h \) and \( E[\theta|s_1 = \alpha_{en}, s_2 \geq \alpha_{en}] > E[\theta|s_1 = \beta_{en}, s_2 \geq \alpha_{en}] = h \). When evaluated at \( \alpha_{en} \), the left-hand side of (30) exceeds the right-hand side. Moreover, from proposition 1, the left-hand side of (30) crosses \( h \) from below at a unique point \( \alpha_{ex} \), and hence \( \alpha_{ex} < \alpha_{en} \). \( \square \)
10 References


